Optimization Assignment - 2

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Problem Statement **- Show that the surface area of a closed cuboid with square base and a given volume is minimum when it is a cube**

Solution

0.1 Considerations

Symbol	Value	Description
l	x	length of cuboid
b	x	breadth of cuboid
h	$y = \frac{V}{x^2}$	height of cuboid
V	8	Volume of cuboid

$$S = 2x^2 + 4xy \tag{1}$$

$$S = 2x^2 + \frac{4V}{x} \tag{2}$$

minima using conventional method

$$\frac{dS}{dx} = 4x - \frac{4V}{x^2} \tag{3}$$

To find minima

$$\frac{dS}{dx} = 0 \tag{4}$$

$$4x - \frac{4V}{x^2} = 0 \tag{5}$$

$$V = x^3 \tag{6}$$

$$x^2 y = x^3 \tag{7}$$

$$y = \frac{x^3}{x^2} \tag{8}$$

$$y = x \tag{9}$$

Hence the height of the cuboid should be equal to the length of square base

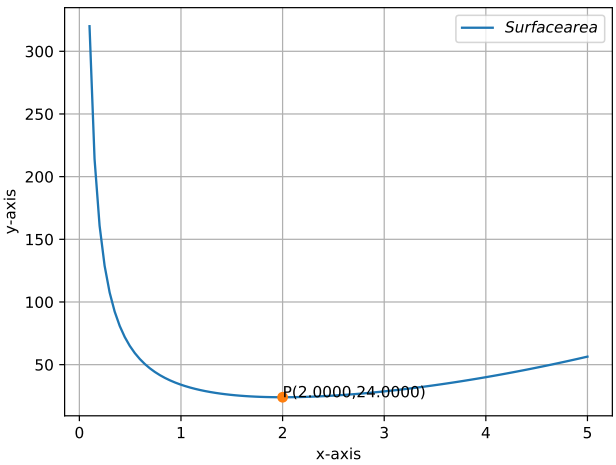

Figure 1: Graph of Surface area

Gradient descent

Using gradient descent method we can find its minima ,

$$x_{n+1} = x_n - \alpha \nabla f(x_n) \tag{10}$$

$$\implies x_{n+1} = x_n - \alpha \left(4x_n - \frac{32}{x_n^2}\right) \tag{11}$$

Taking $x_0 = 0.1, \alpha = 0.001$ and precision $= 0.00000001$, values obtained using python are:

$$\boxed{\text{Minima} = 24.00} \tag{12}$$

$$\boxed{\text{Minima Point} = 2.00} \tag{13}$$