PARALLELOGRAM

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Contents

1 Problem

ABCD is a parallelogram and AP and CQ are perpendiculars from vertices A and C on diagonal BD . Show that
(i) $\Delta APB \cong \Delta$ CQD
(ii) AP = CQ

2 Solution

Theory:
In parallelogram $AB \,||\, CD$ and $AD \,||\, BC$
BD and AC are diagonals of the parallelogram which are also the transversals for $AB \,||\, CD$
To Prove: $\Delta APB \cong \Delta$ CQD
$\angle ABD$ and $\angle CDB$ are equal since they are alternate interior angles
so $\angle ABP$ and $\angle CDQ$ are equal
$\angle APB$ and $\angle CQD$ are perpendicular angles

$\therefore$ due to AAS (angle angle side) rule
$\Delta APB \cong \Delta$ CQD

To Prove: AP = CQ
$\Delta APB \cong \Delta$ CQD

$\therefore$ AP = CQ
Hence, Proved

The input parameters for this construction are

Symbol	Value
a	1
b	2
c	3
A	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$
B	$\begin{pmatrix} c \\ 0 \end{pmatrix}$
C	$\begin{pmatrix} c+a \\ b \end{pmatrix}$
D	$\begin{pmatrix} a \\ b \end{pmatrix}$

To Prove: AP = CQ

Points P and Q are drawn from the vertices A and C to the diagonal BD respectively
The line equation for diagonal BD is mx-y+c = 0
where m = $\frac{b}{c-a}$

s = -(c+a+mb)

then,
$$P = \begin{pmatrix} \frac{m^2 c}{1+m^2} \\ \frac{-mc}{1+m^2} \end{pmatrix}$$

$$Q = \begin{pmatrix} \frac{mc-s}{1+m^2} \\ \frac{-m(c+s)}{1+m^2} \end{pmatrix}$$

distance between A and P is $\|A - P\|$
distance between C and Q is $\|C - Q\|$
if $\|A - P\| = \|C - Q\|$
then AP = CQ..........(1)

To Prove: $\Delta APB \cong \Delta$ CQD
to prove $\angle APD$ and $\angle CQD$ are 90°
m1 = A-P
m2 = P-B
if θ is the angle
then cosθ = $\frac{m1^\intercal m2}{\|\mathbf{m1}\|\|\mathbf{m2}\|}$
for θ = 90°, cosθ = 0
$\therefore m1^\intercal$ m2 = 0
n1 = C-Q
n2 = Q-D
if θ is the angle
then cosθ = $\frac{n1^\intercal n2}{\|\mathbf{n1}\|\|\mathbf{n2}\|}$
for θ = 90°, cosθ = 0
$\therefore n1^\intercal$ n2 = 0

if $m1^\intercal m2 = 0$ and $n1^\intercal n2 = 0$
then, $\angle APD = \angle CQD = 90°$..........(2)

to prove $\angle ABP$ and $\angle CDQ$ are equal
m2 = P-B
m3 = A-B
let $\theta 1$ be the $\angle ABP$
then $\theta 1 = \cos^- 1 \frac{\mathbf{m2 \cdot m3}}{\|\mathbf{m2}\|\|\mathbf{m3}\|}$
n2 = C-D
n3 = Q-D
let $\theta 2$ be the $\angle CDQ$
then $\theta 2 = \cos^- 1 \frac{\mathbf{n2 \cdot n3}}{\|\mathbf{n2}\|\|\mathbf{n3}\|}$

if $\theta 1 = \theta 2$

then $\angle ABP = \angle CQD$..........(3)

$\therefore$ from (1),(2) and (3) $\Delta APB \cong \Delta$ CQD

The below python code realizes the above construction:

https://github.com/sravani21vunnava/sravani21vunnava/blob/main/Matrices_line/codes/matrix_line.py

3 Construction