OM-S20-25: Nonlinear Constrained (Optional)

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http://preon.iiit.ac.in/om_quiz

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Announcements

- **Mid 1** Total of 15 requests are received with specific queries. Will be closing these before the next session. PoC: Jawahar
- **HW and Assignments** Head TAs are arranging an online session every day to address the individual and system related issues. PoC: Aditya and Kritika
- **Class Reviews** Two extra sessions (today and Fri) to replace any missed/lost due to a valid reason. (form will ask for details). Any minor issues beyond is addressed in the slack (100 out of 120). PoC: Piyush
- **ILM Closing** Start to see marks by this week end or before. PoC: Jawahar
- **Special requests from Acad. Office** Any requests specially sanctioned by academic office will be kept/used in the final closing.
- **Online Issues** Please email: omonlinemodeclass@gmail.com or text/whatapp at the phone number already shared.

Agenda

Agenda for today:

- Announcements
- Two examples for insights in KKT
- Extra Review Session (optional)

Summary: KKT

$$\max f(x)$$

s. t.

$$g_j(x) \leq 0 \quad j = 1, \ldots, m$$

$$h_j(x) = 0 \quad j = 1, \ldots, l$$

Then at x^*:

$$\nabla f = \sum_{j=1}^{m} \lambda_j \nabla g_j(x^*) + \sum_{j=1}^{l} \mu_j \nabla h_j(x^*)$$

$$g_j(x^*) \leq 0 \text{ and } h_j(x^*) = 0$$

$$\lambda_j g_j(x^*) = 0 \quad j = 1, \ldots, m$$

$$\lambda_j \geq 0 \text{ and } \mu_j \in R$$

Example 1: KKT for LP

$$\max \quad x_1 + x_2$$

Subject to:

$$x_1 + 2x_2 \leq 4$$

$$2x_1 + x_2 \leq 6$$

$$x_1, x_2 \geq 0$$

$$f(x) = x_1 + x_2$$

$$g_1(x) = x_1 + 2x_2 - 4 \leq 0$$

$$g_2(x) = 2x_1 + x_2 - 6 \leq 0$$

$$g_3(x) = -x_1 \leq 0$$

$$g_4(x) = -x_2 \leq 0$$

$$\nabla_f = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad \nabla_{g_1} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \quad \nabla_{g_2} = \begin{bmatrix} 2 \\ 1 \end{bmatrix} \quad \nabla_{g_3} = \begin{bmatrix} -1 \\ 0 \end{bmatrix} \quad \nabla_{g_4} = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$$

Cont.

$$\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \lambda_1 \cdot \begin{bmatrix} 1 \\ 2 \end{bmatrix} + \lambda_2 \begin{bmatrix} 2 \\ 1 \end{bmatrix} + \lambda_3 \cdot \begin{bmatrix} -1 \\ 0 \end{bmatrix} + \lambda_4 \cdot \begin{bmatrix} 0 \\ -1 \end{bmatrix}$$

$$\lambda_1 + 2\lambda_2 - \lambda_3 = 1$$

$$2\lambda_1 + \lambda_2 - \lambda_4 = 1$$

or

$$\lambda_1 + 2\lambda_2 \geq 1$$

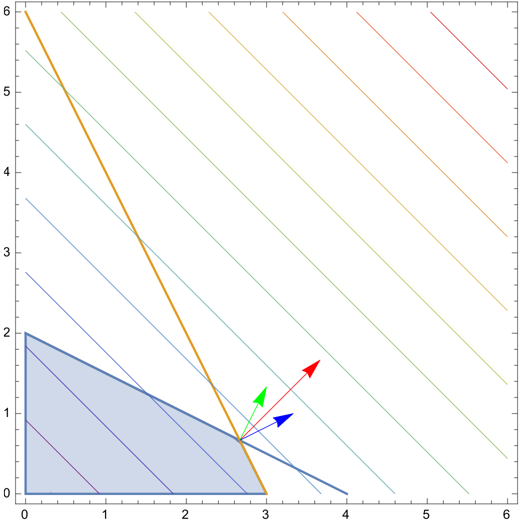
$$2\lambda_1 + \lambda_2 \geq 1$$

or

$$\lambda_1 = \lambda_2 = \frac{1}{3}; \lambda_3 = \lambda_4 = 0$$

How do we solve LP then?

Visualization



Example 2: max area given perimeter=100

$$\max \quad xy$$

subject to:

$$x + y = 50$$

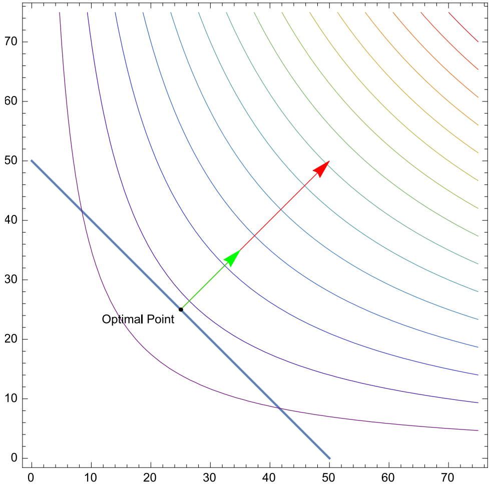
$$x \geq 0; y \geq 0$$

$$\nabla_f = \begin{bmatrix} y \\ x \end{bmatrix} \quad \nabla_h = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad \nabla_{g_1} = \begin{bmatrix} -1 \\ 0 \end{bmatrix} \quad \nabla_{g_2} = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$$

$$\begin{bmatrix} 25 \\ 25 \end{bmatrix} = \lambda_1 \cdot \begin{bmatrix} -1 \\ 0 \end{bmatrix} + \lambda_2 \cdot \begin{bmatrix} 0 \\ -1 \end{bmatrix} + \mu \cdot \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

We know $\lambda_1 = \lambda_2 = 0$; And $\mu = 25$.

Visualization



Next

- Another brief session on Friday, usual time.
- Closing KKT/Nonlinear:
  - Discuss in a structured form like on other channels.
  - Questions in Nonlinear: Objective.