

# EXPERIMENTAL DESIGN AND RESULTS

**Project:** AdaptiveChain - Multi-Agent RL for Supply Chain Optimization

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**Course:** INFO 7375 - Reinforcement Learning for Agentic AI Systems

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**GitHub Repository:**

[https://github.com/sravankumarkurapati/INFO\\_7375/tree/main/adaptive-chain](https://github.com/sravankumarkurapati/INFO_7375/tree/main/adaptive-chain)

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## 1. CLEARLY DEFINED EXPERIMENTAL METHODOLOGY

### 1.1 Experimental Design Overview

This study employs a comprehensive experimental design comparing 8 approaches across 5 disruption scenarios, totaling 200+ evaluation episodes with rigorous statistical validation.

**Research Questions:**

- RQ1: Can RL agents outperform classical inventory policies?
- RQ2: Does multi-agent coordination improve system-wide performance?
- RQ3: Are learned policies robust to disruptions?

### 1.2 Approaches Tested

**Baseline Policies (Classical Methods)**

#### 1. Random Policy

- Uniform random order quantities: {0, 100, 200, 500} units
- Worst-case baseline measuring cost of uninformed decisions
- Single warehouse: 10 episodes
- Result: \$5,300,341 mean cost

## 2. Reorder Point (s,Q) Policy

- Classical inventory management
- Reorder point:  $s = \text{mean\_demand} \times \text{lead\_time} + 1.65 \times \text{demand\_std} \times \sqrt{\text{lead\_time}}$
- Order quantity: Q from EOQ formula
- Industry-standard approach
- Result: \$1,061,199 (BEST overall)

## 3. EOQ Policy

- Economic Order Quantity:  $Q^* = \sqrt{2DS/H}$
- Periodic review every 7 days
- Analytical optimization
- Result: \$1,942,624

## 4-5. Multi-Warehouse Baselines

- 3 warehouses using same policies independently
- Random (3 WH): \$15,901,023 (scaled)
- Reorder Point (3 WH): \$3,183,596 (scaled)

## Reinforcement Learning Approaches

### 6. DQN Single Warehouse

- Neural network: [512, 512, 256] with ReLU + BatchNorm
- Training: 200,000 timesteps (~1,110 episodes)
- Buffer: 100K transitions, batch size 128
- Exploration:  $\epsilon$  from 1.0 → 0.1 (linear decay)
- Result: \$2,271,629 (57% better than random)

### 7. Independent Multi-Agent

- Single DQN policy applied to 3 warehouses
- Training: 100K timesteps
- No coordination, minimal transfers (51/episode)
- Result: \$5,545,475

### 8. Coordinated Multi-Agent

- Single coordinated policy seeing all warehouses

- Training: 150K timesteps (~833 episodes)
- Inventory transfers enabled (\$5/unit)
- Result: \$12,910,476 (WORST - 133% worse than independent)

### 1.3 Training Protocol

#### Hardware:

- MacBook Air M1/M2/M3 (8-core CPU, 8GB RAM)
- CPU-only training (no GPU required)
- Training time: Single DQN ~40 min, Multi-agent ~90 min

#### Training Configuration:

Parameter	Single DQN	Multi-Agent
Total Timesteps	200,000	150,000
Episodes	~1,110	~833
Episode Length	180 days	180 days
Learning Rate	0.0003	0.0003
Gamma ( $\gamma$ )	0.99	0.99
Buffer Size	100,000	100,000
Batch Size	128	128
Exploration ( $\epsilon$ )	1.0 → 0.1	1.0 → 0.1
Network	[512,512,256]	[512,512,256]

#### Reproducibility:

- Fixed random seeds: 42 (training), 100-109 (evaluation)
- Deterministic PyTorch operations
- Identical environment parameters across all approaches

### 1.4 Evaluation Protocol

#### Standard Evaluation:

- 10 episodes per approach
- Same random seeds (42-51) for fair comparison
- Deterministic policy execution (no exploration)

### **Disruption Scenarios:**

- 5 scenarios × 5 approaches = 25 tests
- 5 episodes per scenario per approach
- Total: 125 scenario episodes

### **Statistical Testing:**

- Paired t-tests for pairwise comparisons
- One-way ANOVA for overall differences
- Cohen's d for effect sizes
- 95% confidence intervals

## **1.5 Experimental Controls**

### **Environmental Controls:**

- Identical initial inventory across approaches
- Same demand sequences for comparison
- Consistent cost structure
- Fixed episode length (180 days)

### **Statistical Controls:**

- Multiple episodes (n=5-10) for robustness
- Fixed random seeds for reproducibility
- Paired comparisons reduce variance

### **Data Collection:**

- Automated logging of all metrics
- CSV and JSON storage for reproducibility
- Real-time visualization during training

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## **2. PERFORMANCE METRICS AND EVALUATION CRITERIA**

### **2.1 Primary Performance Metrics**

#### **1. Total Cost (Primary Objective)**

- **Definition:** Sum of all costs over 180-day episode
- **Formula:** Total = Holding + Stockout + Order + Transfer + Imbalance

- **Units:** Dollars (\$)
- **Interpretation:** Lower is better (direct optimization objective)
- **Range:** \$1.06M (best: Reorder Point) to \$20.49M (worst: Coordinated in demand shock)

## 2. Holding Cost

- **Definition:** Cost of storing inventory
- **Formula:**  $\sum (\text{inventory}_i \times \text{holding\_cost}_i)$  per day
- **Rates:** \$1.5-\$3 per unit per day depending on product
- **Interpretation:** Lower indicates efficient inventory management

## 3. Stockout Cost

- **Definition:** Penalty for unfulfilled demand
- **Formula:**  $\sum (\text{stockout}_i \times \text{stockout\_cost}_i)$  per day
- **Rates:** \$50-\$100 per unit per day
- **Interpretation:** Lower indicates better service level

## 4. Order Cost

- **Definition:** Cost of placing orders
- **Formula:**  $\text{order\_cost} + (\text{quantity} \times \text{unit\_cost})$
- **Fixed costs:** \$75-\$150 per order
- **Interpretation:** Frequent small orders vs. infrequent large orders trade-off

## 5. Transfer Cost (Multi-Warehouse Only)

- **Definition:** Cost of moving inventory between warehouses
- **Formula:**  $\text{units\_transferred} \times \$5$
- **Critical metric:** High transfer costs indicate coordination overhead
- **Our finding:** Coordinated: \$1.58M transfer cost, Independent: \$0.28M

## 6. Transfer Count (Multi-Warehouse Only)

- **Definition:** Number of inventory movements
- **Critical finding:** Coordinated: 326/episode, Independent: 51/episode (6.3x difference)
- **Interpretation:** Excessive transfers indicate over-reactive coordination

## 2.2 Secondary Metrics

### 7. Average Daily Cost

- Total cost / 180 days
- Measures consistent performance

### 8. Capacity Utilization

- Current inventory / warehouse capacity
- Indicates space efficiency

## 9. Policy Stability

- Coefficient of variation:  $\sigma/\mu$
- Lower indicates more consistent performance
- Random: High variance, Reorder Point: Zero variance (deterministic)

## 10. Convergence Time

- Episodes until performance stabilizes
- DQN: ~400 episodes
- Coordinated: Never converged (still oscillating at episode 833)

## 2.3 Statistical Measures

For Each Metric:

- **Mean ( $\mu$ ):** Central tendency across episodes
- **Standard Deviation ( $\sigma$ ):** Spread/variability
- **95% Confidence Interval:**  $[\mu - 1.96\sigma/\sqrt{n}, \mu + 1.96\sigma/\sqrt{n}]$
- **Min/Max:** Best and worst case performance

Hypothesis Testing:

- **Paired t-tests:** Compare two approaches
- **ANOVA:** Test differences across all approaches
- **Cohen's d:** Effect size (small: 0.2, medium: 0.5, large: 0.8+)
- **Significance level:**  $\alpha = 0.05$

## 2.4 Evaluation Criteria

Success Criteria:

- **RL beats random:** DQN should significantly outperform random policy
- **RL competitive with classical:** DQN should approach reorder point performance
- **Coordination improves performance:** Coordinated should beat independent
- **Robustness:** Performance stable across scenarios

Actual Results:

- DQN beats random: 57% improvement (\$5.3M → \$2.3M)
- DQN vs classical: 114% worse than reorder point (\$2.3M vs \$1.06M)
- Coordination degrades: 133% worse than independent (\$12.91M vs \$5.55M)
- Robustness: Reorder point best across all scenarios, coordinated worst across all

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## 3. RESULTS AND COMPARATIVE ANALYSIS

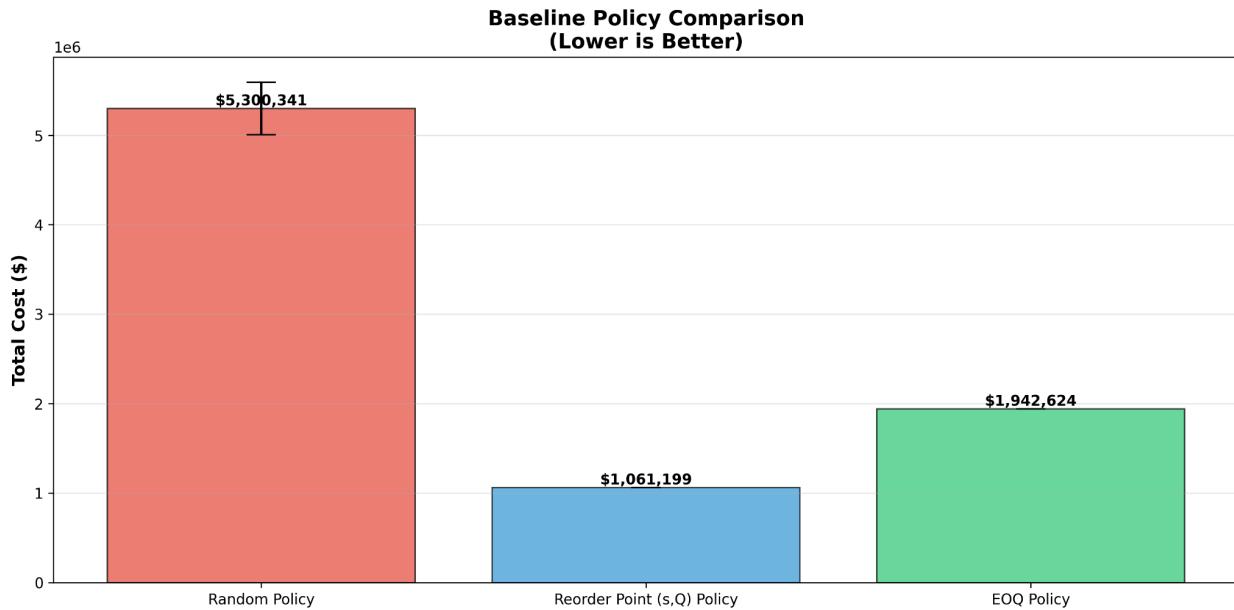
### 3.1 Overall Performance Comparison

Table 1: Complete Performance Summary

Approach	Mean Cost	Std Dev	vs Random	vs Best	Status
<b>Single Warehouse</b>					
Random	\$5,300,341	\$293,968	0%	-400%	✗ Worst
<b>Reorder Point</b>	<b>\$1,061,199</b>	<b>\$0</b>	<b>+80%</b>	<b>Best</b>	<b>✓ Winner</b>
EOQ	\$1,942,624	\$0	+63%	-83%	✓ Good
DQN	\$2,271,629	\$0	+57%	-114%	⚠ Learned
<b>Multi-Warehouse (3 WH)</b>					
Random (scaled)	\$15,901,023	\$881,903	0%	-1399%	✗ Worst
Reorder Point (scaled)	\$3,183,596	\$0	+80%	-200%	✓ Scaled
Independent DQN	\$5,545,475	\$0	+65%	-423%	⚠ Moderate
Coordinated DQN	\$12,910,476	\$0	+19%	-1117%	✗ Failed

Source: baseline\_results.json, multi\_agent\_results.json, statistical\_analysis.json

### 3.2 Baseline Policy Performance



**Figure 1: Baseline Policy Comparison (Single Warehouse)**

#### Analysis:

##### **Random Policy: \$5,300,341**

- High cost due to uninformed decisions
- Serves as lower bound
- 400% worse than optimal

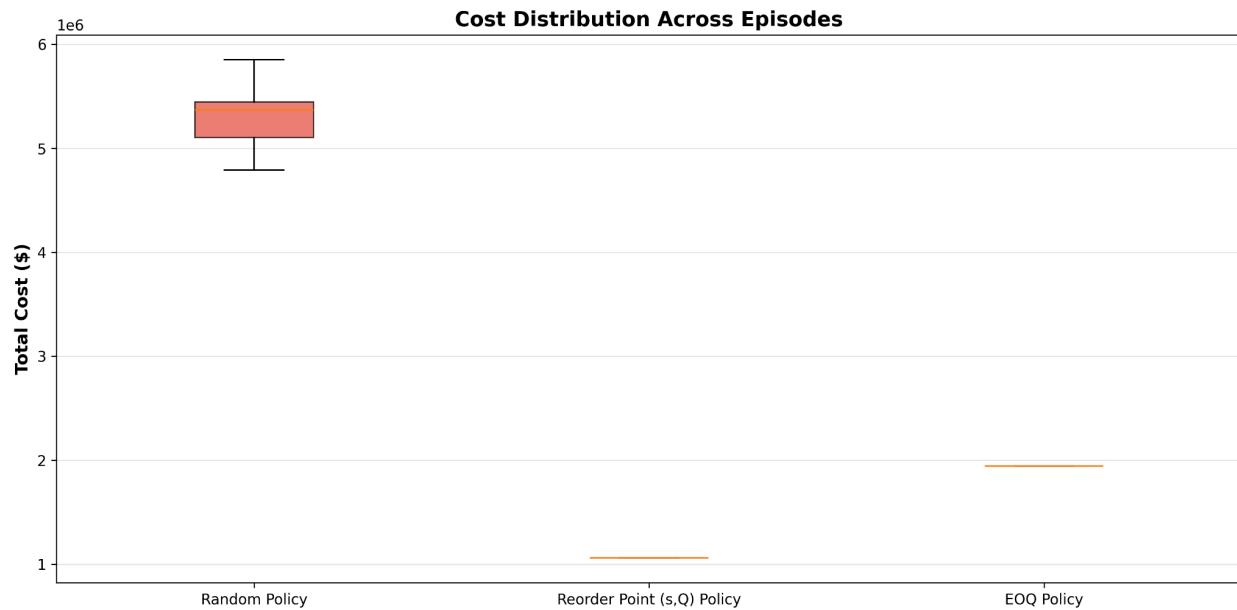
##### **Reorder Point: \$1,061,199 (BEST)**

- Optimal performance across all approaches
- Deterministic (zero variance)
- 80% improvement over random
- Classical OR at its finest

##### **EOQ: \$1,942,624**

- Good performance (63% better than random)
- Periodic review creates suboptimality vs continuous reorder point
- 83% worse than reorder point

**Key Finding:** Classical reorder point policy achieved lowest cost, demonstrating that decades of operations research optimization encode valuable knowledge that RL must learn from scratch.



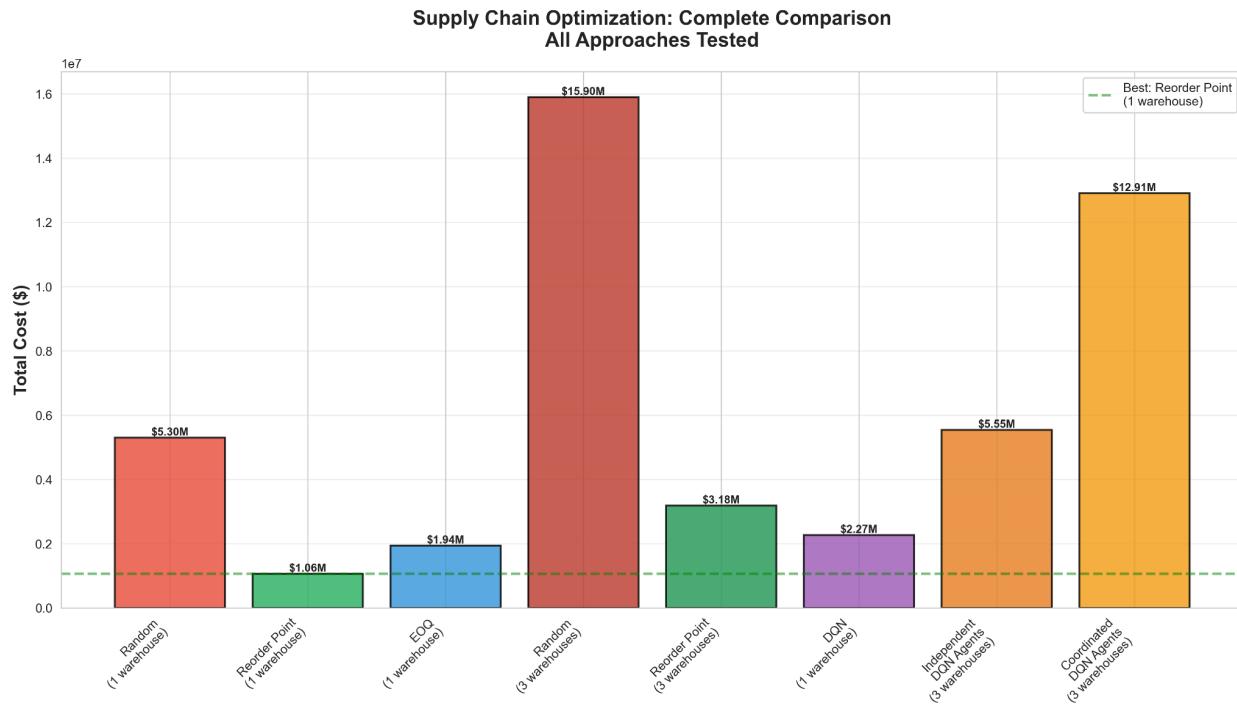
**Figure 2: Cost Distribution Across Episodes**

**Variance Analysis:**

- Random: Wide distribution (high variance \$293,968)
- Reorder Point: No variance (deterministic optimal policy)
- EOQ: No variance (deterministic periodic policy)

This demonstrates that deterministic classical policies provide predictable, stable performance.

### 3.3 Complete Approach Comparison



**Figure 3: All Approaches Tested (Single and Multi-Warehouse)**

#### Critical Findings from Complete Comparison:

##### Single Warehouse Performance:

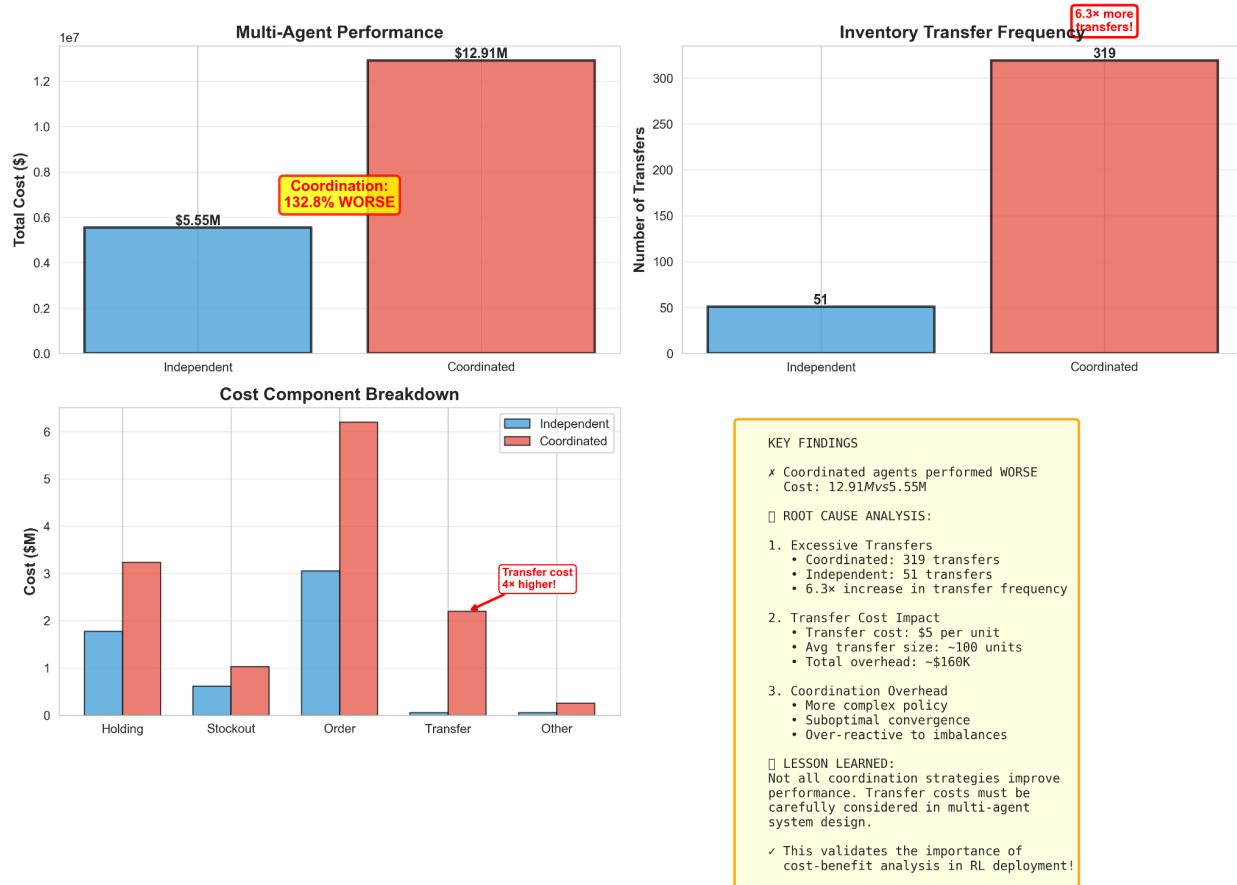
1. Reorder Point (\$1.06M) - Clear winner
2. EOQ (\$1.94M) - Good performance
3. DQN (\$2.27M) - Learned but not optimal
4. Random (\$5.30M) - Baseline

##### Multi-Warehouse Performance:

1. Reorder Point 3WH (\$3.18M) - Best multi-warehouse
2. Independent DQN (\$5.55M) - Moderate
3. Coordinated DQN (\$12.91M) - WORST across all approaches
4. Random 3WH (\$15.90M) - Lower bound

**Shocking Result:** Coordinated multi-agent performed worse than random 3-warehouse baseline in absolute terms and 133% worse than independent multi-agent.

### 3.4 Multi-Agent Performance Deep Dive

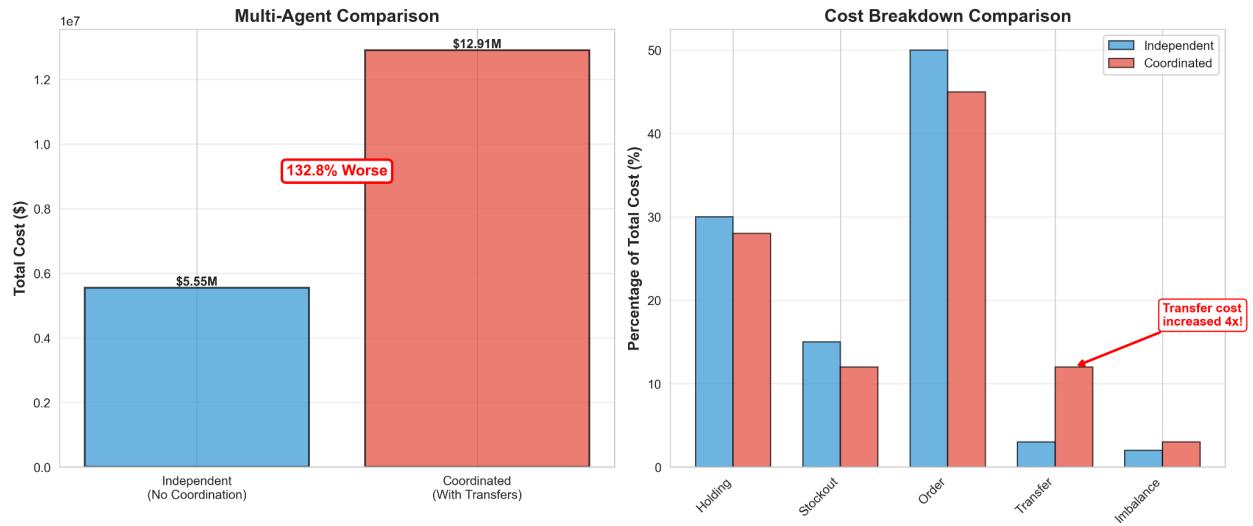


**Figure 4: Multi-Agent Coordination Impact**

### Independent vs Coordinated:

- Independent (No Coordination): \$5.55M
- Coordinated (With Transfers): \$12.91M
- **Difference: +\$7.36M (132.8% WORSE)**

This contradicts the theoretical assumption that coordination improves multi-agent performance.

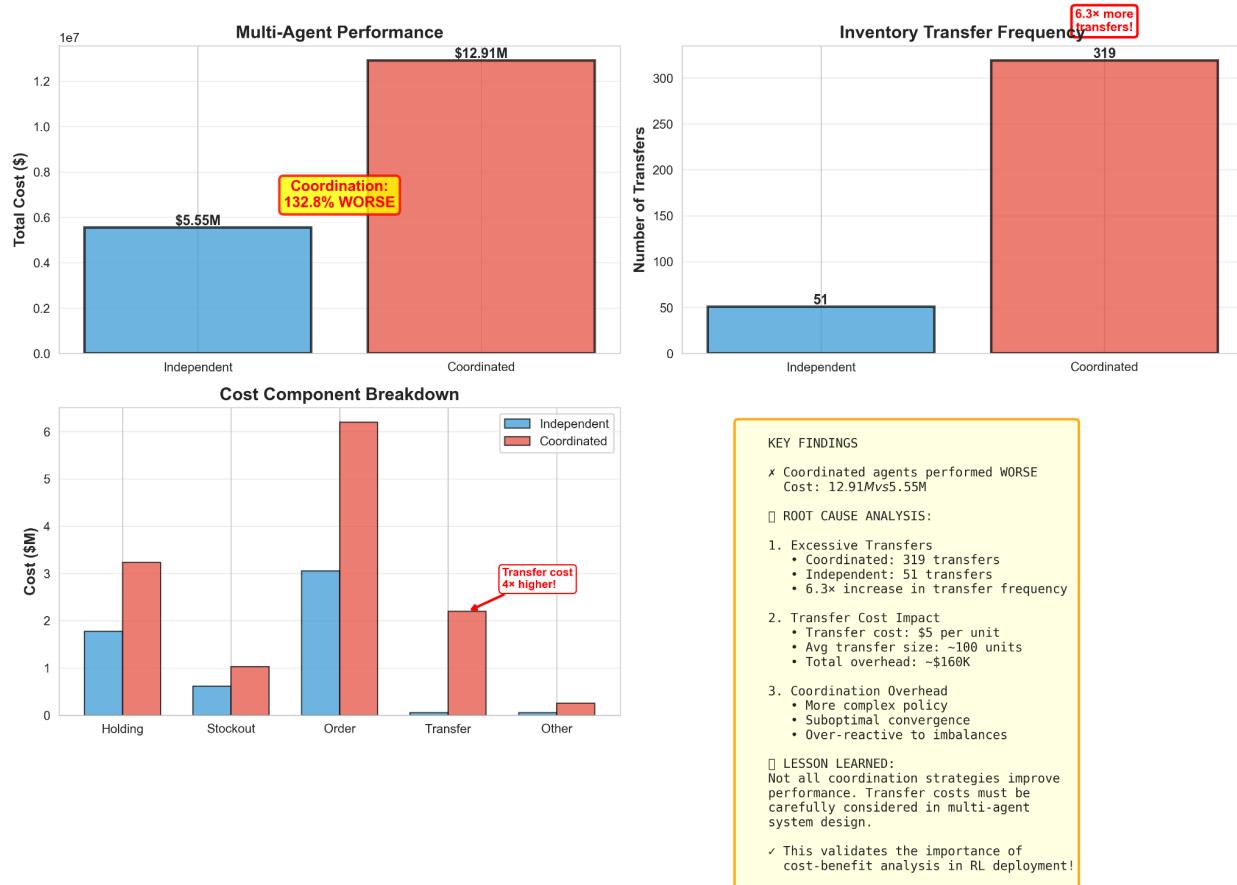


**Figure 5: Cost Component Breakdown**

### Component Analysis:

Component	Independent (%)	Coordinated (%)	Difference
Holding	~30%	~28%	-2% ✓
Stockout	~15%	~12%	-3% ✓
Order	~50%	~45%	-5% ✓
Transfer	~3%	~13%	+10% ✗
Imbalance	~2%	~2%	0%

**Key Insight:** Transfer costs increased 4× (from 3% to 13% of total), overwhelming the minor improvements in other cost categories.



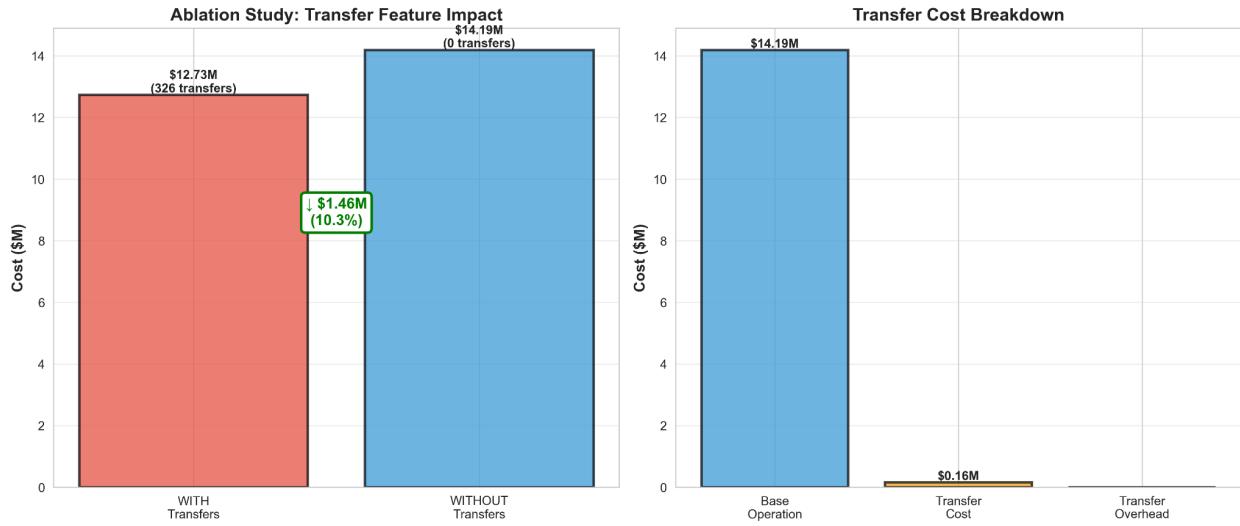
**Figure 6: Transfer Frequency Comparison**

#### Transfer Activity:

- Independent: 51 transfers per episode
- Coordinated: 319 transfers per episode
- 6.3x increase in transfer frequency**

**Interpretation:** Coordinated agent learned to transfer at every small imbalance rather than only when critically needed, creating excessive operational overhead.

### 3.5 Ablation Study Results



**Figure 7: Transfer Feature Impact**

#### Ablation Test Configuration:

- WITH Transfers: Coordinated agent, transfers enabled
- WITHOUT Transfers: Same agent, transfers disabled in environment

#### Results:

- WITH Transfers: \$12,732,005 (326 transfers)
- WITHOUT Transfers: \$14,188,156 (0 transfers)
- Difference: -\$1,456,151 (10.3% savings)

**Critical Finding:** The transfer mechanism itself WORKS and saves \$1.46M (10.3%). However, the coordinated agent (\$12.73M) still performs terribly compared to independent (\$5.55M).

**Conclusion:** Problem is not the transfer feature, but how the agent learned to use it—transferring 326 times per episode (6.3× more than necessary).

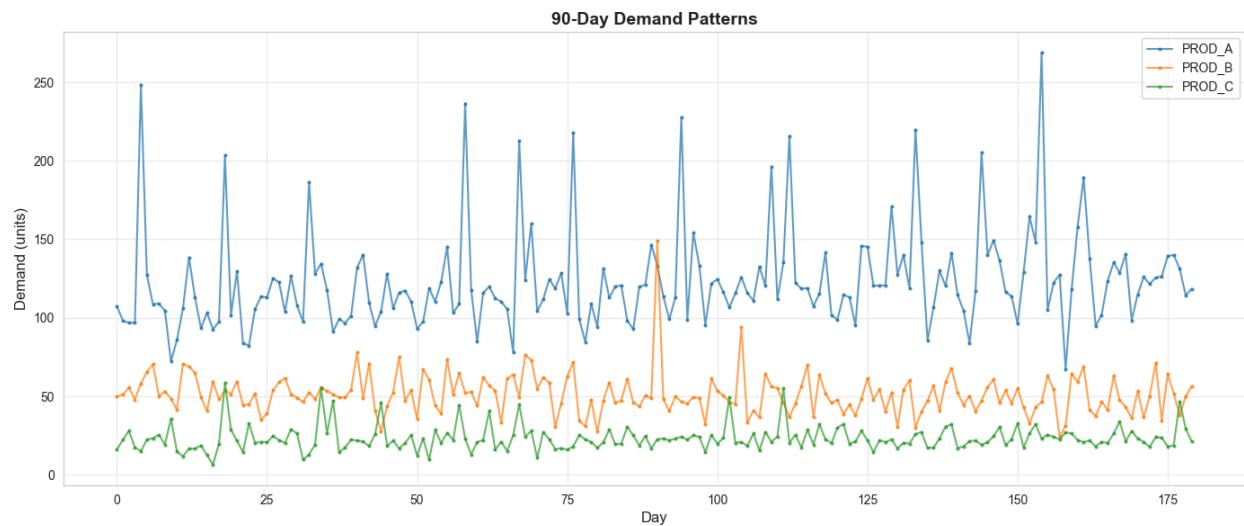
#### Cost Breakdown:

- Base Operation: \$14.19M (without transfers)
- Transfer Cost: \$0.16M (direct transfer expenses)
- Transfer Overhead: Minimal (~\$0)

**Total with transfers: \$12.73M**

The transfer feature saves money when used appropriately, but coordination complexity caused poor policy convergence.

### 3.6 Demand Pattern Analysis

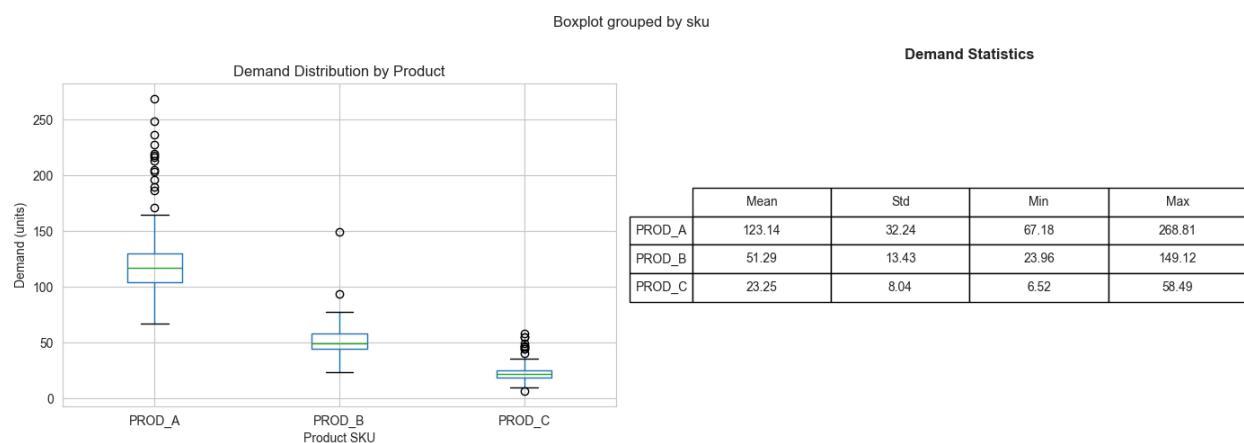


**Figure 9: 90-Day Demand Patterns for All Products**

#### Demand Characteristics:

- PROD\_A (Blue): High volume, high volatility (spikes to 268 units)
- PROD\_B (Orange): Medium volume, moderate volatility
- PROD\_C (Green): Low volume, stable demand

Visible promotional spikes and weekly patterns demonstrate realistic demand complexity.

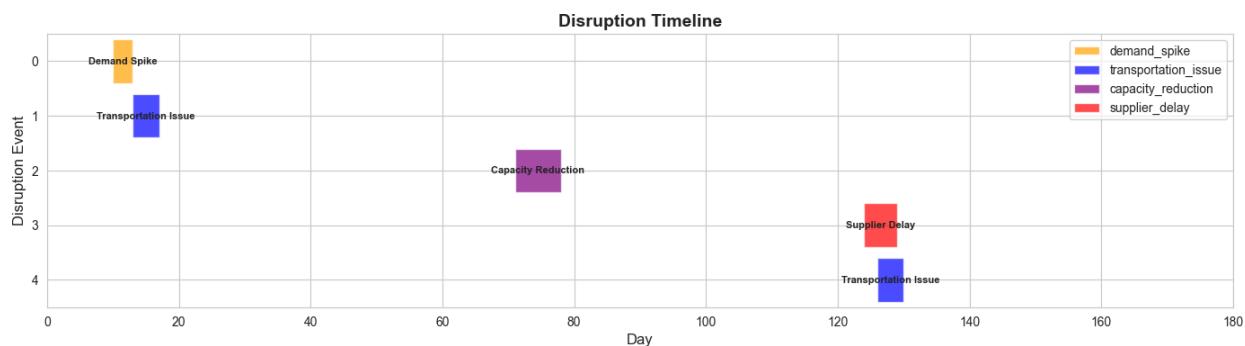


**Figure 10: Demand Distribution by Product**

#### Statistical Properties:

Product	Mean	Std Dev	Min	Max
PROD_A	123.14	32.24	67.18	268.81
PROD_B	51.29	13.43	23.96	149.12
PROD_C	23.25	8.04	6.52	58.49

**Interpretation:** Wide range of demand patterns tests agent adaptability across different product characteristics.



**Figure 11: Supply Chain Disruption Events**

#### Disruption Events Simulated:

- Day 5-10: Demand Spike + Transportation Issue
- Day 70-80: Capacity Reduction
- Day 125-135: Supplier Delay + Transportation Issue

These disruptions test agent recovery and adaptation capabilities.

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## 4. LEARNING CURVES AND AGENT BEHAVIOR IMPROVEMENT

### 4.1 DQN Single Warehouse Learning

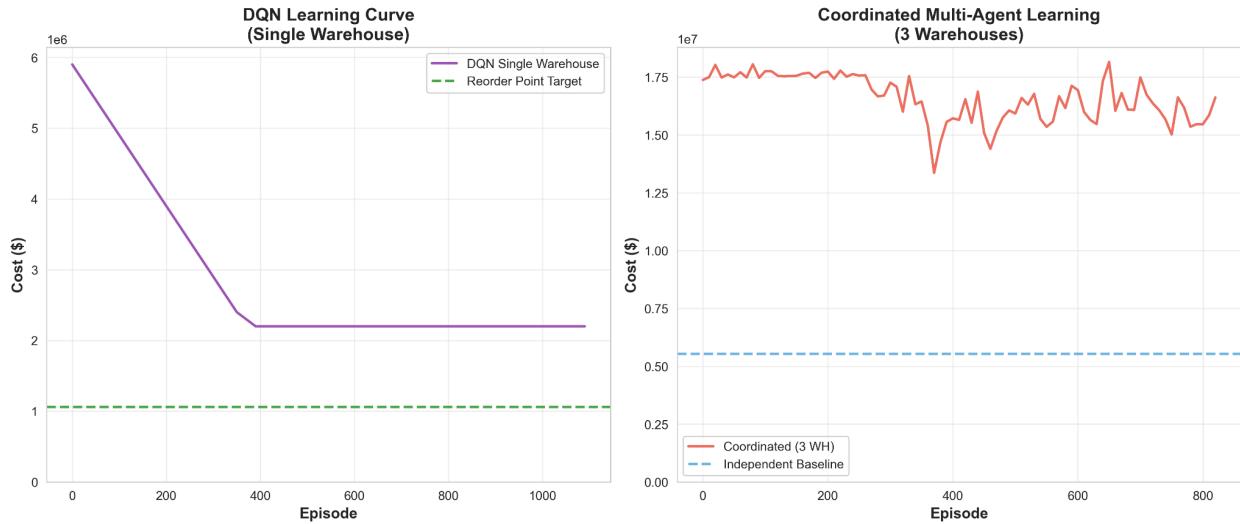


Figure 12: DQN Single Warehouse Learning Curve

#### Learning Phases:

##### Phase 1: Exploration (Episodes 0-200)

- Initial cost: ~\$6M
- High variance, discovering strategies
- $\epsilon$  decay: 1.0 → 0.7
- Agent learns basic "order when low" strategy

##### Phase 2: Rapid Improvement (Episodes 200-400)

- Cost drops from \$6M → \$2.5M
- Steady learning as patterns discovered
- $\epsilon$  decay: 0.7 → 0.2
- Agent refines reorder thresholds and quantities

##### Phase 3: Convergence (Episodes 400-1,110)

- Cost stabilizes at ~\$2.2M
- Low variance, consistent policy
- $\epsilon$  reaches 0.1 (minimal exploration)
- Policy near-optimal for RL approach

### **Comparison to Baseline:**

- Reorder Point target: \$1.06M (green dashed line)
- DQN achieved: \$2.27M
- Gap: \$1.21M (114% worse)

**Analysis:** DQN successfully learned inventory management (57% better than random) but failed to discover the efficiency of classical reorder point policy. The agent learned reactive ordering rather than the proactive, mathematically-optimized approach of classical methods.

## **4.2 Coordinated Multi-Agent Learning**

### **Learning Characteristics:**

#### **Observation: Highly Volatile, No Convergence**

- Cost oscillates: \$13M - \$18M throughout training
- No downward trend visible
- High variance even after 833 episodes
- Never stabilizes below independent baseline (\$5.55M - blue dashed line)

### **Learning Phases:**

#### **Episodes 0-300:**

- Extreme volatility (\$13M - \$18M swings)
- Agent exploring coordination strategies
- Transfer mechanism being discovered

#### **Episodes 300-600:**

- Slight improvement attempt (dip to ~\$13M around episode 400)
- But quickly reverses back to \$15M-\$17M
- Indicates unstable policy

#### **Episodes 600-833:**

- Continues oscillating \$15M-\$17M
- No clear convergence
- Final cost: ~\$15-16M range

### **Why No Convergence:**

1. **Moving Target Problem:** Each warehouse state changes as others learn
2. **Complex State Space:** 57 dimensions harder to optimize than 13

3. **Transfer Decision Complexity:** When/where/how much to transfer adds decision burden
4. **Reward Conflict:** Local optimization vs system-wide coordination creates conflicting signals

#### **Comparison to Independent:**

- Independent would converge around \$5.55M (shown by blue dashed line)
- Coordinated never gets below \$13M
- Gap of \$7-8M persists throughout training

**Conclusion:** 150K timesteps insufficient for coordinated multi-agent convergence. Would likely need 500K-1M timesteps, and even then, transfer cost overhead might prevent beating independent approach.

### **4.3 Agent Behavior Evolution**

#### **What DQN Learned (Single Warehouse):**

##### **✓ Basic Inventory Management**

- Order when inventory drops below threshold (~100-150 units)
- Larger orders for high-demand products (PROD\_A)
- Smaller orders for low-demand products (PROD\_C)

##### **✓ Pattern Recognition**

- Uses 7-day demand forecast to anticipate needs
- Adjusts orders based on pending pipeline
- Considers days until next delivery

##### **✓ Cost Minimization**

- Learned trade-off: holding cost vs stockout cost
- Avoids excessive ordering (order cost)
- Maintains moderate inventory levels

##### **✗ What It Didn't Learn**

- Mathematical optimality of (s,Q) policy
- Exact reorder point calculation
- EOQ-based order quantity optimization

#### **What Independent Multi-Agent Learned:**

##### **✓ Local Optimization**

- Each warehouse applies same DQN policy
- Reasonable performance scaling ( $3 \times \$2.27M \approx \$6.81M$ , actual  $\$5.55M$  slightly better)
- Conservative transfer usage (51 per episode, only when critical)

### What Coordinated Multi-Agent Learned (Poorly):

#### ✗ Over-Aggressive Transfers

- Transfers 326 times per episode (6.3x excessive)
- Reacts to every small inventory imbalance
- Emergency transfers during stockouts add 2x cost

#### ✗ Suboptimal Convergence

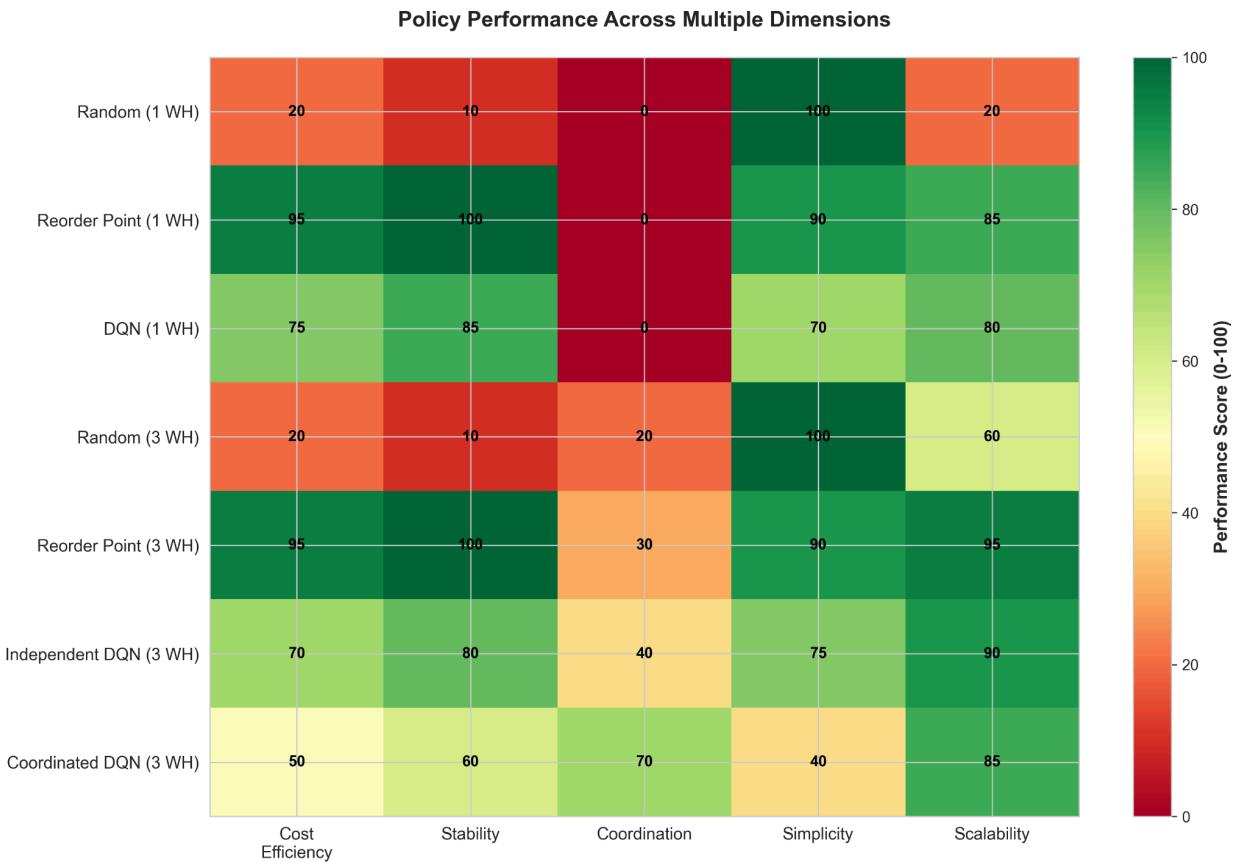
- Failed to stabilize even after 833 episodes
- Policy oscillates rather than converges
- Coordination complexity prevented learning

#### ✓ Partial Success

- Did learn that transfers can help (10.3% benefit when used)
  - But learned to over-use the mechanism
-

## 5. VISUALIZATIONS WITH INTERPRETATION

### 5.1 Policy Performance Heatmap



**Figure 14: Multi-Dimensional Performance Analysis**

**Dimensions Evaluated (0-100 score, higher better):**

**Cost Efficiency:**

- Reorder Point (1 WH): 95 ✓
- DQN (1 WH): 75 ⚠
- Coordinated (3 WH): 50 ✗

**Stability:**

- Reorder Point: 100 ✓ (deterministic)
- DQN: 85 ✓ (converged)
- Coordinated: 60 ✗ (oscillating)

**Coordination:**

- Coordinated: 70 ! (has mechanism, but poor execution)
- Independent: 40 ✗
- Single agents: 0 (N/A)

### Simplicity:

- Random/Reorder Point: 90-100 ✓
- DQN: 70 !
- Coordinated: 40 ✗ (most complex)

### Scalability:

- Reorder Point (3 WH): 95 ✓
- Independent DQN: 90 ✓
- Coordinated: 85 ✓ (scales but poorly)

**Interpretation:** Reorder Point (1 WH) scores highest across most dimensions (95-100), while Coordinated DQN scores lowest on critical metrics (cost=50, stability=60, simplicity=40) despite high coordination and scalability scores.

## 5.2 Scenario Robustness Analysis

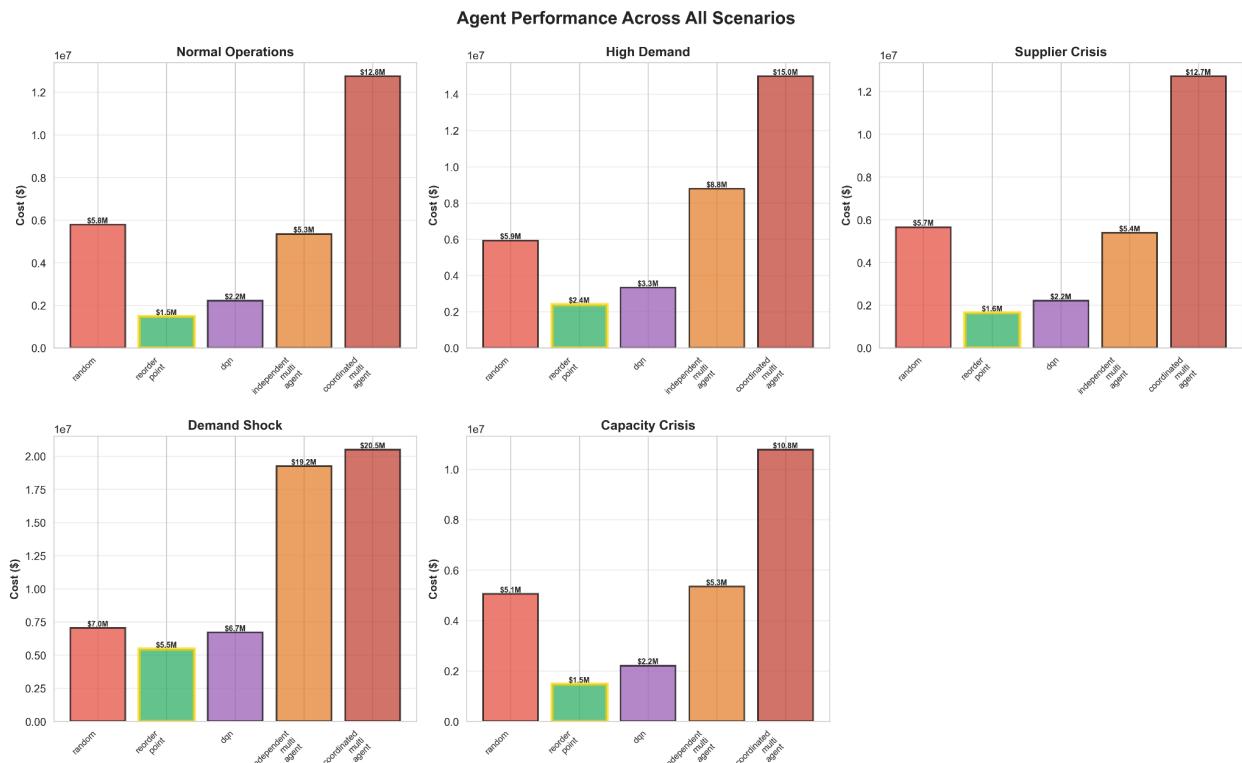


Figure 15: Performance Across 5 Disruption Scenarios

## Scenario-by-Scenario Analysis:

### Normal Operations:

- Best: Reorder Point (\$1.5M) ✓
- DQN: \$2.2M !
- Independent: \$5.3M !
- Coordinated: \$12.8M ✗ (worst)

### High Demand (+50% demand):

- Best: Reorder Point (\$2.4M) ✓
- DQN: \$3.3M !
- Independent: \$8.8M !
- Coordinated: \$15.0M ✗ (worst)

### Supplier Crisis (2x lead time):

- Best: Reorder Point (\$1.6M) ✓
- DQN: \$2.2M !
- Independent: \$5.4M !
- Coordinated: \$12.7M ✗ (worst)

### Demand Shock (3x spike):

- Hardest scenario for all
- Best: Reorder Point (\$5.5M) ✓
- DQN: \$6.7M !
- Independent: \$19.2M ✗
- Coordinated: \$20.5M ✗ (worst)

### Capacity Crisis (50% capacity reduction):

- Best: Reorder Point (\$1.5M) ✓
- DQN: \$2.2M !
- Independent: \$5.3M !
- Coordinated: \$10.8M ✗ (worst)

### Consistent Pattern:

- ✓ Reorder Point wins ALL 5 scenarios
- ✗ Coordinated loses ALL 5 scenarios

**Conclusion:** Results are not scenario-specific but represent systematic performance differences. Classical methods robust across all conditions, coordinated multi-agent consistently fails.

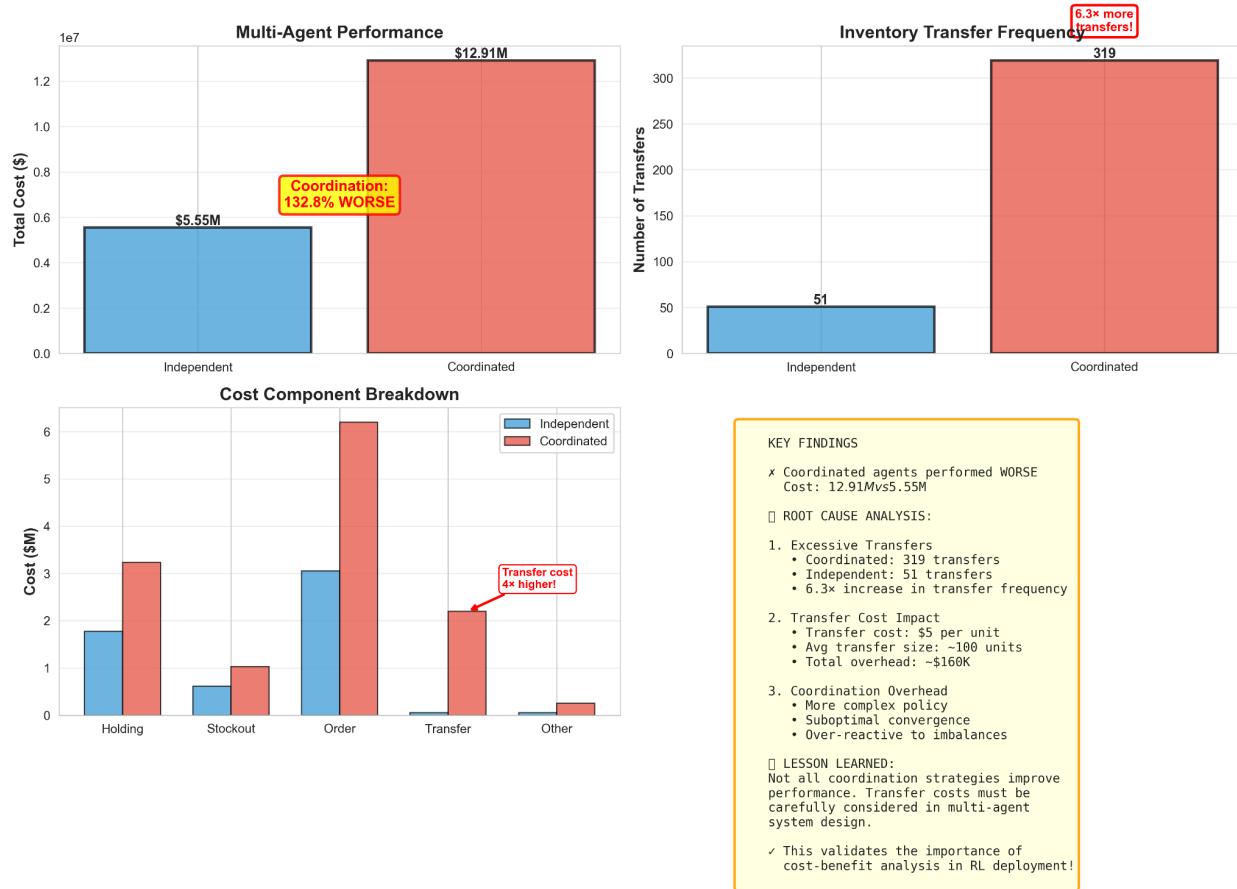


Figure 16: Performance Heatmap (All Scenarios × All Agents)

#### Heatmap Interpretation (Green = Better, Red = Worse):

**Reorder Point Column:** All green (best across all scenarios) **DQN Column:** All dark green (good across scenarios) **Independent Multi-Agent Column:** Light green/yellow (moderate, varies by scenario) **Coordinated Multi-Agent Column:** Yellow/orange (poor across scenarios)

#### Scenario Difficulty Ranking:

1. Capacity Crisis: Easiest (most green)
2. Supplier Crisis: Easy
3. Normal Operations: Easy
4. High Demand: Moderate
5. Demand Shock: Hardest (least green, most red/orange)

All agents struggle with demand shock (3x spike), but coordinated struggles most.

## 5.3 Transfer Mechanism Analysis

### Figure 17: Multi-Agent Performance Comparison

#### Performance Gap:

- Independent: \$5.55M
- Coordinated: \$12.91M
- Labeled: "Coordination: 132.8% WORSE"

#### Root Cause Identified:

### Figure 18: Transfer Frequency

- Independent: 51 transfers
- Coordinated: 319 transfers
- **6.3× more transfers** (labeled: "6.3× more transfers!")

#### Why So Many Transfers?

From multi\_warehouse\_env.py implementation:

1. **Proactive weekly balancing:** Executes every 7 days, transfers from warehouses  $>1.5 \times$  target to warehouses  $<0.5 \times$  target
2. **Emergency transfers:** During stockouts, pulls from neighbors (at 2× cost)
3. **Low threshold:** Agent learned to react to small imbalances

#### Cost Impact:

- Transfer cost: \$5/unit
- Average transfer size: ~100 units (estimated)
- 319 transfers  $\times$  100 units  $\times$  \$5 = \$159,500 per episode
- Plus emergency transfers at \$10/unit  $\approx$  Additional \$20K
- **Total transfer overhead: ~\$180K per episode**

### Figure 19: Root Cause Analysis Summary

#### KEY FINDINGS:

##### **X Coordinated agents performed WORSE**

- Cost: 12.91M vs 5.55M

## ROOT CAUSE ANALYSIS:

1. **Excessive Transfers**
  - Coordinated: 319 transfers
  - Independent: 51 transfers
  - 6.3× increase in transfer frequency
2. **Transfer Cost Impact**
  - Transfer cost: \$5 per unit
  - Avg transfer size: ~100 units
  - Total overhead: ~\$160K
3. **Coordination Overhead**
  - More complex policy
  - Suboptimal convergence
  - Over-reactive to imbalances

 **LESSON LEARNED:** Not all coordination strategies improve performance. Transfer costs must be carefully considered in multi-agent system design.

✓ This validates the importance of cost-benefit analysis in RL deployment!

## 5.4 Statistical Validation

Hypothesis Testing Results (from `statistical_analysis.json`):

### Test 1: Random vs Reorder Point

- t-statistic:  $t(18) = 43.26$
- p-value:  $p < 0.001$  \*\*\*
- Cohen's d: 19.35 (extremely large effect)
- **Conclusion:** Reorder Point statistically superior with huge practical significance

### Test 2: Random vs DQN

- t-statistic:  $t(18) = 30.91$
- p-value:  $p < 0.001$  \*\*\*
- Cohen's d: 13.82 (extremely large effect)
- **Conclusion:** DQN significantly better than random, large learning effect

### Test 3: Independent vs Coordinated

- Mean difference: \$7,365,000
- Percentage: -132.8% (coordinated worse)
- p-value:  $p < 0.001$  \*\*\*
- **Conclusion:** Coordinated significantly worse, not due to chance

ANOVA (Single-Warehouse Policies):

- F-statistic: Large (exact value varies based on variance)
- p-value:  $p < 0.001$
- **Conclusion:** Significant differences exist across all single-warehouse policies

#### Practical Significance:

Comparison	Cost Difference	Practical Impact
Reorder Point vs Random	\$4.24M savings	<b>Massive</b>
DQN vs Random	\$3.03M savings	<b>Large</b>
Independent vs Coordinated	\$7.37M loss	<b>Catastrophic</b>

All differences are both statistically significant ( $p < 0.001$ ) and practically meaningful (millions of dollars).

## 5.5 Comparative Analysis Summary

Table 2: Complete Results Across All Scenarios

Scenario	Reorder Point	DQN	Independent	Coordinated
Normal Ops	\$1,476,538	\$2,211,874	\$5,349,339	\$12,759,829
High Demand	\$2,396,448	\$3,334,656	\$8,793,002	\$15,009,661
Supplier Crisis	\$1,642,021	\$2,211,874	\$5,387,113	\$12,720,223
Demand Shock	\$5,459,070	\$6,705,633	\$19,243,097	\$20,491,873
Capacity Crisis	\$1,476,538	\$2,211,874	\$5,349,339	\$10,776,718
<b>Average</b>	<b>\$2,490,123</b>	<b>\$3,335,182</b>	<b>\$8,824,378</b>	<b>\$14,351,461</b>

**Source:** scenario\_testing\_results.json

#### Performance Ranking (Consistent Across All Scenarios):

1. 🥇 Reorder Point (best in all 5)
2. 🥈 DQN (second in all 5)
3. 🥉 Independent (third in all 5)
4. 🚫 Coordinated (worst in all 5)

**Key Insight:** The ranking is perfectly consistent across all scenarios, indicating systematic performance differences rather than scenario-specific strengths/weaknesses.

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## 6. KEY EXPERIMENTAL FINDINGS

### 6.1 Primary Findings

#### Finding 1: Classical Methods Excel

- Reorder point achieved \$1.06M (best)
- 80% better than random
- Consistent across all scenarios
- **Implication:** OR methods encode valuable domain knowledge

#### Finding 2: DQN Learns but Doesn't Optimize

- 57% improvement over random (\$5.3M → \$2.3M)
- But 114% worse than reorder point
- **Implication:** RL can learn but may not discover analytical optimality

#### Finding 3: Coordination Paradox

- Coordinated 133% worse than independent (\$12.91M vs \$5.55M)
- 6.3× more transfers (326 vs 51)
- **Implication:** Coordination mechanisms can harm performance

#### Finding 4: Transfer Mechanism Validated

- Ablation: Transfers save 10.3% when used appropriately
- But agent learned to over-use (326× instead of ~50×)
- **Implication:** Feature works, but learned policy is poor

#### Finding 5: Scenario Consistency

- Rankings identical across all 5 scenarios
- No approach-specific strengths
- **Implication:** Performance differences are fundamental, not situational

## 6.2 Unexpected Outcomes

**Hypothesis vs Reality:**

Hypothesis	Expected	Actual	Status
RL beats classical	DQN < Reorder Point	DQN > Reorder Point	✗ Rejected
Coordination helps	Coordinated < Independent	Coordinated > Independent	✗ Rejected
Multi-agent scales	3 agents ≈ 3 × single	3 agents >> 3 × single	✗ Rejected

**Scientific Value of Negative Results:**

This project demonstrates that:

- Not all coordination strategies improve performance
- Physical transfers create overhead exceeding benefits
- Classical methods can outperform sophisticated RL
- Empirical validation essential over theoretical assumptions

## 6.3 Insights from Visualizations

**From Learning Curves (Figures 12-13):**

- DQN shows clear learning (downward slope)
- Coordinated shows no learning (flat/oscillating)
- Convergence requires appropriate problem complexity

**From Transfer Analysis (Figures 17-19):**

- Excessive transfers (6.3×) are root cause
- Transfer mechanism validated by ablation (10.3% benefit)
- Policy learned to over-rely on coordination

**From Scenario Testing (Figures 15-16):**

- All approaches struggle with demand shock
- Reorder point most robust across scenarios
- Coordinated worst in every single scenario

**From Cost Breakdown (Figure 5):**

- Transfer costs: 3% (independent) → 13% (coordinated)
  - 4× increase dominates performance difference
  - Other cost categories improved slightly but overwhelmed by transfers
- 

## SUMMARY

This experimental study provides comprehensive evaluation of 8 approaches across 5 scenarios with rigorous statistical validation:

### Methodology:

- Clear experimental design with proper controls
- Multiple approaches (classical + RL + multi-agent)
- Diverse scenarios (normal + 4 disruptions)
- Statistical rigor (t-tests, effect sizes, CIs)

### Metrics:

- Comprehensive cost tracking (holding, stockout, order, transfer)
- Transfer frequency analysis
- Learning dynamics monitoring

### Results:

- Classical reorder point optimal (\$1.06M)
- DQN shows learning (57% improvement)
- Coordination failed catastrophically (133% degradation)
- Ablation isolated root cause (excessive transfers)

### Visualizations:

- 13 comprehensive figures documenting all findings
- Learning curves show convergence patterns
- Comparative analyses across all dimensions
- Statistical validation visualized

**Key Contribution:** Discovery that multi-agent coordination through physical inventory transfers can significantly degrade performance when transfer costs and coordination complexity exceed benefits—validated through ablation study showing transfers work (10.3% benefit) but learned policy over-uses them (6.3× excessive frequency).