

Question: Suppose that 90% of people are right-handed. What is the probability that at most 6 of a random sample of 10 people are right-handed?

Solution: To calculate the probability of having at most 6 right-handed individuals out of a random sample of 10 people, given that the probability of an individual being right-handed is 90% (0.9), we can use the binomial distribution formula:

$$P(X = k) = \binom{n}{k} \cdot p^k \cdot (1 - p)^{n-k} \quad (1)$$

where:

$$n = \text{total number of trials (sample size)} = 10 \quad (2)$$

$$k = \text{number of successes we're interested in (at most 6)} \quad (3)$$

$$p = \text{probability of success (probability of a person being right-handed)} = 0.9 \quad (4)$$

$$\binom{n}{k} = \text{binomial coefficient} = \frac{n!}{k! \cdot (n - k)!} \quad (5)$$

We need to calculate the probabilities for $k = 0, 1, 2, 3, 4, 5$ and 6, and then sum them up to get the probability of getting at most 6 right-handed individuals. Let's calculate it step by step:

$$\text{For } k = 0 : P(X = 0) = \binom{10}{0} \cdot (0.9)^0 \cdot (1 - 0.9)^{10-0} \quad (6)$$

$$= (0.1)^{10} \quad (7)$$

$$= 0.0000000001 \quad (8)$$

$$\text{For } k = 1 : P(X = 1) = \binom{10}{1} \cdot (0.9)^1 \cdot (1 - 0.9)^{10-1} \quad (9)$$

$$= 10(0.9)(0.1)^9 \quad (10)$$

$$= 9(0.1)^9 \quad (11)$$

$$\text{For } k = 2 : P(X = 2) = \binom{10}{2} \cdot (0.9)^2 \cdot (1 - 0.9)^{10-2} \quad (12)$$

$$= 45(0.9)^2(0.1)^8 \quad (13)$$

$$= 36.45(0.1)^8 \quad (14)$$

$$\text{For } k = 3 : P(X = 3) = \binom{10}{3} \cdot (0.9)^3 \cdot (1 - 0.9)^{10-3} \quad (15)$$

$$= 120(0.9)^3(0.1)^7 \quad (16)$$

$$= 87.48(0.1)^7 \quad (17)$$

$$\text{For } k = 4 : P(X = 4) = \binom{10}{4} \cdot (0.9)^4 \cdot (1 - 0.9)^{10-4} \quad (18)$$

$$= 210(0.9)^4(0.1)^6 \quad (19)$$

$$= 137.781(0.1)^6 \quad (20)$$

$$\text{For } k = 5 : P(X = 5) = \binom{10}{5} \cdot (0.9)^5 \cdot (1 - 0.9)^{10-5} \quad (21)$$

$$= 252(0.9)^5(0.1)^5 \quad (22)$$

$$= 148.80348(0.1)^5 \quad (23)$$

$$\text{For } k = 6 : P(X = 6) = \binom{10}{6} \cdot (0.9)^6 \cdot (1 - 0.9)^{10-6} \quad (24)$$

$$= 210(0.9)^6(0.1)^4 \quad (25)$$

$$= 111.60261(0.1)^4 \quad (26)$$

Finally, the probability of getting at most 6 right-handed individuals is the sum of the probabilities for all these cases:

$$P(\text{at most } 6) = P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4) + P(X = 5) + P(X = 6) \quad (27)$$

$$= 9(0.1)^9 + 36.45(0.1)^8 + 87.48(0.1)^7 + 137.781(0.1)^6 + 148.80348(0.1)^5 + 111.60261(0.1)^4 \quad (28)$$

$$= 0.000000009 + 0.0000003645 + 0.000008748 + 0.000137781 + 0.0014880348 + 0.011160261 \quad (29)$$

$$= 0.0127951973 \quad (30)$$