

Question: Suppose that 90% of people are right-handed. What is the probability that at most 6 of a random sample of 10 people are right-handed?

Solution: Calculating Probability Using Bernoulli Distribution

Given that 90% of people are right-handed, let $p = \frac{9}{10}$ be the probability of success (right-handed), and $(1 - p) = \frac{1}{10}$ be the probability of failure (left-handed).

$$X \sim \text{Ber}(p) \quad (1)$$

Suppose $X_i, 1 \leq i \leq n$ represent each of the n draws. Define Y as

$$Y = \sum_{i=1}^n X_i \quad (2)$$

Then, since the X_i are iid, the pmf of Y is given by

$$Y \sim \text{Bin}(n, p) \quad (3)$$

The cdf of Y is given by

$$F_Y(k) = \Pr(Y \leq k) \quad (4)$$

$$= \begin{cases} 0 & k < 0 \\ \sum_{i=1}^k \binom{n}{i} p^i (1-p)^{n-i} & 1 \leq k \leq n \\ 1 & k \geq n \end{cases} \quad (5)$$

In this case,

$$p = \frac{9}{10}, \quad n = 10 \quad (6)$$

1) We require $\Pr(Y \leq 6)$. Since $n = 10$,

$$\Pr(Y \leq 6) = 1 - \Pr(Y > 6) \quad (7)$$

$$= F_Y(6) \quad (8)$$

$$= \sum_{k=0}^6 p_Y(k) \quad (9)$$

$$= \sum_{k=0}^6 \binom{n}{k} p^k (1-p)^{n-k} \quad (10)$$

$$= 0.0128 \quad (11)$$