Question:Suppose than 90% of people are right-handed. What is the probability that atmost 6 of a random sample of 10 people are right-handed?
Solution: :Calculating Probability Using Bernoulli Distribution
Given that 90% of people are right-handed, let $p = 0.90$ be the probability of success (right-handedness), and $q = 0.10$ be the probability of failure (left-handedness). We want to find the probability that at most 6 out of 10 people in a random sample are right-handed.

$$P(X = k) = \binom{n}{k} \cdot p^k \cdot q^{n-k} \tag{1}$$

where:

$$n = \text{total number of trials (sample size)} = 10 \tag{2}$$
$$k = \text{number of successes we're interested in (at most 6)} \tag{3}$$
$$p = \text{probability of success (probability of a person being right-handed)} = 0.9 \tag{4}$$
$$\binom{n}{k} = \text{binomial coefficient} = \frac{n!}{k! \cdot (n-k)!} \tag{5}$$

We need to calculate the probabilities for $k = 0, 1, 2, 3, 4, 5$ and 6, and then sum them up to get the probability of getting at most 6 right-handed individuals. Let's calculate it step by step:

$$P(X \leq 6) = \sum_{k=0}^{6} \binom{10}{k} \cdot (0.90)^k \cdot (0.10)^{10-k} \tag{6}$$

$$\begin{aligned}
\text{For } k = 0: \quad P(X = 0) &= \binom{10}{0} \cdot (0.9)^0 \cdot (1 - 0.9)^{10-0} && (7)\\
&= (0.1)^{10} && (8)\\
&= 0.0000000001 && (9)\\
\text{For } k = 1: \quad P(X = 1) &= \binom{10}{1} \cdot (0.9)^1 \cdot (1 - 0.9)^{10-1} && (10)\\
&= 10(0.9)(0.1)^9 && (11)\\
&= 9(0.1)^9 && (12)\\
\text{For } k = 2: \quad P(X = 2) &= \binom{10}{2} \cdot (0.9)^2 \cdot (1 - 0.9)^{10-2} && (13)\\
&= 45(0.9)^2(0.1)^8 && (14)\\
&= 36.45(0.1)^8 && (15)\\
\text{For } k = 3: \quad P(X = 3) &= \binom{10}{3} \cdot (0.9)^3 \cdot (1 - 0.9)^{10-3} && (16)\\
&= 120(0.9)^3(0.1)^7 && (17)\\
&= 87.48(0.1)^7 && (18)\\
\text{For } k = 4: \quad P(X = 4) &= \binom{10}{4} \cdot (0.9)^4 \cdot (1 - 0.9)^{10-4} && (19)\\
&= 210(0.9)^4(0.1)^6 && (20)\\
&= 137.781(0.1)^6 && (21)\\
\text{For } k = 5: \quad P(X = 5) &= \binom{10}{5} \cdot (0.9)^5 \cdot (1 - 0.9)^{10-5} && (22)\\
&= 252(0.9)^5(0.1)^5 && (23)\\
&= 148.80348(0.1)^5 && (24)\\
\text{For } k = 6: \quad P(X = 6) &= \binom{10}{6} \cdot (0.9)^6 \cdot (1 - 0.9)^{10-6} && (25)\\
&= 210(0.9)^6(0.1)^4 && (26)\\
&= 111.60261(0.1)^4 && (27)
\end{aligned}$$

Finally, the probability of getting at most 6 right-handed individuals is the sum of the probabilities for all these cases:

$$\begin{aligned}
P(\text{at most } 6) &= P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4) + P(X = 5) + P(X = 6) && (28)\\
&= 9(0.1)^9 + 36.45(0.1)^8 + 87.48(0.1)^7 + 137.781(0.1)^6 + 148.80348(0.1)^5 + 111.60261(0.1)^4 && (29)\\
&= 0.000000009 + 0.0000003645 + 0.000008748 + 0.000137781 + 0.0014880348 + 0.011160261 && (30)\\
&= 0.0127951973 && (31)
\end{aligned}$$