Question: Three persons, A, B and C, fire at a target in turn, starting with A. Their probability of hitting the target are 0.4, 0.3 and 0.2 respectively. The probability of two hits is
(A) 0.024
(B) 0.188
(C) 0.336
(D) 0.452
Solution:
Let X, Y and Z be random variables with definition given as under: we want to find the probability of

Random Variable	Values	Description
X	probability of A hitting the target	0.4
Y	probability of A hitting the target	0.3
Z	probability of A hitting the target	0.2

TABLE 0
DEFINITION OF RANDOM VARIABLES

two hits, which corresponds to $S = X + Y + Z$being equal to 2 PMF of S using z-transform: applying the z-transform on both the sides

$$M_S(z) = M_{X+Y+Z}(z) \tag{1}$$

Using the expectation operator:

$$E\left[z^{-S}\right] = E\left[z^{-X-Y-Z}\right] \tag{2}$$
$$= M_X(z) \cdot M_Y(z) \cdot M_Z(z) \tag{3}$$

Extracting the PMF by considering the defenition of z-transform

$$P_S(s) = \sum_{k=-\infty}^{\infty} P_X(k)P_Y(s-k)P_Z(s-k) \tag{4}$$

Substituting $s = 2$

$$P_S(s) = \sum_{k=-\infty}^{\infty} P_X(k)P_Y(s-k)P_Z(s-k) \tag{5}$$

For a Bernoulli random variable X with parameter p, the Z-transform is given by:

$$M_X(z) = E[z^X] = (1-p) + pz \tag{6}$$

$$\Pr(A) = 0.4 \tag{7}$$
$$\Pr(B) = 0.3 \tag{8}$$
$$\Pr(C) = 0.2 \tag{9}$$

$$\Pr(A') = 1 - \Pr(A) = 0.6 \tag{10}$$
$$\Pr(B') = 1 - \Pr(B) = 0.7 \tag{11}$$
$$\Pr(C') = 1 - \Pr(C) = 0.8 \tag{12}$$

$$\text{Probability that A wins,B wins and C misses} = \Pr(A) \times \Pr(B) \times \Pr(C') \tag{13}$$
$$= 0.4 \times 0.3 \times 0.8 \tag{14}$$
$$= 0.096 \tag{15}$$

$$\begin{aligned}
\text{Probability that A wins,B misses and C wins} &= \Pr(A) \times \Pr(B) \times \Pr(C') && (16)\\
&= 0.4 \times 0.7 \times 0.2 && (17)\\
&= 0.096 && (18)
\end{aligned}$$

$$\begin{aligned}
\text{Probability that A misses,B wins and C wins} &= \Pr(A) \times \Pr(B) \times \Pr(C') && (19)\\
&= 0.6 \times 0.3 \times 0.2 && (20)\\
&= 0.036 && (21)
\end{aligned}$$

$$\therefore \text{The probability of two hits } = 0.188 \qquad (22)$$