Question: An urn contains 25 balls of which 10 balls bear a mark X and the remaining 15 bear a mark Y. A ball is drawn at random from the urn, its mark is noted down and it is replaced. If 6 balls are drawn in this way, find the probability that

1) all will bear X mark.
2) not more than 2 will bear Y mark.
3) at least one ball will bear Y mark.
4) the number of balls with X mark and Y mark will be equal.

Solution:

Parameter	Values	Description
n	6	Number of draws
p_X	0.4	Probability that ball bears X mark
p_Y	0.6	Probability that ball bears X mark

TABLE 0
DEFINITION OF PARAMETERS

1) all will bear 'X mark Mean is given by

$$\mu = np_X \tag{1}$$
$$= 2.4 \tag{2}$$

Standard Deviation is given by

$$\sigma = \sqrt{np_X(1-p_X)} \tag{3}$$
$$= \sqrt{6 \times 0.4 \times (1-0.4)} \tag{4}$$
$$= \sqrt{6 \times 0.4 \times 0.6} \tag{5}$$
$$= 1.2 \tag{6}$$

We have,

$$Z = \frac{X-\mu}{\sigma} \tag{7}$$
$$= 3 \tag{8}$$

Hence, from standard distribution table of Z, we have

$$\Pr(Z \leq 2) = 0.49865 \tag{9}$$
$$= 49.865\% \tag{10}$$

2) not more than 2 will bear Y mark.
Mean is given by

$$\mu = np_Y \tag{11}$$
$$= 3.6 \tag{12}$$

Standard Deviation is given by

$$\sigma = \sqrt{np_Y(1-p_Y)} \quad (13)$$
$$= \sqrt{6 \times 0.6 \times 0.4} \quad (14)$$
$$= 1.2 \quad (15)$$

We need to find

$$Z_0 = -3 \quad (16)$$
$$Z_1 = -1.166 \quad (17)$$
$$Z_2 = -0.333 \quad (18)$$
$$\quad (19)$$

We have,

$$Z = \sum_{i=0}^{2} \frac{X_i - \mu}{\sigma} \quad (20)$$
$$= -3.5 \quad (21)$$

Hence, from standard distribution table of Z, we have

$$\Pr(Z \leq -3.5) = 0.49865 \quad (22)$$
$$= 49.865\% \quad (23)$$

3) at least one ball will bear Y mark.
Mean is given by

$$\mu = np_Y \quad (24)$$
$$= 3.6 \quad (25)$$

Standard Deviation is given by

$$\sigma = \sqrt{np_Y(1-p_Y)} \quad (26)$$
$$= \sqrt{6 \times 0.6 \times 0.4} \quad (27)$$
$$= 1.2 \quad (28)$$

$$\Pr(X \geq 1) = 0.98487 \quad (29)$$

4) the number of balls with X mark and Y mark will be equal.
Mean is given by

$$\mu = np_X \quad (30)$$
$$= 2.4 \quad (31)$$

Standard Deviation is given by

$$\sigma = \sqrt{np_X p_Y} \quad (32)$$
$$= \sqrt{6 \times 0.4 \times 0.6} \quad (33)$$
$$= 1.2 \quad (34)$$

We have,

$$Z = \frac{X - \mu}{\sigma} \tag{35}$$

$$= 3 \tag{36}$$

$$\Pr(2.4 \leq X \leq 3.6) \tag{37}$$

From standard distribution table of Z, we have

$$\Pr(2.4 \leq Z \leq 3.6) = 0.49865 \tag{38}$$

$$= 49.865\% \tag{39}$$