Question : An urn contains 25 balls of which 10 balls bear a mark X and the remaining 15 bear a mark Y. A ball is drawn at random from the urn, its mark is noted down and it is replaced. If 6 balls are drawn in this way, find the probability that

(a) all will bear X mark.

(b) not more than 2 will bear Y mark.

(c) at least one ball will bear Y mark.

(d) the number of balls with X mark and Y mark will be equal.

Solution:

Parameter	Values	Description
n	6	Number of draws
p	0.4	Probability that ball bears X mark
q	0.6	Probability that ball bears Y mark
$\mu = np$	2.4	mean of the distribution
$\sigma = npq$	1.2	variance of the distribution
X		Number of cards bear mark X
Y		Number of cards bear mark Y

TABLE 4
DEFINITION OF PARAMETERS

using Gaussian

$$Y \sim \mathcal{N}\left(\mu, \sigma^2\right) \tag{1}$$

The CDF of Y:

$$F_Y(y) = 1 - \Pr(Y > y) \tag{2}$$
$$= 1 - \Pr\left(\frac{Y-\mu}{\sigma} > \frac{y-\mu}{\sigma}\right) \tag{3}$$

But,

$$\frac{Y-\mu}{\sigma} \sim \mathcal{N}(2.4, 1.44) \tag{4}$$
$$\tag{5}$$

the Q-function is defined as:

$$Q(x) = \Pr(Y > x)\ \forall x \in Y \sim \mathcal{N}(2.4, 1.44) \tag{6}$$

therefore the cdf will be:

$$F_Y(y) = \begin{cases} 1 - Q\left(\frac{y-\mu}{\sigma}\right), & y > \mu \\ Q\left(\frac{\mu-y}{\sigma}\right), & y < \mu \end{cases} \tag{7}$$

(a) all will bear X mark.
using Gaussian
considering 0.5 as the correction term,

$$\Pr(X > 5.5) = 1 - F_X(5.5) \tag{8}$$
$$= Q\left(\frac{5.5-\mu}{\sigma}\right) \tag{9}$$
$$= Q\left(\frac{3.1}{1.2}\right) \tag{10}$$
$$= Q(2.583) \tag{11}$$
$$= 0.00489 \tag{12}$$

(b) not more than 2 will bear Y mark.
using Gaussian
considering 0.5 as the correction term,

$$\Pr(Y < 2.5) = 1 - Q\left(\frac{2.5-\mu}{\sigma}\right) \tag{13}$$
$$= 1 - Q\left(\frac{-1.1}{1.2}\right) \tag{14}$$
$$= 1 - Q(-0.9166) \tag{15}$$
$$= Q(0.9166) \qquad = 0.1796 \tag{16}$$

(c) at least one ball will bear Y mark.
using Gaussian
considering 0.5 as the correction term,

$$\Pr(Y < 0.5) = 1 - Q\left(\frac{0.5-\mu}{\sigma}\right) \tag{17}$$
$$= 1 - Q\left(\frac{-1.1}{1.2}\right) \tag{18}$$
$$= 1 - Q(-2.588) \tag{19}$$
$$= 1 - 0.0048 \tag{20}$$
$$= 0.9952 \tag{21}$$

(d) the number of balls with X mark and Y mark will be equal.
using Gaussian
considering 0.5 as the correction term,
the gaussian distribution function is defined as:

$$p_Y(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \tag{22}$$

the probability of the person winning the prize exactly once is given by:

$$p_X(3) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(3-\mu)^2}{2\sigma^2}} \tag{23}$$
$$= \frac{0.882}{\sqrt{2\pi\sigma^2}} \tag{24}$$
$$= 0.2933 \tag{25}$$
$$p_Y(3) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(3-\mu)^2}{2\sigma^2}} \tag{26}$$
$$= 0.2933 \tag{27}$$
$$\Pr(X = 3 + Y = 3) = p_X(3)\, p_Y(3) \tag{28}$$
$$= 0.086 \tag{29}$$

Gaussian vs Binomial Table

Question	Gaussian	Binomial
all will bear X mark	0.00489	0.00409
not more than 2 will bear Y mark	0.1796	0.1792
at least one ball will bear Y mark	0.9952	0.9959
the number of balls with X mark and Y mark will be equal	2.4	0.2764

TABLE 4
DEFINITION OF PARAMETERS

Gaussian vs Binomial graph

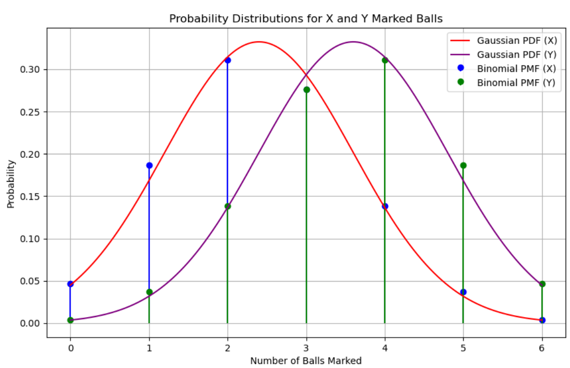

Fig. 4. pmf of binomial and pdf of Gaussian of X and Y marked balls