Question An urn contains 25 balls of which 10 balls bear a mark X and the remaining 15 bear a mark Y. A ball is drawn at random from the urn, its mark is noted down and it is replaced. If 6 balls are drawn in this way, find the probability that

1) all will bear X mark.
2) not more than 2 will bear Y mark.
3) at least one ball will bear Y mark.
4) the number of balls with X mark and Y mark will be equal.

Solution:

Parameter	Values	Description
n	6	Number of draws
p	0.4	Probability that ball bears X mark
q	0.6	Probability that ball bears Y mark
μ	2.4	np
σ	1.2	$\sqrt{np(1-p)}$
X		Number of cards bear mark X
Y		Number of cards bear mark X

TABLE 0
DEFINITION OF PARAMETERS

1) all will bear 'X mark We know that Q-function is given as

$$Q(x) = \Pr(X > x) \tag{1}$$

Now, we want to find probability that all will bear X mark.

$$\Pr(X = 0) = 1 - \Pr(X > 0) \tag{2}$$
$$= 1 - \Pr\left(\frac{X-\mu}{\sigma} > \frac{0 - 2.4}{1.2}\right) \tag{3}$$
$$= 1 - Q(-2) \tag{4}$$
$$= 1 - \int_{-2}^{\infty} \frac{1}{\sqrt{2\pi}} \times e^{-\frac{x^2}{2}} dx \tag{5}$$
$$= 1 - 0.9958 \tag{6}$$
$$= 0.004082 \tag{7}$$

Binomial

$$\Pr(X = 0) = 1 - \Pr(X > 0) \tag{8}$$
$$= {}^{n}C_k p^k (1-p)^{n-k} \tag{9}$$
$$= 0.004096 \tag{10}$$

2) not more than 2 will bear Y mark.
We know that Q-function is given as

$$Q(x) = \Pr(X > x) \tag{11}$$
$$= \int_{x}^{\infty} \frac{1}{\sqrt{2\pi}} \times e^{-\frac{x^2}{2}} dx \tag{12}$$

We need to find probability that not more than 2 will bear Y mark

$$\Pr(X \leq 2) = 1 - \Pr(X > 2) \tag{13}$$
$$= 1 - \Pr\left(\frac{X-\mu}{\sigma} > \frac{2 - 3.6}{1.2}\right) \tag{14}$$
$$= 1 - Q(-1.33) \tag{15}$$
$$= 1 - \int_{-1.33}^{\infty} \frac{1}{\sqrt{2\pi}} \times e^{-\frac{x^2}{2}} dx \tag{16}$$
$$= 1 - 0.921 \tag{17}$$
$$= 0.179 \tag{18}$$

Binomial

$$\Pr(X \leq 2) = 1 - \Pr(X > 2) \tag{19}$$
$$= 1 - \sum_{k=3}^{6} {}^{n}C_k p^k (1-p)^{n-k} \tag{20}$$
$$= 0.1792 \tag{21}$$

3) at least one ball will bear Y mark.
We know that Q-function is given as

$$Q(x) = \Pr(X > x) \tag{22}$$

We need to find the probability that at least one ball will bear Y mark.

$$\Pr(X \geq 1) = \Pr(X > 0) \tag{23}$$
$$= \Pr\left(\frac{X-\mu}{\sigma} > \frac{0 - 3.6}{\sqrt{1.2}}\right) \tag{24}$$
$$= \Pr\left(\frac{X-\mu}{\sigma} > -3\right) \tag{25}$$
$$= 0.99586 \tag{26}$$
$$\tag{27}$$

Binomial

$$\Pr(X \geq 1) = 1 - \Pr(X = 0) \tag{28}$$
$$= 1 - {}^{n}C_k p^k (1-p)^{n-k} \tag{29}$$
$$= 0.995904 \tag{30}$$

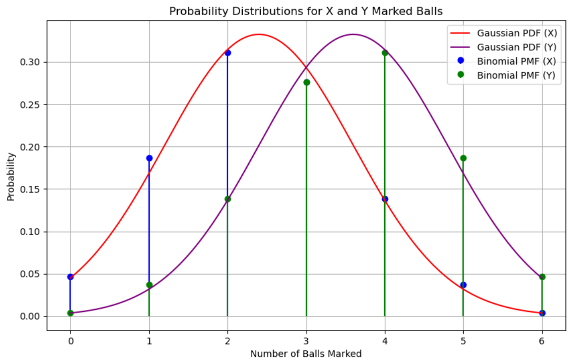

Fig. 4. pmf of binomial and pdf of Gaussian of X and Y marked balls

4) the number of balls with X mark and Y mark will be equal.
   Here, Probability that the number of balls with X mark and Y mark will be equal.

$$\Pr(X = 3) = 1 - Q(x) \tag{31}$$
$$= 1 - \frac{X - \mu}{\sigma} \tag{32}$$
$$= 0.2763 \tag{33}$$
$$\tag{34}$$

Binomial

$$\Pr(X = 3) = {}^{n}C_{k} p^{k} (1 - p)^{n-k} \tag{35}$$
$$= 0.2764 \tag{36}$$