Question: Three persons, A, B and C, fire at a target in turn, starting with A. Their probability of hitting the target are 0.4, 0.3 and 0.2 respectively. The probability of two hits is
(A) 0.024
(B) 0.188
(C) 0.336
(D) 0.452
Solution:
Let X, Y and Z be random variables with definition given as under:

Random Variable	Description
X	A hitting the target
Y	B hitting the target
Z	C hitting the target

TABLE 0
DEFINITION OF RANDOM VARIABLES

Bernouli probability	Value	Description
p_1	Probability of A hitting the target	0.4
p_2	Probability of B hitting the target	0.3
p_3	Probability of C hitting the target	0.2

TABLE 0
DEFINITION OF RANDOM VARIABLES

We want to find the probability of two hits, which corresponds to $S = X + Y + Z$ being equal to 2
PMF of S using z-transform:
Applying the z-transform on both the sides

$$M_S(z) = M_{X+Y+Z}(z) \tag{1}$$

Using the expectation operator:

$$E\left[z^{-S}\right] = M_X(z) \cdot M_Y(z) \cdot M_Z(z) \tag{2}$$

For a Bernoulli random variable X with parameter p, the Z-transform is given by:

$$M_X(z) = (1 - p_i) + zp_i \tag{3}$$

For the random variables

$$M_X(z) = (1 - 0.4) + 0.4z = 0.6 + 0.4z \tag{4}$$
$$M_Y(z) = (1 - 0.3) + 0.3z = 0.7 + 0.3z \tag{5}$$
$$M_Z(z) = (1 - 0.2) + 0.2z = 0.8 + 0.2z \tag{6}$$

Now, we can find the z-transform of S, denoted as $M_S(z)$, which is the product of the Z-transforms of X,Y and Z since they are independent

Coefficient of the z^2 term in $M_S(z)$ is corresponds to the probability of two hits.

$$M_S(z) = M_X(z)M_Y(z)M_Z(z) \tag{7}$$

$$= \prod_{k=1}^{3}(1 - p_i) + zp_i \tag{8}$$

$$= (0.6 + 0.4z)(0.7 + 0.3z)(0.8 + 0.2z) \tag{9}$$

$$= 0.188z^2 + 1.208z + .. \tag{10}$$

$$\therefore \text{The probability of two hits} = 0.188 \tag{11}$$