Question: An urn contains 25 balls of which 10 balls bear a mark X and the remaining 15 bear a mark Y. A ball is drawn at random from the urn, its mark is noted down and it is replaced. If 6 balls are drawn in this way, find the probability that

1) all will bear X mark.
2) not more than 2 will bear Y mark.
3) at least one ball will bear Y mark.
4) the number of balls with X mark and Y mark will be equal.

Solution:

Parameter	Values	Description
n	6	Number of items
p_X	0.4	Probability that ball bears X mark
p_Y	0.6	Probability that ball bears X mark

TABLE 0
DEFINITION OF PARAMETERS

1) all will bear 'X mark Mean is given by

$$\mu = np_X \tag{1}$$
$$= 2.4 \tag{2}$$

Standard Deviation is given by

$$\sigma = \sqrt{np_X(1-p_X)} \tag{3}$$
$$= \sqrt{6 \times 0.4 \times (1-0.4)} \tag{4}$$
$$= \sqrt{6 \times 0.4 \times 0.6} \tag{5}$$
$$= 1.2 \tag{6}$$

We have,

$$Z = \frac{X-\mu}{\sigma} \tag{7}$$
$$= 3 \tag{8}$$

Hence, from standard distribution table of Z, we have

$$\Pr(Z \leq 3) = \tag{9}$$
$$= \% \tag{10}$$

2) not more than 2 will bear Y mark.
Mean is given by

$$\mu = np_X \tag{11}$$
$$= 2.4 \tag{12}$$

Standard Deviation is given by

$$\sigma = \sqrt{np_X(1-p_X)} \tag{13}$$
$$= 1.2 \tag{14}$$

We need to find

$$\Pr(X \le 2) = \Pr(X < 1.5) \tag{15}$$

We have,

$$Z = \sum_{i=0}^{2} \frac{X_i - \mu}{\sigma} \tag{16}$$
$$= 3 \tag{17}$$

3) at least one ball will bear Y mark.
   Mean is given by

$$\mu = np_Y \tag{18}$$
$$= 2.4 \tag{19}$$

   Standard Deviation is given by

$$\sigma = \sqrt{np_Y(1-p_Y)} \tag{20}$$
$$= \sqrt{6 \times 0.6 \times (1-0.6)} \tag{21}$$
$$= \sqrt{6 \times 0.6 \times 0.4} \tag{22}$$
$$= 1.2 \tag{23}$$

4) the number of balls with X mark and Y mark will be equal.