Question An urn contains 25 balls of which 10 balls bear a mark X and the remaining 15 bear a mark Y. A ball is drawn at random from the urn, its mark is noted down and it is replaced. If 6 balls are drawn in this way, find the probability that

(a) all will bear X mark.

(b) not more than 2 will bear Y mark.

(c) at least one ball will bear Y mark.

(d) the number of balls with X mark and Y mark will be equal.

Solution:

Parameter	Values	Description
n	6	Number of draws
p	0.4	Probability that ball bears X mark
q	0.6	Probability that ball bears Y mark
μ	2.4	np
σ	1.2	$\sqrt{np(1-p)}$
X		Number of cards bear mark X
Y		Number of cards bear mark Y

TABLE 4
DEFINITION OF PARAMETERS

(i) **Gaussian Distribution**

$$X \approx Y \sim \mathcal{N}(2.4, 1.2) \tag{1}$$

(a) all will bear X mark

$$X = 5 \tag{2}$$

i) Without correction:

$$\Pr(X = 5) = Q\left(\frac{5-\mu}{\sigma}\right) \tag{3}$$
$$= Q\left(\frac{5-1.2}{2.4}\right) \tag{4}$$
$$= Q(1.583) \tag{5}$$
$$= 0.005671 \tag{6}$$

ii) With a 0.5 correction:

$$\Pr(X \geq 5) = Q\left(\frac{5+0.5-\mu}{\sigma}\right) \tag{7}$$
$$= Q\left(\frac{5+0.5-1.2}{2.4}\right) \tag{8}$$
$$= Q\left(\sqrt{2.5}\right) \tag{9}$$
$$= 0.004 \tag{10}$$

Binomial

$$\Pr(X = 5) = {}^{n}C_{k} p^{k} (1-p)^{n-k} \tag{11}$$
$$= 0.004096 \tag{12}$$

(b) not more than 2 will bear Y mark.

$$Y \leq 2 \tag{13}$$

i) Without correction:

$$\Pr(Y \leq 2) = Q\left(\frac{2-\mu}{\sigma}\right) \tag{14}$$
$$= Q\left(\frac{2-1.2}{2.4}\right) \tag{15}$$
$$= Q(0.33) \tag{16}$$
$$= 0.3707 \tag{17}$$

ii) With a 0.5 correction:

$$\Pr(Y \leq 2) = Q\left(\frac{2+0.5-\mu}{\sigma}\right) \tag{18}$$
$$= Q\left(\frac{2+0.5-1.2}{2.4}\right) \tag{19}$$
$$= Q(0.541) \tag{20}$$
$$= 0.1781 \tag{21}$$

Binomial

$$\Pr(Y \leq 2) = 1 - \Pr(Y > 2) \tag{22}$$
$$= 1 - \sum_{k=3}^{6} {}^{n}C_{k} p^{k} (1-p)^{n-k} \tag{23}$$
$$= 0.1792 \tag{24}$$

(c) at least one ball will bear Y mark.

$$Y \leq 2 \tag{25}$$

i) Without correction:

$$\Pr(\geq 1) = Q\left(\frac{1-\mu}{\sigma}\right) \tag{26}$$
$$= Q\left(\frac{1-1.2}{2.4}\right) \tag{27}$$
$$= Q(-0.833) \tag{28}$$
$$= 0.791462 \tag{29}$$

ii) With a 0.5 correction:

$$\Pr(Y \geq 1) = Q\left(\frac{1+0.5-\mu}{\sigma}\right) \tag{30}$$
$$= Q\left(\frac{1+0.5-1.2}{2.4}\right) \tag{31}$$
$$= Q\left(\sqrt{1.125}\right) \tag{32}$$
$$= Q(1.5811) \tag{33}$$
$$= 0.981 \tag{34}$$

Binomial

$$\Pr(Y \geq 1) = 1 - \Pr(Y = 0) \tag{35}$$
$$= 1 - {}^{n}C_{k} p^{k} (1-p)^{n-k} \tag{36}$$
$$= 0.995904 \tag{37}$$

(d) the number of balls with X mark and Y mark will be equal.

$$Y = 3 \tag{38}$$

i) Without correction:

$$\Pr(X = 3) = Q\left(\frac{3-\mu}{\sigma}\right) \tag{39}$$
$$= Q\left(\frac{3-1.2}{2.4}\right) \tag{40}$$
$$= Q(0.75) \tag{41}$$
$$= 0.2276 \tag{42}$$

ii) With a 0.5 correction:

$$\Pr(X = 3) = Q\left(\frac{3+0.5-\mu}{\sigma}\right) \tag{43}$$
$$= Q\left(\frac{3+0.5-1.2}{2.4}\right) \tag{44}$$
$$= Q(0.85) \tag{45}$$
$$= 0.274 \tag{46}$$

Binomial

$$\Pr(X = 3) = {}^{n}C_{k} p^{k} (1-p)^{n-k} \tag{47}$$
$$= 0.2764 \tag{48}$$

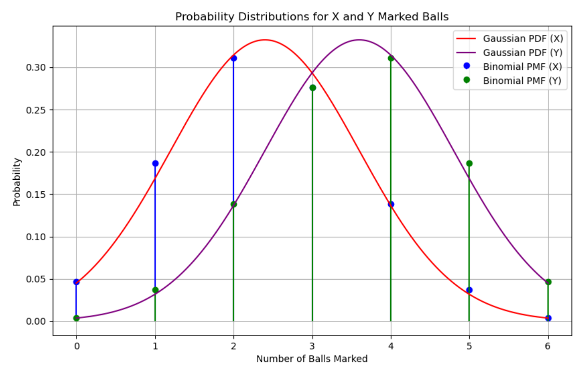

Fig. 1. pmf of binomial and pdf of Gaussian of X and Y marked balls