Question An urn contains 25 balls of which 10 balls bear a mark X and the remaining 15 bear a mark Y. A ball is drawn at random from the urn, its mark is noted down and it is replaced. If 6 balls are drawn in this way, find the probability that

1) all will bear X mark.
2) not more than 2 will bear Y mark.
3) at least one ball will bear Y mark.
4) the number of balls with X mark and Y mark will be equal.

Solution:

Parameter	Values	Description
n	6	Number of draws
p	0.4	Probability that ball bears X mark
q	0.6	Probability that ball bears X mark
μ	2.4	np
μ	3.6	nq
σ	1.2	$\sqrt{np(1-p)}$

TABLE 0
DEFINITION OF PARAMETERS

1) all will bear 'X mark We know that Q-function is given as

$$Q(x) = \Pr(X > x) \tag{1}$$

Now, we want to find probability that all will bear X mark.

$$\begin{align} \Pr(X = 0) &= 1 - \Pr(X > 0) \tag{2} \\ &= 1 - \Pr\left(\frac{X-\mu}{\sigma} > \frac{0-2.4}{1.2}\right) \tag{3} \\ &= 1 - Q(-2) \tag{4} \\ &= 1 - \int_{-2}^{\infty} \frac{1}{\sqrt{2\pi}} \times e^{-\frac{x^2}{2}} dx \tag{5} \\ &= 1 - 0.9958 \tag{6} \\ &= 0.004082 \tag{7} \end{align}$$

Binomial

$$\begin{align} \Pr(X = 0) &= 1 - \Pr(X > 0) \tag{8} \\ &= {}^{n}C_k p^k (1-p)^{n-k} \tag{9} \\ &= 0.004096 \tag{10} \end{align}$$

2) not more than 2 will bear Y mark.
We know that Q-function is given as

$$\begin{align} Q(x) &= \Pr(X > x) \tag{11} \\ &= \int_{x}^{\infty} \frac{1}{\sqrt{2\pi}} \times e^{-\frac{x^2}{2}} dx \tag{12} \end{align}$$

We need to find probability that not more than 2 will bear Y mark

$$\begin{align} \Pr(X \le 2) &= 1 - \Pr(X > 2) \tag{13} \\ &= 1 - \Pr\left(\frac{X-\mu}{\sigma} > \frac{2-3.6}{1.2}\right) \tag{14} \\ &= 1 - Q(-1.33) \tag{15} \\ &= 1 - \int_{-1.33}^{\infty} \frac{1}{\sqrt{2\pi}} \times e^{-\frac{x^2}{2}} dx \tag{16} \\ &= 1 - 0.921 \tag{17} \\ &= 0.179 \tag{18} \end{align}$$

Binomial

$$\begin{align} \Pr(X \le 2) &= 1 - \Pr(X > 2) \tag{19} \\ &= 1 - \sum_{k=3}^{6} {}^{n}C_k p^k (1-p)^{n-k} \tag{20} \\ &= 0.1792 \tag{21} \end{align}$$

3) at least one ball will bear Y mark.
We know that Q-function is given as

$$Q(x) = \Pr(X > x) \tag{22}$$

We need to find the probability that at least one ball will bear Y mark.

$$\begin{align} \Pr(X \ge 1) &= \Pr(X > 0) \tag{23} \\ &= \Pr\left(\frac{X-\mu}{\sigma} > \frac{0-3.6}{\sqrt{1.2}}\right) \tag{24} \\ &= \Pr\left(\frac{X-\mu}{\sigma} > -3\right) \tag{25} \\ &= 0.99586 \tag{26} \\ & \tag{27} \end{align}$$

Binomial

$$\begin{align} \Pr(X \ge 1) &= 1 - \Pr(X = 0) \tag{28} \\ &= 1 - {}^{n}C_k p^k (1-p)^{n-k} \tag{29} \\ &= 0.995904 \tag{30} \end{align}$$

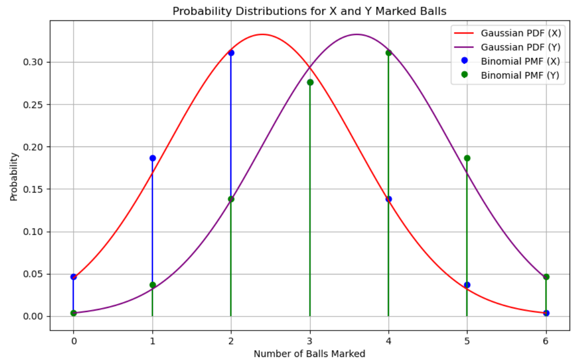

Fig. 4. pmf of binomial and pdf of Gaussian of X and Y marked balls

4) the number of balls with X mark and Y mark will be equal.
Here, Probability that the number of balls with X mark and Y mark will be equal.

$$\Pr(X = 3) = 1 - Q(x) \tag{31}$$
$$= 1 - \frac{X - \mu}{\sigma} \tag{32}$$
$$= 0.2763 \tag{33}$$
$$\tag{34}$$

Binomial

$$\Pr(X = 3) = {}^{n}C_k p^k (1 - p)^{n-k} \tag{35}$$
$$= 0.2764 \tag{36}$$