Question An urn contains 25 balls of which 10 balls bear a mark X and the remaining 15 bear a mark Y. A ball is drawn at random from the urn, its mark is noted down and it is replaced. If 6 balls are drawn in this way, find the probability that

1) all will bear X mark.
2) not more than 2 will bear Y mark.
3) at least one ball will bear Y mark.
4) the number of balls with X mark and Y mark will be equal.

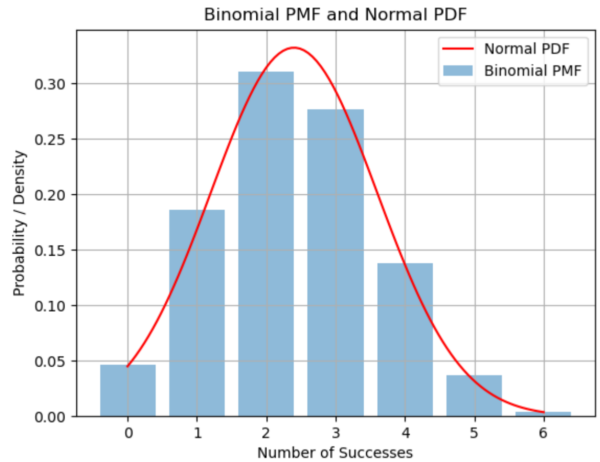

Fig. 1. pmf of binomial and pdf of Gaussian

Solution:

Parameter	Values	Description
n	6	Number of draws
p	0.4	Probability that ball bears X mark
q	0.6	Probability that ball bears X mark

TABLE 0
DEFINITION OF PARAMETERS

1) all will bear 'X mark Mean is given by

$$\mu = np \tag{1}$$
$$= 2.4 \tag{2}$$

Standard Deviation is given by

$$\sigma = \sqrt{np(1-p)} \tag{3}$$
$$= \sqrt{6 \times 0.4 \times (1-0.4)} \tag{4}$$
$$= \sqrt{6 \times 0.4 \times 0.6} \tag{5}$$
$$= 1.2 \tag{6}$$

We know that Q-function is given as

$$Q(x) = \Pr(X > x) \tag{7}$$
$$= \int_x^\infty \frac{1}{\sqrt{2\pi}} \times e^{-\frac{x^2}{2}} dx \tag{8}$$

Now, we want to find probability that all will bear X mark.

$$\Pr(X > 2) = Q(x) \tag{9}$$
$$= \Pr\left(\frac{X-\mu}{\sigma} > \frac{0-2.4}{\sqrt{1.2}}\right) \tag{10}$$
$$= \Pr(Z > -2) \tag{11}$$
$$= Q(-2) \tag{12}$$
$$= \int_{-2}^\infty \frac{1}{\sqrt{2\pi}} \times e^{-\frac{x^2}{2}} dx \tag{13}$$
$$= 0.97725 \tag{14}$$

Hence, from standard distribution table of Z, we have

$$\Pr(X > 2) = 97.725\% \tag{15}$$

2) not more than 2 will bear Y mark.
Mean is given by

$$\mu = nq \tag{16}$$
$$= 3.6 \tag{17}$$

Standard Deviation is given by

$$\sigma = \sqrt{nq(1-q)} \tag{18}$$
$$= \sqrt{6 \times 0.6 \times 0.4} \tag{19}$$
$$= 1.2 \tag{20}$$

We know that Q-function is given as

$$Q(x) = \Pr(X > x) \tag{21}$$
$$= \int_x^\infty \frac{1}{\sqrt{2\pi}} \times e^{-\frac{x^2}{2}} dx \tag{22}$$

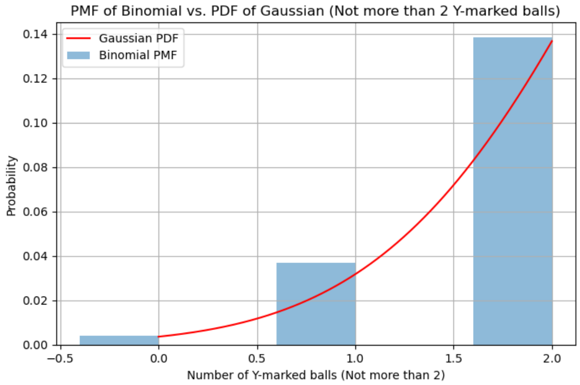

Fig. 2. pmf of binomial and pdf of Gaussian

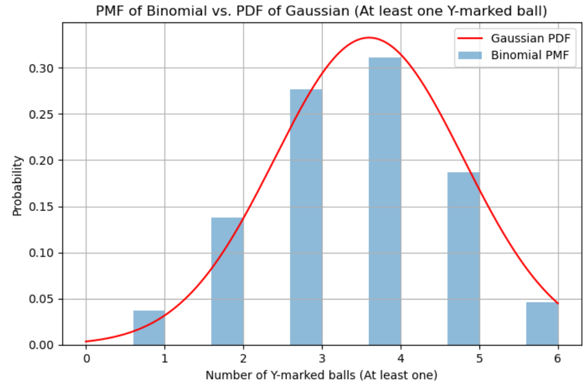

Fig. 3. pmf of binomial and pdf of Gaussian

We need to find probability that not more than 2 will bear Y mark

$$\begin{aligned}
\Pr(X \le 2) &= 1 - \Pr(X > 2) \tag{23}\\
&= 1 - \Pr\left(\frac{X-\mu}{\sigma} > \frac{2-3.6}{\sqrt{1.2}}\right) \tag{24}\\
&= 1 - \Pr(Z > -1.33) \tag{25}\\
&= 1 - Q(-1.33) \tag{26}\\
&= 1 - \int_{-1.33}^{\infty} \frac{1}{\sqrt{2\pi}} \times e^{-\frac{x^2}{2}} dx \tag{27}\\
&= 1 - 0.9082 \tag{28}\\
&= 0.0918 \tag{29}
\end{aligned}$$

Hence, from standard distribution table of Z, we have

$$\Pr(X \le 2) = 9.18\% \tag{30}$$

3) at least one ball will bear Y mark.
Mean is given by

$$\begin{aligned}
\mu &= nq \tag{31}\\
&= 3.6 \tag{32}
\end{aligned}$$

Standard Deviation is given by

$$\begin{aligned}
\sigma &= \sqrt{nq(1-q)} \tag{33}\\
&= \sqrt{6 \times 0.6 \times 0.4} \tag{34}\\
&= 1.2 \tag{35}
\end{aligned}$$

We know that Q-function is given as

$$Q(x) = \Pr(X > x) \tag{36}$$

We need to find the probability that at least one ball will bear Y mark.

$$\begin{aligned}
\Pr(X \ge 1) &= \Pr(X > 0) \tag{37}\\
&= \Pr\left(\frac{X-\mu}{\sigma} > \frac{0-3.6}{\sqrt{1.2}}\right) \tag{38}\\
&= \Pr(Z > -3) \tag{39}\\
&= 0.9986 \tag{40}\\
& \tag{41}
\end{aligned}$$

Hence, from standard distribution table of Z, we have

$$\Pr(X \ge 1) = 99.86\% \tag{42}$$

4) the number of balls with X mark and Y mark will be equal.
Mean is given by

$$\begin{aligned}
\mu &= np \tag{43}\\
&= 2.4 \tag{44}
\end{aligned}$$

Standard Deviation is given by

$$\begin{aligned}
\sigma &= \sqrt{npq} \tag{45}\\
&= \sqrt{6 \times 0.4 \times 0.6} \tag{46}\\
&= 1.2 \tag{47}
\end{aligned}$$

Here, Probability that the number of balls with X mark and Y mark will be equal.

$$\begin{aligned}
\Pr(X = 3) &= 1 - Q(x) \tag{48}\\
&= 1 - \frac{X-\mu}{\sigma} \tag{49}\\
&= 00134 \tag{50}\\
& \tag{51}
\end{aligned}$$

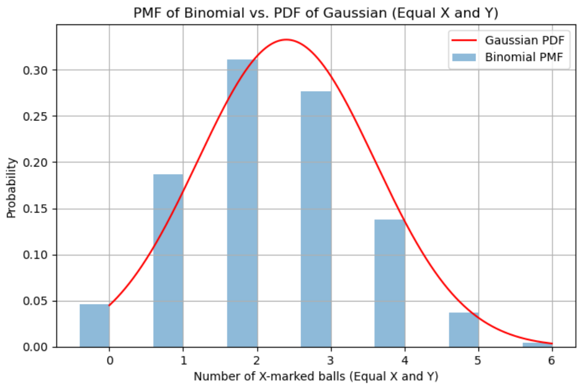

Fig. 4. pmf of binomial and pdf of Gaussian

From standard distribution table of Z, we have

$$\Pr(X = 3) = 0.134\% \tag{52}$$