Question An urn contains 25 balls of which 10 balls bear a mark X and the remaining 15 bear a mark Y. A ball is drawn at random from the urn, its mark is noted down and it is replaced. If 6 balls are drawn in this way, find the probability that

1) all will bear X mark.
2) not more than 2 will bear Y mark.
3) at least one ball will bear Y mark.
4) the number of balls with X mark and Y mark will be equal.

Solution:

Parameter	Values	Description
n	6	Number of draws
p	0.4	Probability that ball bears X mark
q	0.6	Probability that ball bears X mark

TABLE 0
DEFINITION OF PARAMETERS

1) all will bear 'X mark Mean is given by

$$\mu = np \tag{1}$$
$$= 2.4 \tag{2}$$

Standard Deviation is given by

$$\sigma = \sqrt{np(1-p)} \tag{3}$$
$$= \sqrt{6 \times 0.4 \times (1-0.4)} \tag{4}$$
$$= \sqrt{6 \times 0.4 \times 0.6} \tag{5}$$
$$= 1.2 \tag{6}$$

We know that Q-function is given as

$$Q(x) = \Pr(X > x) \tag{7}$$
$$= \int_x^{\infty} \frac{1}{\sqrt{2\pi}} \times e^{-\frac{x^2}{2}} dx \tag{8}$$

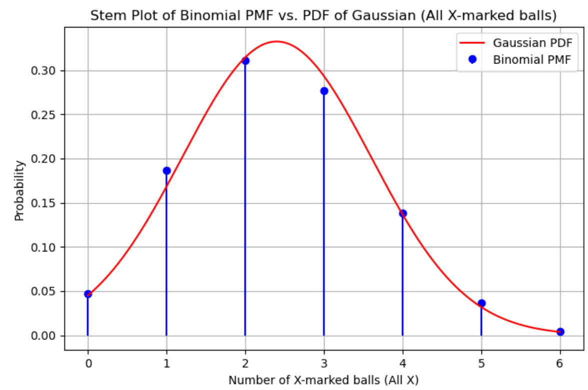

Fig. 1. pmf of binomial and pdf of Gaussian

Now, we want to find probability that all will bear X mark.

$$\Pr(X = 0) = 1 - \Pr(X > 0) \tag{9}$$
$$= 1 - \Pr\left(\frac{X-\mu}{\sigma} > \frac{0-2.4}{1.2}\right) \tag{10}$$
$$= 1 - \Pr(Z > -2) \tag{11}$$
$$= 1 - Q(-2) \tag{12}$$
$$= 1 - \int_{-2}^{\infty} \frac{1}{\sqrt{2\pi}} \times e^{-\frac{x^2}{2}} dx \tag{13}$$
$$= 1 - 0.9958 \tag{14}$$
$$= 0.004082 \tag{15}$$

Binomial

$$\Pr(X = 0) = 1 - \Pr(X > 0) \tag{16}$$
$$= \binom{n}{k} p^k (1-p)^{n-k} \tag{17}$$
$$= 0.004096 \tag{18}$$

2) not more than 2 will bear Y mark.
Mean is given by

$$\mu = nq \tag{19}$$
$$= 3.6 \tag{20}$$

Standard Deviation is given by

$$\sigma = \sqrt{nq(1-q)} \tag{21}$$
$$= \sqrt{6 \times 0.6 \times 0.4} \tag{22}$$
$$= 1.2 \tag{23}$$

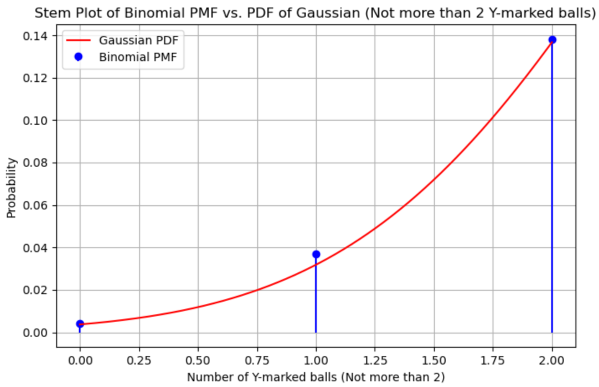

Fig. 2. pmf of binomial and pdf of Gaussian

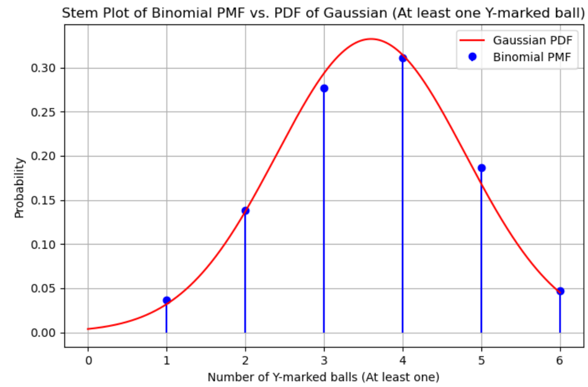

Fig. 3. pmf of binomial and pdf of Gaussian

We know that Q-function is given as

$$Q(x) = \Pr(X > x) \tag{24}$$
$$= \int_x^\infty \frac{1}{\sqrt{2\pi}} \times e^{-\frac{x^2}{2}} dx \tag{25}$$

We need to find probability that not more than 2 will bear Y mark

$$\Pr(X \leq 2) = 1 - \Pr(X > 2) \tag{26}$$
$$= 1 - \Pr\left(\frac{X-\mu}{\sigma} > \frac{2-3.6}{1.2}\right) \tag{27}$$
$$= 1 - \Pr(Z > -1.33) \tag{28}$$
$$= 1 - Q(-1.33) \tag{29}$$
$$= 1 - \int_{-1.33}^\infty \frac{1}{\sqrt{2\pi}} \times e^{-\frac{x^2}{2}} dx \tag{30}$$
$$= 1 - 0.921 \tag{31}$$
$$= 0.179 \tag{32}$$

Binomial

$$\Pr(X \leq 2) = 1 - \Pr(X > 2) \tag{33}$$
$$= 1 - \sum_{k=3}^{6} \binom{n}{k} p^k (1-p)^{n-k} \tag{34}$$
$$= 0.1792 \tag{35}$$

3) at least one ball will bear Y mark.
Mean is given by

$$\mu = nq \tag{36}$$
$$= 3.6 \tag{37}$$

Standard Deviation is given by

$$\sigma = \sqrt{nq(1-q)} \tag{38}$$
$$= \sqrt{6 \times 0.6 \times 0.4} \tag{39}$$
$$= 1.2 \tag{40}$$

We know that Q-function is given as

$$Q(x) = \Pr(X > x) \tag{41}$$

We need to find the probability that at least one ball will bear Y mark.

$$\Pr(X \geq 1) = \Pr(X > 0) \tag{42}$$
$$= \Pr\left(\frac{X-\mu}{\sigma} > \frac{0-3.6}{\sqrt{1.2}}\right) \tag{43}$$
$$= \Pr(Z > -3) \tag{44}$$
$$= 0.99586 \tag{45}$$
$$\tag{46}$$

Binomial

$$\Pr(X \geq 1) = 1 - \Pr(X = 0) \tag{47}$$
$$= 1 - \binom{n}{k} p^k (1-p)^{n-k} \tag{48}$$
$$= 0.995904 \tag{49}$$

4) the number of balls with X mark and Y mark will be equal.
Mean is given by

$$\mu = np \tag{50}$$
$$= 2.4 \tag{51}$$

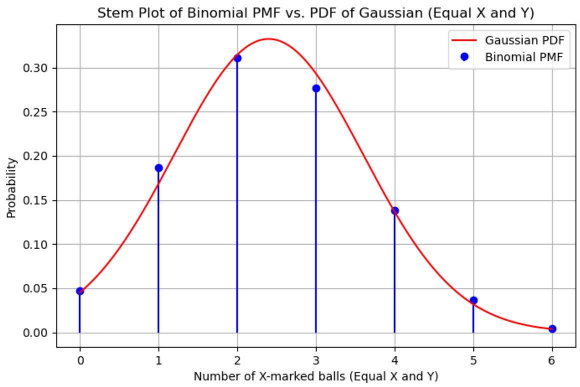

Fig. 4. pmf of binomial and pdf of Gaussian

Standard Deviation is given by

$$\sigma = \sqrt{npq} \tag{52}$$
$$= \sqrt{6 \times 0.4 \times 0.6} \tag{53}$$
$$= 1.2 \tag{54}$$

Here, Probability that the number of balls with X mark and Y mark will be equal.

$$\Pr(X = 3) = 1 - Q(x) \tag{55}$$
$$= 1 - \frac{X - \mu}{\sigma} \tag{56}$$
$$= 2763 \tag{57}$$
$$\tag{58}$$

Binomial

$$\Pr(X = 3) = \binom{n}{k} p^k (1 - p)^{n-k} \tag{59}$$
$$= 0.2764 \tag{60}$$