Question: Two probability distributions of the discrete random variable X and Y are given below.

TABLE 0
TABLE-1

X	0	1	2	3
$P(X)$	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{1}{5}$	$\frac{1}{5}$

TABLE 0
TABLE-2

Y	0	1	2	3
$P(Y)$	$\frac{1}{5}$	$\frac{3}{10}$	$\frac{2}{5}$	$\frac{1}{10}$

Prove that $E\left(Y^2\right) = 2E\left(X\right)$

Solution:

The probability distribution function of X is:

$$p_Y(k) = \begin{cases} \frac{1}{5} & k = 0 \\ \frac{3}{10} & k = 1 \\ \frac{2}{5} & k = 2 \\ \frac{1}{10} & k = 3 \end{cases} \tag{1}$$

$$E\left(Y^2\right) = \sum_k (k)^2 \times p_Y(k) \tag{2}$$

$$= 0 \times \frac{1}{5} + 1^2 \times \frac{3}{10} + (2)^2 \times \frac{2}{5} + (3)^2 \times \frac{1}{10} \tag{3}$$

$$= \frac{14}{5} \tag{4}$$

The probability distribution function of Y is:

$$p_Y(k) = \begin{cases} \frac{1}{5} & k = 0 \\ \frac{2}{5} & k = 1 \\ \frac{1}{5} & k = 2 \\ \frac{1}{5} & k = 3 \end{cases} \tag{5}$$

$$E\left(X\right) = \sum_k k \times p_X(k) \tag{6}$$

$$= 0 \times \frac{1}{5} + 1 \times \frac{2}{5} + 2 \times \frac{1}{5} + 3 \times \frac{1}{5} \tag{7}$$

$$= \frac{7}{5} \tag{8}$$

From (4) and (8);

$$\frac{14}{5} = 2 \times \frac{7}{5} \tag{9}$$

$$\therefore E\left(Y^2\right) = 2E\left(X\right) \tag{10}$$

Hence proved (11)