Question: An urn contains 25 balls of which 10 balls bear a mark $X$ and the remaining 15 bear a mark $Y$. A ball is drawn at random from the urn, its mark is noted down and it is replaced. If 6 balls are drawn in this way, find the probability that

1) all will bear $X$ mark.
2) not more than 2 will bear $Y$ mark.
3) at least one ball will bear $Y$ mark.
4) the number of balls with X mark and $Y$ mark will be equal.

**Solution:**

| Parameter | Values | Description |
|---|---|---|
| $n$ | 6 | Number of draws |
| $p_X$ | 0.4 | Probability that ball bears $X$ mark |
| $p_Y$ | 0.6 | Probability that ball bears $X$ mark |

TABLE 0

DEFINITION OF PARAMETERS

1) all will bear '$X$ mark Mean is given by

$$\mu = np_X \tag{1}$$
$$= 2.4 \tag{2}$$

Standard Deviation is given by

$$\sigma = \sqrt{np_X(1-p_X)} \tag{3}$$
$$= \sqrt{6 \times 0.4 \times (1-0.4)} \tag{4}$$
$$= \sqrt{6 \times 0.4 \times 0.6} \tag{5}$$
$$= 1.2 \tag{6}$$

We have,

$$Z = \frac{X-\mu}{\sigma} \tag{7}$$
$$= 3 \tag{8}$$

Hence, from standard distribution table of $Z$, we have

$$\Pr(Z \leq 2) = 0.49865 \tag{9}$$
$$= 49.865\% \tag{10}$$

2) not more than 2 will bear $Y$ mark.
Mean is given by

$$\mu = np_Y \tag{11}$$
$$= 3.6 \tag{12}$$

Standard Deviation is given by

$$\sigma = \sqrt{np_Y(1-p_Y)} \tag{13}$$
$$= \sqrt{6 \times 0.6 \times 0.4} \tag{14}$$
$$= 1.2 \tag{15}$$

We need to find

$$Z_0 = -3 \tag{16}$$
$$Z_1 = -1.166 \tag{17}$$
$$Z_2 = -0.333 \tag{18}$$
$$\tag{19}$$

We have,

$$Z = \sum_{i=0}^{2} \frac{X_i - \mu}{\sigma} \tag{20}$$
$$= -3.5 \tag{21}$$

Hence, from standard distribution table of $Z$, we have

$$\Pr(Z \leq -3.5) = 0.49865 \tag{22}$$
$$= 49.865\% \tag{23}$$

3) at least one ball will bear $Y$ mark.
   Mean is given by

$$\mu = np_Y \tag{24}$$
$$= 3.6 \tag{25}$$

Standard Deviation is given by

$$\sigma = \sqrt{np_Y(1-p_Y)} \tag{26}$$
$$= \sqrt{6 \times 0.6 \times 0.4} \tag{27}$$
$$= 1.2 \tag{28}$$

$$\Pr(X \geq 1) = 0.98487 \tag{29}$$

4) the number of balls with X mark and $Y$ mark will be equal.
   Mean is given by

$$\mu = np_X \tag{30}$$
$$= 2.4 \tag{31}$$

Standard Deviation is given by

$$\sigma = \sqrt{np_X p_Y} \tag{32}$$
$$= \sqrt{6 \times 0.4 \times 0.6} \tag{33}$$
$$= 1.2 \tag{34}$$

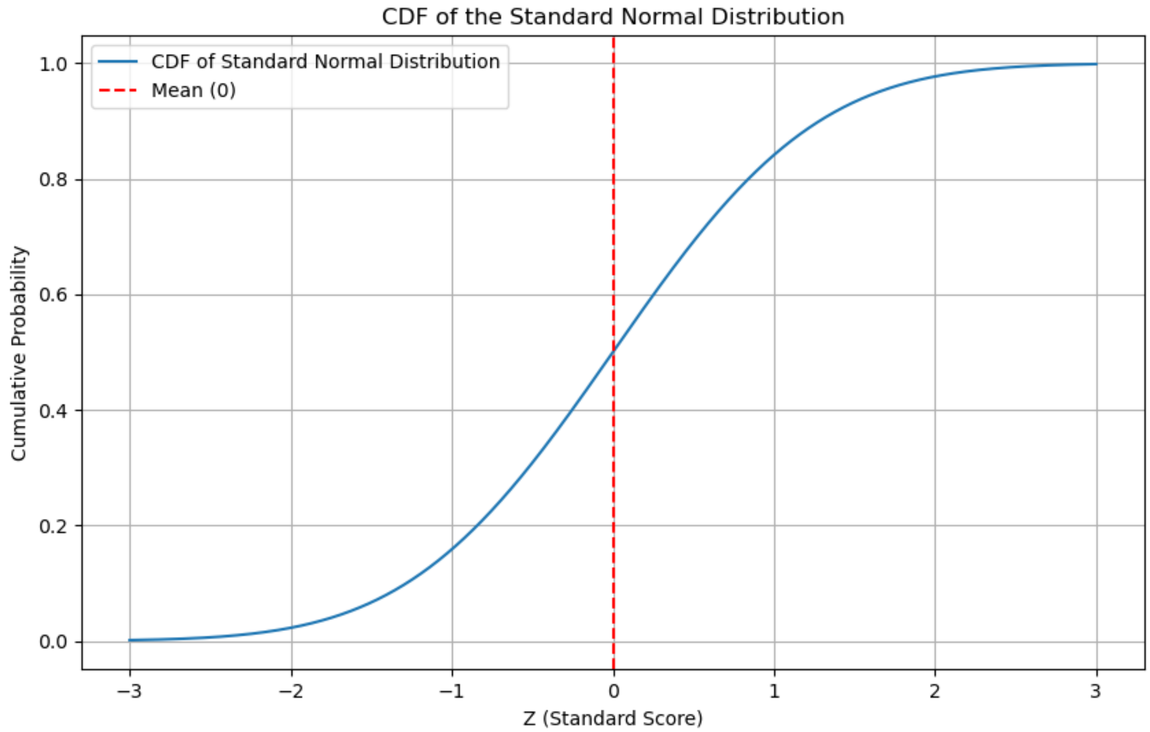


Fig. 4. CDF of Z

We have,

$$Z = \frac{X - \mu}{\sigma} \tag{35}$$
$$= 3 \tag{36}$$
$$\Pr(2.4 \leq X \leq 3.6) \tag{37}$$

From standard distribution table of $Z$, we have

$$\Pr(2.4 \leq Z \leq 3.6) = 0.49865 \tag{38}$$
$$= 49.865\% \tag{39}$$