

Question An urn contains 25 balls of which 10 balls bear a mark X and the remaining 15 bear a mark Y . A ball is drawn at random from the urn, its mark is noted down and it is replaced. If 6 balls are drawn in this way, find the probability that

- 1) all will bear X mark.
- 2) not more than 2 will bear Y mark.
- 3) at least one ball will bear Y mark.
- 4) the number of balls with X mark and Y mark will be equal.

Solution:

Parameter	Values	Description
n	6	Number of draws
p	0.4	Probability that ball bears X mark
q	0.6	Probability that ball bears Y mark
μ	2.4	np
σ	1.2	$\sqrt{np(1-p)}$
X		Number of cards bear mark X
Y		Number of cards bear mark Y

TABLE 0
DEFINITION OF PARAMETERS

- 1) all will bear ' X mark We know that Q-function is given as

$$Q(x) = \Pr(X > x) \quad (1)$$

Now, we want to find probability that all will bear X mark.

$$\Pr(X = 0) = 1 - \Pr(X > 0) \quad (2)$$

$$= 1 - \Pr\left(\frac{X - \mu}{\sigma} > \frac{0 - 2.4}{1.2}\right) \quad (3)$$

$$= 1 - Q(-2) \quad (4)$$

$$= 1 - \int_{-2}^{\infty} \frac{1}{\sqrt{2\pi}} \times e^{-\frac{x^2}{2}} dx \quad (5)$$

$$= 1 - 0.9958 \quad (6)$$

$$= 0.004082 \quad (7)$$

Binomial

$$\Pr(X = 0) = 1 - \Pr(X > 0) \quad (8)$$

$$= {}^nC_k p^k (1-p)^{n-k} \quad (9)$$

$$= 0.004096 \quad (10)$$

- 2) not more than 2 will bear Y mark.
We know that Q-function is given as

$$Q(x) = \Pr(X > x) \quad (11)$$

$$= \int_x^{\infty} \frac{1}{\sqrt{2\pi}} \times e^{-\frac{x^2}{2}} dx \quad (12)$$

We need to find probability that not more than 2 will bear Y mark

$$\Pr(X \leq 2) = 1 - \Pr(X > 2) \quad (13)$$

$$= 1 - \Pr\left(\frac{X - \mu}{\sigma} > \frac{2 - 3.6}{1.2}\right) \quad (14)$$

$$= 1 - Q(-1.33) \quad (15)$$

$$= 1 - \int_{-1.33}^{\infty} \frac{1}{\sqrt{2\pi}} \times e^{-\frac{x^2}{2}} dx \quad (16)$$

$$= 1 - 0.921 \quad (17)$$

$$= 0.179 \quad (18)$$

Binomial

$$\Pr(X \leq 2) = 1 - \Pr(X > 2) \quad (19)$$

$$= 1 - \sum_{k=3}^6 {}^nC_k p^k (1-p)^{n-k} \quad (20)$$

$$= 0.1792 \quad (21)$$

- 3) at least one ball will bear Y mark.
We know that Q-function is given as

$$Q(x) = \Pr(X > x) \quad (22)$$

We need to find the probability that at least one ball will bear Y mark.

$$\Pr(X \geq 1) = \Pr(X > 0) \quad (23)$$

$$= \Pr\left(\frac{X - \mu}{\sigma} > \frac{0 - 3.6}{\sqrt{1.2}}\right) \quad (24)$$

$$= \Pr\left(\frac{X - \mu}{\sigma} > -3\right) \quad (25)$$

$$= 0.99586 \quad (26)$$

$$= 0.99586 \quad (27)$$

Binomial

$$\Pr(X \geq 1) = 1 - \Pr(X = 0) \quad (28)$$

$$= 1 - {}^nC_k p^k (1-p)^{n-k} \quad (29)$$

$$= 0.995904 \quad (30)$$

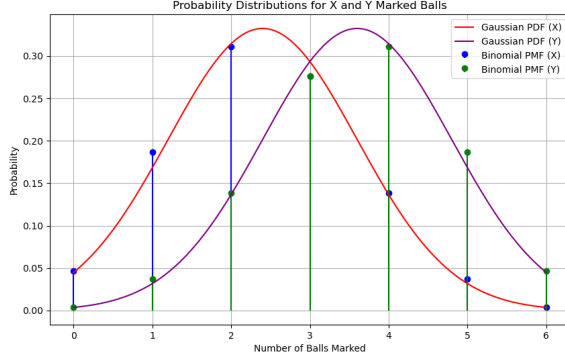


Fig. 4. pmf of binomial and pdf of Gaussian of X and Y marked balls

4) the number of balls with X mark and Y mark will be equal.

Here, Probability that the number of balls with X mark and Y mark will be equal.

$$\Pr(X = 3) = 1 - Q(x) \quad (31)$$

$$= 1 - \frac{X - \mu}{\sigma} \quad (32)$$

$$= 0.2763 \quad (33)$$

$$(34)$$

Binomial

$$\Pr(X = 3) = {}^nC_k p^k (1 - p)^{n-k} \quad (35)$$

$$= 0.2764 \quad (36)$$