

Question : An urn contains 25 balls of which 10 balls bear a mark X and the remaining 15 bear a mark Y . A ball is drawn at random from the urn, its mark is noted down and it is replaced. If 6 balls are drawn in this way, find the probability that

- (a) all will bear X mark.
- (b) not more than 2 will bear Y mark.
- (c) at least one ball will bear Y mark.
- (d) the number of balls with X mark and Y mark will be equal.

Solution:

Parameter	Values	Description
n	6	Number of draws
p	0.4	Probability that ball bears X mark
q	0.6	Probability that ball bears Y mark
$\mu = np$	2.4	mean of the distribution
$\sigma = npq$	1.2	variance of the distribution
X		Number of cards bear mark X
Y		Number of cards bear mark Y

TABLE 4
DEFINITION OF PARAMETERS

using Gaussian

$$Y \sim \mathcal{N}(\mu, \sigma^2) \quad (1)$$

The CDF of Y :

$$F_Y(y) = 1 - \Pr(Y > y) \quad (2)$$

$$= 1 - \Pr\left(\frac{Y - \mu}{\sigma} > \frac{y - \mu}{\sigma}\right) \quad (3)$$

But,

$$\frac{Y - \mu}{\sigma} \sim \mathcal{N}(2.4, 1.44) \quad (4)$$

(5)

the Q-function is defined as:

$$Q(x) = \Pr(Y > x) \quad \forall x \in Y \sim \mathcal{N}(2.4, 1.44) \quad (6)$$

therefore the cdf will be:

$$F_Y(y) = \begin{cases} 1 - Q\left(\frac{y - \mu}{\sigma}\right), & y > \mu \\ Q\left(\frac{\mu - y}{\sigma}\right), & y < \mu \end{cases} \quad (7)$$

- (a) all will bear X mark.
using Gaussian

considering 0.5 as the correction term,

$$\Pr(X > 5.5) = 1 - F_X(5.5) \quad (8)$$

$$= Q\left(\frac{5.5 - \mu}{\sigma}\right) \quad (9)$$

$$= Q\left(\frac{3.1}{1.2}\right) \quad (10)$$

$$= Q(2.583) \quad (11)$$

$$= 0.00489 \quad (12)$$

- (b) not more than 2 will bear Y mark.

using Gaussian

considering 0.5 as the correction term,

$$\Pr(Y < 2.5) = 1 - Q\left(\frac{2.5 - \mu}{\sigma}\right) \quad (13)$$

$$= 1 - Q\left(\frac{-1.1}{1.2}\right) \quad (14)$$

$$= 1 - Q(-0.9166) \quad (15)$$

$$= Q(0.9166) \quad (16)$$

$$= 0.1796 \quad (17)$$

- (c) at least one ball will bear Y mark.

using Gaussian

considering 0.5 as the correction term,

$$\Pr(Y < 0.5) = 1 - Q\left(\frac{0.5 - \mu}{\sigma}\right) \quad (18)$$

$$= 1 - Q\left(\frac{-1.1}{1.2}\right) \quad (19)$$

$$= 1 - Q(-2.588) \quad (20)$$

$$= 1 - 0.0048 \quad (21)$$

$$= 0.9952 \quad (22)$$

- (d) the number of balls with X mark and Y mark will be equal.

using Gaussian

Calculate the mean(μ_D) and variance(σ_D) of the difference variance D ,

$$\mu_D = -1.2 \quad (23)$$

$$\sigma_D = 2.4 \quad (24)$$

$$\Pr(D = 0) = Q\left(\frac{0 - \mu_D}{\sigma_D}\right) \quad (25)$$

$$= Q\left(\frac{1.2}{1.697}\right) \quad (26)$$

$$= Q(0.71) \quad (27)$$

$$= 0.2388 \quad (28)$$

Gaussian vs Binomial Table

Question	Gaussian	Binomial
all will bear X mark	0.00489	0.00409
not more than 2 will bear Y mark	0.1796	0.1792
at least one ball will bear Y mark	0.9952	0.9959
the number of balls with X mark and Y mark will be equal	0.2388	0.2764

TABLE 4
DEFINITION OF PARAMETERS

Gaussian vs Binomial graph

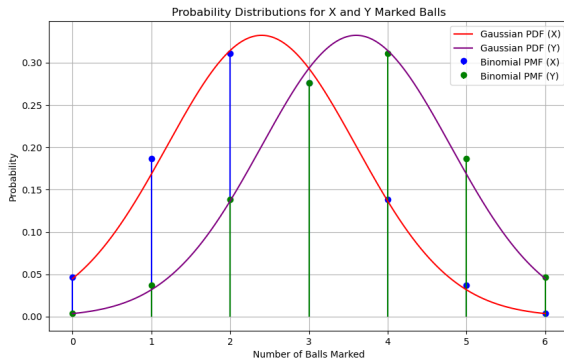


Fig. 4. pmf of binomial and pdf of Gaussian of X and Y marked balls