Question: Two probability distributions of the discrete random variable X and Y are given below.

TABLE 0
Table-1

X	0	1	2	3
$P(X)$	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{1}{5}$	$\frac{1}{5}$

TABLE 0
Table-2

Y	0	1	2	3
$P(X)$	$\frac{1}{5}$	$\frac{3}{10}$	$\frac{2}{5}$	$\frac{1}{10}$

Prove that $E(Y^2) = 2E(X)$

Solution:

$$E(Y^2) = \sum_{i=0}^{3}(Y_i)^2.P(Y_i) \tag{1}$$
$$= 0 \times \frac{1}{5} + 1^2 \times \frac{3}{10} + (2)^2 \times \frac{2}{5} + (3)^2 \times \frac{1}{10} \tag{2}$$
$$= 0 + \frac{3}{10} + \frac{8}{5} + \frac{9}{10} \tag{3}$$
$$= \frac{14}{5} \tag{4}$$

$$E(X) = \sum_{i=0}^{3} X_i.P(X_i) \tag{5}$$
$$= 0 \times \frac{1}{5} + 1 \times \frac{2}{5} + 2 \times \frac{1}{5} + 3 \times \frac{1}{5} \tag{6}$$
$$= 0 + \frac{2}{5} + \frac{2}{5} + \frac{3}{5} \tag{7}$$
$$= \frac{7}{5} \tag{8}$$

From (4) and (8);

$$\frac{14}{5} = 2 \times \frac{7}{5} \tag{9}$$
$$\therefore E(Y^2) = 2E(X) \tag{10}$$
$$\text{Hence proved} \tag{11}$$