

Question: Verify that

$$\frac{BG}{GE} = \frac{CG}{GF} = \frac{AG}{GD} = 2$$

Solution: :

Let **D,E,F** be the midpoints of BC,CA,AB respectively, then

$$\mathbf{D} = \begin{pmatrix} \frac{-7}{2} \\ \frac{1}{2} \end{pmatrix} \quad (1)$$

$$\mathbf{E} = \begin{pmatrix} -1 \\ -3 \end{pmatrix} \quad (2)$$

$$\mathbf{F} = \begin{pmatrix} \frac{-3}{2} \\ \frac{5}{2} \end{pmatrix} \quad (3)$$

From the previous question 1.2.3, we got

$$\mathbf{G} = \begin{pmatrix} -2 \\ 0 \end{pmatrix} \quad (4)$$

Direction vectors are

$$\mathbf{BG} = \mathbf{G} - \mathbf{B} = \begin{pmatrix} 2 \\ -6 \end{pmatrix} \quad (5)$$

$$\mathbf{GE} = \mathbf{E} - \mathbf{G} = \begin{pmatrix} 1 \\ -3 \end{pmatrix} \quad (6)$$

$$\mathbf{CG} = \mathbf{G} - \mathbf{C} = \begin{pmatrix} 1 \\ 5 \end{pmatrix} \quad (7)$$

$$\mathbf{GF} = \mathbf{F} - \mathbf{G} = \begin{pmatrix} \frac{1}{2} \\ \frac{5}{2} \end{pmatrix} \quad (8)$$

$$\mathbf{AG} = \mathbf{G} - \mathbf{A} = \begin{pmatrix} -3 \\ 1 \end{pmatrix} \quad (9)$$

$$\mathbf{GD} = \mathbf{D} - \mathbf{G} = \begin{pmatrix} \frac{-3}{2} \\ \frac{1}{2} \end{pmatrix} \quad (10)$$

The ratios can be calculated as follows:

1)

$$\frac{BG}{GE} = \frac{2\sqrt{10}}{\sqrt{10}} = 2 \quad (11)$$

2)

$$\frac{CG}{GF} = \frac{\sqrt{26}}{\frac{1}{2}\sqrt{26}} = 2 \quad (12)$$

3)

$$\frac{AG}{GD} = \frac{\sqrt{10}}{\frac{1}{2}\sqrt{10}} = 2 \quad (13)$$

Hence, we have verified that

$$\frac{BG}{GE} = \frac{CG}{GF} = \frac{AG}{GD} = 2$$