Question: Verify that
$\frac{BG}{GE} = \frac{CG}{GF} = \frac{AG}{GD} = 2$
Solution: :
Let $\mathbf{D}, \mathbf{E}, \mathbf{F}$ be the midpoints of BC, CA, AB respectively, then

$$\mathbf{D} = \begin{pmatrix} \frac{-7}{2} \\ \frac{1}{2} \end{pmatrix} \tag{1}$$

$$\mathbf{E} = \begin{pmatrix} -1 \\ -3 \end{pmatrix} \tag{2}$$

$$\mathbf{F} = \begin{pmatrix} \frac{-3}{2} \\ \frac{5}{2} \end{pmatrix} \tag{3}$$

From the previous question 1.2.3, we got

$$\mathbf{G} = \begin{pmatrix} -2 \\ 0 \end{pmatrix} \tag{4}$$

1) For $BG : GE$ ratio:
   Direction vectors of BG and GE are

$$\mathbf{G} - \mathbf{B} = \begin{pmatrix} 2 \\ -6 \end{pmatrix} \tag{5}$$

$$\mathbf{E} - \mathbf{G} = \begin{pmatrix} 1 \\ -3 \end{pmatrix} \tag{6}$$

   Norm of BG and GE:

$$\|BG\| = \sqrt{2^2 + (-6)^2} \tag{7}$$

$$= 2\sqrt{10} \tag{8}$$

$$\|GE\| = \sqrt{1^2 + (-3)^2} \tag{9}$$

$$= \sqrt{10} \tag{10}$$

$$\therefore \frac{BG}{GE} = \frac{2\sqrt{10}}{\sqrt{10}} = 2 \tag{11}$$

2) For $CG : GF$ ratio:
   Direction vectors of CG and GF are

$$\mathbf{G} - \mathbf{C} = \begin{pmatrix} 1 \\ 5 \end{pmatrix} \tag{12}$$

$$\mathbf{F} - \mathbf{G} = \begin{pmatrix} \frac{1}{2} \\ \frac{5}{2} \end{pmatrix} \tag{13}$$

Norm of CG and GF:

$$\|CG\| = \sqrt{1^2 + (5)^2} \tag{14}$$
$$= \sqrt{26} \tag{15}$$
$$\|GF\| = \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{5}{2}\right)^2} \tag{16}$$
$$= \frac{1}{2}\sqrt{26} \tag{17}$$
$$\therefore \frac{CG}{GF} = \frac{\sqrt{26}}{\frac{1}{2}\sqrt{26}} = 2 \tag{18}$$

3) For $AG : GD$ ratio:
Direction vectors of AG and GD are

$$\mathbf{G} - \mathbf{A} = \begin{pmatrix} -3 \\ 1 \end{pmatrix} \tag{19}$$
$$\mathbf{D} - \mathbf{G} = \begin{pmatrix} \frac{-3}{2} \\ \frac{1}{2} \end{pmatrix} \tag{20}$$

Norm of AG and GD:

$$\|AG\| = \sqrt{(-3)^2 + (1)^2} \tag{21}$$
$$= \sqrt{10} \tag{22}$$
$$\|GD\| = \sqrt{\left(\frac{-3}{2}\right)^2 + \left(\frac{1}{2}\right)^2} \tag{23}$$
$$= \frac{1}{2}\sqrt{10} \tag{24}$$
$$\therefore \frac{AG}{GD} = \frac{\sqrt{10}}{\frac{1}{2}\sqrt{10}} = 2 \tag{25}$$

Hence, from the above ratios we have verified that
$\frac{BG}{GE} = \frac{CG}{GF} = \frac{AG}{GD} = 2$