Question: Verify that
$\frac{BG}{GE} = \frac{CG}{GF} = \frac{AG}{GD} = 2$
Solution: :
Let **D**,**E**,**F** be the midpoints of BC,CA,AB respectively, then

$$\mathbf{D} = \begin{pmatrix} \frac{-7}{2} \\ \\ \frac{1}{2} \end{pmatrix} \tag{1}$$

$$\mathbf{E} = \begin{pmatrix} -1 \\ -3 \end{pmatrix} \tag{2}$$

$$\mathbf{F} = \begin{pmatrix} \frac{-3}{2} \\ \\ \frac{5}{2} \end{pmatrix} \tag{3}$$

From the previous question 1.2.3, we got

$$\mathbf{G} = \begin{pmatrix} -2 \\ 0 \end{pmatrix} \tag{4}$$

Direction vectors as follows:

$$\mathbf{BG} = \mathbf{G} - \mathbf{B} = \begin{pmatrix} 2 \\ -6 \end{pmatrix} \tag{5}$$

$$\mathbf{GE} = \mathbf{E} - \mathbf{G} = \begin{pmatrix} 1 \\ -3 \end{pmatrix} \tag{6}$$

$$\mathbf{CG} = \mathbf{G} - \mathbf{C} = \begin{pmatrix} 1 \\ 5 \end{pmatrix} \tag{7}$$

$$\mathbf{GF} = \mathbf{F} - \mathbf{G} = \begin{pmatrix} \frac{1}{2} \\ \frac{5}{2} \end{pmatrix} \tag{8}$$

$$\mathbf{AG} = \mathbf{G} - \mathbf{A} = \begin{pmatrix} -3 \\ 1 \end{pmatrix} \tag{9}$$

$$\mathbf{GD} = \mathbf{D} - \mathbf{G} = \begin{pmatrix} \frac{-3}{2} \\ \frac{1}{2} \end{pmatrix} \tag{10}$$

Norm of BG and GE:

$$\|\mathbf{BG}\| = \|\mathbf{G} - \mathbf{B}\| = \sqrt{2^2 + (-6)^2} \tag{11}$$

$$= 2\sqrt{10} \tag{12}$$

$$\|\mathbf{GE}\| = \|\mathbf{G} - \mathbf{B}\| = \sqrt{1^2 + (-3)^2} \tag{13}$$

$$= \sqrt{10} \tag{14}$$

Norm of CG and GC:

$$\|\mathbf{CG}\| = \|\mathbf{G} - \mathbf{C}\| = \sqrt{1^2 + (5)^2} \tag{15}$$

$$= \sqrt{26} \tag{16}$$

$$\|\mathbf{GC}\| = \|\mathbf{C} - \mathbf{G}\| = \sqrt{\frac{1}{2}^2 + \frac{5}{2}^2} \tag{17}$$

$$= \frac{1}{2}\sqrt{26} \tag{18}$$

Norm of AG and GD:

$$\|\mathbf{AG}\| = \|\mathbf{G} - \mathbf{A}\| = \sqrt{(-3)^2 + (1)^2} \tag{19}$$

$$= \sqrt{10} \tag{20}$$

$$\|\mathbf{GD}\| = \|\mathbf{D} - \mathbf{G}\| = \sqrt{\frac{-3}{2}^2 + \frac{1}{2}^2} \tag{21}$$

$$= \frac{1}{2}\sqrt{10} \tag{22}$$

The ratios can be calculated as follows:

1)

$$\frac{BG}{GE} = \frac{2\sqrt{10}}{\sqrt{10}} = 2 \tag{23}$$

2)

$$\frac{CG}{GF} = \frac{\sqrt{26}}{\frac{1}{2}\sqrt{26}} = 2 \tag{24}$$

3)

$$\frac{AG}{GD} = \frac{\sqrt{10}}{\frac{1}{2}\sqrt{10}} = 2 \tag{25}$$

Hence,we have verified that
$\frac{BG}{GE} = \frac{CG}{GF} = \frac{AG}{GD} = 2$