Question: Verify that
$\frac{BG}{GE} = \frac{CG}{GF} = \frac{AG}{GD} = 2$
Solution: :
Let $\mathbf{D}$,$\mathbf{E}$,$\mathbf{F}$ be the midpoints of BC,CA,AB respectively, then

$$\mathbf{D} = \begin{pmatrix} \frac{-7}{2} \\ \\ \frac{1}{2} \end{pmatrix} \tag{1}$$

$$\mathbf{E} = \begin{pmatrix} -1 \\ -3 \end{pmatrix} \tag{2}$$

$$\mathbf{F} = \begin{pmatrix} \frac{-3}{2} \\ \\ \frac{5}{2} \end{pmatrix} \tag{3}$$

From the previous question 1.2.3, we got

$$\mathbf{G} = \begin{pmatrix} -2 \\ 0 \end{pmatrix} \tag{4}$$

Direction vectors as follows:

$$\mathbf{BG} = \mathbf{G} - \mathbf{B} \tag{5}$$
$$= \begin{pmatrix} 2 \\ -6 \end{pmatrix} \tag{6}$$
$$\mathbf{GE} = \mathbf{E} - \mathbf{G} \tag{7}$$
$$= \begin{pmatrix} 1 \\ -3 \end{pmatrix} \tag{8}$$
$$\mathbf{CG} = \mathbf{G} - \mathbf{C} \tag{9}$$
$$= \begin{pmatrix} 1 \\ 5 \end{pmatrix} \tag{10}$$
$$\mathbf{GF} = \mathbf{F} - \mathbf{G} \tag{11}$$
$$= \begin{pmatrix} \frac{1}{2} \\ \frac{5}{2} \end{pmatrix} \tag{12}$$
$$\mathbf{AG} = \mathbf{G} - \mathbf{A} \tag{13}$$
$$= \begin{pmatrix} -3 \\ 1 \end{pmatrix} \tag{14}$$
$$\mathbf{GD} = \mathbf{D} - \mathbf{G} \tag{15}$$
$$= \begin{pmatrix} \frac{-3}{2} \\ \frac{1}{2} \end{pmatrix} \tag{16}$$

Norm of BG and GE:

$$||\mathbf{BG}|| = ||\mathbf{G} - \mathbf{B}|| \tag{17}$$
$$= \sqrt{2^2 + (-6)^2} \tag{18}$$
$$= 2\sqrt{10} \tag{19}$$
$$||\mathbf{GE}|| = ||\mathbf{G} - \mathbf{B}|| \tag{20}$$
$$= \sqrt{1^2 + (-3)^2} \tag{21}$$
$$= \sqrt{10} \tag{22}$$

Norm of CG and GC:

$$\|\mathbf{CG}\| = \|\mathbf{G} - \mathbf{C}\| = \sqrt{1^2 + (5)^2} \tag{23}$$
$$= \sqrt{26} \tag{24}$$
$$\|\mathbf{GC}\| = \|\mathbf{C} - \mathbf{G}\| = \sqrt{\frac{1}{2}^2 + \frac{5}{2}^2} \tag{25}$$
$$= \frac{1}{2}\sqrt{26} \tag{26}$$

Norm of AG and GD:

$$\|\mathbf{AG}\| = \|\mathbf{G} - \mathbf{A}\| = \sqrt{(-3)^2 + (1)^2} \tag{27}$$
$$= \sqrt{10} \tag{28}$$
$$\|\mathbf{GD}\| = \|\mathbf{D} - \mathbf{G}\| = \sqrt{\frac{-3}{2}^2 + \frac{1}{2}^2} \tag{29}$$
$$= \frac{1}{2}\sqrt{10} \tag{30}$$

The ratios can be calculated as follows:

1)
$$\frac{BG}{GE} = \frac{2\sqrt{10}}{\sqrt{10}} = 2 \tag{31}$$

2)
$$\frac{CG}{GF} = \frac{\sqrt{26}}{\frac{1}{2}\sqrt{26}} = 2 \tag{32}$$

3)
$$\frac{AG}{GD} = \frac{\sqrt{10}}{\frac{1}{2}\sqrt{10}} = 2 \tag{33}$$

Hence,we have verified that
$\frac{BG}{GE} = \frac{CG}{GF} = \frac{AG}{GD} = 2$