

Question: Verify that

$$\frac{BG}{GE} = \frac{CG}{GF} = \frac{AG}{GD} = 2$$

Solution: :

Let **D**, **E**, **F** be the midpoints of BC, CA, AB respectively, then

$$\mathbf{D} = \begin{pmatrix} -7 \\ 2 \\ 1 \\ 2 \end{pmatrix} \quad (1)$$

$$\mathbf{E} = \begin{pmatrix} -1 \\ -3 \end{pmatrix} \quad (2)$$

$$\mathbf{F} = \begin{pmatrix} -3 \\ 2 \\ 5 \\ 2 \end{pmatrix} \quad (3)$$

From the previous question 1.2.3, we got

$$\mathbf{G} = \begin{pmatrix} -2 \\ 0 \end{pmatrix} \quad (4)$$

1) For $BG : GE$ ratio:

Direction vectors of **BG** and **GE** are

$$\mathbf{BG} = \mathbf{G} - \mathbf{B} \quad (5)$$

$$= \begin{pmatrix} 2 \\ -6 \end{pmatrix} \quad (6)$$

$$\mathbf{GE} = \mathbf{E} - \mathbf{G} \quad (7)$$

$$= \begin{pmatrix} 1 \\ -3 \end{pmatrix} \quad (8)$$

Norm of **BG** and **GE**:

$$\|\mathbf{BG}\| = \sqrt{2^2 + (-6)^2} \quad (9)$$

$$= 2\sqrt{10} \quad (10)$$

$$\|\mathbf{GE}\| = \sqrt{1^2 + (-3)^2} \quad (11)$$

$$= \sqrt{10} \quad (12)$$

$$\therefore \frac{BG}{GE} = \frac{2\sqrt{10}}{\sqrt{10}} = 2 \quad (13)$$

2) For $CG : GF$ ratio:

Direction vectors of **CG** and **GF** are

$$\mathbf{CG} = \mathbf{G} - \mathbf{C} \quad (14)$$

$$= \begin{pmatrix} 1 \\ 5 \end{pmatrix} \quad (15)$$

$$\mathbf{GF} = \mathbf{F} - \mathbf{G} \quad (16)$$

$$= \begin{pmatrix} \frac{1}{2} \\ \frac{5}{2} \end{pmatrix} \quad (17)$$

Norm of **CG** and **GF**:

$$\|\mathbf{CG}\| = \sqrt{1^2 + (5)^2} \quad (18)$$

$$= \sqrt{26} \quad (19)$$

$$\|\mathbf{GF}\| = \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{5}{2}\right)^2} \quad (20)$$

$$= \frac{1}{2} \sqrt{26} \quad (21)$$

$$\therefore \frac{CG}{GF} = \frac{\sqrt{26}}{\frac{1}{2}\sqrt{26}} = 2 \quad (22)$$

3) For $AG : GD$ ratio:

Direction vectors of **AG** and **GD** are

$$\mathbf{AG} = \mathbf{G} - \mathbf{A} \quad (23)$$

$$= \begin{pmatrix} -3 \\ 1 \end{pmatrix} \quad (24)$$

$$\mathbf{GD} = \mathbf{D} - \mathbf{G} \quad (25)$$

$$= \begin{pmatrix} -3 \\ \frac{1}{2} \end{pmatrix} \quad (26)$$

Norm of **AG** and **GD**:

$$\|\mathbf{AG}\| = \sqrt{(-3)^2 + (1)^2} \quad (27)$$

$$= \sqrt{10} \quad (28)$$

$$\|\mathbf{GD}\| = \sqrt{\left(\frac{-3}{2}\right)^2 + \left(\frac{1}{2}\right)^2} \quad (29)$$

$$= \frac{1}{2} \sqrt{10} \quad (30)$$

$$\therefore \frac{AG}{GD} = \frac{\sqrt{10}}{\frac{1}{2}\sqrt{10}} = 2 \quad (31)$$

Hence, from the above ratios we have verified that

$$\frac{BG}{GE} = \frac{CG}{GF} = \frac{AG}{GD} = 2$$