2019101101

100 questions | Automata Theory

Phase-1 15 questions

Question 1: Given language of DFA L(A) where A is DFA $A=(Q, \Sigma, \delta, q_0, F)$ then $L(A)=\{w \mid \hat{\delta}(q_0, w) \text{ is in } F\}$ here ~~δ~~ w is strings start state q_0

show $\hat{\delta}(q, xy) = \hat{\delta}(\hat{\delta}(q, x), y)$

where, $\hat{\delta}$ - breaks string (input) with certain string of labels

q - any state

x, y - strings.

Answer:-

Let say length of string $y = \epsilon$

and then ~~q~~ $\hat{\delta}(q, x) = \hat{\delta}(\hat{\delta}(q, x), \epsilon)$ ($\because$ from $\hat{\delta}$ def)

lets says a string t which is shorter than y

then $y = ta$ on ~~b~~giving 'a' as input symbol of y

Now, $\hat{\delta}(\hat{\delta}(q, x), y)$ RHS

$\Rightarrow \hat{\delta}(\hat{\delta}(q, x), ta)$

$\Rightarrow \delta(\hat{\delta}(\hat{\delta}(q, x), t), a)$ (breaking ta)

$\Rightarrow \delta(\hat{\delta}(q, xt), a)$

$\Rightarrow \hat{\delta}(q, xta)$ ($\hat{\delta}$ definition)

$\Rightarrow \hat{\delta}(q, xy)$ ($\because ta = y$ earlier assumption)

$=$ LHS

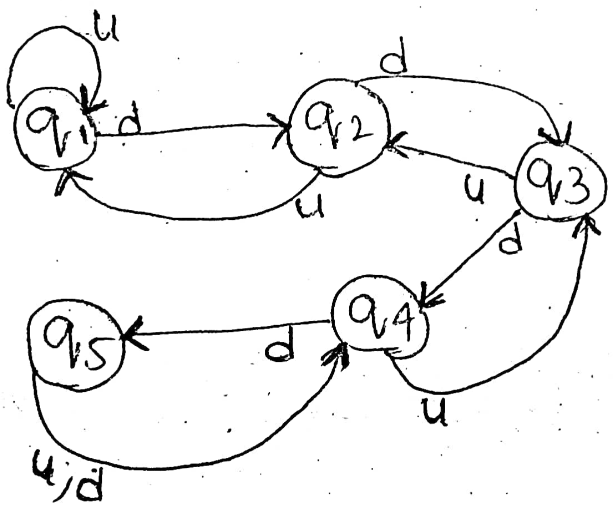
Thus LHS = RHS is proven.

Question-2: Provide the state diagram for the following DFA M where $(\{q_1, q_2, q_3, q_4, q_5\}, \{u, d\}, \delta, q_3, \{q_3\})$, and $\delta \Rightarrow$

	u	d
q_1	q_1	q_2
q_2	q_1	q_3
q_3	q_2	q_4
q_4	q_3	q_5
q_5	q_4	q_4

Answer:-

from transition table

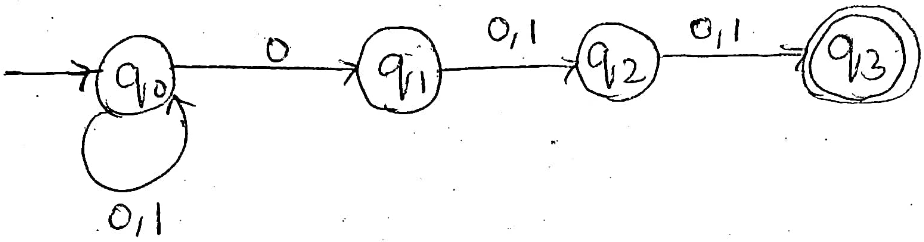


Question3: Design an NFA, where NFA A and its language L(A), $A = (Q, \Sigma, q_0, F, \delta)$ where it accepts a state give set of possible next states (empty too)

(a) $\Sigma = \{0, 1\}$ accepts all strings when third symbol from right is 0.

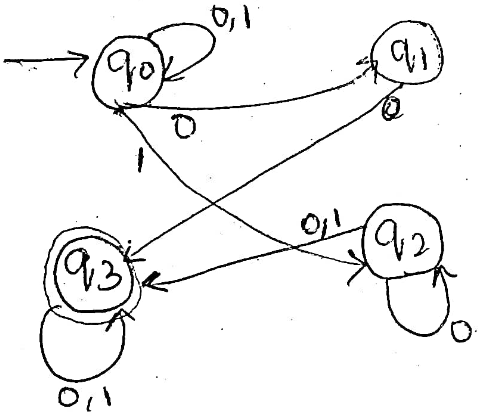
Answer:

To make NFA, firstly the start state could be $q_0 \xrightarrow{0} q_0$, $q_0 \xrightarrow{a} q_1$ then, third tight be zero when q_1 is obtained on taking 0 action/input. Thus we have four states designed as following



Thus NFA is designed

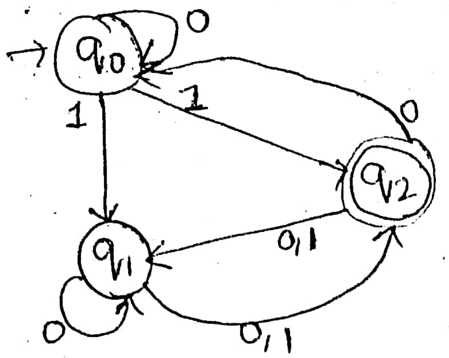
(b) Obtain the transistion table for following NFA state diagram for NFA M



Answer: NFA $M = (\{q_0, q_1, q_2, q_3\}, \{0,1\}, \delta, q_0, \{q_3\})$

δ	0	1
$\rightarrow q_0$	q_0, q_1	q_0, q_2
q_1	$*q_3$	ϕ
q_2	$q_2, *q_3$	$*q_3$
$*q_3$	$*q_3$	$*q_3$

Question 4: Convert the NFA to DFA



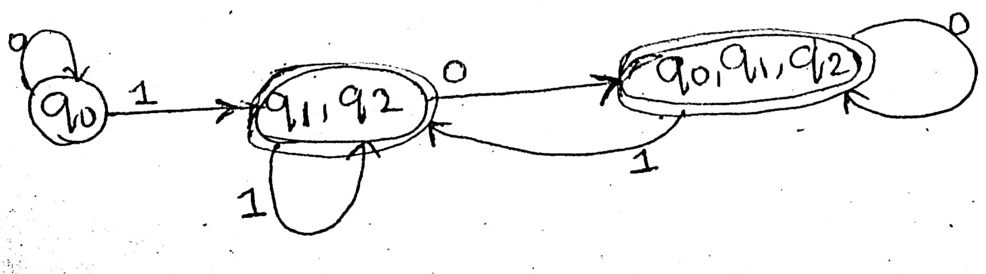
Answer:- For the given NFA state diagram the transistion state table is

δ	0	1
→ q0	q0	q1, *q2
q1	q1, *q2	*q2
*q2	q0, q1	q1

Let write down every state transition one by one, initial state q0.

δ	0	1
→ q0	q0	{q1, q2}
*{q1, q2}	{q0, q1, q2}	{q1, q2}
*{q0, q1, q2}	*{q0, q1, q2}	*{q1, q2}

Thus DFA state diagram is :-

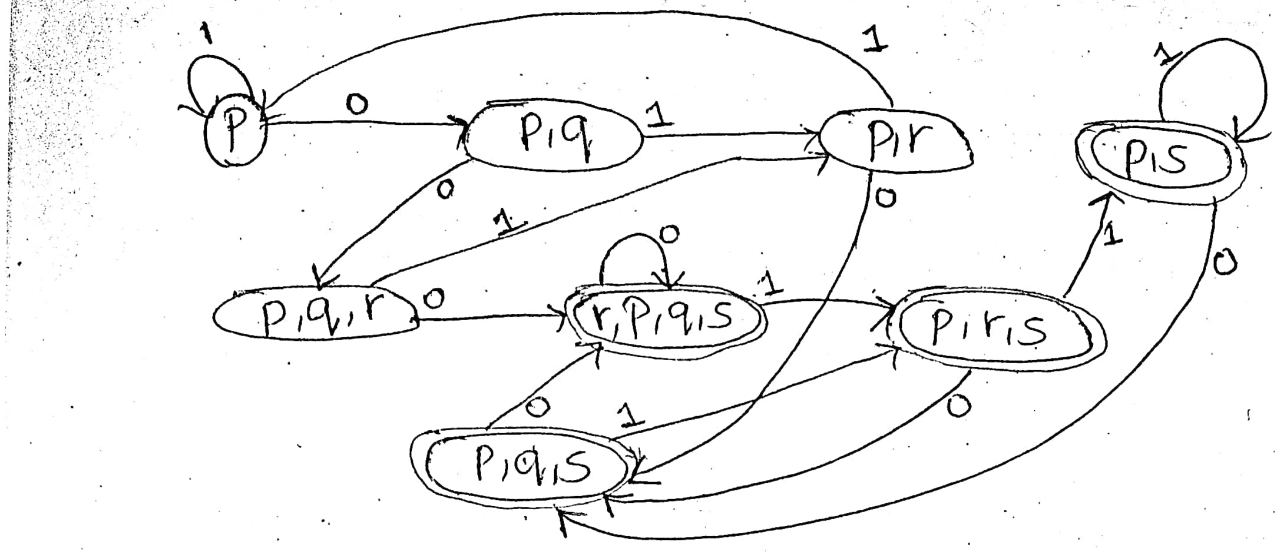


Question 5: Convert to a DFA given the transistion function table for NFA and state diagram

δ	0	1
$\rightarrow p$	$\{p,q\}$	$\{p\}$
q	$\{r\}$	$\{r\}$
r	$\{s\}$	ϕ
$*s$	$\{s\}$	$\{s\}$

Solution: transistion table for DFA will be

δ	0	1
$\rightarrow \{p\}$	$\{p,q\}$	$\{p\}$
$\{p,q\}$	$\{p,q,r\}$	$\{p,r\}$
$\{p,r\}$	$\{p,q,s\}$	$\{p\}$
$\{p,q,r\}$	$\{p,q,r,s\}$	$\{p,r\}$
$*\{p,q,s\}$	$\{p,q,r,s\}$	$\{p,q,r,s\}$
$*\{p,q,r,s\}$	$\{p,q,r,s\}$	$\{p,r,s\}$
$*\{p,r,s\}$	$\{p,q,s\}$	$\{p,s\}$
$*\{p,s\}$	$\{p,q,s\}$	$\{p,s\}$



is the DFA state diagram for given NFA.

Question 6: Prove that every NFA can be converted to an equivalent one which has a single accept state.

We know that NFA's has power set of states for next state from any given state (possibility)

Now, to construct an NFA with single accept state be N_S and this N_S should able to recognise $N = (Q, \Sigma, \delta, F, q_0)$

$$N_S = (Q \cup \{q_{accept}\}, \Sigma, \delta', \{q_{accept}\}, q_0)$$

Since, q_{accept} has no apparent transition, total ε-transistions

Transition function

$$\delta'(q, a) = \begin{cases} \delta(q, a) & a \neq \varepsilon \text{ or } q \notin F \\ \delta(q, a) \cup \{q_{accept}\} & a = \varepsilon \text{ and } q \in F \end{cases}$$

where $a \in \Sigma_\varepsilon$

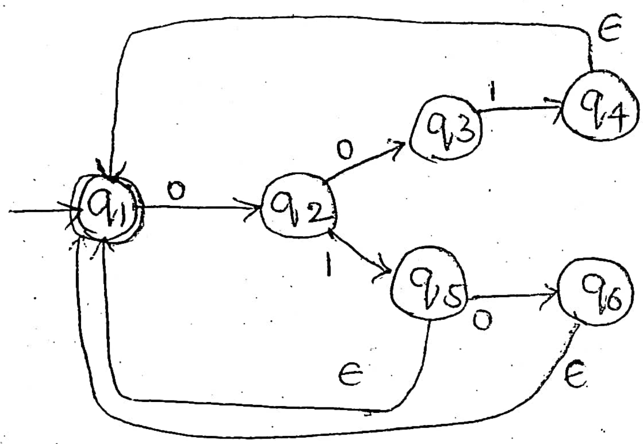
and then $\delta'(q_{accept}, a) = \phi \quad \forall a \in \Sigma_\varepsilon$

Question 7: (a) Give an NFA which recognizing the language $(01 \cup 001 \cup 010)^*$

(b) Convert this NFA to equivalent DFA. Only one portion of DFA reachable to star state.

Solutions:-

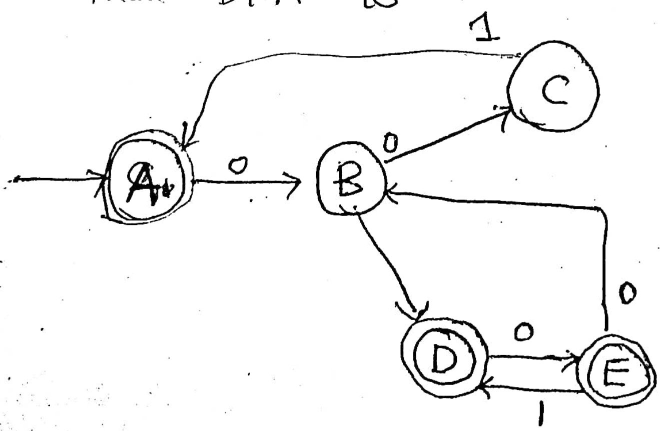
(a) Observe the no. of strings in the expression (3) and apply the theorem (class of regular languages closed under star operation)



(b) We know maximum states in DFA is $2^n = 2^6 = 64$

here, the fragment is selected (∵ other part won't be computed by machine)

Then DFA is



A ≠
B ≠
C ≠ different states
D ≠ (possible/happened from NFA transitions)
E ≠

Question 8:

(a) Describe the language for following regular expression : $re = (b^* (aaa)^* b^*)^*$

(b) Language L, $\Sigma = \{0,1\}$ where all strings don't contain substring 01. the write RE

(c) Write RE for L, $\Sigma = \{0,1\}$ where ~~has~~ at least two occurences of 1's between any two occurences of 1's between any two occurences of 0's.

Answer:-

(a) Now, to describe the language.

Firstly given RE means (any combination of b's)

$(aaa)^*$ (any combination of b's)

Thus Language is string with triplets of a's and with zero restriction on count of b's.

(b) Language L over $\Sigma = \{0,1\}$

so, $L = \{\varepsilon, 0, 1, 00, 01, 100, 11, \dots\}$

Regular Expression $RE = (1^* 0^*)$

(c) From question L is string with atleast two 1's between two occurrence of 0's.

$\Rightarrow (0111^*0)^*$

Now, $(1 + (0111^* 0))^*$ (since no occurrence of 0's thus many no. of 1's)

Question 9: Let $C_n = \{x \mid x$ is a binary number that is multiple of $n\}$. Show that for each $n >= 1$ language C_n is regular.

Answer: Let M be a DFA with n states that recognises C_n.

Here M is defined as simulating binary division. then n states are n possible remainders from that division.

so, M gets MSB first thus goes on till LSB as input string.

Claim to prove M results C_n as regular is to end the input string with accept state (remainder 0)

Proof:-

For Each input bit, M doubles the remainder its current and adds the input bits.

If new state is sum modulo n

double remainder (∵ left shift)

Thus, we left with no remainder on dividing the with n.

Therefore, member of C_n

$$M = (\{q_0, q_1, q_2 \dots, q_{n-1}\}, \{0,1\}, \delta, q_0, \{q_0\})$$

$$\forall q_i \in Q \quad a \in \{0,1\} \quad \delta(q_i, a) = q_j$$

$$(\because j = (2i + a) \bmod n)$$

—*—

Question 10: Give an algorithm that takes a DFA A and computes the number of strings of length n (for some given n) accepted by A. Should be polynomial in both n & no. of states of A.

Answer:-

Now, claim:- No. of paths from state (any state)

say $R^{(K)}_{ijm}$ where i - state i
j - state j (length m)

through, no states numbered higher than K is found.

Now, by induction for states we will have the following sequence

Basis:- R^{0}_{ij1} is no of arcs from state i to j.

$R^{0}_{ij0} = 1$ and R^{0}_{ijm}'s $= 0$ $(m \neq 0)$

Claim:- For all states i, j for m no greater than n by induction on K

Induction:- $R^{(K)}_{ijm} = R^{(K-1)}_{ijm} + \sum_{q=1}^{r} P_q$ (P = positive Integers $r = m$)

$R^{(K-1)}_{ikp_1} * R^{(K-1)}_{kkp_2} * R^{(K-1)}_{kkp_3} * R^{(K-1)}_{kkp(r-1)} * R^{(K-1)}_{kjp_r}$ $(r \geq 2$ $r \leq m)$

So, Sum of $R^{(K)}_{1jn}$ is the required

where K is no. of states
1 is start state
j is accepting state

Question 11: Use the pumping lemma to show that following are not regular:

(a) $A_1 = \{0^n 1^n 2^n \mid n \geq 0\}$

(b) $A_2 = \{a^{2^n} \mid n \geq 0\}$ (a^{2^n} is string of a's with 2^n b...

Solution:

(a) In contradiction proof say A_1 is regular

p from pumping lemma (p be pumping length)

say length of b is m ($m \geq p$) then b is member of A_1 ($\because$ pumping lemma)

We have $b = 0^p 1^p 2^p$ also

then from pumping lemma $b = xyz$

and xy^iz for $i \geq 0$ in A_1

⇒ String y - only 0's (or) only 1's (or) only 2's	⇒ sting has more than 1 kind of symbol then xyyz have no ~~more~~ repeated So, contradicts (not a member)

then xyyz is not a member of A_1

Contradicts

Thus A_1 contradicts and proven to be not regular.

(b) Given $A_3 = \{a^{2n} \mid n \geq 0\}$

Assume regular and claim contradiction

Let b is a^{2^p} where p is pumping length

Since b is member of A_2 and b is longer than p

$b = xyz$ (∵ pumping lemma)

third condition of pumping lemma

For $|xy| \leq p$ and $p < 2^p$ we have $|y| < 2^p$

then $|xyyz| = |xyz| + |y| < 2^p + 2^p = 2^{p+1}$

second cond. of pumping lemma

For $|y| > 0$ so, $2^p < |xyyz| < 2^{p+1}$

but $xyyz$ length can't be power of 2

then contradicts as can't be a member of A_2

Proven A_2 is not regular.

*

$\boxed{\text{Question 12:}}$ Using pumping lemma show

$A = \{www \mid w \in \{a,b\}^*\}$ is not regular

Solution:-

Proving through contradiction

Assume A is regular

p is pumping length (from pumping lemma)

say q is string such that $q = a^p b a^p b a^p b$

so then q is member where length is longer than p

then, $q = xyz$ (satisfying all three condition of pumping lemma)

But observe,

third condition y consists only if

$xyyz \notin A_2$. Thus other violated

Proved A_2 is not regular.

Question 143: Give an algorithm to tell whether a regular language L is infinite (using pumping lemma)

Answer:

Let p be the pumping length

Now string s be of length atleast $2n$

And L is defined for strings with length atleast $2n$. Then,

Claim:- s is member of L

proof:- As length of s is as short as length of strings of L atleast $2n$.

$s = xyz$ ($\because$ pumping lemma)

xz is in L

Now,

length of xz can't be $2n$ ($\because$ s is shorter than L and xz is shorter than s)

It is n ($\because n < 2n$ or S)

So, It is between n and $2n-1$

If we assume L has no strings between n and 2n-1, then this claim contradicts as s is member. Thus from pumping lemma if one string exists then it applies to such a string and pumped to show infinite seq of strings in L.

As the claim is resulted then, L can't be empty thus contradicts wrong assumption of its emptiness.

Thus L is infinite (& L is regular here)

Question 14: For the following languages give the minimum pumping length & justify.

(a) $r = 001 + 0^*1^*$

(b) $r = 0^*1^*$

(c) $r = (01)^*$

(d) $r = \epsilon$

Answers:-

Given $r = 001 + 0^*1^*$

then elements can be $L = \{\epsilon, 0, 1, 00, 11, 01, 000, \ldots\}$

length can't be zero ($\because \epsilon \in L$, can't be pumped)

Thus also non-empty (remaining strings)

can be splitted using pumping lemma into 3.

Thus, Minimum pumping length is 1.

(b) Given $r = 0^* 1^*$

Same as (a) $L = \{\epsilon, 0, 1, 00, 11, 01, 000, 111, 001, \ldots\}$

For any string in this language, first symbol is possible, Thus, all strings of length are greater than 1.

Thus minimum pumping length = 1

(c) $r = (01)^*$

Then L can be $\{\epsilon, 01, 0101, 010101, \ldots\}$

Observe, you can take any string $|w| \geq 2$ thus first symbols i.e 01 as y, pump it

Min pumping length is 2.

(d) $r = \epsilon$

As ~~ϵ be pumped to~~ the finite language

Because we couldn't find infinite seq (no atleast 1 string is greater than 0)

Thus, By a fact if language is finite then minimum pumping length $x+1$ (x - length of string)

Here, $x = 0$

Thus Minimum pumping length = 1

Question 15 : Let $A|B = \{w \mid wx \in A$, for some $x \in B\}$

(a) If A is any language and B is regular, is A/B necessarily any language? Explain.

(b) If A is regular and B is any language. Show $A|B$ is regular.

Solution :-

(a) Given, a is any language the from pumping lemma, a string s in A and $|s| \geq P$ the $s = uvxyz$

So, let M_1, M_2 be machine for A, B then it guesses a string x and processes x by M_1 and M_2 ($\because$ NFA)

Thus machine will accept only if both machines simultinaously accepts. Thus $A|B$ is any laguaglue or context-free.

(b) To show; Let $M = (Q, \Sigma, \delta, q_0, F)$ be DFA for A. Then its language for state $q \in Q$

$$L_q = \{x \mid \delta(q, x) \in F\}$$

Then other $M' = (Q, \Sigma, \delta, q_0, F')$ is a DFA

where $A|B$ for $F' = \{q \mid L_q \cap B \neq \phi\}$

i.e when intersection of lanpuape of A and B is non-empty.

Thus $A|B$ is repular is possible.