MDL Assignment 3, Part 1

Roll numbers – 2019101101 , 2019115009

Considered Number – 2019101101

$x = 1 - (((1101 \% 30) + 1) / 100)$

$= 1 - (21 + 1 / 100)$

$= 1 - 0.22$

$= 0.78$

$y = (01 \% 4) + 1$

$= 1 + 1$

$= 2$

Probability of Desired direction = 0.78

Probability of Opposite of desired direction = 1 - 0.78

= 0.22

Since y = 2,

P(Observation = Red | State = Red) = 0.9

P(Observation = Green | State = Red) = 0.1

P(Observation = Green | State = Green) = 0.85

P(Observation = Red | State = Green) = 0.15

Initially, agent in one of the three states S1, S3, S6

So belief states of

$(S1, S2, S3, S4, S5, S6) = \left(\frac{1}{3}, 0, \frac{1}{3}, 0, 0, \frac{1}{3}\right)$

To compute the next belief states, we use formula

$$b'(s') = \alpha P(e|s') \sum P(s'|s,a)\, b(s)$$

Where b(s) is the current belief state of state s which upon taking action 'a' reaches state s'

and previous evidence e

$P(s'|s,a) \rightarrow$ transition probability

$P(e|s') \rightarrow$ sensory model

$\alpha \rightarrow$ Normalization factor

Step 1 :- Agent takes action Right and Observed Green

Since $b' = \sum P(e|s')\, P(s'|s,a)\, b(s)$

We compute where $b(s) \neq 0$, i.e $S1, S3, S6$

for $S1$

$= P(\text{Red} | \text{Left}) \times P(\text{Green} | \text{Red}) \times b(S1) +$

$P(\text{Green} | \text{Right}) \times P(\text{Green} | \text{Green}) \times b(S1)$

$= \cancel{0.78} \times$

$= 0.22 \times 0.1 \times \frac{1}{3} + 0.78 \times 0.85 \times \frac{1}{3}$

$= \dfrac{0.685}{3}$

for $S3$

$= P(\text{Green} | \text{Left}) \times P(\text{Green} | \text{Green}) \times b(S3) +$

$P(\text{Green} | \text{Right}) \times P(\text{Green} | \text{Green}) \times b(S3)$

$= 0.85 \times \frac{1}{3} (0.22 + 0.78)$

$= \dfrac{0.85}{3}$

for S6

$= P(\text{Red} \mid \text{Right}) \times P(\text{Green} \mid \text{Red}) \times b(S6) +$

$P(\text{Green} \mid \text{Left}) \times P(\text{Green} \mid \text{Green}) \times b(S6)$

$= 0.78 \times 0.1 \times \frac{1}{3} + 0.22 \times 0.85 \times \frac{1}{3}$

$= \frac{0.265}{3}$

Adding S1+S3+S6

$= \frac{0.685 + 0.85 + 0.265}{3}$

$= \frac{1.8}{3} = 0.6$

$\alpha = 0.6$

$b'(S1) = \frac{0.22 \times 0.1 \times \frac{1}{3}}{\alpha}$

$= \frac{0.022}{1.8} = 0.0122\bar{2}$

$= 0.0122$

$b'(S2) = \frac{0.78 * 0.85 \times \frac{1}{3} + 0.22 \times 0.85 \times \frac{1}{3}}{\alpha}$

$= \frac{0.663 + 0.185}{1.8} = \frac{0.85}{1.8} = 0.4722\bar{2}$

$= 0.4722$

$b'(S3) = 0$

No action of Right / Left leads to S3

$$b'(S4) = \frac{0.78 \times 0.85 \times \frac{1}{3}}{0.6}$$

$$= \frac{0.663}{1.8} = 0.3683\overline{3}$$

$$= 0.3683$$

$$b'(S5) = \frac{0.22 \times 0.85 \times \frac{1}{3}}{0.6}$$

$$= \frac{0.187}{1.8} = 0.1038\overline{8}$$

$$= 0.1039$$

$$b'(S6) = \frac{0.78 \times 0.1 \times \frac{1}{3}}{0.6} = \frac{0.078}{1.8} = 0.0433\overline{3}$$

$$= 0.0433$$

The belief states now are

$(S1, S2, S3, S4, S5, S6) = (0.0122, 0.4722, 0.0, 0.3683, 0.1039, 0.0433)$

And summation is almost $1 \approx 0.999$ due to round off

For next step : Action Left and observed Red

for S1

P(Red | Left) × P(Red | Red) × b(S1) +

P(Green | Right) × P(Red | green) × b(S1)

$= 0.78 \times 0.9 \times 0.012222 + 0.22 \times 0.15 \times 0.01222$

$= 0.00882$

for S2

P(~~g~~ Red | Right) × P(Red | Red) × b(S2) +

P(Red | Left) × P(Red | Red) × b(S2)

$= 0.472222\,(0.22 \times 0.9 + 0.78 \times 0.9)$

$= 0.42498$

for S3

P(Green | Left) P(Red | Green) × b(S3) +

P(Green | Right) × P(Red | green) × b(S3)

$= 0 \quad (\because b(S3) = 0)$

for S4

P(Red | Left) P(Red | Red) × b(S4) +
P(Green | Right) × P(Red | green) × b(S4)

= 0.3683 (0.78 × 0.9 + 0.22 × 0.15)

= 0.33147

for S5

P(Green | Left) × P(Red | Green) × b(S5) +
P(Red | Right) × P(Red | Red) × b(S5)

= 0.103882 (0.78 × 0.15 + 0.22 × 0.9)

= 0.0327285

for S6

P(Green | Left) × P(Red | Green) × b(S6) +
P(Red | Right) × P(~~g~~ Red | Red) × b(S6)

= 0.0433 (0.78 × 0.15 + 0.22 × 0.9)

= 0.0136395

Now d = 0.00882 + 0.42498 + 0.33147 + 0.0327285
+ 0.0136395

= 0.7510467

$$b'(S1) = \frac{0.78 \times 0.9 \times (0.01222 + 0.47222)}{d}$$

$$= \frac{0.702\,(0.01222 + 0.47222)}{0.7510467}$$

$$= 0.452888$$

$$b'(S2) = \frac{0.22 \times 0.15 \times 0.01222 \times}{0.7510467}$$

$$= 0.0005368$$

$$= 0.0005$$

$$b'(S3) = \frac{(0.9 \times 0.22 \times 0.47222 + 0.78 \times 0.9 \times 0.3683)}{0.7510467}$$

$$= 0.4687325$$

$$= 0.4687$$

$$b'(S4) = \frac{0.78 \times 0.15 \times 0.1039}{0.7510467}$$

$$= 0.0162181 = 0.0162$$

$$b'(S5) = \frac{(0.22 \times 0.15 \times 0.3683) + (0.78 \times 0.15 + 0.04333)}{0.7510467}$$

$$= 0.0228946 = 0.0229$$

$$b'(S6) = \frac{(0.22 \times 0.9 \times 0.10388) + (0.22 \times 0.9 \times 0.0433)}{0.7510467}$$

$$= 0.0387643$$

$$= 0.0388$$

The belief states now are

[0.4528, 0.0005, 0.4687, 0.0162, 0.0229, 0.0388]

Next step: Agent took Action left and Observed Green.

for S1:-

P(Red | left) x P(Green | Red) x b(S1) +

P(Green | Right) x P(Green | Green) x b(S1)

= 0.78 x 0.1 x 0.452888 + 0.22 x 0.85 x 0.452888

= 0.12001532

for S2:-

P(Red | left) x P(Green | Red) x b(S2) +

P(Red | Right) x P (Green | Red) x b(S2)

= 0.78 x 0.1 x 0.0005368 + 0.22 x 0.1 x 0.0005368

= 0.00005368

for S3:-

P(Green | left) x P(Green | Green) x b(S3) +

P(Green | Right) + P(Green | Green) x b(S3)

= 0.78 x 0.85 x 0.4687325 + 0.22 x 0.85 x 0.4687325

= 0.398422625

for S4:-

P(Red | Left) × P(Green | Red) × b(S4) +

P(Green | Right) × P(Green | Green) × b(S4)

= 0.78 × 0.1 × 0.0162181 + 0.22 × 0.85 × 0.0162181

= 0.0042977

for S5:-

P(Green | Left) × P(Green | Green) × b(S5) +

P(Red | Right) × P(Green | Red) × b(S5)

= 0.78 × 0.85 × 0.022894 + 0.22 × 0.1 × 0.022894

= 0.01568239

for S6:-

P(Green | Left) × P(Green | Green) × b(S6) +

P(Red | Right) × P(Green | Red) × b(S6).

= 0.78 × 0.85 × 0.0387643 + 0.22 × 0.1 × 0.0387643

= 0.0264248

Normalization factor α =

0.12001532 + 0.0005368 + 0.398422625 + 0.0042977 +

0.01568239 + 0.0264248

= 0.5649701

Baling fct S1

$$b'(S1) = \frac{0.78 \times 0.1 \times 0.452888 + 0.78 \times 0.1 \times 0.0005368}{0.5649701}$$

$= 0.06261923$

$= 0.0626$

$$b'(S2) = \frac{0.22 \times 0.85 \times 0.452888 + 0.78 \times 0.85 \times 0.4687325}{0.5649701}$$

$= 0.699827315$

$= 0.6998$

$$b'(S3) = \frac{0.22 \times 0.1 \times 0.0005368 + 0.78 \times 0.1 \times 0.0162181}{0.5649701}$$

$= 0.00234721$

$= 0.0023$

$$b'(S4) = \frac{0.85 \times 0.22 \times 0.4687325 + 0.78 \times 0.85 \times 0.022894}{0.569701}$$

$= 0.18202314$

$= 0.1820$

$$b'(S5) = \frac{0.22 \times 0.85 \times 0.016218 + 0.78 \times 0.85 \times 0.0387643}{0.569701}$$

$= 0.05088279$

$= 0.0509$

$$b'(S6) = \frac{0.78 \times 0.1 \times 0.0387643 + 0.22 \times 0.1 \times 0.022894}{0.5649701}$$

$$= 0.00243279\,1$$

$$= 0.0024$$

Hence final belief states

$(S1, S2, S3, S4, S5, S6) = (0.0626, 0.6998, 0.0023, 0.1820, 0.0509, 0.0024)$