DATA SCIENCE

STATISTICAL CONCEPTS

What does Statistics cover?

Sample v/s Population

Probability Theory

Probability Distribution Concepts

Types of Distributions

1. **Discrete**
2. Continuous

Discrete Distribution

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Discrete Distribution

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Examples:

- A coin flip

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Binomial Distribution

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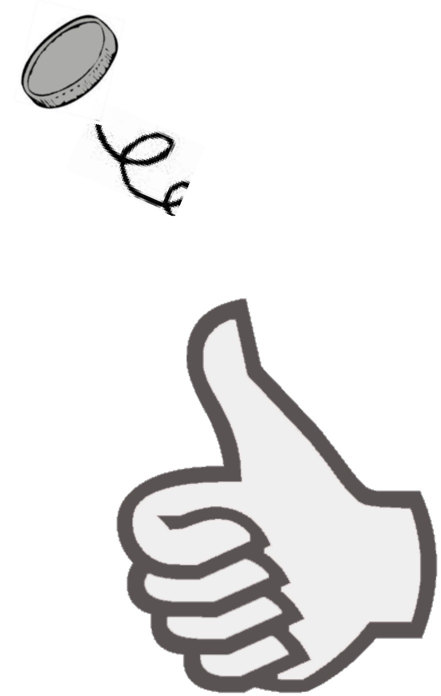
<u>The common feature across all these is:</u>

1. There are only two possible outcomes: Win or Lose, 1 or 0, Male or Female
2. There are no external factors influencing the probability of each outcome over time
3. The chances of each outcome are independent of previous results

Binomial Distribution

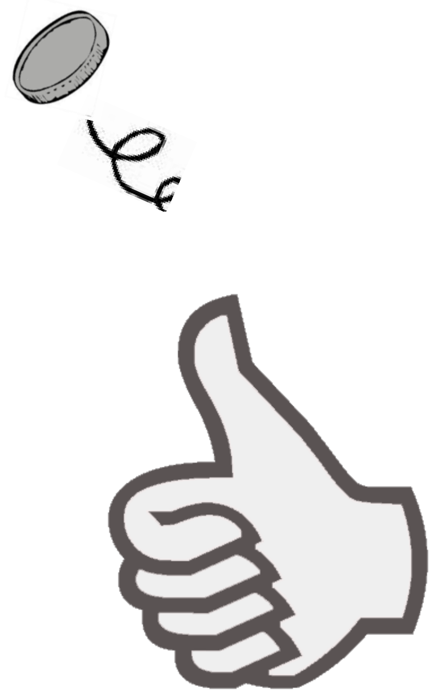
Simple non-business example: a coin toss



Binomial Distribution

Simple non-business example: a coin toss

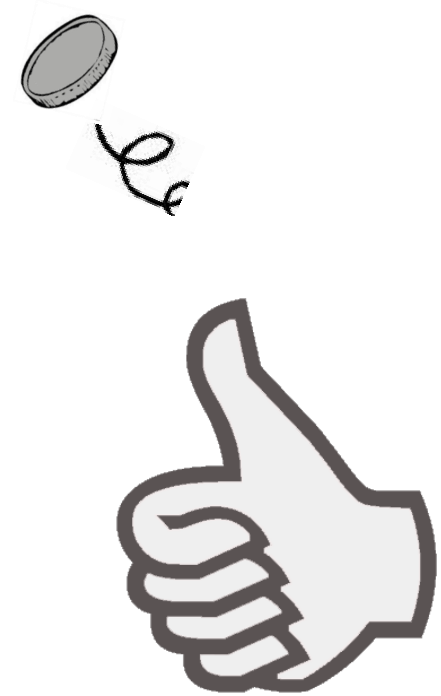
- Outcome: **Random**



Binomial Distribution

Simple non-business example: a coin toss

- Outcome: **Random**
- Possible outcomes: **Two**



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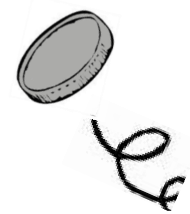
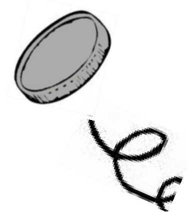


- Outcome: **Random**
- Possible outcomes: **Two**
- External factors influencing outcome probability over time: **No**

Binomial Distribution

Simple non-business example: a coin toss

- Outcome: **Random**
- Possible outcomes: **Two**
- External factors influencing outcome probability over time: **No**
- Chances of each outcome independent of previous trials: **Yes**



Binomial Distribution

Toss a coin once: Bernoulli trial

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Toss a coin 10 times:

- Measure # of *heads* observed: outcome follows a binomial distribution

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- How many *heads* will we see? 0? 1? 2? …10?

Binomial Distribution

Toss a coin once: Bernoulli trial

Toss a coin 10 times:

- Measure # of *heads* observed: outcome follows a binomial distribution
- How many *heads* will we see? 0? 1? 2? …10?
- Are all of these outcomes equally likely?

Binomial Distribution

We calculate these probabilities using a mathematical formula:

probability distribution function

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For a binomial distribution:

PDF: $$P(x) = \frac{n!}{x!\,(n-x)!}\,p^{x}\,(1-p)^{n-x}$$

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Where:

x = Outcome
n = Trials
p = Probability of success on each trial

Binomial Distribution

Probability of 4 *heads*?

$$P(x) = \frac{n!}{x!\,(n-x)!}\,p^{x}(1-p)^{n-x}$$

Binomial Distribution

Probability of 4 *heads*?

$$P(x) = \frac{n!}{x!\,(n-x)!}\, p^x (1-p)^{n-x}$$

p(4) = (10!/ 4! * 6!) * (0.5)^4 * (0.6)^6 = 0.205

Binomial Distribution

Probability of 4 *heads*?

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There are built in functions in statistical tools that generate these calculations

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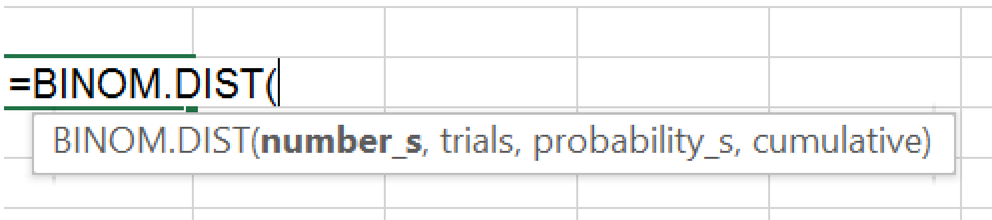
In Excel: **BINOM.DIST**

Binomial Distribution



Binomial Distribution

=BINOM.DIST(

BINOM.DIST(**number_s**, trials, probability_s, cumulative)

In our example:

number_s = 4
trials = 10
probability_s = 0.5
cumulative = False

Binomial Distribution

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BINOM.DIST(**number_s**, trials, probability_s, cumulative)

In our example:

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Binomial Distribution

Example: Quality Control

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Supposing from a random lot you get 2 defects,

how likely is this outcome due to random chance?

Binomial Distribution

Example: Quality Control

In a manufacturing unit, the process has a defect rate of 10%.

As part of the QC process, you randomly evaluate products in lots of 10.

Supposing from a random lot you get 2 defects,

how likely is this outcome due to random chance?

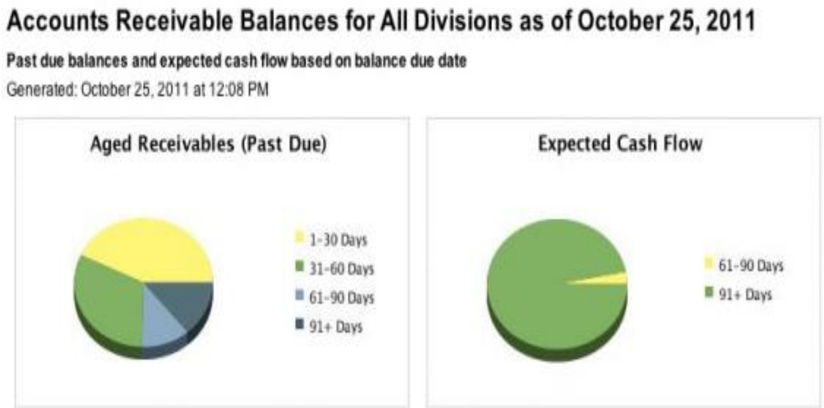
=BINOM.DIST(2,10,0.1,FALSE) = 0.19

Coming Up

Binomial Distribution Example:

How a Finance Manager may use Binomial Distribution to create a contingency fund based on likelihood

Binomial Distribution



Month	Paid	AR Balances	Running AR Total
2012 January	$44,220.30	$188,809.50	$213,696.46
2012 February	$114,300.00	$768,978.05	$982,674.51
2012 March	$271,091.61	$1,568,852.37	$2,551,526.88
2012 April	$147,800.00	$905,515.20	$3,457,042.08
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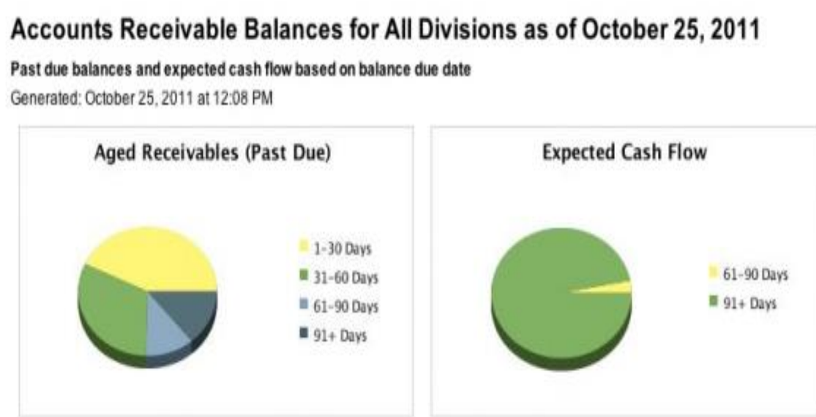
Binomial Distribution

As Finance Manager you are reviewing AR balances:



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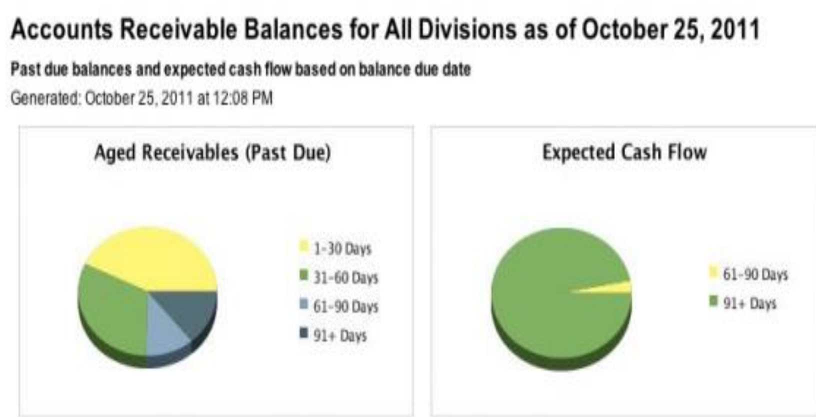
As Finance Manager you are reviewing AR balances:

- Based on past data, on average 40% of customers are more than 60 days late with payments



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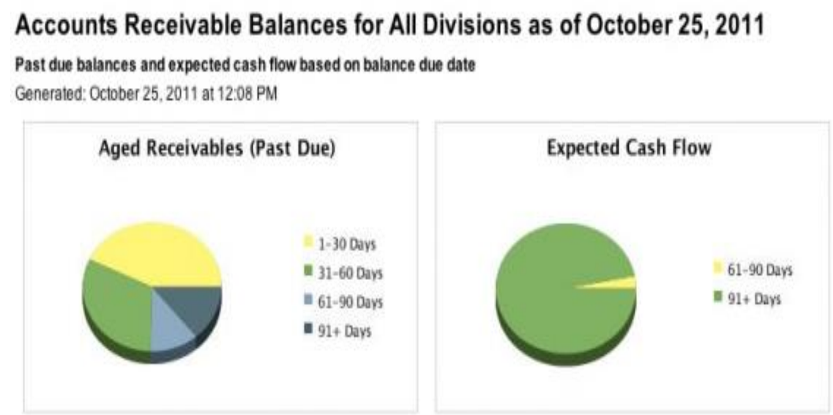
- Assuming a total of 150 customers, you want to create a contingency where > 50% of customers are late



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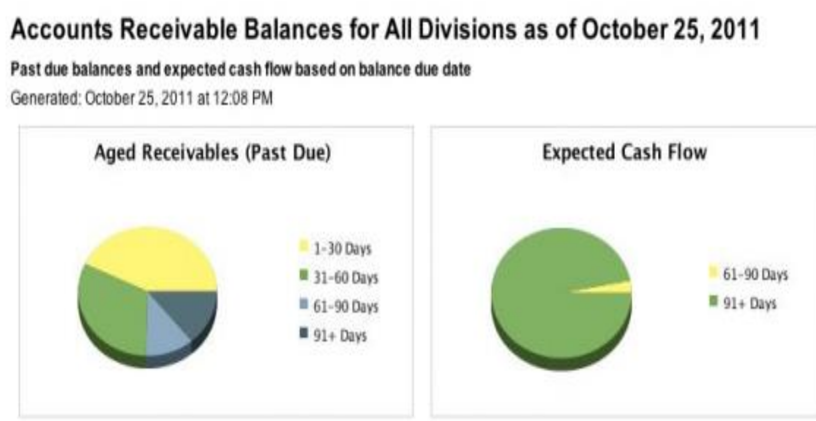
- The size of the contingency fund is directly proportional to the probability of > 50% of customers being late



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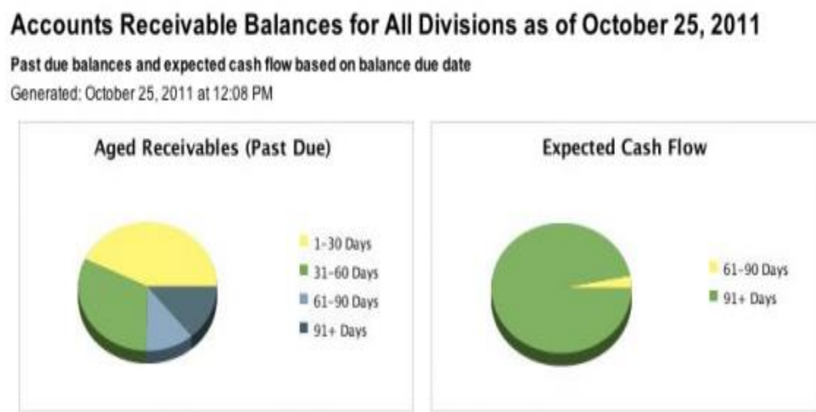
- Assuming a total of 150 customers, you want to create a contingency where > 50% of customers are late
- The size of the contingency fund is directly proportional to the probability of > 50% of customers being late
- You need a sense of the likelihood of > 50% being late in any given month



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Binomial Distribution

- Probability of > 50% of customers being late in any month?

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P(76 being late) + P (77 being late) + P (78 being late) + … P(150 being late)

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- This is a tedious manual calculation

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- This is a tedious manual calculation

- A faster method is to use the concept of **cumulative probability**

Binomial Distribution

In Excel:

=BINOM.DIST(S,T,P, **FALSE**) :

Binomial Distribution

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=BINOM.DIST(S,T,P, **FALSE**) :

the random chance probability of seeing EXACTLY S successes in T Bernoulli trials, when the probability of success on any trial is P

Binomial Distribution

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=BINOM.DIST(S,T,0, **TRUE**):
the random chance probability of seeing LESS THAN EQUAL TO S successes in T Bernoulli trials, when the probability of success on any trial is P

Binomial Distribution

Probability of seeing > 50% of customers being late

Binomial Distribution

Probability of seeing > 50% of customers being late

= Probability of seeing > 75 customers being late

Binomial Distribution

Probability of seeing > 50% of customers being late

= Probability of seeing > 75 customers being late

= 1 – BINOM.DIST(75,150,0.4,TRUE)

Binomial Distribution

Probability of seeing > 50% of customers being late

= Probability of seeing > 75 customers being late

= 1 – BINOM.DIST(75,150,0.4,TRUE)

= 0.005

Coming Up

Discrete Distributions:

Hypergeometric Distribution

Hypergeometric Distribution

A hypergeometric distribution is generated when you have Bernoulli trials, **but selections are not replaced**

$$P\{X = k\} = \frac{\binom{K}{k}\binom{N-K}{n-k}}{\binom{N}{n}}$$

Where

$$\binom{A}{B} = \frac{A!}{(A-B)!B!}$$

https://controls.engin.umich.edu/wiki/index.php/Discrete_Distributions:_hypergeometric,_binomial,_and_poisson#Hypergeometric_Distribution

Hypergeometric Distribution

Example: HR Policies and Diversity

A company wants to encourage diversity in its management ranks. Of the 18 employees eligible for promotion into middle management, 9 are women.

Hypergeometric Distribution

<u>Example: HR Policies and Diversity</u>

A company wants to encourage diversity in its management ranks. Of the 18 employees eligible for promotion into middle management, 9 are women.

The promotions are announced: 8 are promoted of which 3 are women.
Is there a problem?

PROMOTED
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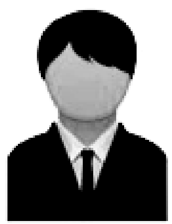

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In this case, we use Hypergeometric Distribution rather than Binomial

Hypergeometric Distribution

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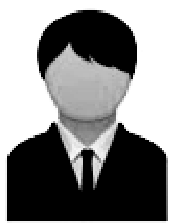

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=hypgeom.dist(3,8,9,18,false) = 0.24

Other Binomial Type Distributions

Negative Binomial

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Negative Binomial

- Used to find out the number of trials needed to get X successes

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- Example: What is the probability that the 30^{th} purchase in my store will happen with the 100^{th} customer, when the probability of purchase for any customer is 20%?

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=NEGBINOM.DIST (number_f, number_s, probability_s, cumulative)

Other Binomial Type Distributions

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- Example: What is the probability that the 30th purchase in my store will happen with the 100th customer, when the probability of purchase for any customer is 20%?

=NEGBINOM.DIST (number_f, number_s, probability_s, cumulative)

=NEGBINOM.DIST(70,30,0.2,FALSE) = 0.001

Other Binomial Type Distributions

Geometric Distribution

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Geometric Distribution

- Probability of the first success in the rth trial

Other Binomial Type Distributions

Geometric Distribution

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=NEGBINOM.DIST(19,1,0.02,true) = 0.33

Coming Up

Discrete Distributions:

Poisson Distribution

Poisson Distribution

Poisson Distribution

Another discrete probability distribution used to model number of events occurring in a time frame

Poisson Distribution

Another discrete probability distribution used to model number of events occurring in a time frame

Examples:

- ❑ Number of insurance claims in a month

Poisson Distribution

Another discrete probability distribution used to model number of events occurring in a time frame

Examples:

- ❑ Number of insurance claims in a month
- ❑ Disease spread in a day

Poisson Distribution

Another discrete probability distribution used to model number of events occurring in a time frame

Examples:

- Number of insurance claims in a month
- Disease spread in a day
- Number of telephone calls in an hour

Poisson Distribution

Another discrete probability distribution used to model number of events occurring in a time frame

Examples:

- ❑ Number of insurance claims in a month
- ❑ Disease spread in a day
- ❑ Number of telephone calls in an hour
- ❑ Number of patients needing emergency services in a day

Poisson Distribution

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3. Average frequency of occurrence for the given time period is known
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Notice here that there is no n (sample size) impact

Poisson Distribution

Example: Call Centre Management

You are a Manager in a call center with a staff of 55 people, who on average handle 330 calls in an hour. A holiday is coming up and 5 resources want leave. You estimate the 50 remaining resources can manage 20% greater calls, but want to plan for the chance of greater than 20% increased call volume.

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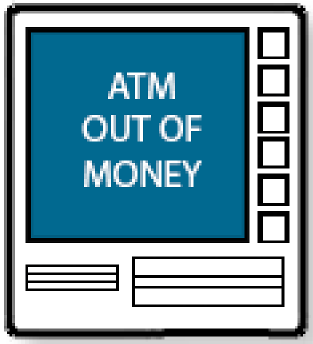
- We need **Probability of seeing 8** or more calls an hour when **average is 6**. Calculated manually or by referencing the Poisson Distribution table

Poisson Distribution

Example: ATM Machine



Poisson Distribution

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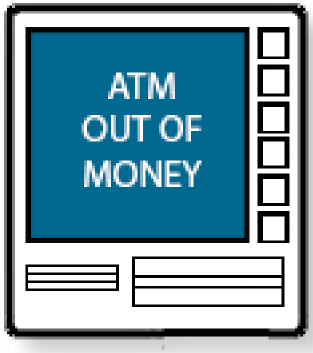

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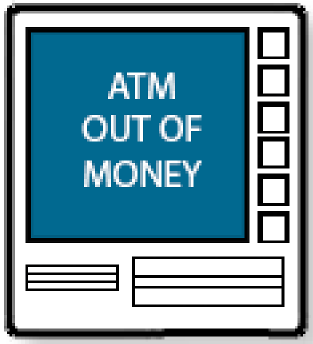
- A customer service KRA is to keep customer complaints to less than 10%



Binomial or Poisson?

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1. If a *mean / average probability* of an event happening per unit time / per page / per mile cycled etc., is given, and you are asked to calculate a probability of *n* events happening in a given time / number of pages / number of miles cycled, then the Poisson Distribution is used.

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2. If an *exact probability* of an event happening is given, or implied, in the question, and you are asked to calculate the probability of this event happening k times out of n, then the Binomial Distribution must be used.

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Binomial Distribution

describes the distribution of binary data from a finite sample. Thus it gives the probability of getting r events out of n trials.

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describes the distribution of binary data from an infinite sample. Thus it gives the probability of getting r events in a population.

Recap

Types of Distributions

Discrete:

Recap

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Types of Distributions

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Recap

Types of Distributions

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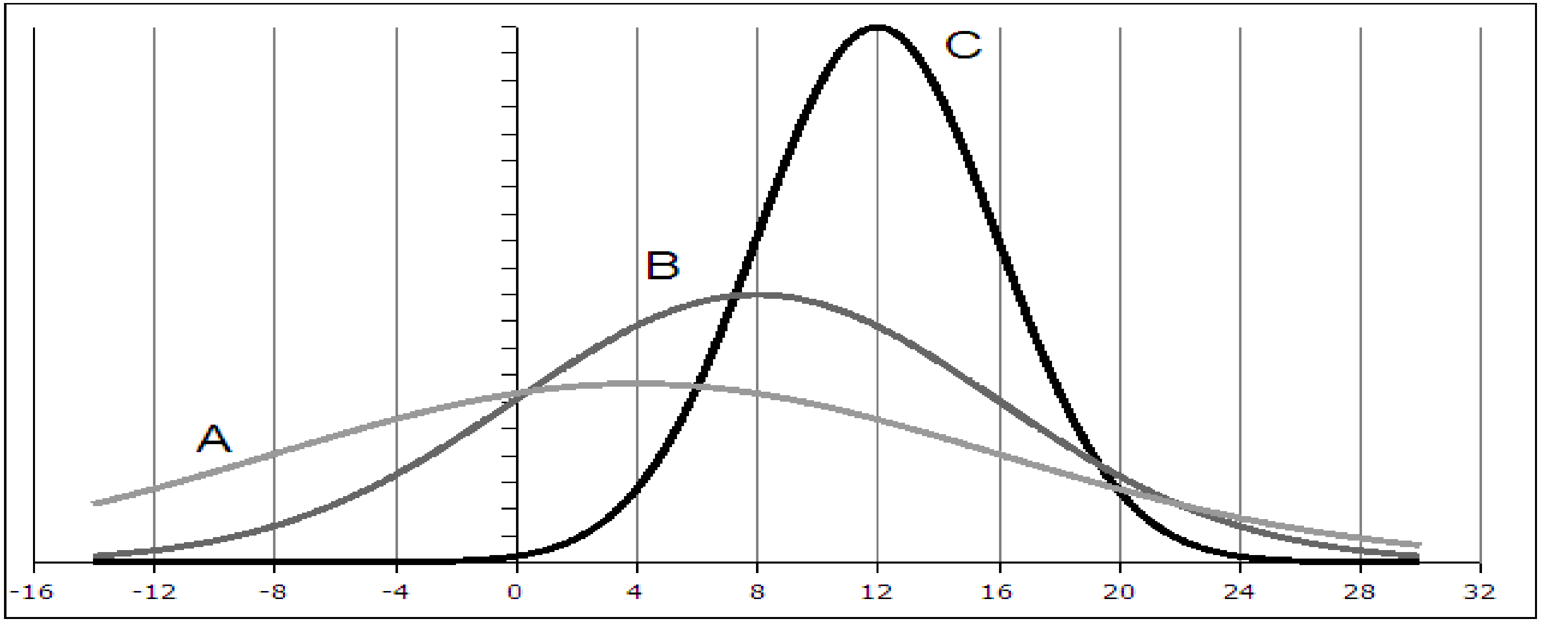
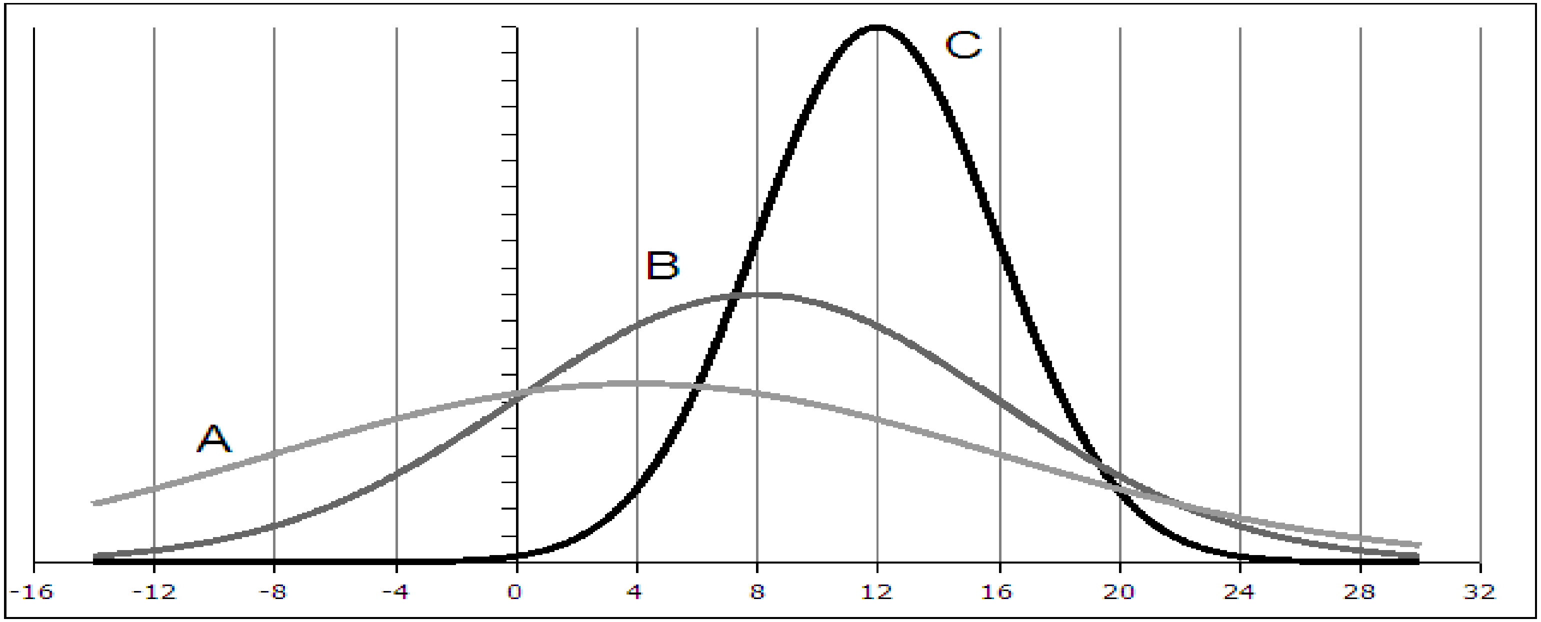
Normal Distribution is the most common kind of a continuous probability distribution due to its useful applications in Statistics

Normal Probability Distribution



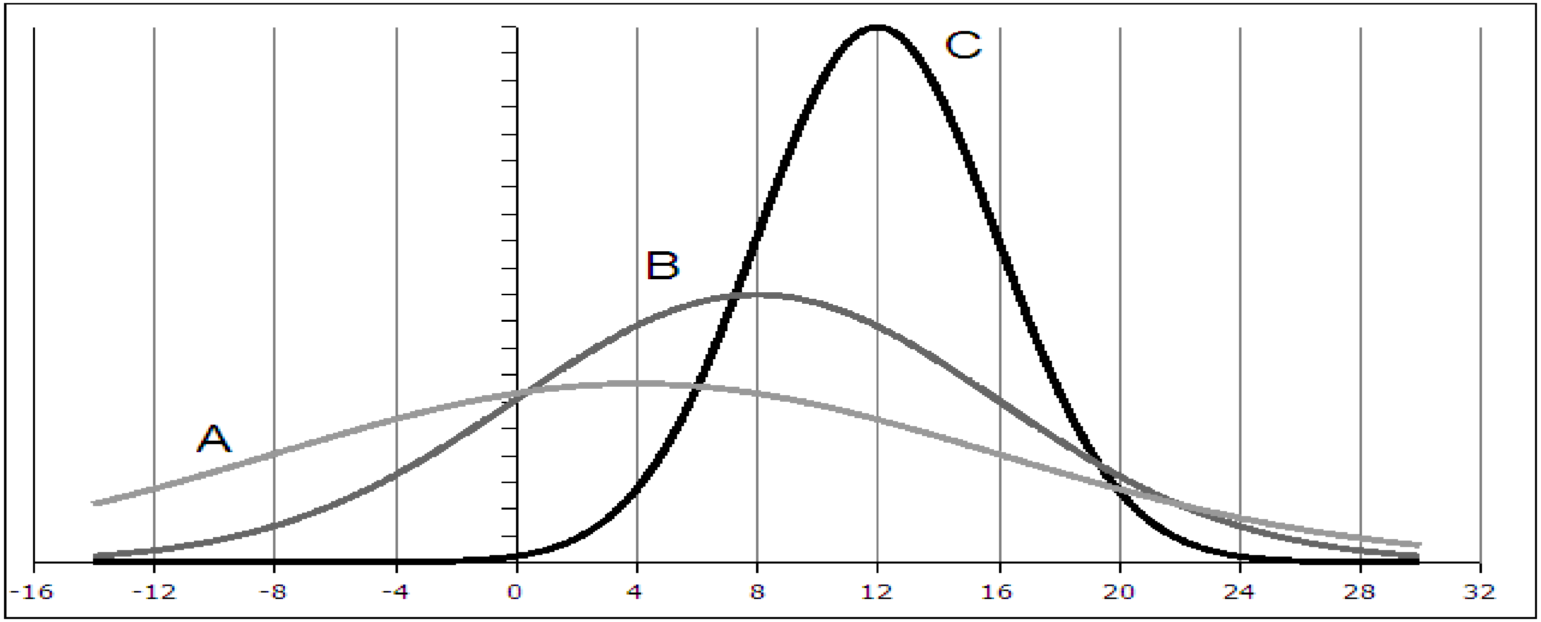
Normal Probability Distribution

1. Symmetric about the (single) mean



Normal Probability Distribution

1. Symmetric about the (single) mean
2. Mean = Median = Mode



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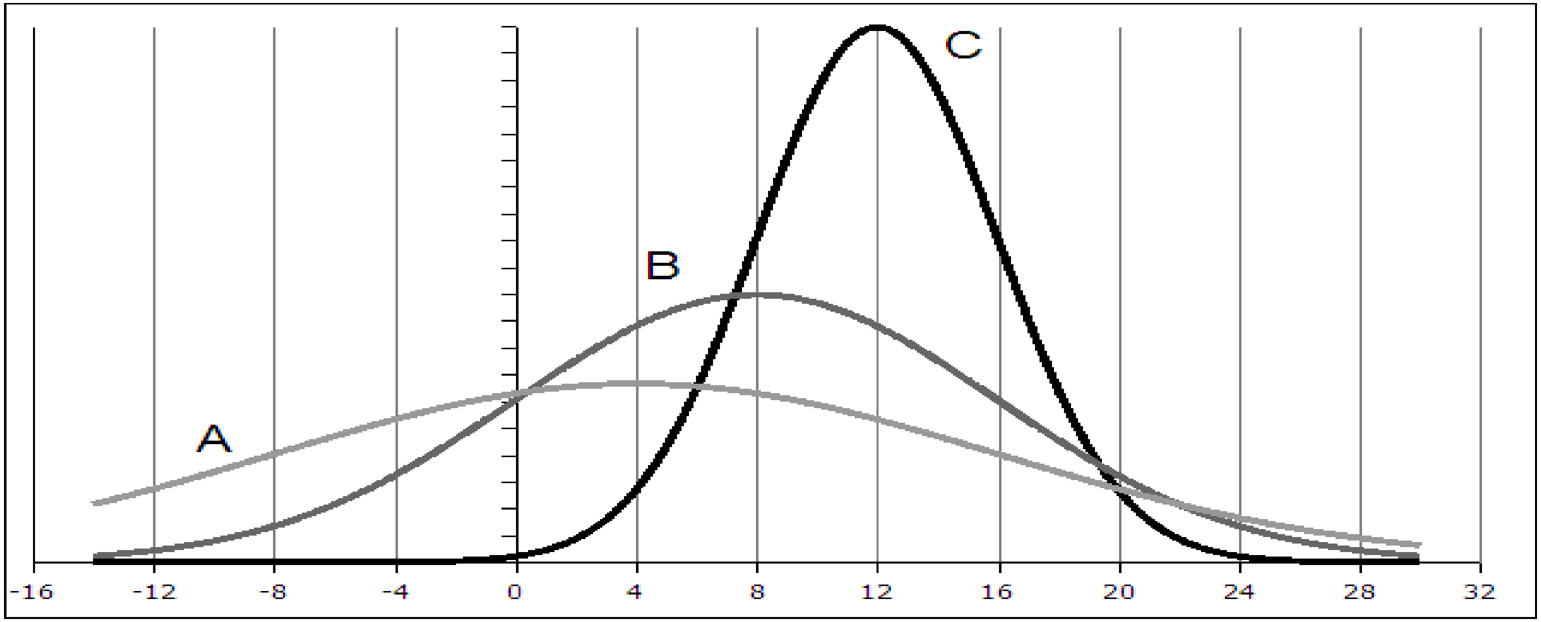

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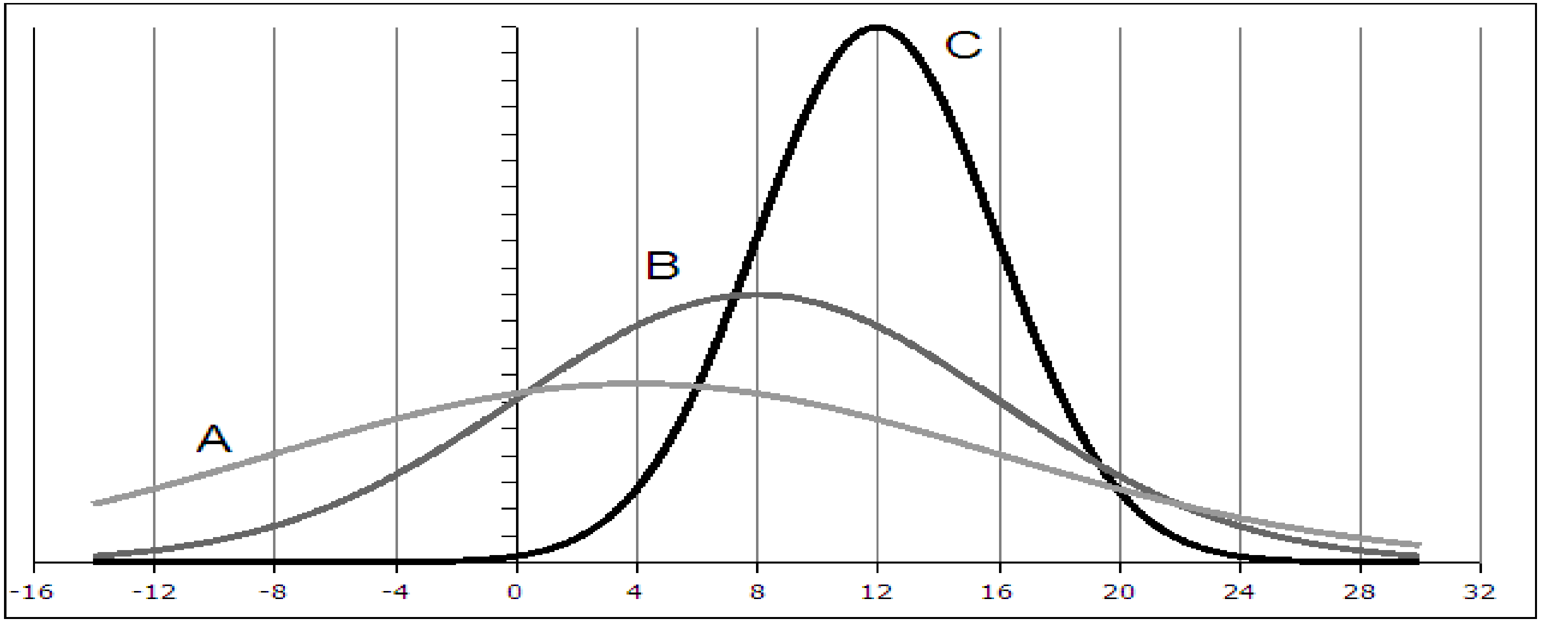


What are the differences between the curves?

Normal Probability Distribution

$$y = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}}$$

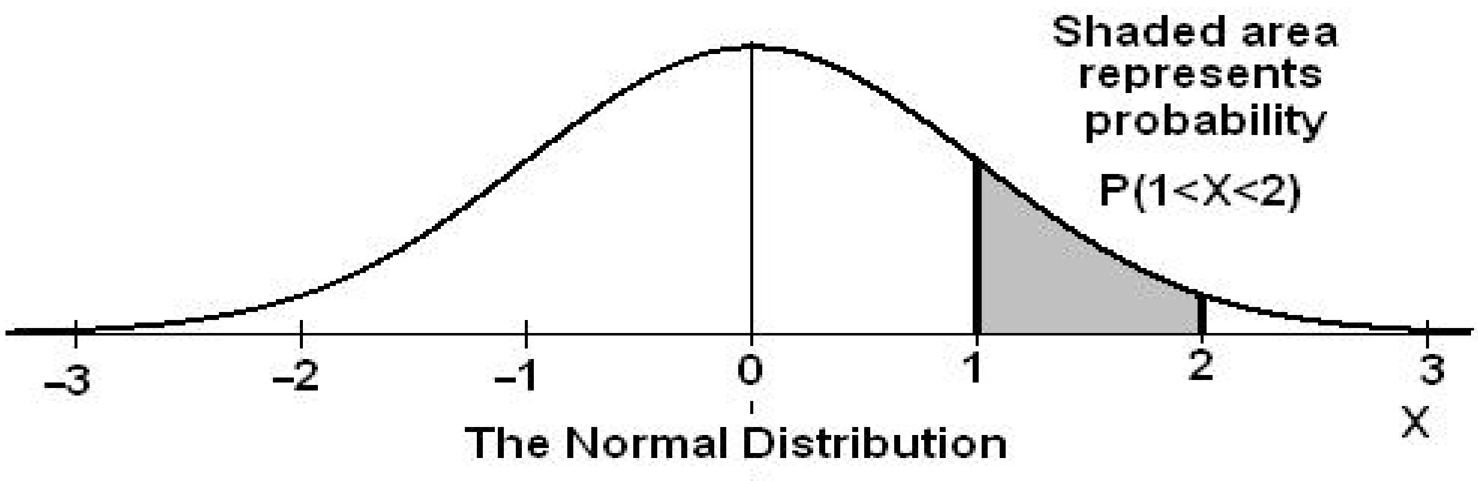
where:	y	=	vertical height of a point on the normal distribution
	x	=	distance along the horizontal axis
	σ	=	standard deviation of the data distribution
	μ	=	mean of the data distribution
	e	=	exponential constant = 2.71828...
	π	=	pi = 3.14159....

http://www.pavementinteractive.org/index.php?title=Normal_Distribution

Normal Distribution

Area under the curve:

The total area under a normal probability curve is always 1. This property allows us to think of the area as probability, and therefore we can compute probability two values on the curve



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IQ Score: Random variable

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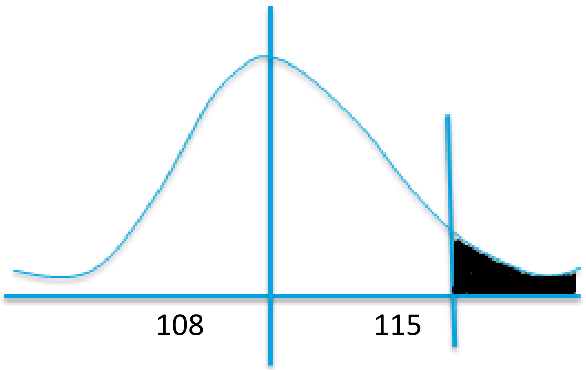

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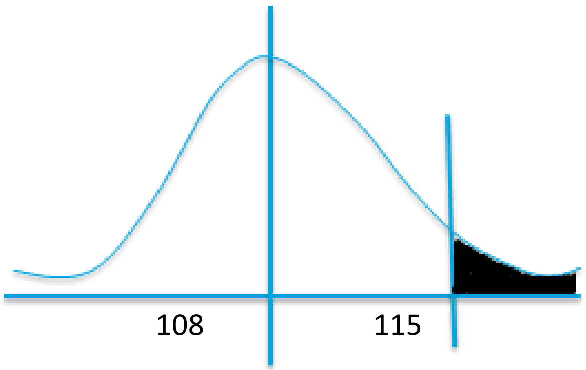

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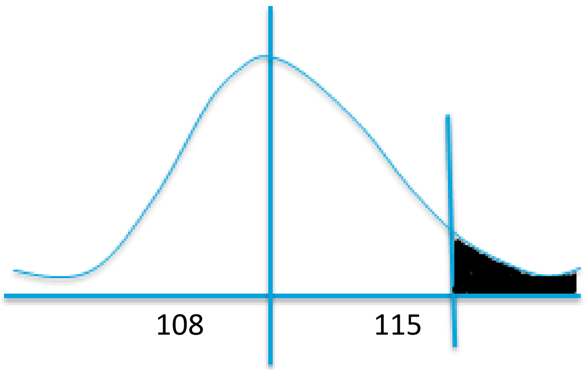
We can of course rely on Excel:
NORMDIST(Outcome, Mean, Std Dev, Cuml)



Normal Distribution

Example: Product Guarantee

A manufacturer wants to state a guarantee for performance of their product, in hours, so that product failure rates on the basis of hours of performance are restricted to less than 5%.

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What number of performance hours should they guarantee for failure rates to be less than 5%?

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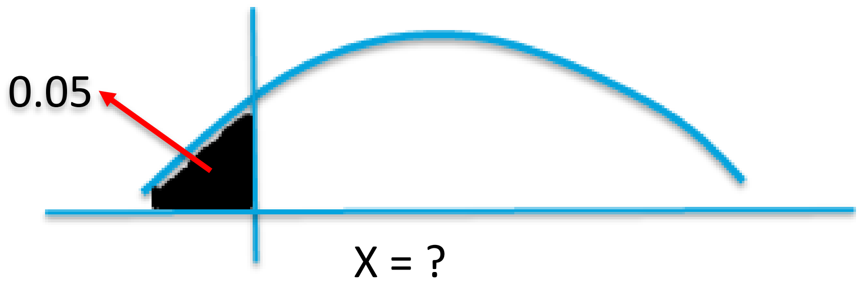
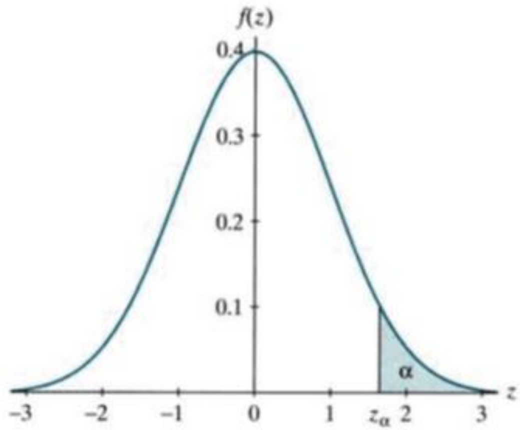
Normal Distribution

We could also calculate probabilities of multiple values of hours and identify the hours at which probability is less than 5%



Normal Distribution

But, there is an easier way to calculate the X value that will give us a specific probability – using a **standard normal probability table**



$$P(Z > z_\alpha) = \alpha$$

$$P(Z > z) = 1 - \Phi(z) = \Phi(-z)$$

z_α	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641
0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867

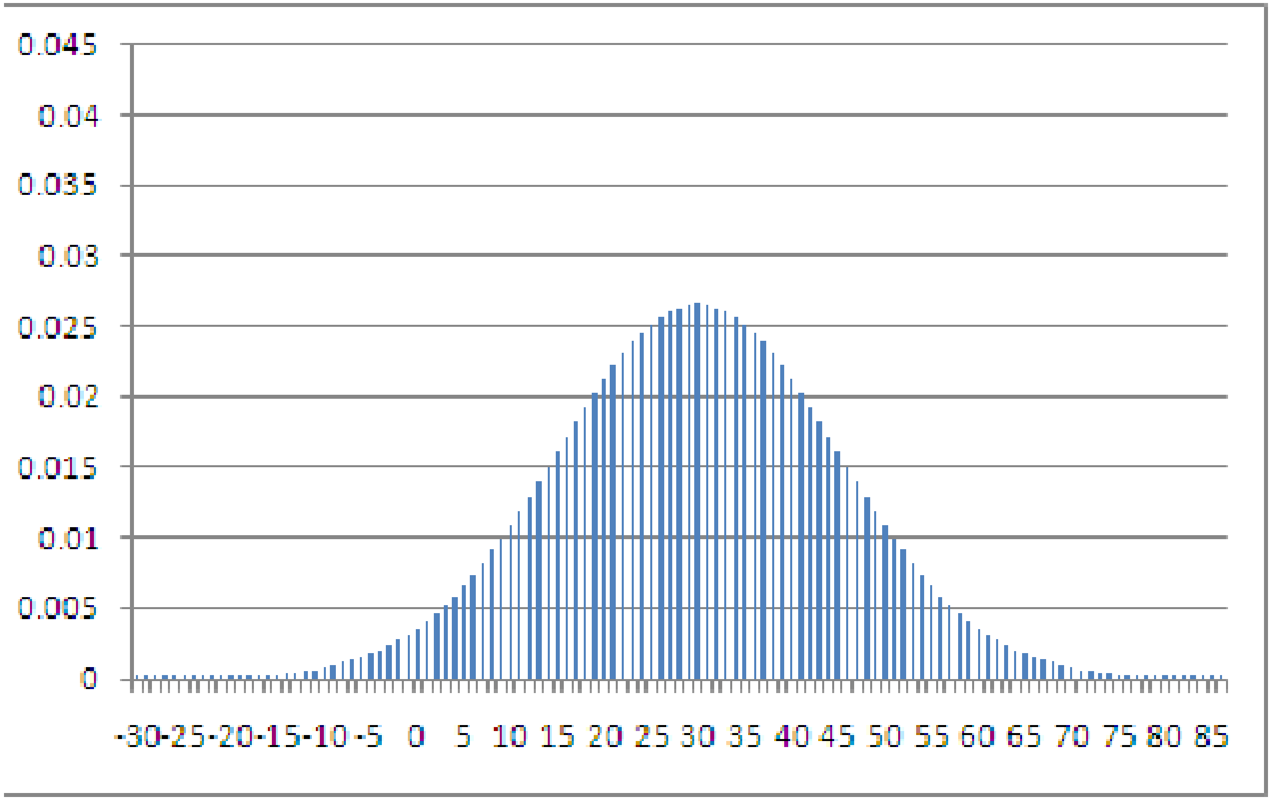
Coming Up

Continuous Distributions:

Standard Normal Distribution

Normal Distribution

We can compute the probabilities for any Mean, Std Dev and an X



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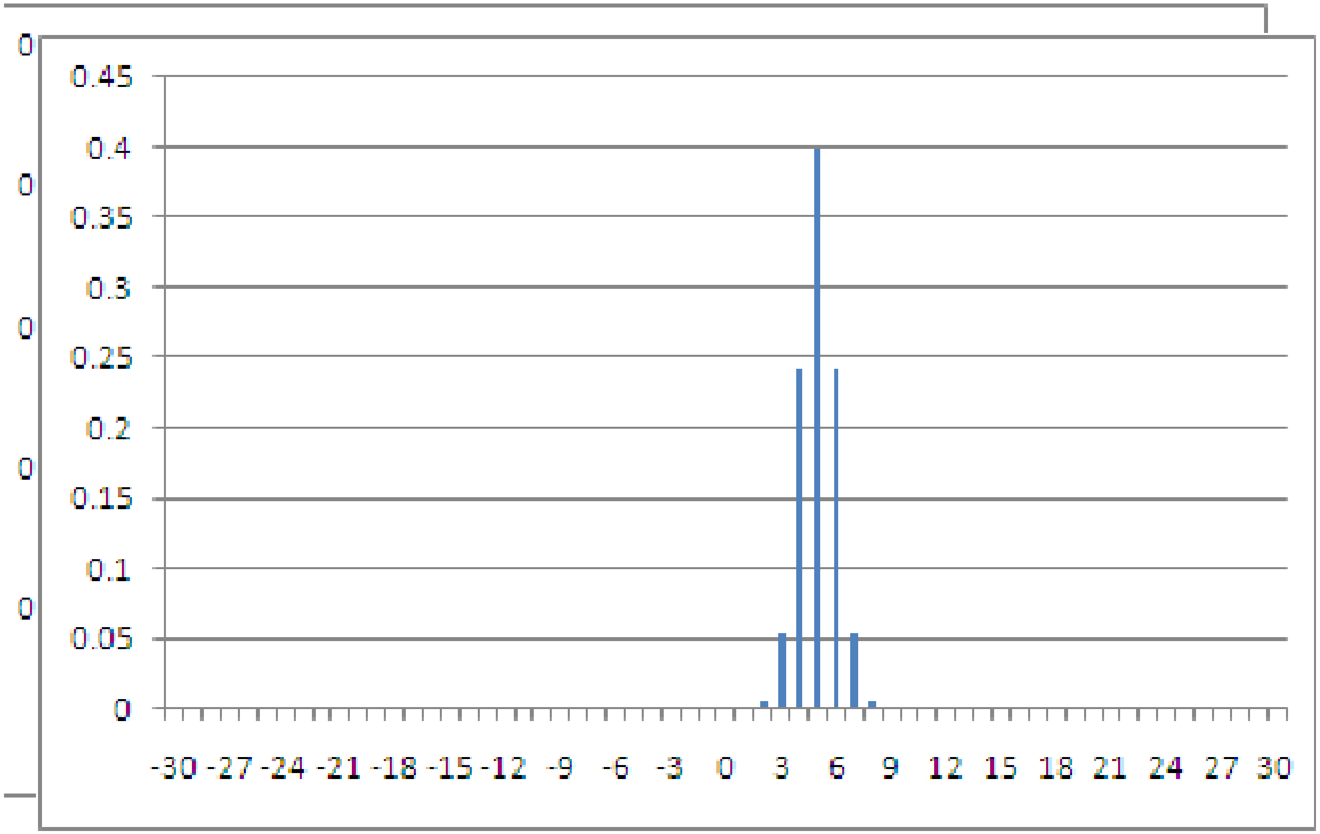

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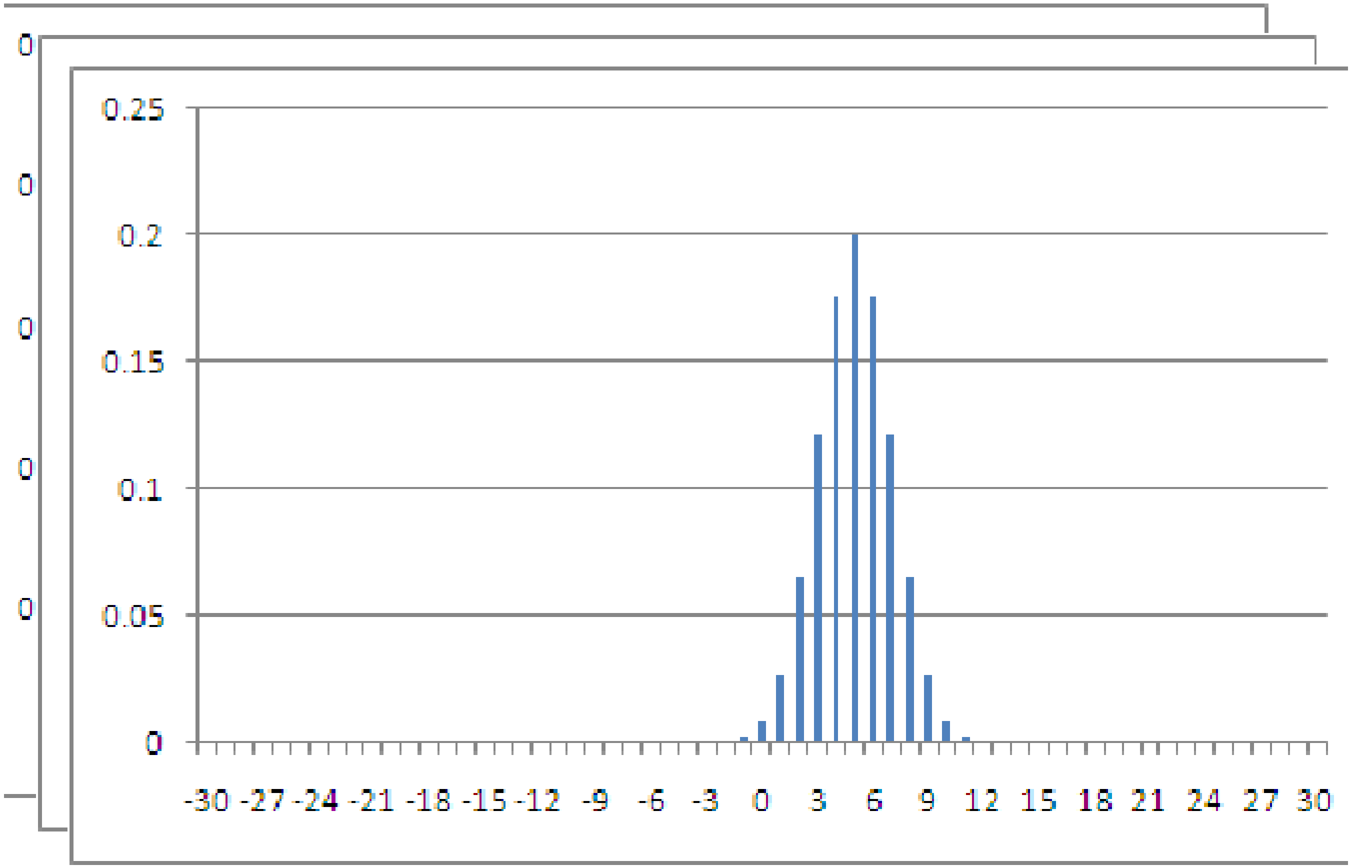

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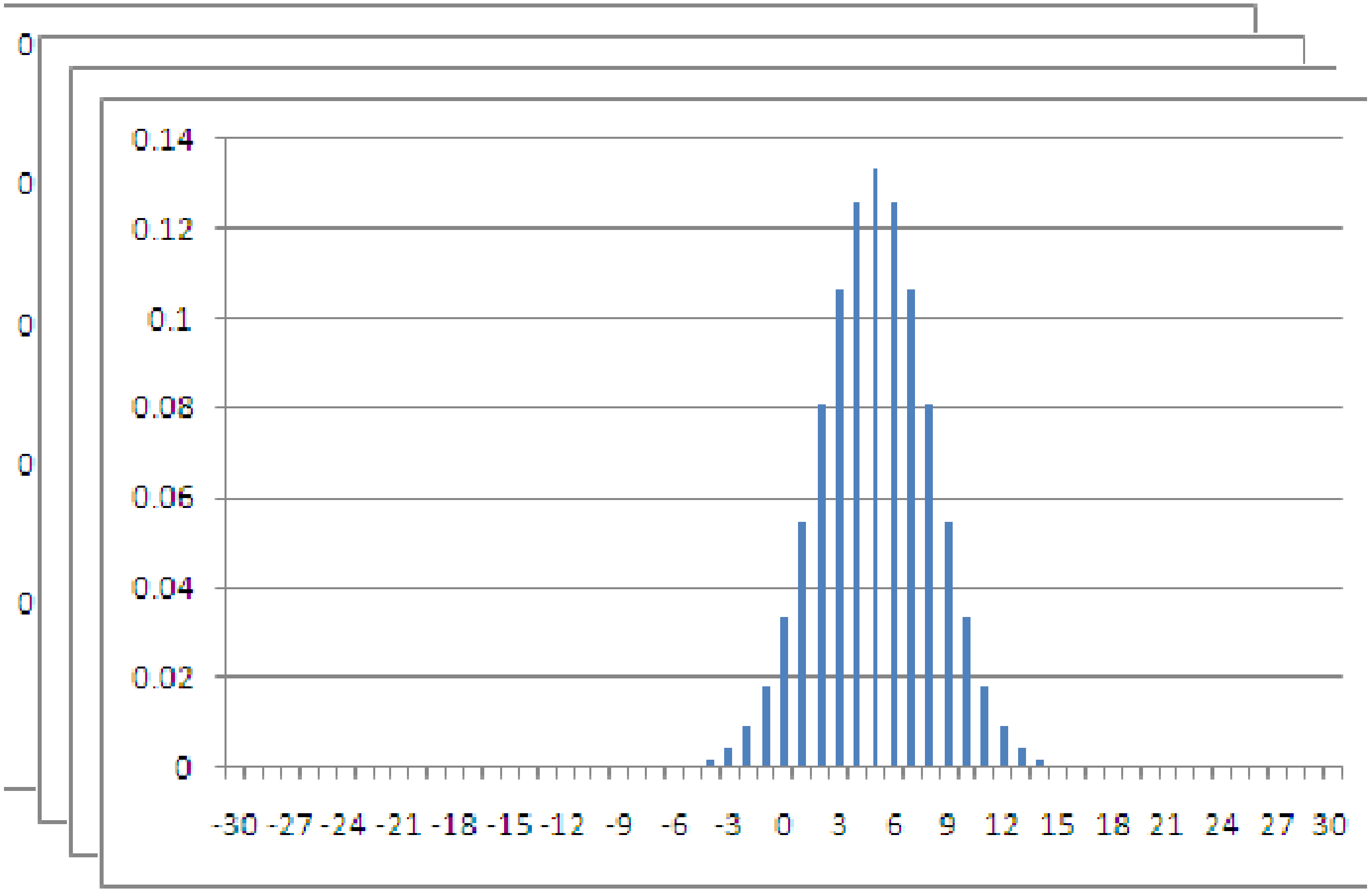

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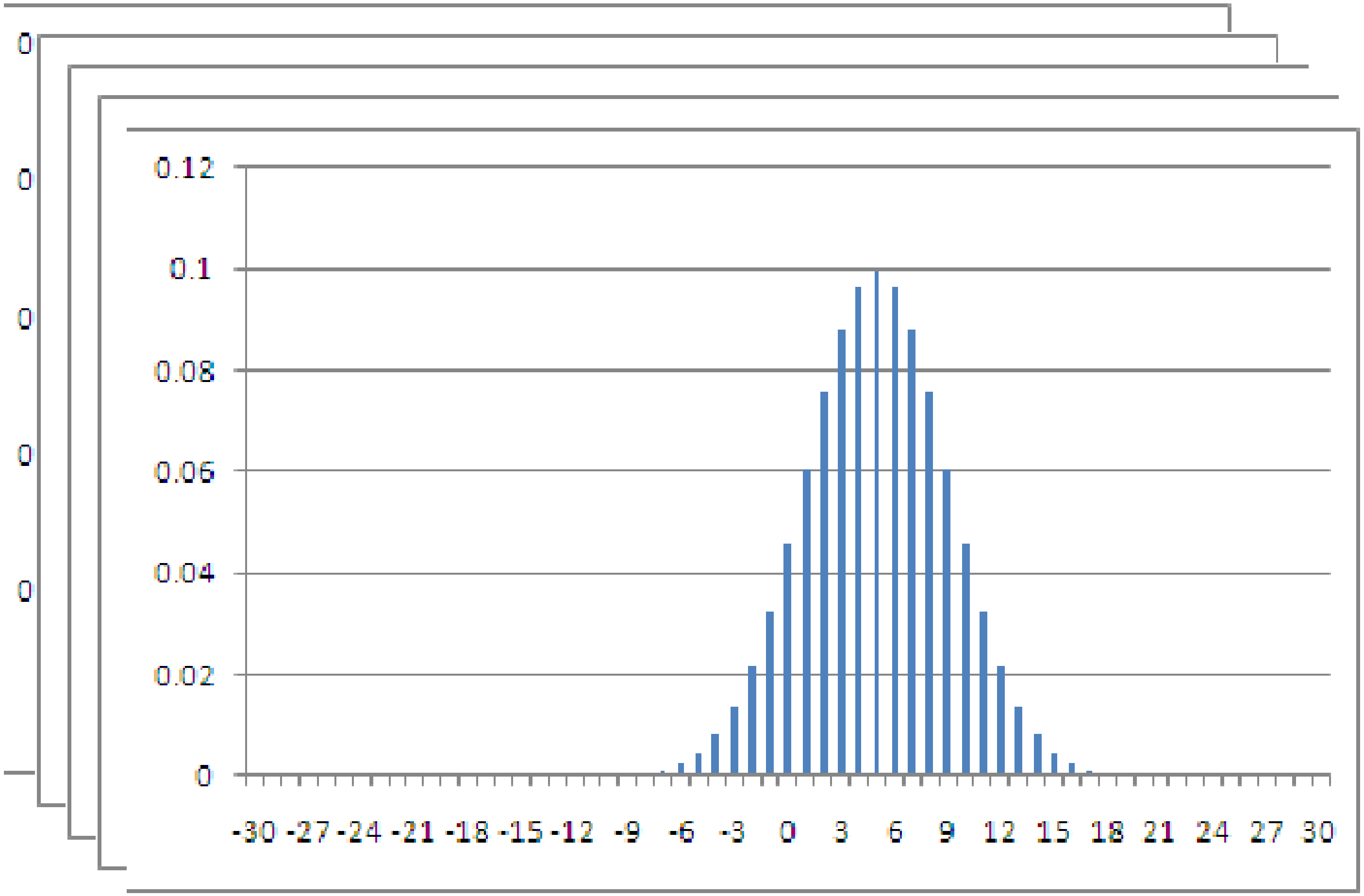

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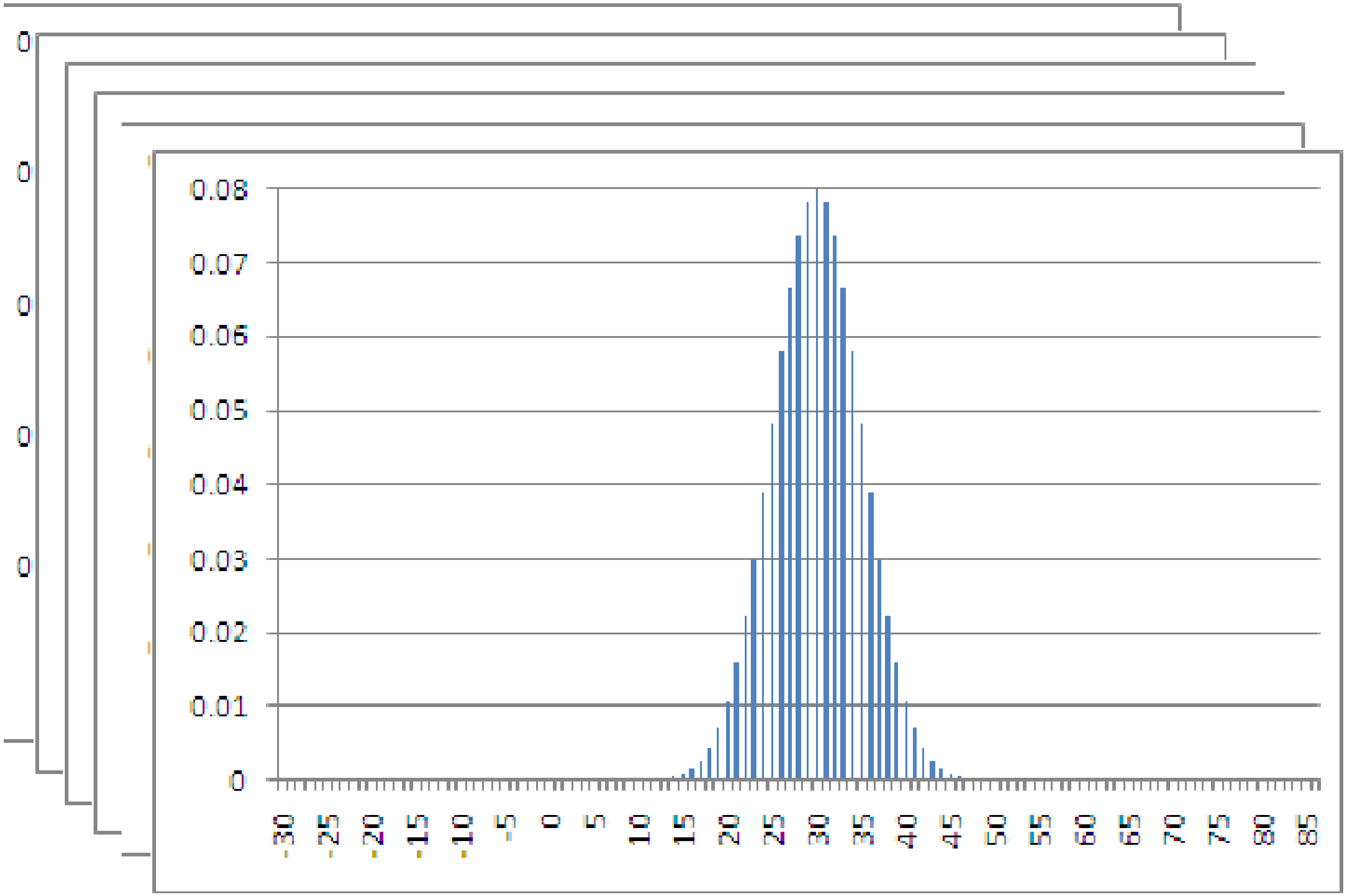

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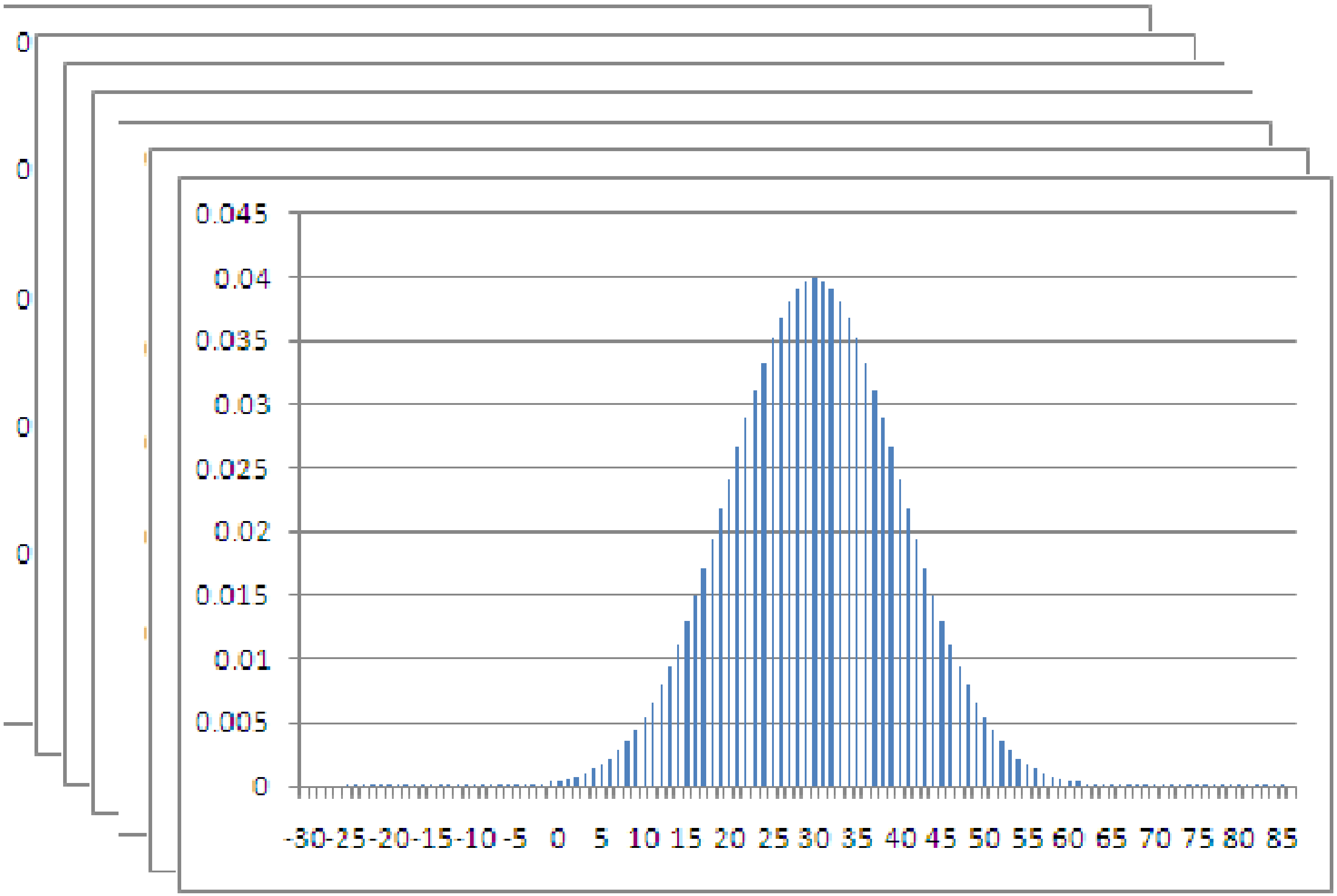

Standard Normal Distribution

Supposing we were looking at three different normal distributions:

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Dist A (Mean = 250, Std Dev = 20)

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B: Outcome X <= 1620

C: Outcome X <= 0.2

Standard Normal Distribution

The probability of each of these outcomes = 0.023 – why?

=NORM.DIST(0.2,0.26,0.03,TRUE)

B	C	D	E	F	G
A	0.023				
B	0.023				
C	0.023				

Is it a coincidence?

Standard Normal Distribution

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- For any normal distribution, an outcome X can be expressed in standardized units – number of its std deviations away from its mean

Standard Normal Distribution

Why is it standardized units?

We are converting any normal distribution into a normal distribution with a mean of 0, and a std deviation of 1

$$z = \frac{x - \mu}{\sigma}$$

Standard Normal Distribution

What does this imply?

Standard Normal Distribution

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As long as something is the same deviation from any mean, p values will be the same

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As long as something is the same deviation from any mean, p values will be the same

In other words,

If an outcome in a normal distribution is say + 1 std deviation away from its mean (irrespective of the actual values of mean and std dev), its probability will be the same

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- The standard normal distribution is a special case of the normal distribution with a mean of 0 and a std deviation of 1 (why is mean 0?)

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$$Z = \frac{X - \mu}{\sigma}$$

Where X is the value of the random variable in the original normal distribution,
μ *is the mean, and* σ *is the std. deviation of the original normal distribution*

Standard Normal Distribution

- The standard normal distribution is a special case of the normal distribution with a mean of 0 and a std deviation of 1 (why is mean 0?)

- All normal distributions can be converted to std normal by the following formula

$$Z = \frac{X - \mu}{\sigma}$$

Where X is the value of the random variable in the original normal distribution,
μ *is the mean, and* σ *is the std. deviation of the original normal distribution*

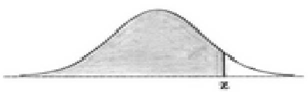
$$y_z = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-z^2}{2}}$$

where: y_z = vertical height on the standard normal distribution

z = as defined above

Standard Normal Distribution

Tables of the Normal Distribution



Probability Content from -oo to Z

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0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
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Standard Normal Distribution

IQ Example:

Average IQ scores for Jigsaw students is 108, std deviation is 7.

Standard Normal Distribution

IQ Example:

Average IQ scores for Jigsaw students is 108, std deviation is 7.

1. What is the probability that a random student will have a score < 120?

Standard Normal Distribution

IQ Example:

Average IQ scores for Jigsaw students is 108, std deviation is 7.

1. What is the probability that a random student will have a score < 120?

2. What is the probability that a random student will have a score between 110 and 115?

Standard Normal Distribution

<u>IQ Example:</u>

Average IQ scores for Jigsaw students is 108, std deviation is 7.

1. What is the probability that a random student will have a score < 120?

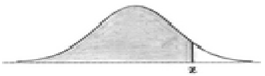
2. What is the probability that a random student will have a score between 110 and 115?

3. What is the probability that a random student will have a score of < 105?

Standard Normal Distribution

What is the probability that a random student will have a score < 120?

Tables of the Normal Distribution



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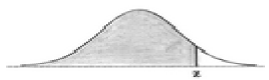
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986

Standard Normal Distribution

What is the probability that a random student will have a score < 120?

$Z = (120 - 108)/7 = 1.71$

Tables of the Normal Distribution



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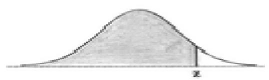
Standard Normal Distribution

Probability that a random student will have a score between 110 and 115?

Tables of the Normal Distribution



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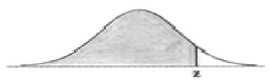
Standard Normal Distribution

Probability that a random student will have a score between 110 and 115?

Z = between (110-108)/7 and (115-108)/7 = between 0.28 and 1

Tables of the Normal Distribution



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0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
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0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
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2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
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2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986

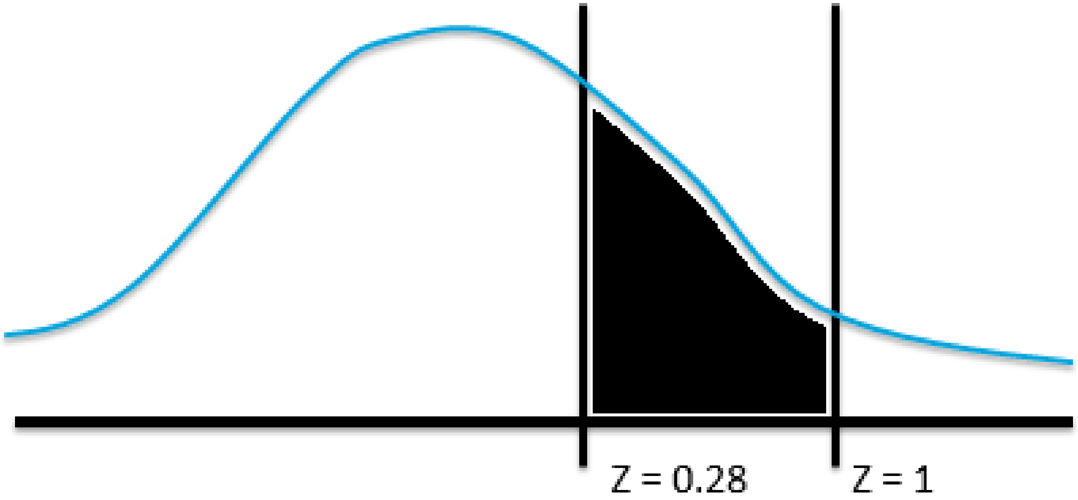
Standard Normal Distribution

What is the probability a random student will have a score between 110 and 115?

Based on the table:

$P(z < 1) = 0.8413$

$P(z < 0.28) = 0.6103$



Standard Normal Distribution

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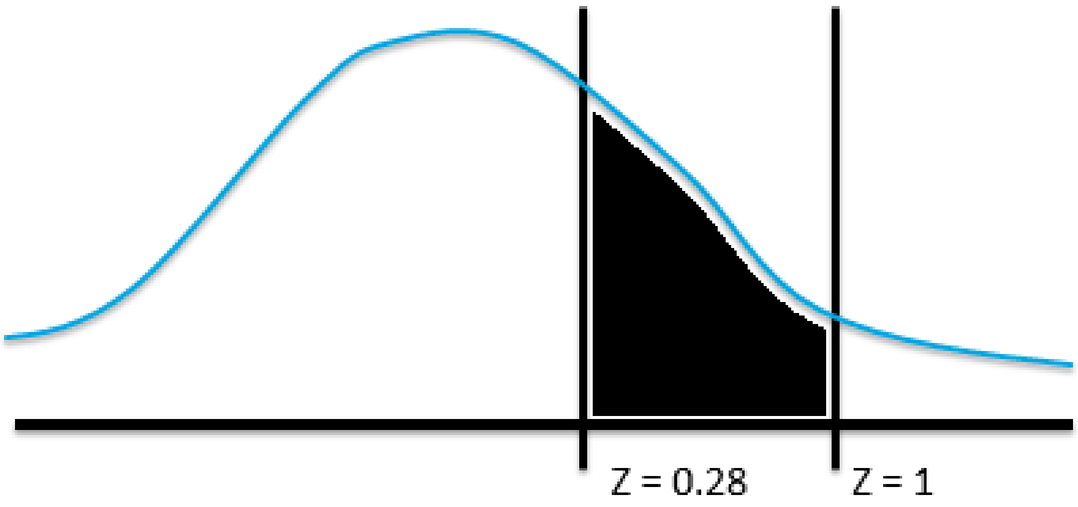
$P(z < 1) = 0.8413$

$P(z < 0.28) = 0.6103$

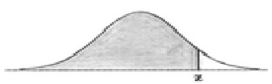
So P(z between 0.28 and 1) = 0.8413 – 0.6103 = **0.23**



Standard Normal Distribution

What is the probability that a random student will have a score of < 105?

Tables of the Normal Distribution



Probability Content from -oo to Z

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0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
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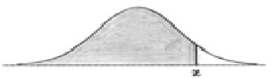
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986

Standard Normal Distribution

What is the probability that a random student will have a score of < 105?

Z = (105– 108)/7 = -0.42

Tables of the Normal Distribution



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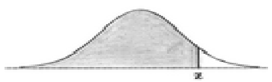
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986

Standard Normal Distribution

P (z < -0.42) = p (z > 0.42)

Tables of the Normal Distribution



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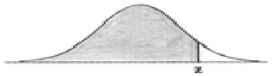
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986

Standard Normal Distribution

P (z < -0.42) = p (z > 0.42)

p(z >0.42) = 1- p(z <0.42) = 1- 0.6628

Tables of the Normal Distribution



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0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986

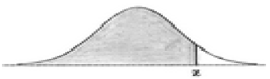
Standard Normal Distribution

P (z < -0.42) = p (z > 0.42)

p(z >0.42) = 1- p(z <0.42) = 1- 0.6628

So, p(score < 105) = 0.3372

Tables of the Normal Distribution



Probability Content from -oo to Z

Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
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1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
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1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
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2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986

Probability Distributions

Case Study Examples

Recap

- What does Statistics cover?

Recap

- What does Statistics cover?
  - Summary Statistics
  - Inferential Statistics

Recap

- What does Statistics cover?
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  - Inferential Statistics
- Sample v/s Population

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- Probability Theory

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- Probability Distribution Concepts
- Types of Distributions
  - Discrete

Recap

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  - Inferential Statistics
- Sample v/s Population
- Probability Theory
- Probability Distribution Concepts
- Types of Distributions
  - Discrete
  - Continuous