

University of Dhaka
Department of Computer Science and Engineering
3rd Year 1st Semester Final Examination, 2022
MATH-3105: Multivariable Calculus and Geometry (3 Credits)

Time: 3 hours

Total Marks: 70

Answer any five (5) out of the following seven (7) questions. Marks are given in the right margin

1. (a) Given two vectors v and w , proof that $\|v + w\| \leq \|v\| + \|w\|$. [4]
- (b) Let there be a supershop that sells three items pen, pencil, and bottle. John is a buyer who bought two pens and two pencils. We represent the items bought by a person using a three dimensional vector where the values represent the number of pen, pencil, and bottles bought, respectively. The items bought by John are represented as $[2, 2, 0]$. Analyze why Cosine Similarity will be more suitable than Euclidean Distance to define similarity between the buying patterns of two persons. [5]
- (c) Let the vector representation items bought by three persons John, Bob and Alice be $[1, 1, 0]$, $[2, 2, 0]$ and $[100, 100, 0]$, respectively. In case of cosine similarity, similarity between John and Bob is equal to the similarity between John and Alice. But Bob is more similar to John than Alice. Define a new similarity measure modifying cosine similarity that incorporate such differences. [5]
2. (a) Let $L_1(t) = c + ta$ and $L_2(t) = c + tb$ be two parametric line equations. Here, a , b , and c be three vectors. [6]
- I. Find if the lines intersect or not.
- II. If $a \cdot b < 0$, what can we infer about the angle between the lines?
- (b) Find a vector perpendicular to the plane that passes through the points $P(1, 4, 6)$, $Q(2, 5, 1)$ and $R(1, 1, 1)$. [4]
- (c) Find vector equations of the lines passing through the pairs of points listed below. [4]
- $(5, 2, 1, 3)$ and $(1, 3, 4, 2)$
 - $(1, 1, 0)$ and $(2, 4, 2)$
3. (a) Let L_1 and L_2 be two lines lying on a plane $2x + 3y - 4z = 22$. Let the angle between L_1 and L_2 be 30 degrees. Proof that the lines intersect with each other. [4]
- (b) Find an equation of the plane that passes through point $(2, 4, -1)$ and is parallel to the vector $(2, 3, 4)$. [5]
- (c) Find the shortest distance between the plane $x_1 + x_2 + x_3 - 1 = 0$ and point $(2, 2, 3)$. [3]
- (d) Find the shortest distance between the lines $L_1(t) = (t, 0, 0)$ and $L_2(t) = (0, 1+t, 1)$. [2]
4. (a) Find the unit tangent vector for the vector equation $r(t) = (1+t^3, te-t, \cos 2t)$ at $t = 0$. [4]
- (b) Find the arc length L for the curve $r(t) = (\cos t, \sin t, t)$ for $0 \leq t \leq 2\pi$. [3]
- (c) Professor Mosby wanted to find the arc length L for the curve $r(t) = (t^2, t^3, t^4)$ from point $(0, 0, 0)$ to $(4, 8, 32)$ but failed to do so. Explain why. [3]
- (d) Calculate the volume under the surface $z = 1 - x^2 - y^2$ over the region D defined by $0 \leq x \leq 1$ and $0 \leq y \leq 1$. [4]

5. (a) Let $T: \mathbb{R}^5 \rightarrow \mathbb{R}^5$ be defined by $T(x) = A(x)$, where x is \mathbb{R}^5 and

[6]

$$A = \begin{bmatrix} 1 & 2 & 0 & 1 & -1 \\ 2 & 1 & 3 & 1 & 0 \\ -1 & 0 & -2 & 0 & 1 \\ 0 & 0 & 0 & 2 & 8 \end{bmatrix}$$

Find a basis for the range of T .

- (b) Let $f(x, y)$ be a function. Discuss the points where $f_x(a, b) = 0$ and $f_y(a, b) = 0$.

[4]

- (c) Find the maximum value of $f(x, y) = x^4 + y^4 - 4xy + 1$.

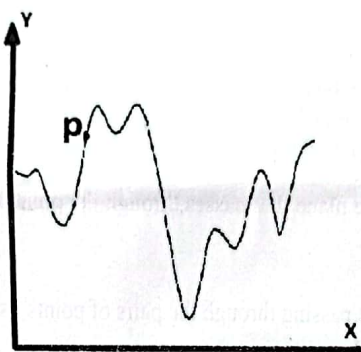
[4]

6. (a) Let $f(x, y) = x^2y + xy^2 + x^2y^2$. Starting at $(x, y) = (3, 5)$, demonstrate how gradient descent algorithm maximizes $f(x, y)$ with learning rate, $\alpha = 0.002$. You must report the values of x , y , and $f(x, y)$ after each iteration for the first four iterations. Show one iteration with $\alpha = 1.0$ and comment on the choice of learning rate.

[6]

- (b) Let $y = f(x)$ be a function plotted in the graph below. Is the point P a good choice as starting point for minimizing using gradient descent algorithm? What about in case of maximizing?

[3]



- (c) Find the maximum and minimum values of $f(x, y, z) = xyz$ subject to the constraint $x + y + z = 1$. Assume that $x, y, z \geq 0$.

[5]

7. (a) Find $\frac{dY}{dX}$ where $Y = [x^5, x^6, x^7]$ and $X = [x, x^2]$.

[4]

- (b) Let f be a function. Explain the difference between df and Δf .

[2]

- (c) Find the tangent plane of surface $z = 2x^2 + y^2$ at point $(1, 1, 3)$.

[4]

- (d) Find the area inside the ellipse $\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = 1$ using double integral.

[4]

$$x^2 + y^2 = 9 - k^2$$

University of Dhaka
Department of Computer Science and Engineering
3rd Year 1st Semester Final Examination, 2021
MATH-3105: Multivariable Calculus and Geometry (3 Credits)

Time: 3 hours

Total Marks: 70

Answer any five (5) out of the following seven (7) questions. Marks are given in the right margin.

- 1 (a) What region in R^3 is represented by the following inequalities? [4]
 $1 \leq x^2 + y^2 + z^2 \leq 4, z \geq 0$

- (b) Using the idea of Change of Basis, find the transition matrix from B to B' for the bases for R^3 below. [5]
 $B = \{(-3, 2), (4, -2)\}$ and $B' = \{(-1, 2), (2, -2)\}$

- (c) Find the scalar and vector projections of: $b = \langle 1, 1, 2 \rangle$ onto $a = \langle -2, 3, 1 \rangle$ [5]

- 2 (a) What is the relationship between the *spanning set* and *basis* of vector space? [2]

- (b) Consider the linear transformation $T: R^n \rightarrow R^m$ represented by $T(x) = Ax$. Find the nullity and rank of T , and determine whether T is one-to-one, onto, or neither. [6]

a. $A = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ [1]

b. $A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$ [1]

c. $A = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & -1 \end{bmatrix}$ [0]

d. $A = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ [0]

- 3 (c) Let $T: R^5 \rightarrow R^4$ be defined by $T(x) = A(x)$, where x is R^5 and [6]

$$A = \begin{bmatrix} 1 & 2 & 0 & 1 & -1 \\ 2 & 1 & 3 & 1 & 0 \\ -1 & 0 & -2 & 0 & 1 \\ 0 & 0 & 0 & 2 & 8 \end{bmatrix}$$

1, 2, 4

Find a basis for the range of T .

- 3 (a) Show that the matrix A is diagonalizable. [6]

$$A = \begin{bmatrix} 1 & -1 & -1 \\ 1 & 3 & 1 \\ -3 & 1 & -1 \end{bmatrix}$$

2 -2 3

Then find a matrix P such that $P^{-1}AP$ is diagonal.

- (b) Sketch the level curves of the function: [4]

$$g(x, y) = \sqrt{9 - x^2 - y^2} \quad \text{for } k = 0, 1, 2, 3$$

- (c) Find the coordinate matrix of $x = \langle 1, 2, -1 \rangle$ in R^3 relative to the non-standard basis: [4]

$$B' = \{u_1, u_2, u_3\} = \{(1, 0, 1), (0, -1, 2), (2, 3, -5)\}$$

5
-2
-2

- 4 (a) Find an equation of the plane that passes through the points: $P(1, 3, 2)$, $Q(3, -1, 6)$ and $R(5, 2, 0)$. [5]

- (b) Find parametric equations and symmetric equations of the line that passes through the points $A(2, 4, -3)$ and $B(3, -1, 1)$. [4]

2+4
9-5+
-3+4+

- (c) Find the distance between the parallel planes $x + 2y - 3z = 4$ and $2x + 4y - 6z = 3$. [5]
- 5 (a) What is the difference between standard basis and nonstandard basis? Explain with an example. [2]
- (b) Find the directional derivative of the function at the given point in the direction of the vector \mathbf{v} : [6]
- $f(x, y) = e^x \sin y, \quad (0, \pi/3), \quad \mathbf{v} = \langle -6, 8 \rangle$ $\frac{8 - 3\sqrt{3}}{10} = 0.11$
- (c) If $f(x, y) = xe^y$, find the rate of change of f at the point $P(2, 0)$ in the direction from P to $Q(1/2, 2)$. [6]
- 6 (a) Find the local maximum and minimum values and saddle point(s) of the function [7]
- $f(x, y) = x^2 + xy + y^2 + y$
- (b) Find the absolute maximum and minimum values of the function $f(x, y) = x^2 - 2xy + 2y$ on the rectangle $D = \{(x, y) \mid 0 \leq x \leq 3, 0 \leq y \leq 2\}$. [7]
- 7 (a) Find the points on the sphere $x^2 + y^2 + z^2 = 4$ that are closest to and farthest from the point $(3, 1, -1)$. [6]
- (b) Compute the following double integral: [4]
- $\iint_R 6xy^2 dA, R = [2, 4] \times [1, 2]$
- (c) Find the tangent plane to the elliptic paraboloid $z = 2x^2 + y^2$ at the point $(1, 1, 3)$. [4]

University of Dhaka
Department of Computer Science and Engineering
3rd Year 1st Semester Final Examination, 2020
Course Code: CSE-3105

Course Title: Multivariable Calculus and Geometry (3 Credits)

Time: 2 hours

Total Marks: 70

Answer any three (3) out of the following five (5) questions. Marks are given in the right margin.

- 1 (a)** Find a vector perpendicular to the plane that passes through the points $P(1, 4, 6)$, $Q(2, 5, 1)$ and $R(1, 1, 1)$. [5]
- (b)** Find the scalar and vector projections of: $\mathbf{b} = \langle 1, 1, 2 \rangle$ onto $\mathbf{a} = \langle -2, 3, 1 \rangle$. [7]
- (c)** What conditions must be satisfied by a subset of a vector space to be a subspace? [3.33]
- (d)** Find the Kernel of the following linear transformation: [5]

$$T(\mathbf{x}) = A\mathbf{x} = \begin{bmatrix} 1 & -1 & -2 \\ -1 & 2 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \quad (T: R^3 \rightarrow R^2)$$

- (e)** Let $T: R^5 \rightarrow R^7$ be a linear transformation. Find the dimension of the kernel of T when the dimension of the range is 2. [3]
- 2 (a)** Let $S = \{v_1, v_2, \dots, v_n\}$ be a set of vectors in vector space V . What are the conditions which make S a basis for V ? What do you mean by the standard basis? Give an example. [2+3]
- (b)** Let $T: R^5 \rightarrow R^4$ be defined by $T(\mathbf{x}) = A(\mathbf{x})$, where \mathbf{x} is R^5 and [6.33]

$$A = \begin{bmatrix} 1 & 2 & 0 & 1 & -1 \\ 2 & 1 & 3 & 1 & 0 \\ -1 & 0 & -2 & 0 & 1 \\ 0 & 0 & 0 & 2 & 8 \end{bmatrix}$$

Find a basis for the range of T .

- (c)** Find the null space of A , where [12]

$$A = \begin{bmatrix} 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & -2 & 0 \\ 4 & 2 & 0 & 0 & 3 \\ 1 & 1 & 1 & -2 & 1 \\ 2 & 2 & 0 & 0 & 2 \\ 1 & 1 & 2 & -4 & 1 \end{bmatrix}$$

3 (a) What do you mean by a normal vector? How do you decide whether two vectors lie on the same plane? **[2+2]**

(b) Find the angle between the planes $x + y + z = 1$ and $x - 2y + 3z = 1$. Also find symmetric equations for the line of intersection L of these two planes. **[5+5]**

(c) Find the distance between the parallel planes $10x + 2y - 2z = 5$ and $5x + y - z = 1$. **[5]**

(d) Using the idea of Change of Basis, find the transition matrix from \mathbf{B} to \mathbf{B}' for the bases for R^2 below: **[4.33]**

$$\mathbf{B} = \{(-3, 2), (4, -2)\} \text{ and } \mathbf{B}' = \{(-1, 2), (2, -2)\}.$$

4 (a) Find the eigenvalues and corresponding eigenvectors of **[5]**

$$A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

(b) Explain the concept of Tangent Planes with an example. Find the tangent plane to the elliptic paraboloid $z = 2x^2 + y^2$ at the point $(1, 1, 3)$. **[3+4]**

(c) Find the directional derivative of the function $f(x, y) = x^2y^3 - 4y$ at the point $(2, -1)$ in the direction of the vector $\mathbf{v} = 2\mathbf{i} + 5\mathbf{j}$. **[7]**

(d) What is the second derivative test? Why is it used? **[4.33]**

5 (a) A rectangular box without a lid is to be made from 12m^2 of cardboard. Find the maximum volume of such a box. **[6]**

(b) Compute the following double integral: **[7.33]**

$$\iint_R 6xy^2 dA, R = [2, 4] \times [1, 2]$$

(c) Find the maximum and minimum values of $f(x, y, z) = xyz$ subject to the constraint $x + y + z = 1$. Assume that $x, y, z \geq 0$. **[10]**

University of Dhaka
Department of Computer Science and Engineering
3rd Year 1st Semester B. Sc. (Hons) Final Examination, 2019
MATH-3105: Multivariable Calculus and Geometry

Total Marks: 60

Time: 3 Hours

(Answer any four (4) of the following questions)

1. a) Determine whether each statement is true or false. Explain your answer briefly. [6]
 Assume everything is described in 3D.
 - i. Two lines parallel to a third line are parallel.
 - ii. Two lines perpendicular to a third line are parallel.
 - iii. Two planes parallel to a third plane are parallel.
 - iv. Two lines perpendicular to a plane are parallel.
 - v. Two lines either intersect or are parallel.
 - vi. A plane and a line either intersect or are parallel.
 - b) Find parametric equations and symmetric equations for the line through the origin [5]
 and the point (4, 3, -1).
 - c) Find equations of the planes that are parallel to the plane $x + 2y - 2z = 1$ and two [4]
 units away from it.
2. a) Is $\lambda = 4$ an eigenvalue of $\begin{bmatrix} 3 & 0 & -1 \\ 2 & 3 & 1 \\ -3 & 4 & 5 \end{bmatrix}$? If so, find one corresponding eigenvector. [6]
 - b) Diagonalize $\begin{bmatrix} 2 & 2 & -1 \\ 1 & 3 & -1 \\ -1 & -2 & 2 \end{bmatrix}$. Show full process in details. [9]
3. a) Find the domain and range of $g(x, y) = \sqrt{9 - x^2 - y^2}$ and sketch the level curves [3]
 of the function for $k=0, 1, 2, 3$.
 - b) Show if the following polynomials are linearly independent [4]

$$p_1(t) = 1 + 2t^2 \rightarrow [p_1(t)]_B = (1, 0, 2)$$

$$p_2(t) = 4 + t + 5t^2 \rightarrow [p_2(t)]_B = (4, 1, 5)$$

$$p_3(t) = 3 + 2t \rightarrow [p_3(t)]_B = (3, 2, 0)$$
 - c) Let $A = \begin{bmatrix} 3 & -6 & 0 \\ -6 & 0 & 6 \\ 0 & 6 & -3 \end{bmatrix}$. Show that A is orthogonally diagonalizable by finding an [6]
 orthogonal matrix U and a diagonal matrix D such that $A = UDU^T$.
 - d) Sketch the graph of the function $f(x, y) = 6 - 3x - 2y$ [2]

4. a) Find the local maximum and minimum values and saddle point(s) of the given [9]
functions.
- $x^2 + xy + y^2 + y$
 - $y^3 - 3x^2y - 6x^2 - 6y^2 + 2$
 - $e^x \cos y$
- b) Use Lagrange multipliers to find the maximum and minimum values of the [6]
function subject to the given constraint:
- $f(x, y) = 3x + y; x^2 + y^2 = 10$
 - $f(x, y) = e^{xy}; x^3 + y^3 = 16$
5. a) Calculate the iterated integral: [6]
- $\int_1^4 \int_0^2 (6x^2y - 2x) dy dx$
 - $\int_0^2 \int_0^4 y^3 e^{2x} dy dx$
- b) Evaluate the double integral: [9]
- $\iint_D y^2 dA, D = \{(x, y) \mid -1 \leq y \leq 1, -y - 2 \leq x \leq y\}$
 - $\iint_D x \cos y dA, D$ is bounded by $y = 0, y = x^2$ and $x = 1$
 - $\iint_D y^2 dA, D$ is a triangular region with vertices $(0, 1), (1, 2), (4, 1)$.
6. a) Use polar coordinates to find the volume of the given solid under the cone [5]
 $z = \sqrt{x^2 + y^2}$ and above the disk $x^2 + y^2 \leq 4$
- A cylindrical drill with radius r is used to bore a hole through the center of a sphere [5]
of radius R . Find the volume of the ring-shaped solid that remains.
 - A swimming pool is circular with a 40-ft diameter. The depth is constant along [5]
east-west lines and increases linearly from 2 ft at the south end to 7 ft at the north
end. Find the volume of water in the pool.