University of Dhaka
Department of Computer Science and Engineering
3rd Year 1st Semester Final Examination, 2022
MATH-3105: Multivariable Calculus and Geometry (3 Credits)

Time: 3 hours **Total Marks: 70**

Answer any five (5) out of the following seven (7) questions. Marks are given in the right margin

1. (a) Given two vectors v and w, proof that $\| v + w \| <= \|v\| + \|w\|$. [4]

(b) Let there be a supershop that sells three items pen, pencil, and bottle. John is a buyer who bought two pens and two pencils. We represent the items bought by a person using a three dimensional vector where the values represent the number of pen, pencil, and bottles bought, respectively. The items bought by John are represented as [2, 2, 0]. Analyze why Cosine Similarity will be more suitable than Euclidean Distance to define similarity between the buying patterns of two persons. [5]

(c) Let the vector representation items bought by three persons John, Bob and Alice be [1, 1, 0], [2, 2, 0] and [100, 100, 0], respectively. In case of cosine similarity, similarity between John and Bob is equal to the similarity between John and Alice. But Bob is more similar to John than Alice. Define a new similarity measure modifying cosine similarity that incorporate such differences. [5]

2. (a) Let $L_1(t) = c + ta$ and $L_2(t) = c + tb$ be two parametric line equations. Here, a, b, and c be three vectors. [6]
   I. Find if the lines intersect or not.
   II. If $a.b < 0$, what can we infer about the angle between the lines?

(b) Find a vector perpendicular to the plane that passes through the points P (1, 4, 6), Q (2, 5, 1) and R (1, 1, 1). [4]

(c) Find vector equations of the lines passing through the pairs of points listed below. [4]
- (5, 2, 1, 3) and (1, 3, 4, 2)
- (1, 1, 0) and (2, 4, 2)

3. (a) Let L_1 and L_2 be two lines lying on a plane $2x + 3y - 4z = 22$. Let the angle between L_1 and L_2 be 30 degrees. Proof that the lines intersect with each other. [4]

(b) Find an equation of the plane that passes through point (2, 4, -1) and is parallel to the vector (2, 3, 4). [5]

(c) Find the shortest distance between the plane $x_1 + x_2 + x_3 - 1 = 0$ and point (2, 2, 3). [3]

(d) Find the shortest distance between the lines $L_1(t) = (t, 0, 0)$ and $L_2(t) = (0, 1+t, 1)$. [2]

4. (a) Find the unit tangent vector for the vector equation $r(t) = (1+t^3, te\text{-}t, \cos 2t)$ at $t = 0$. [4]

(b) Find the arc length L for the curve $r(t) = (\cos t, \sin t, t)$ for $0 \leq t \leq 2\pi$. [3]

(c) Professor Mosby wanted to find the arc length L for the curve $r(t) = (t^2, t^3, t^4)$ from point (0, 0, 0) to (4, 8, 32) but failed to do so. Explain why. [3]

(d) Calculate the volume under the surface $z = 1 - x^2 - y^2$ over the region D defined by $0 \leq x \leq 1$ and $0 \leq y \leq 1$. [4]

5. (a) Let $T: R^5 \rightarrow R^4$ be defined by $T(x) = A(x)$, where x is R^5 and **[6]**

$$A = \begin{bmatrix} 1 & 2 & 0 & 1 & -1 \\ 2 & 1 & 3 & 1 & 0 \\ -1 & 0 & -2 & 0 & 1 \\ 0 & 0 & 0 & 2 & 8 \end{bmatrix}$$

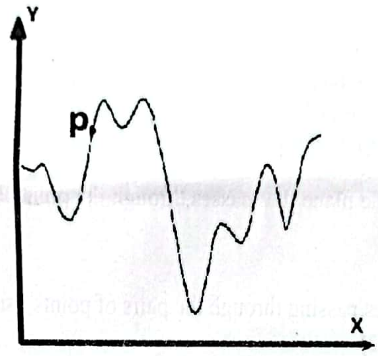
Find a basis for the range of T.

(b) Let $f(x, y)$ be a function. Discuss the points where $f_x(a, b) = 0$ and $f_y(a, b) = 0$. **[4]**

(c) Find the maximum value of $f(x, y) = x^4 + y^4 - 4xy + 1$. **[4]**

6. (a) Let $f(x, y) = x^2y + xy^2 + x^2y^2$. Starting at $(x, y) = (3, 5)$, demonstrate how gradient descent algorithm maximizes $f(x, y)$ with learning rate, $\underline{\alpha} = 0.002$. You must report the values of x, y, and $f(x, y)$ after each iteration for the first four iterations. Show one iteration with $\underline{\alpha} = 1.0$ and comment on the choice of learning rate. **[6]**

(b) Let $y = f(x)$ be a function plotted in the graph below. Is the point P a good choice as starting point for minimizing using gradient descent algorithm? What about in case of maximizing? **[3]**



(c) Find the maximum and minimum values of $f(x,y,z)=xyz$ subject to the constraint $x+ y+ z= 1$. Assume that $x,y,z>=0$. **[5]**

7. (a) Find $\frac{dY}{dX}$ where $Y = [x^5, x^6, x^7]$ and $X = [x, x^2]$. **[4]**

(b) Let f be a function. Explain the difference between df and Δf. **[2]**

(c) Find the tangent plane of surface $z = 2x^2 + y^2$ at point (1, 1, 3). **[4]**

(d) Find the area inside the ellipse $(\frac{x}{a})^2 + (\frac{y}{b})^2 = 1$ using double integral. **[4]**

$x^2 + y^2 = 9 - k^2$

University of Dhaka

Department of Computer Science and Engineering

3rd Year 1st Semester Final Examination, 2021

MATH-3105: Multivariable Calculus and Geometry (3 Credits)

Time: 3 hours **Total Marks: 70**

Answer any five (5) out of the following seven (7) questions. Marks are given in the right margin.

1 (a) What region in R^3 is represented by the following inequalities? [4]

$$1 \le x^2 + y^2 + z^2 \le 4,\ z \le 0$$

(b) Using the idea of Change of Basis, find the transition matrix from B to B′ for the bases for R^2 below. [5]

$$B = \{(-3, 2), (4, -2)\} \text{ and } B' = \{(-1, 2), (2, -2)\}$$

$\begin{bmatrix} -1 & 2 \\ -2 & 3 \end{bmatrix}$ $\quad -\frac{3}{7}i, \frac{9}{14}j + \frac{3}{14}k$

(c) Find the scalar and vector projections of: b = < 1, 1, 2 > onto a = < -2, 3, 1 > [5] $\quad \frac{3}{\sqrt{14}}$

2 (a) What is the relationship between the *spanning set* and *basis* of vector space? [2]

(b) Consider the linear transformation $T: R^n \to R^m$ represented by $T(\mathbf{x}) = A\mathbf{x}$. Find the nullity and rank of T, and determine whether T is one-to-one, onto, or neither. [6]

a. $A = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ 11 $\qquad$ b. $A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$ 10

c. $A = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & -1 \end{bmatrix}$ 01 $\qquad$ d. $A = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ 00

(c) Let $T: R^5 \to R^4$ be defined by $T(\mathbf{x}) = A(x)$, where x is R^5 and [6]

$$A = \begin{bmatrix} 1 & 2 & 0 & 1 & -1 \\ 2 & 1 & 3 & 1 & 0 \\ -1 & 0 & -2 & 0 & 1 \\ 0 & 0 & 0 & 2 & 8 \end{bmatrix}$$

1, 2, 4

Find a basis for the range of T.

3 (a) Show that the matrix A is diagonalizable. [6]

$$A = \begin{bmatrix} 1 & -1 & -1 \\ 1 & 3 & 1 \\ -3 & 1 & -1 \end{bmatrix}$$

2 $\quad$ -2 $\quad$ 3

Then find a matrix P such that $P^{-1}AP$ is diagonal.

(b) Sketch the level curves of the function: [4]

$$g(x, y) = \sqrt{9 - x^2 - y^2} \quad \text{for } k = 0, 1, 2, 3$$

(c) Find the coordinate matrix of x = <1, 2, −1> in R^3 relative to the non-standard basis: [4]

B′ = {**u1, u2, u3**} = {(1, 0, 1), (0, −1, 2), (2, 3, −5)} $\quad$ 5, -8, -2

4 (a) Find an equation of the plane that passes through the points : $P(1,3,2)$, $Q(3,-1,6)$ and $R(5,2,0)$. [5]

(b) Find parametric equations and symmetric equations of the line that passes through the points $A(2,4,-3)$ and $B(3,-1,1)$. [4]

2+t
4-5t
-3+4t

(c) Find the distance between the parallel planes $x + 2y - 3z = 4$ and $2x + 4y - 6z = 3$. [5]

5 (a) What is the difference between standard basis and nonstandard basis? Explain with an example. [2]

(b) Find the directional derivative of the function at the given point in the direction of the vector **v**: [6]

$$f(x, y) = e^x \sin y, \quad (0, \pi/3), \quad \mathbf{v} = \langle -6, 8 \rangle \qquad \frac{9-3\sqrt{3}}{10} = 0.11$$

(c) If $f(x, y) = xe^y$, find the rate of change of f at the point $P(2,0)$ in the direction from P to $Q(1/2, 2)$. [6]

6 (a) Find the local maximum and minimum values and saddle point(s) of the function [7]

$$f(x, y) = x^2 + xy + y^2 + y.$$

(b) Find the absolute maximum and minimum values of the function $f(x, y) = x^2 - 2xy + 2y$ on the rectangle $D = \{(x, y) \mid 0 \le x \le 3, 0 \le y \le 2\}$ [7]

7 (a) Find the points on the sphere $x^2 + y^2 + z^2 = 4$ that are closest to and farthest from the point $(3,1,-1)$. [6]

(b) Compute the following double integral: [4]

$$\iint_R 6xy^2 \, dA, \; R = [2, 4] \times [1, 2]$$

(c) Find the tangent plane to the elliptic paraboloid $z = 2x^2 + y^2$ at the point $(1,1,3)$. [4]

University of Dhaka
Department of Computer Science and Engineering
3rd Year 1st Semester Final Examination, 2020
Course Code: CSE-3105
Course Title: Multivariable Calculus and Geometry (3 Credits)

Time: 2 hours **Total Marks: 70**

Answer any three (3) out of the following five (5) questions. Marks are given in the right margin.

1 (a) Find a vector perpendicular to the plane that passes through the points $P(1, 4, 6)$, $Q(2, 5, 1)$ and $R(1, 1, 1)$. **[5]**

(b) Find the scalar and vector projections of: $\mathbf{b} = \langle 1, 1, 2 \rangle$ onto $\mathbf{a} = \langle -2, 3, 1 \rangle$. **[7]**

(c) What conditions must be satisfied by a subset of a vector space to be a subspace? **[3.33]**

(d) Find the Kernel of the following linear transformation: **[5]**

$$T(\mathbf{x}) = A\mathbf{x} = \begin{bmatrix} 1 & -1 & -2 \\ -1 & 2 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \quad (T : R^3 \to R^2)$$

(e) Let T: $R^5 \to R^7$ be a linear transformation. Find the dimension of the kernel of T when the dimension of the range is 2. **[3]**

2 (a) Let $S = \{v_1, v_2, \ldots, v_n\}$ be a set of vectors in vector space V. What are the conditions which make S a basis for V? What do you mean by the standard basis? Give an example. **[2+3]**

(b) Let T: $R^5 \to R^4$ be defined by $T(\mathbf{x}) = A(\mathbf{x})$, where $\boldsymbol{x}$ is R^5 and **[6.33]**

$$A = \begin{bmatrix} 1 & 2 & 0 & 1 & -1 \\ 2 & 1 & 3 & 1 & 0 \\ -1 & 0 & -2 & 0 & 1 \\ 0 & 0 & 0 & 2 & 8 \end{bmatrix}$$

Find a basis for the range of T.

(c) Find the null space of A, where **[12]**

$$A = \begin{bmatrix} 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & -2 & 0 \\ 4 & 2 & 0 & 0 & 3 \\ 1 & 1 & 1 & -2 & 1 \\ 2 & 2 & 0 & 0 & 2 \\ 1 & 1 & 2 & -4 & 1 \end{bmatrix}$$

3 (a) What do you mean by a normal vector? How do you decide whether two vectors lie on the same plane? **[2+2]**

(b) Find the angle between the planes $x + y + z = 1$ and $x - 2y + 3z = 1$. Also find symmetric equations for the line of intersection L of these two planes. **[5+5]**

(c) Find the distance between the parallel planes $10x + 2y - 2z = 5$ and $5x + y - z = 1$. **[5]**

(d) Using the idea of Change of Basis, find the transition matrix from $\boldsymbol{B}$ to $\boldsymbol{B}'$ for the bases for R^2 below: **[4.33]**

$\boldsymbol{B} = \{(-3, 2), (4, -2)\}$ and $\boldsymbol{B}' = \{(-1, 2), (2, -2)\}$.

4 (a) Find the eigenvalues and corresponding eigenvectors of **[5]**

$$A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

(b) Explain the concept of Tangent Planes with an example. Find the tangent plane to the elliptic paraboloid $z = 2z^2 + y^2$ at the point (1, 1, 3). **[3+4]**

(c) Find the directional derivative of the function $f(x, y) = x^2y^3 - 4y$ at the point (2,-1) in the direction of the vector $\boldsymbol{v} = 2\boldsymbol{i} + 5\boldsymbol{j}$. **[7]**

(d) What is the second derivative test? Why is it used? **[4.33]**

5 (a) A rectangular box without a lid is to be made from 12m^2 of cardboard. Find the maximum volume of such a box. **[6]**

(b) Compute the following double integral: **[7.33]**

$$\iint_R 6xy^2 \, dA, \; R = [2, 4] \times [1, 2]$$

(c) Find the maximum and minimum values of $f(x,y,z) = xyz$ subject to the constraint $x + y + z = 1$. Assume that $x, y, z \geq 0$. **[10]**

University of Dhaka

Department of Computer Science and Engineering

3rd Year 1st Semester B. Sc. (Hons) Final Examination, 2019

MATH-3105: Multivariable Calculus and Geometry

Total Marks: 60 **Time: 3 Hours**

(Answer any four (4) of the following questions)

1. a) Determine whether each statement is true or false. Explain your answer briefly. Assume everything is described in 3D. [6]
   i. Two lines parallel to a third line are parallel.
   ii. Two lines perpendicular to a third line are parallel.
   iii. Two planes parallel to a third plane are parallel.
   iv. Two lines perpendicular to a plane are parallel.
   v. Two lines either intersect or are parallel.
   vi. A plane and a line either intersect or are parallel.

   b) Find parametric equations and symmetric equations for the line through the origin and the point (4, 3, -1). [5]

   c) Find equations of the planes that are parallel to the plane x + 2y - 2z = 1 and two units away from it. [4]

2. a) Is $\lambda = 4$ a eigen~~vector~~ value of $\begin{bmatrix} 3 & 0 & -1 \\ 2 & 3 & 1 \\ -3 & 4 & 5 \end{bmatrix}$? If so, find one corresponding eigenvector. [6]

   b) Diagonalize $\begin{bmatrix} 2 & 2 & -1 \\ 1 & 3 & -1 \\ -1 & -2 & 2 \end{bmatrix}$. Show full process in details. [9]

3. a) Find the domain and range of g(x, y) = $\sqrt{9 - x^2 - y^2}$ and sketch the level curves of the function for k=0, 1, 2, 3. [3]

   b) Show if the following polynomials are linearly independent [4]

   $p1(t) = 1 + 2t^2 \rightarrow [\,p1(t)\,]B = (1, 0, 2)$

   $p2(t) = 4 + t + 5t^2 \rightarrow [\,p2(t)\,]B = (4, 1, 5)$

   $p3(t) = 3 + 2t \rightarrow [\,p3(t)]B = (3, 2, 0)$

   c) Let A=$\begin{bmatrix} 3 & -6 & 0 \\ -6 & 0 & 6 \\ 0 & 6 & -3 \end{bmatrix}$. Show that A is orthogonally diagonalizable by finding an orthogonal matrix U and a diagonal matrix D such that A=UDU^T. [6]

   d) Sketch the graph of the function f(x, y) = 6-3x-2y [2]

4. a) Find the local maximum and minimum values and saddle point(s) of the given functions. [9]

   i. $x^2 + xy + y^2 + y$

   ii. $y^3 - 3x^2y - 6x^2 - 6y^2 + 2$

   iii. $e^x \cos y$

   b) Use Lagrange multipliers to find the maximum and minimum values of the function subject to the given constraint: [6]

   i. $f(x, y) = 3x + y;\ x^2 + y^2 = 10$

   ii. $f(x, y) = e^{xy};\ x^3 + y^3 = 16$

5. a) Calculate the iterated integral: [6]

   i. $\int_1^4 \int_0^2 (6x^2y - 2x)dy\, dx$

   ii. $\int_0^2 \int_0^4 y^3 e^{2x} dy\, dx$

   b) Evaluate the double integral: [9]

   i. $\iint_D y^2 dA,\ D = \{(x, y) \mid -1 \le y \le 1, -y - 2 \le x \le y\}$

   ii. $\iint_D x\cos y dA,\ D \text{ is bounded by } y = 0, y = x^2 \text{ and } x = 1$

   iii. $\iint_D y^2 dA,\ D \text{ is a triangular region with vertices } (0, 1), (1, 2), (4, 1).$

6. a) Use polar coordinates to find the volume of the given solid under the cone $z = \sqrt{x^2 + y^2}$ and above the disk $x^2 + y^2 \le 4$ [5]

   b) A cylindrical drill with radius r is used to bore a hole through the center of a sphere of radius R. Find the volume of the ring-shaped solid that remains. [5]

   c) A swimming pool is circular with a 40-ft diameter. The depth is constant along east-west lines and increases linearly from 2 ft at the south end to 7 ft at the north end. Find the volume of water in the pool. [5]