HOMEWORK 1

1. (a)

$$
\begin{aligned}
&\frac{1}{2}\left(e^{i\theta}+e^{-i\theta}\right)\\
&=\frac{1}{2}\left(\cos\theta+i\sin\theta+\frac{1}{\cos\theta+i\sin\theta}\right)\\
&=\frac{1}{2}\left(\frac{\cos^2\theta+i\sin\theta\cos\theta}{\cos\theta+i\sin\theta}+\frac{i\sin\theta\cos\theta-\sin^2\theta}{\cos\theta i\sin\theta}+\frac{1}{\cos\theta+i\sin\theta}\right)\\
&=\frac{1}{2}\left(\frac{2\cos^2\theta+2i\cos\theta\sin\theta}{\cos\theta+i\sin\theta}\right)\\
&=\frac{2\cos\theta}{2}\left(\frac{\cos\theta+i\sin\theta}{\cos\theta+i\sin\theta}\right)\\
&=\cos\theta
\end{aligned}
$$

$$
\begin{aligned}
&\frac{1}{2i}\left(e^{i\theta}-e^{-i\theta}\right)\\
&=\frac{1}{2i}\left(\cos\theta+i\sin\theta-\frac{1}{\cos\theta+i\sin\theta}\right)\\
&=\frac{1}{2}\left(\frac{\cos^2\theta+i\sin\theta\cos\theta}{\cos\theta+i\sin\theta}+\frac{i\sin\theta\cos\theta-\sin^2\theta}{\cos\theta i\sin\theta}-\frac{1}{\cos\theta+i\sin\theta}\right)\\
&=\frac{1}{2i}\left(\frac{-2\sin^2\theta+2i\sin\theta\cos\theta}{\cos\theta+i\sin\theta}\right)\\
&=\sin\theta\left(\frac{-\sin\theta+i\cos\theta}{-\sin\theta+i\cos\theta}\right)\\
&=\sin\theta
\end{aligned}
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(b)