HOMEWORK 1

1. (a)

$$
\begin{aligned}
&\frac{1}{2}\left(e^{i\theta}+e^{-i\theta}\right)\\
=&\frac{1}{2}\left(\cos\theta+i\sin\theta+\frac{1}{\cos\theta+i\sin\theta}\right)\\
=&\frac{1}{2}\left(\frac{\cos^2\theta+i\sin\theta\cos\theta}{\cos\theta+i\sin\theta}+\frac{i\sin\theta\cos\theta-\sin^2\theta}{\cos\theta i\sin\theta}+\frac{1}{\cos\theta+i\sin\theta}\right)\\
=&\frac{1}{2}\left(\frac{2\cos^2\theta+2i\cos\theta\sin\theta}{\cos\theta+i\sin\theta}\right)\\
=&\frac{2\cos\theta}{2}\left(\frac{\cos\theta+i\sin\theta}{\cos\theta+i\sin\theta}\right)\\
=&\cos\theta
\end{aligned}
$$

$$
\begin{aligned}
&\frac{1}{2i}\left(e^{i\theta}-e^{-i\theta}\right)\\
=&\frac{1}{2i}\left(\cos\theta+i\sin\theta-\frac{1}{\cos\theta+i\sin\theta}\right)\\
=&\frac{1}{2}\left(\frac{\cos^2\theta+i\sin\theta\cos\theta}{\cos\theta+i\sin\theta}+\frac{i\sin\theta\cos\theta-\sin^2\theta}{\cos\theta i\sin\theta}-\frac{1}{\cos\theta+i\sin\theta}\right)\\
=&\frac{1}{2i}\left(\frac{-2\sin^2\theta+2i\sin\theta\cos\theta}{\cos\theta+i\sin\theta}\right)\\
=&\sin\theta\left(\frac{-\sin\theta+i\cos\theta}{-\sin\theta+i\cos\theta}\right)\\
=&\sin\theta
\end{aligned}
$$

(b) The probability density, $P(x)$ is defined:

$$P(x)=\psi^*(x)\,\psi(x)$$

So we have:

$$
\begin{aligned}
P(x) &= \chi(x)e^{-i\varphi}\chi(x)e^{i\varphi}\\
&= [\chi(x)]^2
\end{aligned}
$$

Hence we see that the probability density is independent of the global complex phase factor.

(c)

$$
\begin{aligned}
&\sin\theta\sin\varphi \\
=&\frac{1}{2i}\frac{1}{2i}\left(e^{i\theta}-e^{-i\theta}\right)\left(e^{i\varphi}-e^{-i\varphi}\right) \\
=&-\frac{1}{4}\left[e^{i\theta+i\varphi}-e^{i\theta-i\varphi}-e^{i\varphi-i\theta}+e^{-i\theta-i\varphi}\right] \\
=&-\frac{1}{4}\left[2\cos\left(\theta+\varphi\right)-2\cos\left(\theta-\varphi\right)\right] \\
=&\frac{1}{2}\left[\cos\left(\theta-\varphi\right)-\cos\left(\theta+\varphi\right)\right]
\end{aligned}
$$