Physics 40—Quantum Physics of Matter

HW 4—Due April 25, 2024

1. A beam of electrons enters a uniform 1.20 T magnetic field.

   (a) Find the energy difference between electrons whose spins are“parallel” (up) and “antiparallel” to the field (down).

   (b) Find the wavelength of the radiation that can cause the electronswhose spins are parallel to the field to flip so that their spins are antiparallel.

2. Radio astronomers can detect clouds of hydrogen in our galaxy too cool to radiate in the optical part of the spectrum. The detection is by means of the 21 cm spectral line that corresponds to the flipping of the electron in a hydrogen atom in its ground state from having its spin parallel to the spin of the proton to having it antiparallel. Find the magnetic field experienced by the electron in a hydrogen atom.

3. (ER problem 8.10): Consider the states in which $l = 4$ and $s = 1/2$. For the state with the largest possible j and largest possible m_j, calculate (a) the angle between $\vec{L}$ and $\vec{S}$, (b) the angle between $\vec{\mu}_l$ and $\vec{\mu}_s$, and (c) the angle between $\vec{J}$ and the $+z$ axis.)

4. Consider the $3d$ level of the H atom.

   (a) Discuss the effect of the spin-orbit splitting on the $3d$ level: determine the possible values of j and m_j, and for each value of j give the degeneracy and spectroscopic notation.

   (b) Why is it impossible to have an $2D_{3/2}$ state?

   (c) Why is the ground state of the hydrogen atom not split into two sub-levels by spin-orbit coupling? (Very short answer.)

5. A fine structure smorgasbord:

   (a) Show that

   $$E_I = \frac{e^2}{4\pi\epsilon_0}\frac{1}{2a_0} = \frac{1}{2}\alpha^2 m_e c^2$$

   where

   $$\alpha = \frac{e^2}{4\pi\epsilon_0 \hbar c}$$

   is the fine structure constant.

(b) Show that the lowest order relativistic correction to the classical kinetic energy is

$$\Delta \mathrm{KE} = -\frac{p^4}{8\mu^3 c^2}.$$

(*Hint:* Start with the relativistic energy $E = \sqrt{(\mu c^2)^2 + (pc)^2}$, subtract off the rest energy to get the kinetic energy, and do a Taylor series expansion on the result, treating $pc/\mu c^2$ as a small parameter.)

(c) It can be shown that for $l > 0$ the relativistic and spin-orbit contributions to the fine structure give the correction

$$\langle \hat{H}_{\mathrm{mv}} + \hat{H}_{\mathrm{SO}} \rangle = \frac{1}{4}\alpha^2 E_I \frac{1}{n^4}[3 + 2nf(j,l)]$$

where

$$f(j,l) = \frac{j(j+1) - 3l(l+1) - \frac{3}{4}}{l(l+\frac{1}{2})(l+1)}.$$

Show that $f(j,l)$ is in reality independent of l and equal to

$$f(n,l) = -\frac{2}{j+\frac{1}{2}}.$$

(*Hint*: for each possible j, rewrite l in terms of j and simplify. There are only two possibilities!)