Number System April 13, 2025

**Built-In Types**

Java has built-in numeric types that extend **Number**:

* **Byte**
* **Short**
* **Integer**, **AtomicInteger**
* **Long**, **AtomicLong**, **LongAccumulator**, and **LongAdder**
* **Float**
* **Double**, **DoubleAccumulator**, and **DoubleAdder**
* **BigInteger**
* **BigDecimal**

If we include (which can be represented by the floating-point types), these all represent subsets of the set of rational numbers extended to include infinities. The built-in types are not closed under either addition or multiplication, and no irrational numbers can be represented. This presents problems when working with Trigonometry, for example, where rational multiples of or square roots of integers are common.

**Rational Types**

We add the following rational-valued types, also defined to extend **Number**:

* **Rational**
* **BigRational**

The rational types have integer numerator and denominator, allowing (for example) quantities like 1/3 to be represented exactly. The **Rational** class uses long integers for numerator and denominator, and the **BigRational** class uses **BigInteger** values as numerator and denominator. Values are always stored in a normalized form with positive nonzero denominator and numerator and denominator relatively prime. This effectively completes the built-in types to include the whole set of rational numbers and closes the set with respect to addition and multiplication, making it a true field.

Since all built-in types represent subsets of the rational numbers, utility methods are provided to convert any of the built-in types to one of the rational types.

**Irrational Types**

The full set of irrationals cannot be represented digitally, but we capture common use cases with two irrational-valued classes, each extending **Number**:

* **Irrational**
* **BigIrrational**

The irrational types consist of a rational factor and an irrational factor, which can be , , or a square root of a positive (nonzero) integer.

These types allow trigonometric functions to return exact results where appropriate. For example, evaluating an extended version of the sine function of can return .

The irrationals with irrational factor are numbers of the form , where is rational. This set is closed under addition, but not under multiplication. It is possible, however, to multiple such a value by any rational number (hence any built-in type) and the result will be an irrational number in the same form.

Irrational numbers with irrational factors that are the square roots of integers are closed under multiplication and have multiplicative inverses.

For operations (multiplication, exponentiation, etc.) where the result cannot be represented in a rational form, a rational approximation of the result is used, substituting the built-in double-valued rational constants approximating or .