Recommender Systems

$i = Movie$
$j = User$
$Features = \{x_1(romance), x_2(action)\}$
$n = \#Features = |Features| = 2$
$x_0 = 1$ Is the default Intercept Feature (applied by convention)

$$x^{(i)} \in (\mathbb{R}^{n+1} = \mathbb{R}^3) = \begin{bmatrix} x_0^{(i)} \\ x_1^{(i)} \\ x_2^{(i)} \end{bmatrix}$$

$$\theta^{(j)} \in \mathbb{R}^{n+1} = \begin{bmatrix} \theta_0^{(j)} \\ \theta_1^{(j)} \\ \theta_2^{(j)} \end{bmatrix}$$

Predicting User-Movie Rating of User j rates Movie i:

$$\left(\theta^{(j)}\right)^T x^{(i)} = \underbrace{\begin{bmatrix} \theta_0^{(j)} & \theta_1^{(j)} & \theta_2^{(j)} \end{bmatrix}}_{1\times(n+1)} \cdot \underbrace{\begin{bmatrix} x_0^{(i)} \\ x_1^{(i)} \\ x_2^{(i)} \end{bmatrix}}_{(n+1)\times 1} = \underbrace{\begin{bmatrix} \theta_0^{(j)} \cdot x_0^{(i)} + \theta_1^{(j)} \cdot x_1^{(i)} + \theta_2^{(j)} \cdot x_2^{(i)} \end{bmatrix}}_{1\times 1}$$

Predicting User-Movie Rating of User $j = 1$ rates Movie $i = 3$ and assume $\theta^{(1)}$ is already calculated and available as follows:

$$\theta^{(1)} = \begin{bmatrix} 0 \\ 5 \\ 0 \end{bmatrix}, x^{(3)} = \begin{bmatrix} 1 \\ 0.99 \\ 0 \end{bmatrix} \Rightarrow$$

$$\left(\theta^{(1)}\right)^T x^{(3)} = \begin{bmatrix} 0 & 5 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0.99 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \cdot 1 + 5 \cdot 0.99 + 0 \cdot 0 \end{bmatrix} = 4.95$$