# Relational Algebra

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# Supplemental Materials

## Helpful References

https://en.wikipedia.org/wiki/Relational_algebra

**Relational algebra** is the basis for the most popular **query language on earth**

# What's a Query Language?

Allows manipulation and retrieval of data from a database.

Traditionally: QL != programming language

- Doesn't need to be turing complete
- *Not* designed for computation
- Supports easy, efficient access to (very) large databases

Today

- Scaling to large datasets is a reality
- Powerful way to think about…
  - data algorithms that scale
  - asynchronous/parallel programming

# 2 Formal Relational Query Languages

Relational Algebra

Operational, used to represent execution plans

$\pi_{name}(\sigma_{age<30}(Sailors))$ sailor names younger than 30

Relational Calculus

Logical, describes what data users want (declarative)

{ s: name | s ∈ Sailors ^ s.age < 30 }

(this is shorthand)

# Journey of a Query

| | |
|---|---|
| SQL | SELECT … FROM … |
| Relational Algebra | $\pi_b(P \bowtie Q \bowtie \ldots)$ |
| Query Rewriting | $\pi_b(P \bowtie S) \bowtie Q \ldots)$ |

## Query execution

https://cseweb.ucsd.edu/classes/fa19/cse132A-a/slides/relational-calculus.pdf

# Prelims

Query is a function over relation instances

$$Q(R_1,\ldots,Rn) = R_{result}$$

Schemas of input and output relations are *fixed* and well defined by the query Q.

Positional vs Named field notation

- Position easier for formal defs
  - one-indexed (not 0-indexed!!!)
- Named is more readable
- Both used in SQL

# Prelims

Relation (for this lecture)

- Instance: **set** of tuples (important!)
- Schema: list of field names and types (domains)
- Students(sid int, name text, major text, gpa int)

How are relations different than generic sets ($\mathbb{R}$)?

- Can assume item structure due to schema
- Some algebra operations (x) need to be modified
- Will use this later

# Relational Algebra Overview

Core 5 operations

- PROJECT ($\pi$)
- SELECT ($\sigma$)
- UNION ($\cup$)
- SET DIFFERENCE (-)
- CROSSPRODUCT (x)

Additional operations

- RENAME ($\rho$)
- INTERSECT ($\cap$)
- JOIN ($\bowtie$)
- DIVIDE (/) not on exam

# Instances Used Today: Library

Students, Reservations

Use positional or named field notation

Fields in query results are inherited from input relations (unless specified)

R1

| sid | rid | day |
|---|---|---|
| 1 | 101 | 10/10 |
| 2 | 102 | 11/11 |

S1

| sid | name | gpa | age |
|---|---|---|---|
| 1 | eugene | 4 | 20 |
| 2 | barb | 3 | 21 |
| 3 | tanya | 2 | 88 |

S2

| sid | name | gpa | age |
|---|---|---|---|
| 4 | aziz | 3.2 | 21 |
| 2 | barb | 3 | 21 |
| 3 | tanya | 2 | 88 |
| 5 | rusty | 3.5 | 21 |

# Project

$$\pi_{<attr1,\ldots>}(A) = R_{result}$$

Pick out desired attributes (subset of columns)

Schema is subset of input schema in the projection list

- $\pi_{<a,b,c>}(A)$ has output schema (a,b,c) w/ types carried over

# Project

S2

| sid | name | gpa | age |
|---|---|---|---|
| 4 | aziz | 3.2 | 21 |
| 2 | barb | 3 | 21 |
| 3 | tanya | 2 | 88 |
| 5 | rusty | 3.5 | 21 |

$$\pi_{name,age}(S2) =$$

| name | age |
|---|---|
| aziz | 21 |
| barb | 21 |
| tanya | 88 |
| rusty | 21 |

# Project

S2

| sid | name | gpa | age |
|---|---|---|---|
| 4 | aziz | 3.2 | 21 |
| 2 | barb | 3 | 21 |
| 3 | tanya | 2 | 88 |
| 5 | rusty | 3.5 | 21 |

$\pi_{name,name,age}(S2) =$

| name | name | age |
|---|---|---|
| aziz | aziz | 21 |
| barb | barb | 21 |
| tanya | tanya | 88 |
| rusty | rusty | 21 |

# Project

S2

| sid | name | gpa | age |
|---|---|---|---|
| 4 | aziz | 3.2 | 21 |
| 2 | barb | 3 | 21 |
| 3 | tanya | 2 | 88 |
| 5 | rusty | 3.5 | 21 |

$\pi_{age}(S2) =$

| age |
|---|
| 21 |
| 88 |

Where did all the rows go?
Real systems typically don't remove duplicates by default. Why?

# Project (Positional Notation)

S2

| sid | name | gpa | age |
|---|---|---|---|
| 4 | aziz | 3.2 | 21 |
| 2 | barb | 3 | 21 |
| 3 | tanya | 2 | 88 |
| 5 | rusty | 3.5 | 21 |

$\pi_{S2.4}(S2) =$

| age |
|---|
| 21 |
| 88 |

# Select

$$\sigma_{<p>}(A) = R_{result}$$

Select subset of rows that satisfy condition *p*

*p*: Boolean expr over constants and attributes in A

Won't have duplicates in result. Why?

Result schema same as input

# Select

S1

| sid | name | gpa | age |
|---|---|---|---|
| 1 | eugene | 4 | 20 |
| 2 | barb | 3 | 21 |
| 3 | tanya | 2 | 88 |

$$\sigma_{age<30}(S1) =$$

| sid | name | gpa | age |
|---|---|---|---|
| 1 | eugene | 4 | 20 |
| 2 | barb | 3 | 21 |

$$\pi_{name}(\sigma_{age<30}(S1)) =$$

| name |
|---|
| eugene |
| barb |

# Select (Positional Notation)

S1

| sid | name | gpa | age |
|---|---|---|---|
| 1 | eugene | 4 | 20 |
| 2 | barb | 3 | 21 |
| 3 | tanya | 2 | 88 |

$\sigma_{S1.4<30}(S1) =$

| sid | name | gpa | age |
|---|---|---|---|
| 1 | eugene | 4 | 20 |
| 2 | barb | 3 | 21 |

$\pi_{\$2}(\sigma_{S1.4<30}(S1)) =$

| name |
|---|
| eugene |
| barb |

$ prefix distinguishes
$2 from the number 2

## Commutative Operations

$$A + B = B + A$$

$$A * B = B * A$$

$$A + (B * C) = (B * C) + A$$

## Associative Operations

$$A + (B + C) = (A + B) + C$$

$$A + (B * C) = (A + B) * C$$

## Commutative Operations

$$A + B = B + A$$

$$A * B = B * A$$

$$A + (B * C) = (B * C) + A$$

## Associative Operations

$$A + (B + C) = (A + B) + C$$

$$\cancel{A + (B * C) = (A + B) * C}$$

# Commutatively

$$\pi_{age}(\sigma_{age<30}(S1))$$

$$\underbrace{\sigma_{age<30}\left( \begin{array}{|l|l|l|l|} \hline \text{sid} & \text{name} & \text{gpa} & \text{age} \\ \hline 1 & \text{eugene} & 4 & 20 \\ \hline 2 & \text{barb} & 3 & 21 \\ \hline 3 & \text{tanya} & 2 & 88 \\ \hline \end{array} \right)}$$

| sid | name | gpa | age |
|---|---|---|---|
| 1 | eugene | 4 | 20 |
| 2 | barb | 3 | 21 |

# Commutatively

$\pi_{age}(\sigma_{age<30}(S1))$

$\sigma_{age<30}$ (

| sid | name | gpa | age |
|---|---|---|---|
| 1 | eugene | 4 | 20 |
| 2 | barb | 3 | 21 |
| 3 | tanya | 2 | 88 |

)

$\pi_{age}$ (

| sid | name | gpa | age |
|---|---|---|---|
| 1 | eugene | 4 | 20 |
| 2 | barb | 3 | 21 |

) =

| age |
|---|
| 20 |
| 21 |

# Commutatively

$\sigma_{age<30}(\pi_{age}(S1))$

$\pi_{age}\Bigg($

| sid | name | gpa | age |
|---|---|---|---|
| 1 | eugene | 4 | 20 |
| 2 | barb | 3 | 21 |
| 3 | tanya | 2 | 88 |

$\Bigg)$

| age |
|---|
| 20 |
| 21 |
| 88 |

# Commutatively

$$\sigma_{age<30}(\pi_{age}(S1))$$

$\pi_{age}$ (

| sid | name | gpa | age |
|---|---|---|---|
| 1 | eugene | 4 | 20 |
| 2 | barb | 3 | 21 |
| 3 | tanya | 2 | 88 |

)

$\sigma_{age<30}$ (

| age |
|---|
| 20 |
| 21 |
| 88 |

) =

| age |
|---|
| 20 |
| 21 |

# Commutatively

Does Project and Select **always** commute?

$$\pi_{age}(\sigma_{age<30}(S1)) = \sigma_{age<30}(\pi_{age}(S1))$$

What about

$$\pi_{name}(\sigma_{age<30}(S1))?$$

# Commutatively

Does Project and Select commute?

$$\pi_{age}(\sigma_{age<30}(S1)) = \sigma_{age<30}(\pi_{age}(S1))$$

What about

$$\pi_{name}(\sigma_{age<30}(S1)) \mathrel{!=} \sigma_{age<30}(\pi_{name}(S1))$$

# Commutatively

Does Project and Select commute?

$$\pi_{age}(\sigma_{age<30}(S1)) = \sigma_{age<30}(\pi_{age}(S1))$$

What about

$$\pi_{name}(\sigma_{age<30}(S1)) \mathrel{!=} \sigma_{age<30}(\pi_{name,\ age}(S1))$$

# Commutatively

Does Project and Select commute?

$$\pi_{age}(\sigma_{age<30}(S1)) = \sigma_{age<30}(\pi_{age}(S1))$$

What about

$$\pi_{name}(\sigma_{age<30}(S1)) = \pi_{name}(\sigma_{age<30}(\pi_{name,\ age}(S1)))$$

OK!


What is needed for a good relationship?
Primary Keys

# Union, Set-Difference

$$A \text{ op } B = R_{result}$$

A, B must be *union-compatible*

- Same number of fields
- Field i in each schema have same type

Result Schema taken from first relation (A)

- A(id int, imgid int) U B(blah int, gloop int) = ?

# Union, Set-Difference

$$A \text{ op } B = R_{result}$$

A, B must be *union-compatible*

- Same number of fields
- Field i in each schema have same type

Result Schema taken from first relation (A)

- A(id int, imgid int) U B(blah int, gloop int) = $R_{result}$(id int, imgid int)

# Union, Intersect, Set-Difference

S1

| sid | name | gpa | age |
|---|---|---|---|
| 1 | eugene | 4 | 20 |
| 2 | barb | 3 | 21 |
| 3 | tanya | 2 | 88 |

S2

| sid | name | gpa | age |
|---|---|---|---|
| 4 | aziz | 3.2 | 21 |
| 2 | barb | 3 | 21 |
| 3 | tanya | 2 | 88 |
| 5 | rusty | 3.5 | 21 |

S1 U S2 =

| sid | name | gpa | age |
|---|---|---|---|
| 1 | eugene | 4 | 20 |
| 4 | aziz | 3.2 | 21 |
| 5 | rusty | 3.5 | 21 |
| 3 | tanya | 2 | 88 |
| 2 | barb | 3 | 21 |

# Union, Intersect, Set-Difference

S1

| sid | name | gpa | age |
|---|---|---|---|
| 1 | eugene | 4 | 20 |
| 2 | barb | 3 | 21 |
| 3 | tanya | 2 | 88 |

S2

| sid | name | gpa | age |
|---|---|---|---|
| 4 | aziz | 3.2 | 21 |
| 2 | barb | 3 | 21 |
| 3 | tanya | 2 | 88 |
| 5 | rusty | 3.5 | 21 |

S1 - S2 =

| sid | name | gpa | age |
|---|---|---|---|
| 1 | eugene | 4 | 20 |

# Note on Set Difference & Performance

Notice that most operators are monotonic

- increasing size of inputs → outputs grow
- if $A \supseteq B \rightarrow Q(A,T) \supseteq Q(B,T)$
- can compute *incrementally*

Set Difference is *not monotonic*

- if $A \supseteq B \rightarrow T - A \subseteq T - B$
- e.g., $5 > 1 \rightarrow 9 - 5 < 9 - 1$

Thus, set difference is *blocking*:

- For T – S, must wait for all S tuples before any results

# Cross-Product

$$A(a_1,\ldots,a_n) \times B(a_{n+1},\ldots,a_m) = R_{result}(a_1,\ldots,a_m)$$

Each row of A paired with each row of B

- Result schema **concats** A and B's fields, inherit if possible
- Names of fields found in both A and B are undefined in result (some DBMSes set a default)

$$\{(1),(2)\} \times \{(3,4)\} = \{\ (1,3,4),\ (2,3,4)\ \}$$

Not same as mathematical "X", which returns **nested** results:

- math $A \times B = \{\ (a, b) \mid a \in A \wedge b \in B\ \}$

$$\{(1),(2)\} \times \{(3,4)\} = \{\ ((1),(3,4)),\ ((2),(3,4))\ \}$$

# Cross-Product

S1

| sid | name | gpa | age |
|---|---|---|---|
| 1 | eugene | 4 | 20 |
| 2 | barb | 3 | 21 |
| 3 | tanya | 2 | 88 |

R1

| sid | rid | day |
|---|---|---|
| 1 | 101 | 10/10 |
| 2 | 102 | 11/11 |

S1 x R1 =

| (sid) | name | gpa | age | (sid) | rid | day |
|---|---|---|---|---|---|---|
| 1 | eugene | 4 | 20 | 1 | 101 | 10/10 |
| 2 | barb | 3 | 21 | 1 | 101 | 10/10 |
| 3 | tanya | 2 | 88 | 1 | 101 | 10/10 |
| 1 | eugene | 4 | 20 | 2 | 102 | 11/11 |
| 2 | barb | 3 | 21 | 2 | 102 | 11/11 |
| 3 | tanya | 2 | 88 | 2 | 102 | 11/11 |

# Rename (can use positional notation)

p(<newRelationName>(<mappings>), Q)

Explicitly defines/changes field names of schema
Mappings of the form: <input attr> → <new name>

p(C($1 → sid1, $5 → sid2), S1 x R1)

C =

| **sid1** | name | gpa | age | **sid2** | rid | day |
|---|---|---|---|---|---|---|
| 1 | eugene | 4 | 20 | 1 | 101 | 10/10 |
| 2 | barb | 3 | 21 | 1 | 101 | 10/10 |
| 3 | tanya | 2 | 88 | 1 | 101 | 10/10 |
| 1 | eugene | 4 | 20 | 2 | 102 | 11/11 |
| 2 | barb | 3 | 21 | 2 | 102 | 11/11 |
| 3 | tanya | 2 | 88 | 2 | 102 | 11/11 |

# Rename alternate syntax

<newRelationName>(<attrnames>) = Q

attrnames is list of attributes ($a_1, \ldots, a_n$).

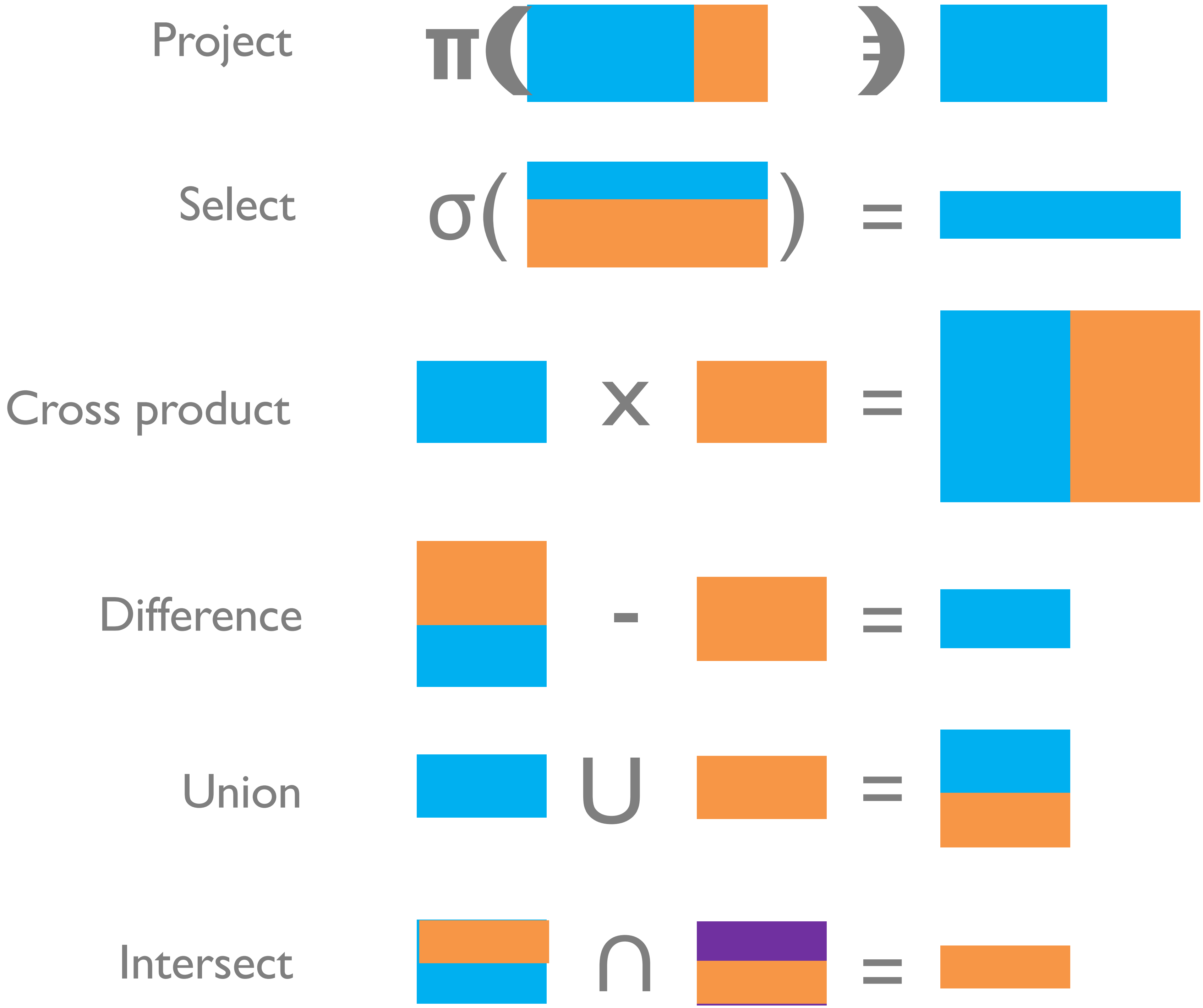
- Same # attrs as output of Q
- $a_i$ will be assigned $i^{th}$ output attribute of Q

C = C(sid,rid,day) = R1

C(foo, bar, baz) = R1

R1

| sid | rid | day |
|---|---|---|
| 1 | 101 | 10/10 |
| 2 | 102 | 11/11 |


Project
Select
Cross product
Difference
Union
Intersect

# Compound/Convenience Operators

INTERSECT ($\cap$)

JOIN ($\bowtie$)

DIVIDE (/)

# Intersect

$$A \cap B = R_{result}$$

A, B must be *union-compatible*

# Intersect

S1

| sid | name | gpa | age |
|---|---|---|---|
| 1 | eugene | 4 | 20 |
| 2 | barb | 3 | 21 |
| 3 | tanya | 2 | 88 |

S2

| sid | name | gpa | age |
|---|---|---|---|
| 4 | aziz | 3.2 | 21 |
| 2 | barb | 3 | 21 |
| 3 | tanya | 2 | 88 |
| 5 | rusty | 3.5 | 21 |

$S1 \cap S2 =$

| sid | name | gpa | age |
|---|---|---|---|
| 2 | barb | 3 | 21 |
| 3 | tanya | 2 | 88 |

# Intersect

$$A \cap B = R_{result}$$

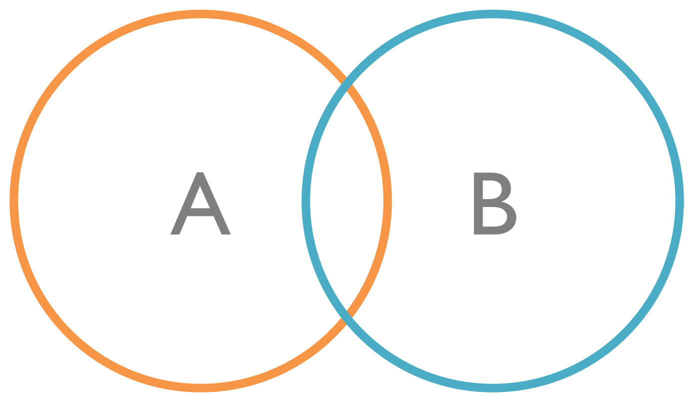
A, B must be *union-compatible*

Can we express using core operators?

$$A \cap B = ?$$

# Intersect

$$A \cap B = R_{result}$$

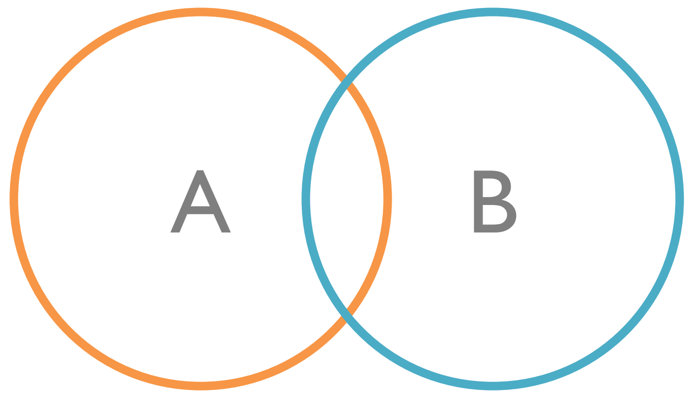



Can we express using core operators?

$A \cap B = A - ?$ (think venn diagram)

# Intersect

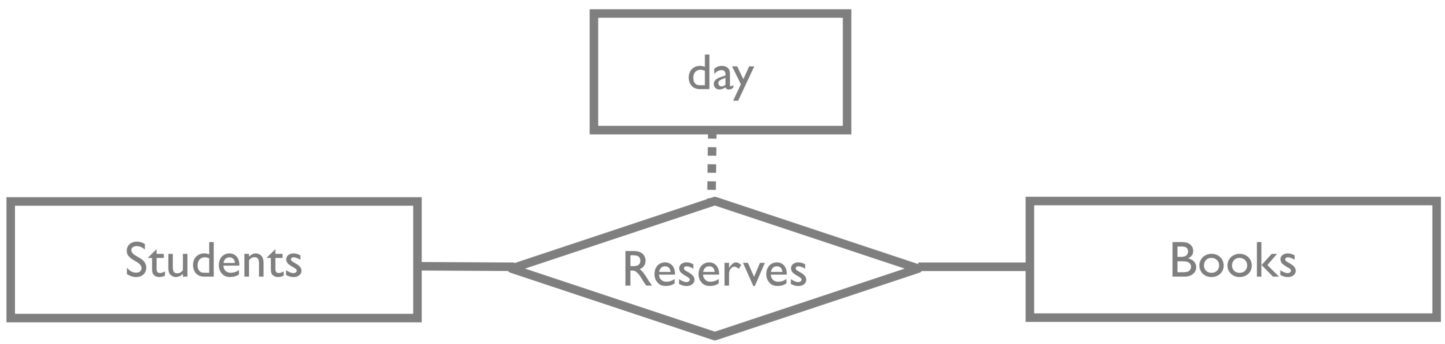
$$A \cap B = R_{result}$$




Can we express using core operators?

$$A \cap B = A - (A - B)$$

# Joins (high level)




relation relation relation

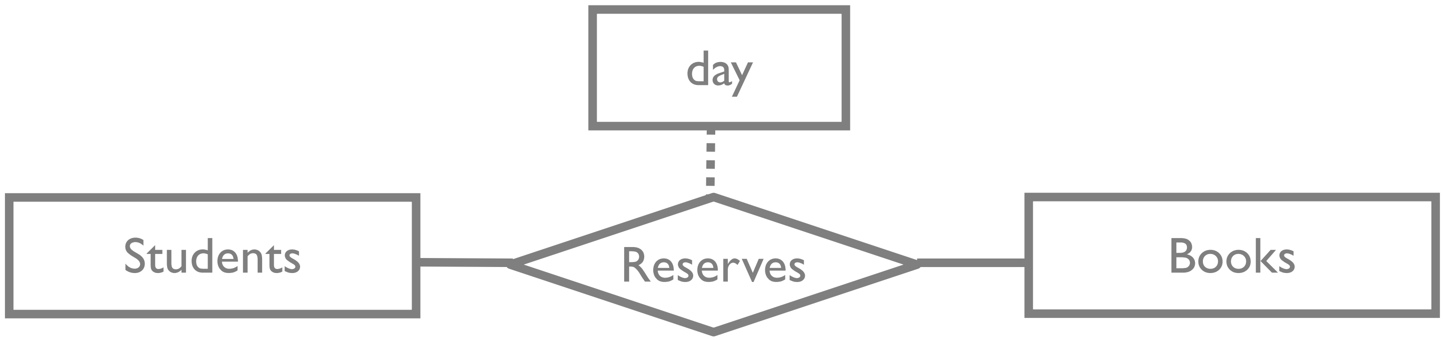
What if you want to query across all three tables?

e.g., all names of students that reserved "The Purple Crayon"

Need to combine these tables

Cross product? But that ignores foreign key references

# Joins (high level)




relation relation relation

S1

| sid | name | gpa | age |
| --- | --- | --- | --- |
| 1 | eugene | 4 | 20 |
| 2 | barb | 3 | 21 |
| 3 | tanya | 2 | 88 |

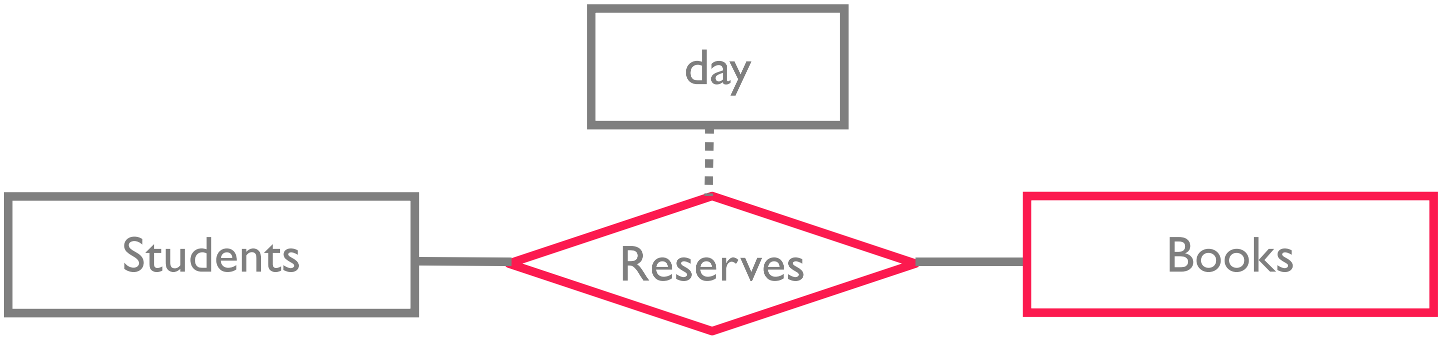
R1

| sid | rid | day |
| --- | --- | --- |
| 1 | 101 | 10/10 |
| 2 | 102 | 11/11 |

B1

| rid | name |
| --- | --- |
| 101 | The Purple Crayon |
| 102 | 1984 |

# Joins (high level)




relation relation relation

S1

| sid | name | gpa | age |
|---|---|---|---|
| 1 | eugene | 4 | 20 |
| 2 | barb | 3 | 21 |
| 3 | tanya | 2 | 88 |

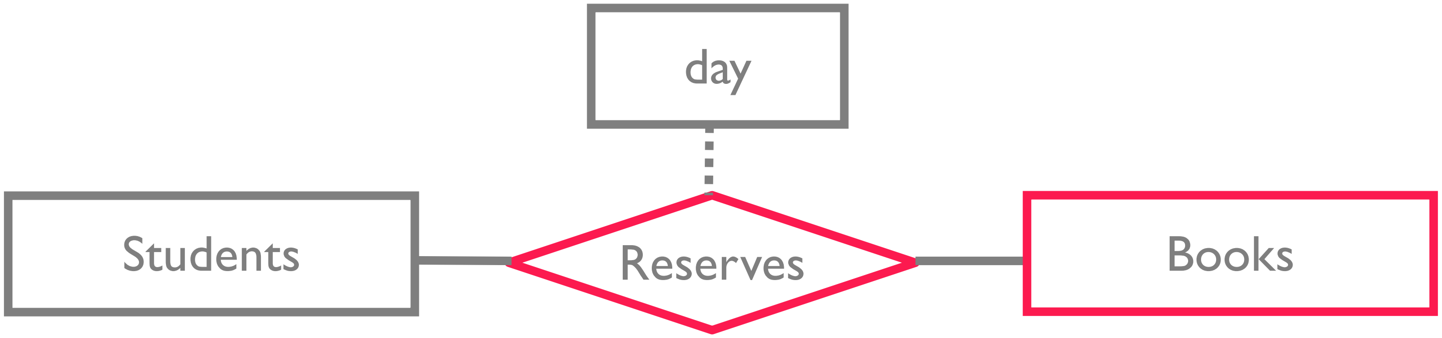
R1

| sid | rid | day |
|---|---|---|
| 1 | 101 | 10/10 |
| 2 | 102 | 11/11 |

B1

| rid | name |
|---|---|
| 101 | The Purple Crayon |
| 102 | 1984 |

# Joins (high level)




relation relation relation

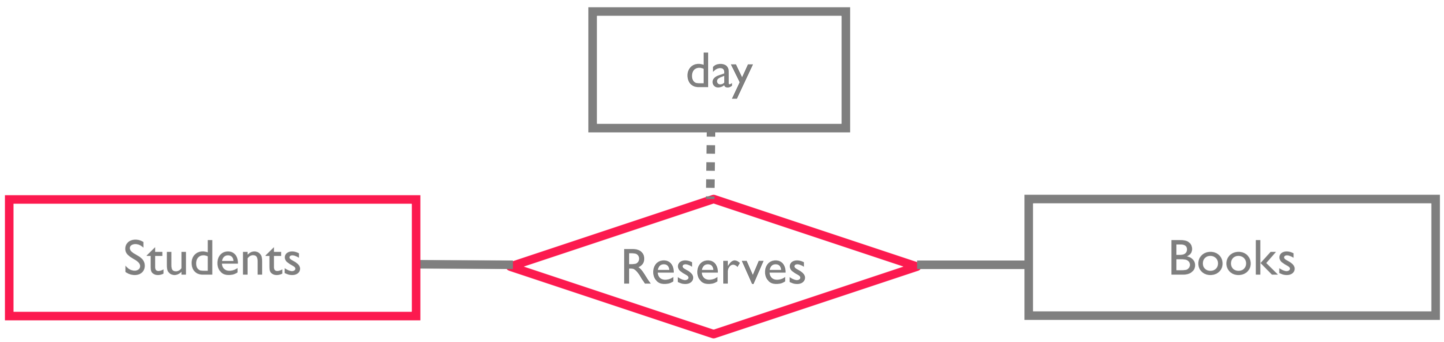
S1

| sid | name | gpa | age |
|---|---|---|---|
| 1 | eugene | 4 | 20 |
| 2 | barb | 3 | 21 |
| 3 | tanya | 2 | 88 |

RB1

| sid | (rid) | day | (rid) | name |
|---|---|---|---|---|
| 1 | 101 | 10/10 | 101 | The Purple Crayon |
| 2 | 102 | 11/11 | 102 | 1984 |

# Joins (high level)




relation

relation

relation

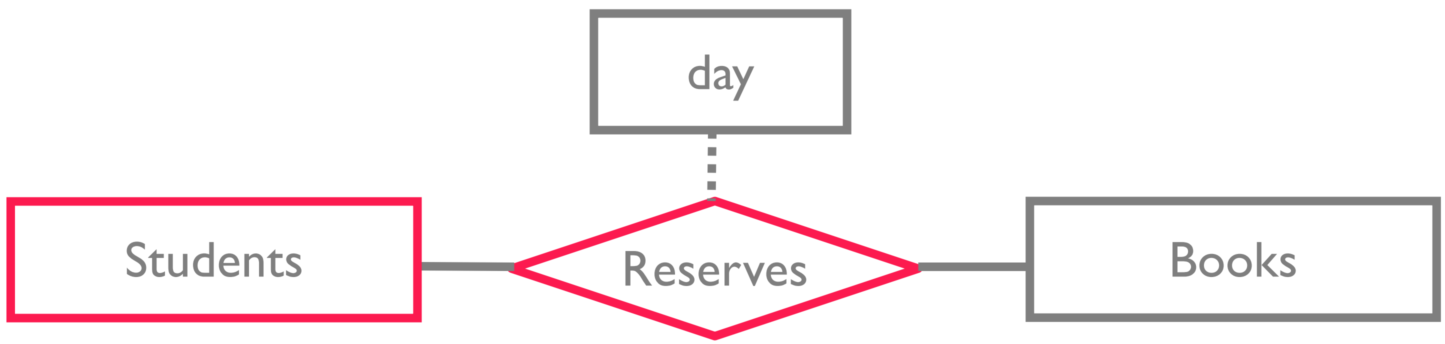
S1

| sid | name | gpa | age |
|---|---|---|---|
| 1 | eugene | 4 | 20 |
| 2 | barb | 3 | 21 |
| 3 | tanya | 2 | 88 |

RB1

| sid | (rid) | day | (rid) | name |
|---|---|---|---|---|
| 1 | 101 | 10/10 | 101 | The Purple Crayon |
| 2 | 102 | 11/11 | 102 | 1984 |

# Joins (high level)




**relation** **relation** **relation**

SRB1

| (sid) | (name) | gpa | age | (sid) | (rid) | day | (rid) | (name) |
|---|---|---|---|---|---|---|---|---|
| 1 | eugene | 4 | 20 | 1 | 101 | 10/10 | 101 | The Purple Crayon |
| 2 | barb | 3 | 21 | 2 | 102 | 11/11 | 102 | 1984 |

# Joins

Theta (θ) Join

Equi-join

# theta (θ) Join

$$A \bowtie_c B = \sigma_c(A \times B)$$

Most general form

Result schema same as cross product

Often *far* more efficient to compute than cross product

Commutative

$$(A \bowtie_c B) \bowtie_c C = A \bowtie_c (B \bowtie_c C)$$

# theta (θ) Join

S1

| sid | name | gpa | age |
|---|---|---|---|
| 1 | eugene | 4 | 20 |
| 2 | barb | 3 | 21 |
| 3 | tanya | 2 | 88 |

R1

| sid | rid | day |
|---|---|---|
| 1 | 101 | 10/10 |
| 2 | 102 | 11/11 |

$S1 \bowtie_{S1.sid \leq R1.sid} R1 =$

$\sigma_{S1.sid \leq R1.sid}(S1 \times R1) =$

| (sid) | name | gpa | age | (sid) | rid | day |
|---|---|---|---|---|---|---|
| 1 | eugene | 4 | 20 | 1 | 101 | 10/10 |
| 2 | barb | 3 | 21 | 1 | 101 | 10/10 |
| 3 | tanya | 2 | 88 | 1 | 101 | 10/10 |
| 1 | eugene | 4 | 20 | 2 | 102 | 11/11 |
| 2 | barb | 3 | 21 | 2 | 102 | 11/11 |
| 3 | tanya | 2 | 88 | 2 | 102 | 11/11 |

# theta (θ) Join

S1

| sid | name | gpa | age |
|---|---|---|---|
| 1 | eugene | 4 | 20 |
| 2 | barb | 3 | 21 |
| 3 | tanya | 2 | 88 |

R1

| sid | rid | day |
|---|---|---|
| 1 | 101 | 10/10 |
| 2 | 102 | 11/11 |

$S1 \bowtie_{S1.sid \leq R1.sid} R1 =$

$\sigma_{S1.sid \leq R1.sid}(S1 \times R1) =$

| (sid) | name | gpa | age | (sid) | rid | day |
|---|---|---|---|---|---|---|
| 1 | eugene | 4 | 20 | 1 | 101 | 10/10 |
| 2 | barb | 3 | 21 | 1 | 101 | 10/10 |
| 3 | tanya | 2 | 88 | 1 | 101 | 10/10 |
| 1 | eugene | 4 | 20 | 2 | 102 | 11/11 |
| 2 | barb | 3 | 21 | 2 | 102 | 11/11 |
| 3 | tanya | 2 | 88 | 2 | 102 | 11/11 |

# theta (θ) Join

S1

| sid | name | gpa | age |
|---|---|---|---|
| 1 | eugene | 4 | 20 |
| 2 | barb | 3 | 21 |
| 3 | tanya | 2 | 88 |

R1

| sid | rid | day |
|---|---|---|
| 1 | 101 | 10/10 |
| 2 | 102 | 11/11 |

$S1 \bowtie_{S1.sid \leq R1.sid} R1 =$

$\sigma_{S1.sid \leq R1.sid}(S1 \times R1) =$

| (sid) | name | gpa | age | (sid) | rid | day |
|---|---|---|---|---|---|---|
| 1 | eugene | 4 | 20 | 1 | 101 | 10/10 |
| 1 | eugene | 4 | 20 | 2 | 102 | 11/11 |
| 2 | barb | 3 | 21 | 2 | 102 | 11/11 |

# Equi-Join

$$A \bowtie_{attr} B = \boldsymbol{\pi}_{\text{all attrs } \textit{except} \text{ B.attr}}(A \bowtie_{A.attr\ =\ B.attr} B)$$

List the attributes that the two relations will be joined on.

$A \bowtie_{x,y} B$ is an equijoin on attributes x and y

Special case where the condition is attribute equality

Result schema only keeps *one copy* of equality fields

Natural Join ($A \bowtie B$):

- Equijoin on *all* shared fields (fields w/ same name)
- *Not recommended* since query results can unexpectedly change if someone changes the schemas (renames an attr)

# Equi-Join

S1

| sid | name | gpa | age |
|---|---|---|---|
| 1 | eugene | 4 | 20 |
| 2 | barb | 3 | 21 |
| 3 | tanya | 2 | 88 |

R1

| sid | rid | day |
|---|---|---|
| 1 | 101 | 10/10 |
| 2 | 102 | 11/11 |

$S1 \bowtie_{sid} R1 =$

| sid | name | gpa | age | rid | day |
|---|---|---|---|---|---|
| 1 | eugene | 4 | 20 | 101 | 10/10 |
| 2 | barb | 3 | 21 | 102 | 11/11 |

# Equi-Join

S1

| sid | name | day | gpa | age |
|---|---|---|---|---|
| 1 | eugene | 10/10 | 4 | 20 |
| 2 | barb | 12/12 | 3 | 21 |
| 3 | tanya | 3/3 | 2 | 88 |

R1

| sid | rid | day |
|---|---|---|
| 1 | 101 | 10/10 |
| 2 | 102 | 11/11 |

$S1 \bowtie_{sid,name} R1$ = INVALID! name not in R1

$S1 \bowtie_{sid,day} R1 = \pi_{S1.*,\, R1.rid}\ S1 \bowtie_{s1.sid=R1.sid\ \wedge\ s1.day\ =\ R1.day} R1$

=

| sid | name | day | gpa | age | rid |
|---|---|---|---|---|---|
| 1 | eugene | 10/10 | 4 | 20 | 101 |

$S1 \bowtie R1 = S1 \bowtie_{sid,day} R1$

# Natural Join Example

S1

| studid | name | date | gpa | age |
|---|---|---|---|---|
| 1 | eugene | 10/10 | 4 | 20 |
| 2 | barb | 12/12 | 3 | 21 |
| 3 | tanya | 3/3 | 2 | 88 |

R1

| sid | rid | day |
|---|---|---|
| 1 | 101 | 10/10 |
| 2 | 102 | 11/11 |

$$S1 \bowtie R1 = S1 \bowtie_{true} R1$$

semantics of query suddenly changed just by renaming input schema attributes

# Different Plans, Same Results

Semantic equivalence:
results are *always* the same

Note that it is independent
of the database instance!

## Names of students that reserved book 2

$$\pi_{name}(\sigma_{rid=2}(R1) \bowtie S1)$$

# Equivalent Queries

$$tmp1 = \sigma_{rid=2}(R1)$$
$$tmp2 = tmp1 \bowtie S1$$
$$\pi_{name}(tmp2)$$

$$\pi_{name}(\sigma_{rid=2}(R1 \bowtie S1))$$

# Names of students that reserved db books

Book(rid, type)    Reserve(sid, rid)    Student(sid, name)

Need to join DB books with reserve and students

$\sigma_{type='db'}$ (Book)

# Names of students that reserved db books

Book(rid, type)     Reserve(sid, rid)     Student(sid, name)

Need to join DB books with reserve and students

$\sigma_{type='db'}$ (Book) $\bowtie_{rid}$ Reserve

# Names of students that reserved db books

Book(rid, type)  Reserve(sid, rid)  Student(sid, name)

Need to join DB books with reserve and students

$\sigma_{type='db'}(\text{Book}) \bowtie_{rid} \text{Reserve} \bowtie_{sid} \text{Student}$

# Names of students that reserved db books

Book(rid, type)    Reserve(sid, rid)    Student(sid, name)

Need to join DB books with reserve and students

$$\pi_{name}(\sigma_{type='db'}(Book) \bowtie_{rid} Reserve \bowtie_{sid} Student)$$

# Names of students that reserved db books

Book(rid, type)     Reserve(sid, rid)     Student(sid, name)

Need to join DB books with reserve and students

$\pi_{name}(\sigma_{type='db'}\ (Book) \bowtie_{rid} Reserve \bowtie_{sid} Student)$

More efficient query

ookrids = $\pi_{rid}\ \sigma_{type='db'}\ (Book)$

$\pi_{sid}(bookrids \bowtie_{rid} Reserve)$

# Names of students that reserved db books

Book(rid, type)     Reserve(sid, rid)     Student(sid, name)

Need to join DB books with reserve and students

$$\pi_{name}(\sigma_{type=\text{'db'}}(\text{Book}) \bowtie_{rid} \text{Reserve} \bowtie_{sid} \text{Student})$$

More efficient query

$$\text{bookrids} = \pi_{rid}\, \sigma_{type=\text{'db'}}(\text{Book})$$

$$\pi_{name}(\pi_{sid}(\text{bookrids} \bowtie_{rid} \text{Reserve}) \bowtie_{sid} \text{Student})$$

Query optimizer can find the more efficient query!

# Students that reserved DB or HCI book

1. Find all DB or HCI books
2. Find students that reserved one of those books

$$tmp = \sigma_{type='DB' \vee type='HCI'} (Book)$$

$$\pi_{name}(tmp \bowtie Reserve \bowtie Student)$$

"v" means logical OR

Alternatives

define tmp using UNION (how?)

# Using UNION

$$tmp1 = \sigma_{type='DB'} (Book)$$

$$dbnames = \pi_{name}(tmp1 \bowtie Reserve \bowtie Student)$$

$$tmp2 = \sigma_{type='HCI'} (Book)$$

$$hcinames = \pi_{name}(tmp2 \bowtie Reserve \bowtie Student)$$

dbnames UNION hcinames

# Students that reserved a DB and HCI book

Can we change v into ^ (AND)?

$$tmp = \sigma_{type='DB' \wedge type='HCI'} (Book)$$

$$\pi_{name}(tmp \bowtie Reserve \bowtie Student)$$

NO

# Why?

$$tmp = \sigma_{type='DB' \wedge type='HCI'}(Book)$$

$$\pi_{name}(tmp \bowtie Reserve \bowtie Student)$$

```
for b in Book:
    if b.type = 'DB' and b.type = 'HCI':        // resolves to FALSE
        for r in Reserve:
            for s in Student:
                if r.sid = s.sid and r.bid = b.bid:
                    yield b.name
```

# Students that reserved a DB and HCI book

Does previous approach work?

1. Find students that reserved DB books
2. Find students that reversed HCI books
3. Intersection

$$tmpDB = \pi_{sid}(\sigma_{type='DB'} \text{ Book}) \bowtie \text{Reserve}$$

$$tmpHCI = \pi_{sid}(\sigma_{type='HCI'} \text{ Book}) \bowtie \text{Reserve}$$

$$\pi_{name}((tmpDB \cap tmpHCI) \bowtie \text{Student})$$

# Students that reserved all books

Students where, **for all books**, the student reserved the book

no concept of "for all" in relational algebra…

Students – Students that didn't reserve all books

# Students that reserved all books

Students where, **for all books**, the student reserved the book

no concept of "for all" in relational algebra…

Students – Students where there is a book that they did not reserve

# Students that reserved all books

Students where, **for all books**, the student reserved the book

no concept of "for all" in relational algebra…

Students – (Students s where (Books – Books s reserved)) (say s is bob)

$$\texttt{s\_reserved} = \pi_{bid}\ \sigma_{sid=bob}(\texttt{Reserve})$$

Students – (Students s where (Books – Books s reserved))

$$\texttt{s\_not\_reserved} = \pi_{bid}(\texttt{Books}) - \texttt{s\_reserved}$$

Students – (Students s where (Books – Books s reserved)) (for each student)

$$\pi_{sid,bid}(\texttt{Students x Books})$$

# Students that reserved all books

Students where, **for all books**, the student reserved the book

no concept of "for all" in relational algebra…

Students – (Students s where (Books – Books s reserved)) (say s is bob)

$$\texttt{s\_reserved} = \pi_{bid}\ \sigma_{sid=bob}(\texttt{Reserve})$$

Students – (Students s where (Books – Books s reserved))

$$\texttt{s\_not\_reserved} = \pi_{bid}(\texttt{Books}) - \texttt{s\_reserved}$$

Students – (Students s where (Books – Books s reserved)) (for each student)

$$\pi_{sid,bid}(\texttt{Students x Books}) - \pi_{sid,bid}(\texttt{Reserve})$$

# Students that reserved all books

Students where, **for all books**, the student reserved the book

no concept of "for all" in relational algebra…

Students – (Students s where (Books – Books s reserved))   (say s is bob)

$$\texttt{s\_reserved} = \pi_{\texttt{bid}}\ \sigma_{\texttt{sid=bob}}(\texttt{Reserve})$$

Students – (Students s where (Books – Books s reserved))

$$\texttt{s\_not\_reserved} = \pi_{\texttt{bid}}(\texttt{Books}) - \texttt{s\_reserved}$$

Students – (Students s where (Books – Books s reserved))   (for each student)

$$\texttt{del\_sids} = \pi_{\texttt{sid}}(\pi_{\texttt{sid,bid}}(\texttt{Students x Books}) - \pi_{\texttt{sid,bid}}(\texttt{Reserve}))$$

# Students that reserved all books

Students where, **for all books**, the student reserved the book

no concept of "for all" in relational algebra…

Students – (Students s where (Books – Books s reserved)) (say s is bob)

$$\texttt{s\_reserved} = \pi_{bid}\ \sigma_{sid=bob}(\texttt{Reserve})$$

Students – (Students s where (Books – Books s reserved))

$$\texttt{s\_not\_reserved} = \pi_{bid}(\texttt{Books}) - \texttt{s\_reserved}$$

Students – (Students s where (Books – Books s reserved)) (for each student)

$$\texttt{del\_sids} = \pi_{sid}(\pi_{sid,bid}(\texttt{Students x Books}) - \pi_{sid,bid}(\texttt{Reserve}))$$

$$\pi_{sid}\ (\texttt{Students}) - \texttt{del\_sids}$$

# Let’s step back

Relational algebra is expressiveness benchmark

- A language that can express relational algebra is “relationally complete”

Limitations

- nulls
- aggregation
- recursion
- duplicates
- can’t really type on keyboard…

# Equi-Joins are everywhere

Matching of two sets based on shared attributes

| | |
|---|---|
| Yelp: | Join between your location and restaurants |
| Market: | Join between consumers and suppliers |
| High five: | Join between two hands on time and space |
| Communication: | Join between minds on ideas/concepts |

Mat


ute

# Who Cares about Relational Alg?

- Clean query semantics & rich program analysis
- Helps/enables optimization
- Opens up rich set of topics
  - Materialized views
  - Data lineage/provenance
  - Query by example
  - Distributed query execution
  - …
- You see its fingerprints EVERYWHERE!

# What can we do with RA?

Query(DB instance) → Relation instance

# What can we do with RA?

Query(DB instance) = Relation instance

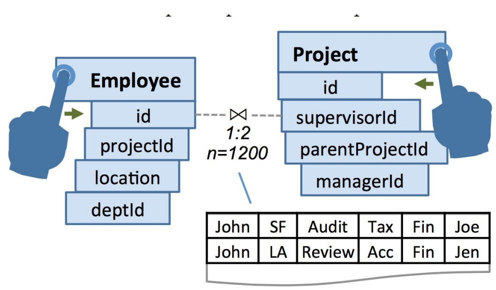
Query by example

*Here's DB instance and result, generate the query*

Data Generation:

*Here's query and result, generate a DB instance*

Novel relationally complete interfaces




GestureDB. Nandi et al.

# Summary

Relational Algebra (RA) operators

Operators are closed

- inputs & outputs are relations

Multiple Relational Algebra queries can be equivalent

- It is operational
- Same semantics but different performance
- Forms basis for optimizations