WKB approximation for broad resonance Regge-poles, $R/M > 3$

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Assume solution of form

$$\phi^{\rm out}_{\omega,\lambda-1/2} \underset{\omega\to\infty}{=} \begin{cases} e^{i\omega(r_*-R_*)} + \mathcal{R}e^{-i\omega(r_*-R_*)} & {\scriptstyle 1/\omega\le r_*<R_*,} \\ \left(1+\mathcal{R}\right)e^{i\omega(r_*-R_*)} & {\scriptstyle R_*\le r_*<\infty.} \end{cases} \tag{1}$$

Here R_* is the tortoise coordinate at the surface of the body, and $\mathcal{R}$ is a reflection coefficient. Assuming the potential has a discontinuity in the $N-th$ derivative at the surface of the compact body (see Zhang 2011 for more details), then

$$\mathcal{R} = -\left(\frac{i}{2\omega}\right)^{N+2}\Delta\frac{d^N V}{dr_*^N} = e^{iN\pi/2}(2\omega)^{-N-2}\Delta\frac{d^N V}{dr_*^N}. \tag{2}$$

A resonance occurs for $\lambda\in\mathbb{C}$ such that the Wronskian of $\phi^{\rm out}_{\omega,\lambda-1/2}$ and the regular interior solution

$$\phi_{\omega,\lambda-1/2} \underset{\omega\to\infty}{=} \omega r_* j_{\lambda-1/2}(\omega r_*), \tag{3}$$

vanishes, i.e.

$$e^{i\pi(\lambda-1/2)-2i\omega R_*} = -\mathcal{R}. \tag{4}$$

Let us write

$$\Delta\frac{d^N V}{dr_*^N} = e^{i\beta}\Delta V^{(N)}, \qquad \Delta V^{(N)} \equiv \left|\Delta\frac{d^N V}{dr_*^N}\right|, \tag{5}$$

Substituting eq. (2) into eq. (4) and assuming $\Delta\frac{d^N V}{dr_*^N}$ is independent of λ, we obtain

$$\lambda_n \approx \frac{2\omega R_*}{\pi} - \left(2n - \frac{N-1}{2} - \beta\right) + \frac{i}{\pi}\ln\left[(2\omega)^{N+2}\left(\Delta V^{(N)}\right)^{-1}\right]. \tag{6}$$

Thus the imaginary part

$$\mathrm{Im}[\lambda_n] \approx \frac{1}{\pi}\left[(N+2)\ln\omega - \ln\left(\Delta V^{(N)}\right)\right]. \tag{7}$$

As the potential becomes 'more smooth' and $N\to\infty$, $\mathrm{Im}[\lambda_n]\to\infty$.