$tmvar$, x, y, f, m, n	variables
$covar$, c	coercion variables
$datacon$, K	
$const$, T, F	
$index$, i	indices

relflag, ρ	::=			relevance flag
	|	$+$		
	|	$-$		
	|	$app\_rho\nu$	S	
	|	(ρ)	S	
appflag, ν	::=			applicative flag
	|	R		
	|	ρ		
role, R	::=			Role
	|	**Nom**		
	|	**Rep**		
	|	$R_1 \cap R_2$	S	
	|	**param** R_1 R_2	S	
	|	$app\_role\nu$	S	
	|	(R)	S	
constraint, ϕ	::=			props
	|	$a \sim_{A/R} b$		
	|	(ϕ)	S	
	|	$\phi\{b/x\}$	S	
	|	$\lvert\phi\rvert$	S	
	|	$a \sim_R b$	S	
tm, a, b, p, v, w, A, B, C	::=			types and kinds
	|	$\star$		
	|	x		
	|	$\lambda^\rho x\!:\!A.b$	bind x in b	
	|	$\lambda^\rho x.b$	bind x in b	
	|	$a\ b^\nu$		
	|	$\Pi^\rho x\!:\!A \to B$	bind x in B	
	|	$\Lambda c\!:\!\phi.b$	bind c in b	
	|	$\Lambda c.b$	bind c in b	
	|	$a[\gamma]$		
	|	$\forall c\!:\!\phi.B$	bind c in B	
	|	$a \triangleright_R \gamma$		
	|	F		
	|	$\square$		
	|	$\mathsf{case}_R\ a\ \mathsf{of}\ F \to b_1 \Vert_{\_} \to b_2$		
	|	K		
	|	**match** a **with** brs		
	|	**sub** R a		
	|	$a\{b/x\}$	S	
	|	$a\{\gamma/c\}$	S	
	|	$a\{b/x\}$	S	
	|	$a\{\gamma/c\}$	S	

	$\vert$	a	S	
	$\vert$	a	S	
	$\vert$	(a)	S	
	$\vert$	a	S	parsing precedence is hard
	$\vert$	$\vert a\vert_R$	S	
	$\vert$	$\mathbf{Int}$	S	
	$\vert$	$\mathbf{Bool}$	S	
	$\vert$	$\mathbf{Nat}$	S	
	$\vert$	$\mathbf{Vec}$	S	
	$\vert$	0	S	
	$\vert$	S	S	
	$\vert$	$\mathbf{True}$	S	
	$\vert$	$\mathbf{Fix}$	S	
	$\vert$	$\mathbf{Age}$	S	
	$\vert$	$a \to b$	S	
	$\vert$	$\phi \Rightarrow A$	S	
	$\vert$	$a\ b$	S	
	$\vert$	$\lambda x.a$	S	
	$\vert$	$\lambda x : A.a$	S	
	$\vert$	$\forall\, x : A \to B$	S	
	$\vert$	$\mathbf{if}\, \phi\, \mathbf{then}\, a\, \mathbf{else}\, b$	S	
brs	$::=$			case branches
	$\vert$	$\mathbf{none}$		
	$\vert$	$K \Rightarrow a; brs$		
	$\vert$	$brs\{a/x\}$	S	
	$\vert$	$brs\{\gamma/c\}$	S	
	$\vert$	(brs)	S	
$co,\ \gamma$	$::=$			explicit coercions
	$\vert$	$\bullet$		
	$\vert$	c		
	$\vert$	$\mathbf{red}\, a\, b$		
	$\vert$	$\mathbf{refl}\, a$		
	$\vert$	$(a \mid=\mid_{\gamma}\ b)$		
	$\vert$	$\mathbf{sym}\, \gamma$		
	$\vert$	$\gamma_1 ; \gamma_2$		
	$\vert$	$\mathbf{sub}\, \gamma$		
	$\vert$	$\Pi^{R,\rho} x : \gamma_1 . \gamma_2$	bind x in γ_2	
	$\vert$	$\lambda^{R,\rho} x : \gamma_1 . \gamma_2$	bind x in γ_2	
	$\vert$	$\gamma_1\ \gamma_2^{R,\rho}$		
	$\vert$	$\mathbf{piFst}\, \gamma$		
	$\vert$	$\mathbf{cpiFst}\, \gamma$		
	$\vert$	$\mathbf{isoSnd}\, \gamma$		
	$\vert$	$\gamma_1 @ \gamma_2$		
	$\vert$	$\forall c : \gamma_1 . \gamma_3$	bind c in γ_3	

	|	$\lambda c{:}\gamma_1.\gamma_3@\gamma_4$	bind c in γ_3	
	|	$\gamma(\gamma_1, \gamma_2)$		
	|	$\gamma@(\gamma_1 \sim \gamma_2)$		
	|	$\gamma_1 \triangleright_R \gamma_2$		
	|	$\gamma_1 \sim_A \gamma_2$		
	|	$\mathbf{conv}\ \phi_1 \sim_\gamma \phi_2$		
	|	$\mathbf{eta}\, a$		
	|	$\mathbf{left}\, \gamma\, \gamma'$		
	|	$\mathbf{right}\, \gamma\, \gamma'$		
	|	(γ)	S	
	|	γ	S	
	|	$\gamma\{a/x\}$	S	
role_context, Ω	::=			role$_c$*ontexts*
	|	$\varnothing$		
	|	$x : R$		
	|	$\Omega, x : R$		
	|	Ω, Ω'	M	
	|	*var_patp*	M	
	|	(Ω)	M	
	|	Ω	M	
roles, Rs	::=			
	|	$\mathbf{nilR}$		
	|	R, Rs		
	|	$\mathbf{range}\, \Omega$	S	
sig_sort	::=			signature classifier
	|	$A@Rs$		
	|	$p \sim a : A/R@Rs$		
sort	::=			binding classifier
	|	$\mathbf{Tm}\, A$		
	|	$\mathbf{Co}\, \phi$		
context, Γ	::=			contexts
	|	$\varnothing$		
	|	$\Gamma, x : A$		
	|	$\Gamma, c : \phi$		
	|	$\Gamma\{b/x\}$	M	
	|	$\Gamma\{\gamma/c\}$	M	
	|	Γ, Γ'	M	
	|	$\|\Gamma\|$	M	
	|	(Γ)	M	
	|	Γ	M	
sig, Σ	::=			signatures

	$\mid$	$\varnothing$	
	$\mid$	$\Sigma \cup \{F : sig\_sort\}$	
	$\mid$	Σ_0	M
	$\mid$	Σ_1	M
	$\mid$	$\lvert\Sigma\rvert$	M
$available\_props,\ \Delta$	$::=$		
	$\mid$	$\varnothing$	
	$\mid$	Δ, x	
	$\mid$	Δ, c	
	$\mid$	$\mathsf{fv}\, a$	M
	$\mid$	Δ, Δ'	M
	$\mid$	$\widetilde{\Gamma}$	M
	$\mid$	$\widetilde{\Omega}$	M
	$\mid$	(Δ)	M
$terminals$	$::=$		
	$\mid$	$\leftrightarrow$	
	$\mid$	$\Leftrightarrow$	
	$\mid$	$\longrightarrow$	
	$\mid$	min	
	$\mid$	$\equiv$	
	$\mid$	$\forall$	
	$\mid$	$\in$	
	$\mid$	$\not\in$	
	$\mid$	$\Leftarrow$	
	$\mid$	$\Rightarrow$	
	$\mid$	$\Rightarrow^*$	
	$\mid$	$\rightarrow$	
	$\mid$	Λ	
	$\mid$		
	$\mid$	$\Box$	
	$\mid$	$\vdash$	
	$\mid$	$\dashv$	
	$\mid$	$\models$	
	$\mid$	$\models$	
	$\mid$	$\neq$	
	$\mid$	$\triangleright$	
	$\mid$	ok	
	$\mid$	$\_$	
	$\mid$	$\rightsquigarrow$	
	$\mid$	$\rightsquigarrow^*$	
	$\mid$	$\rightsquigarrow$	
	$\mid$	$\varnothing$	
	$\mid$	$\circ$	
	$\mid$	fv	

	$\mid$	dom	
	$\mid$	$\sim$	
	$\mid$	$\asymp$	
	$\mid$	$\mid$	
	$\mid$	$\bullet$	
	$\mid$	$\mathbf{fst}$	
	$\mid$	$\mathbf{snd}$	
	$\mid$	$\mathbf{as}$	
	$\mid$	$\mid\Rightarrow\mid$	
	$\mid$	$\vdash_=$	
	$\mid$	$\mathbf{refl_2}$	
	$\mid$	$++$	
	$\mid$	$\{$	
	$\mid$	$\}$	
$\mathit{formula},\ \psi$	$::=$		
	$\mid$	$\mathit{judgement}$	
	$\mid$	$x : A\ \in\ \Gamma$	
	$\mid$	$x : R\ \in\ \Omega$	
	$\mid$	$c : \phi\ \in\ \Gamma$	
	$\mid$	$F : \mathit{sig\_sort}\ \in\ \Sigma$	
	$\mid$	$x\ \in\ \Delta$	
	$\mid$	$c\ \in\ \Delta$	
	$\mid$	$c\,\mathbf{not\,relevant}\ \in\ \gamma$	
	$\mid$	$x\ \notin\ \Delta$	
	$\mid$	$\mathit{uniq}\,\Gamma$	
	$\mid$	$\mathit{uniq}(\Omega)$	
	$\mid$	$c\ \notin\ \Delta$	
	$\mid$	$T\ \notin\ \mathsf{dom}\,\Sigma$	
	$\mid$	$F\ \notin\ \mathsf{dom}\,\Sigma$	
	$\mid$	$R_1 = R_2$	
	$\mid$	$a = b$	
	$\mid$	$\phi_1 = \phi_2$	
	$\mid$	$\Gamma_1 = \Gamma_2$	
	$\mid$	$\gamma_1 = \gamma_2$	
	$\mid$	$\neg\psi$	
	$\mid$	$\psi_1 \wedge \psi_2$	
	$\mid$	$\psi_1 \vee \psi_2$	
	$\mid$	$\psi_1 \Rightarrow \psi_2$	
	$\mid$	(ψ)	
	$\mid$	ψ	
	$\mid$	$c : (a : A \sim b : B)\ \in\ \Gamma$	
	$\mid$		suppress lc hypothesis generated by Ott
$\mathit{JSubRole}$	$::=$		
	$\mid$	$R_1 \leq R_2$	Subroling judgement

$JPath$	$::=$		
	$\vert$	$\mathsf{Path}\ a = F@Rs$	Type headed by constant (partial functi
$JCasePath$	$::=$		
	$\vert$	$\mathsf{CasePath}_R\ a = F$	Type headed by constant (role-sensitive
$JPatCtx$	$::=$		
	$\vert$	$\Omega; \Gamma \vDash p :_F B \Rightarrow A$	Contexts generated by a pattern (variab
$JRename$	$::=$		
	$\vert$	$\mathsf{rename}\ p \to a\ \mathsf{to}\ p' \to a'\ \mathsf{excluding}\ \Delta$	rename with fresh variables
$JMatchSubst$	$::=$		
	$\vert$	$\mathsf{match}\ a_1\ \mathsf{with}\ p \to b_1 = b_2$	match and substitute
$JValuePath$	$::=$		
	$\vert$	$\mathsf{ValuePath}\ a = F$	Type headed by constant (role-sensitive
$JApplyArgs$	$::=$		
	$\vert$	$\mathsf{apply\ args}\ a\ \mathsf{to}\ b \mapsto b'$	apply arguments of a (headed by a cons
$JValue$	$::=$		
	$\vert$	$\mathsf{Value}_R\ A$	values
$JValueType$	$::=$		
	$\vert$	$\mathsf{ValueType}_R\ A$	Types with head forms (erased languag
$Jconsistent$	$::=$		
	$\vert$	$\mathsf{consistent}_R\ a\ b$	(erased) types do not differ in their hea
$Jroleing$	$::=$		
	$\vert$	$\Omega \vDash a : R$	Roleing judgment
$JChk$	$::=$		
	$\vert$	$(\rho = +) \vee (x \notin \mathsf{fv}\ A)$	irrelevant argument check
$Jpar$	$::=$		
	$\vert$	$\Omega \vDash a \Rightarrow_R b$	parallel reduction (implicit language)
	$\vert$	$\Omega \vDash a \Rightarrow^*_R b$	multistep parallel reduction
	$\vert$	$\Omega \vDash a \Leftrightarrow_R b$	parallel reduction to a common term
$Jbeta$	$::=$		
	$\vert$	$\vDash a > b/R$	primitive reductions on erased terms
	$\vert$	$\vDash a \leadsto b/R$	single-step head reduction for implicit la
	$\vert$	$\vDash a \leadsto^* b/R$	multistep reduction
$JBranchTyping$	$::=$		

	$\mid$	$\Gamma \vDash \mathsf{case}_R\ a : A\ \mathsf{of}\ b : B \Rightarrow C \mid C'$	Branch Typing (aligning the types of case)
Jett	$::=$		
	$\mid$	$\Gamma \vDash \phi\ \mathsf{ok}$	Prop wellformedness
	$\mid$	$\Gamma \vDash a : A$	typing
	$\mid$	$\Gamma; \Delta \vDash \phi_1 \equiv \phi_2$	prop equality
	$\mid$	$\Gamma; \Delta \vDash a \equiv b : A/R$	definitional equality
	$\mid$	$\vDash \Gamma$	context wellformedness
Jsig	$::=$		
	$\mid$	$\vDash \Sigma$	signature wellformedness
Jann	$::=$		
	$\mid$	$\Gamma \vdash \phi\ \mathsf{ok}$	prop wellformedness
	$\mid$	$\Gamma \vdash a : A/R$	typing
	$\mid$	$\Gamma; \Delta \vdash \gamma : \phi_1 \sim \phi_2$	coercion between props
	$\mid$	$\Gamma; \Delta \vdash \gamma : A \sim_R B$	coercion between types
	$\mid$	$\vdash \Gamma$	context wellformedness
Jred	$::=$		
	$\mid$	$\Gamma \vdash a \rightsquigarrow b/R$	single-step, weak head reduction to values for a
judgement	$::=$		
	$\mid$	*JSubRole*	
	$\mid$	*JPath*	
	$\mid$	*JCasePath*	
	$\mid$	*JPatCtx*	
	$\mid$	*JRename*	
	$\mid$	*JMatchSubst*	
	$\mid$	*JValuePath*	
	$\mid$	*JApplyArgs*	
	$\mid$	*JValue*	
	$\mid$	*JValueType*	
	$\mid$	*Jconsistent*	
	$\mid$	*Jroleing*	
	$\mid$	*JChk*	
	$\mid$	*Jpar*	
	$\mid$	*Jbeta*	
	$\mid$	*JBranchTyping*	
	$\mid$	*Jett*	
	$\mid$	*Jsig*	
	$\mid$	*Jann*	
	$\mid$	*Jred*	
user_syntax	$::=$		
	$\mid$	*tmvar*	
	$\mid$	*covar*	

| *datacon*
| *const*
| *index*
| *relflag*
| *appflag*
| *role*
| *constraint*
| *tm*
| *brs*
| *co*
| *role_context*
| *roles*
| *sig_sort*
| *sort*
| *context*
| *sig*
| *available_props*
| *terminals*
| *formula*

$\boxed{R_1 \leq R_2}$ Subroling judgement

$$\frac{}{\mathbf{Nom} \leq R} \quad \textsc{NomBot}$$

$$\frac{}{R \leq \mathbf{Rep}} \quad \textsc{RepTop}$$

$$\frac{}{R \leq R} \quad \textsc{Refl}$$

$$\frac{\begin{array}{c} R_1 \leq R_2 \\ R_2 \leq R_3 \end{array}}{R_1 \leq R_3} \quad \textsc{Trans}$$

$\boxed{\mathsf{Path}\ a = F@Rs}$ Type headed by constant (partial function)

$$\frac{F : A@Rs \in \Sigma_0}{\mathsf{Path}\ F = F@Rs} \quad \textsc{Path\_AbsConst}$$

$$\frac{F : p \sim a : A/R_1@Rs \in \Sigma_0}{\mathsf{Path}\ F = F@Rs} \quad \textsc{Path\_Const}$$

$$\frac{\mathsf{Path}\ a = F@R_1, Rs}{\mathsf{Path}\ (a\ b'^{R_1}) = F@Rs} \quad \textsc{Path\_App}$$

$$\frac{\mathsf{Path}\ a = F@Rs}{\mathsf{Path}\ (a\ \square^{-}) = F@Rs} \quad \textsc{Path\_IApp}$$

$$\frac{\mathsf{Path}\ a = F@Rs}{\mathsf{Path}\ (a[\bullet]) = F@Rs} \quad \textsc{Path\_CApp}$$

$\boxed{\mathsf{CasePath}_R\ a = F}$ Type headed by constant (role-sensitive partial function used in case)

$$\frac{F : A@Rs \in \Sigma_0}{\mathsf{CasePath}_R\ F = F} \quad \textsc{CasePath\_AbsConst}$$

$$\frac{F :\ p \sim a : A/R_1@Rs\ \in\ \Sigma_0 \quad \neg(R_1 \leq R)}{\mathsf{CasePath}_R\ F = F}\ \textsc{CasePath\_Const}$$

$$\frac{\mathsf{CasePath}_R\ a = F}{\mathsf{CasePath}_R\ (a\ b'^{\rho}) = F}\ \textsc{CasePath\_App}$$

$$\frac{\mathsf{CasePath}_R\ a = F}{\mathsf{CasePath}_R\ (a[\bullet]) = F}\ \textsc{CasePath\_CApp}$$

$\boxed{\Omega;\Gamma \vDash p :_F B \Rightarrow A}$ Contexts generated by a pattern (variables bound by the pattern)

$$\frac{}{\varnothing;\varnothing \vDash F :_F A \Rightarrow A}\ \textsc{PatCtx\_Const}$$

$$\frac{\Omega;\Gamma \vDash p :_F \Pi^+ x\!:\!A' \to A \Rightarrow B}{\Omega, x : R;\Gamma, x : A' \vDash p\ x^R :_F A \Rightarrow B}\ \textsc{PatCtx\_PiRel}$$

$$\frac{\Omega;\Gamma \vDash p :_F \Pi^- x\!:\!A' \to A \Rightarrow B}{\Omega;\Gamma, x : A' \vDash p\ \Box^- :_F A \Rightarrow B}\ \textsc{PatCtx\_PiIrr}$$

$$\frac{\Omega;\Gamma \vDash p :_F \forall c\!:\!\phi.A \Rightarrow B}{\Omega;\Gamma, c : \phi \vDash p[\bullet] :_F A \Rightarrow B}\ \textsc{PatCtx\_CPi}$$

$\boxed{\mathsf{rename}\ p \to a\ \mathsf{to}\ p' \to a'\ \mathsf{excluding}\ \Delta}$ rename with fresh variables

$$\frac{}{\mathsf{rename}\ F \to a\ \mathsf{to}\ F \to a\ \mathsf{excluding}\ \Delta}\ \textsc{Rename\_Base}$$

$$\frac{\mathsf{rename}\ p_1 \to a_1\ \mathsf{to}\ p_2 \to a_2\ \mathsf{excluding}\ \Delta \quad y \notin \Delta}{\mathsf{rename}\ (p_1\ x^R) \to a_1\ \mathsf{to}\ (p_2\ y^R) \to (a_2\{y/x\})\ \mathsf{excluding}\ (\Delta, y)}\ \textsc{Rename\_AppRel}$$

$$\frac{\mathsf{rename}\ p_1 \to a_1\ \mathsf{to}\ p_2 \to a_2\ \mathsf{excluding}\ \Delta}{\mathsf{rename}\ (p_1\ \Box^-) \to a_1\ \mathsf{to}\ (p_2\ \Box^-) \to a_2\ \mathsf{excluding}\ \Delta}\ \textsc{Rename\_AppIrrel}$$

$$\frac{\mathsf{rename}\ p_1 \to a_1\ \mathsf{to}\ p_2 \to a_2\ \mathsf{excluding}\ \Delta}{\mathsf{rename}\ (p_1[\bullet]) \to a_1\ \mathsf{to}\ (p_2[\bullet]) \to a_2\ \mathsf{excluding}\ \Delta}\ \textsc{Rename\_CApp}$$

$\boxed{\mathsf{match}\ a_1\ \mathsf{with}\ p \to b_1 = b_2}$ match and substitute

$$\frac{}{\mathsf{match}\ F\ \mathsf{with}\ F \to b = b}\ \textsc{MatchSubst\_Const}$$

$$\frac{\mathsf{match}\ a_1\ \mathsf{with}\ a_2 \to b_1 = b_2}{\mathsf{match}\ (a_1\ a^R)\ \mathsf{with}\ (a_2\ x^R) \to b_1 = (b_2\{a/x\})}\ \textsc{MatchSubst\_AppRelR}$$

$$\frac{\mathsf{match}\ a_1\ \mathsf{with}\ a_2 \to b_1 = b_2}{\mathsf{match}\ (a_1\ \Box^-)\ \mathsf{with}\ (a_2\ \Box^-) \to b_1 = b_2}\ \textsc{MatchSubst\_AppIrrel}$$

$$\frac{\mathsf{match}\ a_1\ \mathsf{with}\ a_2 \to b_1 = b_2}{\mathsf{match}\ (a_1[\bullet])\ \mathsf{with}\ (a_2[\bullet]) \to b_1 = b_2}\ \textsc{MatchSubst\_CApp}$$

$\boxed{\mathsf{ValuePath}\ a = F}$ Type headed by constant (role-sensitive partial function used in value)

$$\frac{F :\ A@Rs\ \in\ \Sigma_0}{\mathsf{ValuePath}\ F = F}\ \textsc{ValuePath\_AbsConst}$$

$$\frac{F :\ p \sim a : A/R_1@Rs\ \in\ \Sigma_0}{\mathsf{ValuePath}\ F = F}\ \textsc{ValuePath\_Const}$$

$$\frac{\mathsf{ValuePath}\ a = F}{\mathsf{ValuePath}\ (a\ b'^{\nu}) = F}\ \textsc{ValuePath\_App}$$

$$\frac{\mathsf{ValuePath}\ a = F}{\mathsf{ValuePath}\ (a[\bullet]) = F}\ \textsc{ValuePath\_CApp}$$

$\boxed{\mathsf{apply\ args}\ a\ \mathsf{to}\ b \mapsto b'}$ apply arguments of a (headed by a constant) to b

$$\frac{}{\mathsf{apply\ args}\ F\ \mathsf{to}\ b \mapsto b}\ \textsc{ApplyArgs\_Const}$$

$$\frac{\mathsf{apply\ args}\ a\ \mathsf{to}\ b \mapsto b'}{\mathsf{apply\ args}\ a\ a'^{\rho}\ \mathsf{to}\ b \mapsto b'\ a'^{\rho}}\ \textsc{ApplyArgs\_App}$$

$$\frac{\mathsf{apply\ args}\ a\ \mathsf{to}\ b \mapsto b'}{\mathsf{apply\ args}\ a[\bullet]\ \mathsf{to}\ b \mapsto b'[\bullet]}\ \textsc{ApplyArgs\_CApp}$$

$\boxed{\mathsf{Value}_R\ A}$ values

$$\frac{}{\mathsf{Value}_R\ \star}\ \textsc{Value\_Star}$$

$$\frac{}{\mathsf{Value}_R\ \Pi^{\rho} x\!:\!A \to B}\ \textsc{Value\_Pi}$$

$$\frac{}{\mathsf{Value}_R\ \forall c\!:\!\phi.B}\ \textsc{Value\_CPi}$$

$$\frac{}{\mathsf{Value}_R\ \lambda^{+} x\!:\!A.a}\ \textsc{Value\_AbsRel}$$

$$\frac{}{\mathsf{Value}_R\ \lambda^{+} x.a}\ \textsc{Value\_UAbsRel}$$

$$\frac{\mathsf{Value}_R\ a}{\mathsf{Value}_R\ \lambda^{-} x.a}\ \textsc{Value\_UAbsIrrel}$$

$$\frac{}{\mathsf{Value}_R\ \Lambda c\!:\!\phi.a}\ \textsc{Value\_CAbs}$$

$$\frac{}{\mathsf{Value}_R\ \Lambda c.a}\ \textsc{Value\_UCAbs}$$

$$\frac{\begin{array}{c}\mathsf{ValuePath}\ a = F \\ F : A @ Rs \in \Sigma_0\end{array}}{\mathsf{Value}_R\ a}\ \textsc{Value\_Const}$$

$$\frac{\begin{array}{l}\mathsf{ValuePath}\ a = F \\ F :\ p \sim b : A / R_1 @ Rs \in \Sigma_0 \\ \neg(\mathsf{match}\ a\ \mathsf{with}\ p \to \square = \square)\end{array}}{\mathsf{Value}_R\ a}\ \textsc{Value\_Path}$$

$$\frac{\begin{array}{l}\mathsf{ValuePath}\ a = F \\ F :\ p \sim b : A / R_1 @ Rs \in \Sigma_0 \\ \mathsf{match}\ a\ \mathsf{with}\ p \to \square = \square \\ \neg(R_1 \leq R)\end{array}}{\mathsf{Value}_R\ a}\ \textsc{Value\_PathMatch}$$

$\boxed{\mathsf{ValueType}_R\ A}$ Types with head forms (erased language)

$$\frac{}{\mathsf{ValueType}_R\ \star}\ \textsc{value\_type\_Star}$$

$$\frac{}{\mathsf{ValueType}_R\ \Pi^{\rho} x\!:\!A \to B}\ \textsc{value\_type\_Pi}$$

$$\frac{}{\mathsf{ValueType}_R\ \forall c\!:\!\phi.B}\quad \textsc{value\_type\_CPi}$$

$$\frac{\mathsf{ValuePath}\ \ a = F}{\mathsf{ValueType}_R\ \ a}\quad \textsc{value\_type\_ValuePath}$$

$\boxed{\mathsf{consistent}_R\ a\ b}$ (erased) types do not differ in their heads

$$\frac{}{\mathsf{consistent}_R\ \star\ \star}\quad \textsc{consistent\_a\_Star}$$

$$\frac{}{\mathsf{consistent}_{R'}\ (\Pi^\rho x_1\!:\!A_1 \to B_1)\ (\Pi^\rho x_2\!:\!A_2 \to B_2)}\quad \textsc{consistent\_a\_Pi}$$

$$\frac{}{\mathsf{consistent}_R\ (\forall c_1\!:\!\phi_1.A_1)\ (\forall c_2\!:\!\phi_2.A_2)}\quad \textsc{consistent\_a\_CPi}$$

$$\frac{\begin{array}{l}\mathsf{ValuePath}\ \ a_1 = F\\ \mathsf{ValuePath}\ \ a_2 = F\end{array}}{\mathsf{consistent}_R\ \ a_1\ \ a_2}\quad \textsc{consistent\_a\_ValuePath}$$

$$\frac{\neg\mathsf{ValueType}_R\ \ b}{\mathsf{consistent}_R\ \ a\ \ b}\quad \textsc{consistent\_a\_Step\_R}$$

$$\frac{\neg\mathsf{ValueType}_R\ \ a}{\mathsf{consistent}_R\ \ a\ \ b}\quad \textsc{consistent\_a\_Step\_L}$$

$\boxed{\Omega \vDash a : R}$ Roleing judgment

$$\frac{uniq(\Omega)}{\Omega \vDash \Box : R}\quad \textsc{role\_a\_Bullet}$$

$$\frac{uniq(\Omega)}{\Omega \vDash \star : R}\quad \textsc{role\_a\_Star}$$

$$\frac{\begin{array}{l}uniq(\Omega)\\ x : R\ \in\ \Omega\\ R \le R_1\end{array}}{\Omega \vDash x : R_1}\quad \textsc{role\_a\_Var}$$

$$\frac{\Omega, x : \mathbf{Nom} \vDash a : R}{\Omega \vDash (\lambda^\rho x.a) : R}\quad \textsc{role\_a\_Abs}$$

$$\frac{\begin{array}{l}\Omega \vDash a : R\\ \Omega \vDash b : \mathbf{Nom}\end{array}}{\Omega \vDash (a\ \ b^\rho) : R}\quad \textsc{role\_a\_App}$$

$$\frac{\begin{array}{l}\Omega \vDash a : R\\ \mathsf{Path}\ \ a = F@R_1, Rs\\ \Omega \vDash b : R_1\end{array}}{\Omega \vDash a\ \ b^{R_1} : R}\quad \textsc{role\_a\_TApp}$$

$$\frac{\begin{array}{l}\Omega \vDash A : R\\ \Omega, x : \mathbf{Nom} \vDash B : R\end{array}}{\Omega \vDash (\Pi^\rho x\!:\!A \to B) : R}\quad \textsc{role\_a\_Pi}$$

$$\frac{\begin{array}{l}\Omega \vDash a : R_1\\ \Omega \vDash b : R_1\\ \Omega \vDash A : R_0\\ \Omega \vDash B : R\end{array}}{\Omega \vDash (\forall c\!:\!a \sim_{A/R_1} b.B) : R}\quad \textsc{role\_a\_CPi}$$

$$\frac{\Omega \vDash b : R}{\Omega \vDash (\Lambda c.b) : R} \quad \text{ROLE\_A\_CABS}$$

$$\frac{\Omega \vDash a : R}{\Omega \vDash (a[\bullet]) : R} \quad \text{ROLE\_A\_CAPP}$$

$$\frac{\begin{array}{l} \mathit{uniq}(\Omega) \\ F : A@Rs \in \Sigma_0 \end{array}}{\Omega \vDash F : R} \quad \text{ROLE\_A\_CONST}$$

$$\frac{\begin{array}{l} \mathit{uniq}(\Omega) \\ F : p \sim a : A/R@Rs \in \Sigma_0 \end{array}}{\Omega \vDash F : R_1} \quad \text{ROLE\_A\_FAM}$$

$$\frac{\begin{array}{l} \Omega \vDash a : R \\ \Omega \vDash b_1 : R_1 \\ \Omega \vDash b_2 : R_1 \end{array}}{\Omega \vDash \mathsf{case}_R\ a \ \mathsf{of}\ F \to b_1 \|_{\_} \to b_2 : R_1} \quad \text{ROLE\_A\_PATTERN}$$

$\boxed{(\rho = +) \vee (x \notin \mathsf{fv}\ A)}$ irrelevant argument check

$$\frac{}{(+ = +) \vee (x \notin \mathsf{fv}\ A)} \quad \text{RHO\_REL}$$

$$\frac{x \notin \mathsf{fv} A}{(- = +) \vee (x \notin \mathsf{fv}\ A)} \quad \text{RHO\_IRRREL}$$

$\boxed{\Omega \vDash a \Rightarrow_R b}$ parallel reduction (implicit language)

$$\frac{\Omega \vDash a : R}{\Omega \vDash a \Rightarrow_R a} \quad \text{PAR\_REFL}$$

$$\frac{\begin{array}{l} \Omega \vDash a \Rightarrow_R (\lambda^\rho x.a') \\ \Omega \vDash b \Rightarrow_{\mathbf{Nom}} b' \end{array}}{\Omega \vDash a\ b^\rho \Rightarrow_R a'\{b'/x\}} \quad \text{PAR\_BETA}$$

$$\frac{\begin{array}{l} \Omega \vDash a \Rightarrow_R a' \\ \Omega \vDash b \Rightarrow_{\mathbf{Nom}} b' \end{array}}{\Omega \vDash a\ b^\rho \Rightarrow_R a'\ b'^\rho} \quad \text{PAR\_APP}$$

$$\frac{\Omega \vDash a \Rightarrow_R (\Lambda c.a')}{\Omega \vDash a[\bullet] \Rightarrow_R a'\{\bullet/c\}} \quad \text{PAR\_CBETA}$$

$$\frac{\Omega \vDash a \Rightarrow_R a'}{\Omega \vDash a[\bullet] \Rightarrow_R a'[\bullet]} \quad \text{PAR\_CAPP}$$

$$\frac{\Omega, x : \mathbf{Nom} \vDash a \Rightarrow_R a'}{\Omega \vDash \lambda^\rho x.a \Rightarrow_R \lambda^\rho x.a'} \quad \text{PAR\_ABS}$$

$$\frac{\begin{array}{l} \Omega \vDash A \Rightarrow_R A' \\ \Omega, x : \mathbf{Nom} \vDash B \Rightarrow_R B' \end{array}}{\Omega \vDash \Pi^\rho x\!:\!A \to B \Rightarrow_R \Pi^\rho x\!:\!A' \to B'} \quad \text{PAR\_PI}$$

$$\frac{\Omega \vDash a \Rightarrow_R a'}{\Omega \vDash \Lambda c.a \Rightarrow_R \Lambda c.a'} \quad \text{PAR\_CABS}$$

$$\frac{\begin{array}{c}\Omega \vDash A \Rightarrow_{R_0} A' \\ \Omega \vDash a \Rightarrow_{R_1} a' \\ \Omega \vDash b \Rightarrow_{R_1} b' \\ \Omega \vDash B \Rightarrow_{R} B'\end{array}}{\Omega \vDash \forall c\!:\!a \sim_{A/R_1} b.B \Rightarrow_R \forall c\!:\!a' \sim_{A'/R_1} b'.B'} \quad \textsc{Par\_CPi}$$

$$\frac{\begin{array}{l}F : p \sim b : A/R_1@Rs \in \Sigma_0 \\ \Omega \vDash a : R \\ \mathsf{rename}\ p \to b\ \mathsf{to}\ p' \to b'\ \mathsf{excluding}\ (\widetilde{\Omega}, \mathsf{fv}p) \\ \mathsf{match}\ a\ \mathsf{with}\ p' \to b' = a' \\ R_1 \leq R\end{array}}{\Omega \vDash a \Rightarrow_R a'} \quad \textsc{Par\_Axiom}$$

$$\frac{\begin{array}{c}\Omega \vDash a \Rightarrow_R a' \\ \Omega \vDash b_1 \Rightarrow_{R_0} b_1' \\ \Omega \vDash b_2 \Rightarrow_{R_0} b_2'\end{array}}{\Omega \vDash (\mathsf{case}_R\ a\ \mathsf{of}\ F \to b_1 \| \_ \to b_2) \Rightarrow_{R_0} (\mathsf{case}_R\ a'\ \mathsf{of}\ F \to b_1' \| \_ \to b_2')} \quad \textsc{Par\_Pattern}$$

$$\frac{\begin{array}{l}\Omega \vDash a \Rightarrow_R a' \\ \Omega \vDash b_1 \Rightarrow_{R_0} b_1' \\ \Omega \vDash b_2 \Rightarrow_{R_0} b_2' \\ \mathsf{CasePath}_R\ a' = F \\ \mathsf{apply\ args}\ a'\ \mathsf{to}\ b_1' \mapsto b\end{array}}{\Omega \vDash (\mathsf{case}_R\ a\ \mathsf{of}\ F \to b_1 \| \_ \to b_2) \Rightarrow_{R_0} b[\bullet]} \quad \textsc{Par\_PatternTrue}$$

$$\frac{\begin{array}{l}\Omega \vDash a \Rightarrow_R a' \\ \Omega \vDash b_1 \Rightarrow_{R_0} b_1' \\ \Omega \vDash b_2 \Rightarrow_{R_0} b_2' \\ \mathsf{Value}_R\ a' \\ \neg(\mathsf{CasePath}_R\ a' = F)\end{array}}{\Omega \vDash (\mathsf{case}_R\ a\ \mathsf{of}\ F \to b_1 \| \_ \to b_2) \Rightarrow_{R_0} b_2'} \quad \textsc{Par\_PatternFalse}$$

$\boxed{\Omega \vDash a \Rightarrow^*_R b}$ multistep parallel reduction

$$\frac{}{\Omega \vDash a \Rightarrow^*_R a} \quad \textsc{MP\_Refl}$$

$$\frac{\begin{array}{c}\Omega \vDash a \Rightarrow_R b \\ \Omega \vDash b \Rightarrow^*_R a'\end{array}}{\Omega \vDash a \Rightarrow^*_R a'} \quad \textsc{MP\_Step}$$

$\boxed{\Omega \vDash a \Leftrightarrow_R b}$ parallel reduction to a common term

$$\frac{\begin{array}{c}\Omega \vDash a_1 \Rightarrow^*_R b \\ \Omega \vDash a_2 \Rightarrow^*_R b\end{array}}{\Omega \vDash a_1 \Leftrightarrow_R a_2} \quad \textsc{join}$$

$\boxed{\vDash a > b/R}$ primitive reductions on erased terms

$$\frac{\mathsf{Value}_{R_1}\ (\lambda^\rho x.v)}{\vDash (\lambda^\rho x.v)\ b^\rho > v\{b/x\}/R_1} \quad \textsc{Beta\_AppAbs}$$

$$\frac{}{\vDash (\Lambda c.a')[\bullet] > a'\{\bullet/c\}/R} \quad \textsc{Beta\_CAppCAbs}$$

$$\frac{\begin{array}{l}F : p \sim b : A/R_1@Rs \in \Sigma_0\\ \text{rename } p \to b \text{ to } p_1 \to b_1 \text{ excluding } (\mathsf{fv}\,a, \mathsf{fv}\,p)\\ \text{match } a \text{ with } p_1 \to b_1 = b'\\ R_1 \leq R\end{array}}{\vDash a > b'/R} \quad \textsc{Beta\_Axiom}$$

$$\frac{\begin{array}{c}\mathsf{CasePath}_R\ a = F\\ \text{apply args } a \text{ to } b_1 \mapsto b_1'\end{array}}{\vDash \mathsf{case}_R\ a \text{ of } F \to b_1 \| \_ \to b_2 > b_1'[\bullet]/R_0} \quad \textsc{Beta\_PatternTrue}$$

$$\frac{\begin{array}{c}\mathsf{Value}_R\ a\\ \neg(\mathsf{CasePath}_R\ a = F)\end{array}}{\vDash \mathsf{case}_R\ a \text{ of } F \to b_1 \| \_ \to b_2 > b_2/R_0} \quad \textsc{Beta\_PatternFalse}$$

$\boxed{\vDash a \leadsto b/R}$ single-step head reduction for implicit language

$$\frac{\vDash a \leadsto a'/R_1}{\vDash \lambda^- x.a \leadsto \lambda^- x.a'/R_1} \quad \textsc{E\_AbsTerm}$$

$$\frac{\vDash a \leadsto a'/R_1}{\vDash a\ b^\rho \leadsto a'\ b^\rho/R_1} \quad \textsc{E\_AppLeft}$$

$$\frac{\vDash a \leadsto a'/R}{\vDash a[\bullet] \leadsto a'[\bullet]/R} \quad \textsc{E\_CAppLeft}$$

$$\frac{\vDash a \leadsto a'/R}{\vDash \mathsf{case}_R\ a \text{ of } F \to b_1 \| \_ \to b_2 \leadsto \mathsf{case}_R\ a' \text{ of } F \to b_1 \| \_ \to b_2/R_0} \quad \textsc{E\_Pattern}$$

$$\frac{\vDash a > b/R}{\vDash a \leadsto b/R} \quad \textsc{E\_Prim}$$

$\boxed{\vDash a \leadsto^* b/R}$ multistep reduction

$$\frac{}{\vDash a \leadsto^* a/R} \quad \textsc{Equal}$$

$$\frac{\begin{array}{c}\vDash a \leadsto b/R\\ \vDash b \leadsto^* a'/R\end{array}}{\vDash a \leadsto^* a'/R} \quad \textsc{Step}$$

$\boxed{\Gamma \vDash \mathsf{case}_R\ a : A \text{ of } b : B \Rightarrow C \mid C'}$ Branch Typing (aligning the types of case)

$$\frac{\begin{array}{c}\mathit{uniq}\ \Gamma\\ \texttt{lc\_tm}\ C\end{array}}{\Gamma \vDash \mathsf{case}_R\ a : A \text{ of } b : A \Rightarrow \forall c{:}(a \sim_{A/R} b).C \mid C} \quad \textsc{BranchTyping\_Base}$$

$$\frac{\Gamma, x : A \vDash \mathsf{case}_R\ a : A_1 \text{ of } b\ x^+ : B \Rightarrow C \mid C'}{\Gamma \vDash \mathsf{case}_R\ a : A_1 \text{ of } b : \Pi^+ x{:}A \to B \Rightarrow \Pi^+ x{:}A \to C \mid C'} \quad \textsc{BranchTyping\_PiRel}$$

$$\frac{\Gamma, x : A \vDash \mathsf{case}_R\ a : A_1 \text{ of } b\ \square^- : B \Rightarrow C \mid C'}{\Gamma \vDash \mathsf{case}_R\ a : A_1 \text{ of } b : \Pi^- x{:}A \to B \Rightarrow \Pi^- x{:}A \to C \mid C'} \quad \textsc{BranchTyping\_PiIrrel}$$

$$\frac{\Gamma, c : \phi \vDash \mathsf{case}_R\ a : A \text{ of } b[\bullet] : B \Rightarrow C \mid C'}{\Gamma \vDash \mathsf{case}_R\ a : A \text{ of } b : \forall c{:}\phi.B \Rightarrow \forall c{:}\phi.C \mid C'} \quad \textsc{BranchTyping\_CPi}$$

$\boxed{\Gamma \vDash \phi\ \ \mathsf{ok}}$ Prop wellformedness

$$\frac{\begin{array}{l}\Gamma \vDash a : A \\ \Gamma \vDash b : A \\ \Gamma \vDash A : \star\end{array}}{\Gamma \vDash a \sim_{A/R} b \ \ \mathsf{ok}} \quad \text{E\_WFF}$$

$\boxed{\Gamma \vDash a : A}$ typing

$$\frac{\vDash \Gamma}{\Gamma \vDash \star : \star} \quad \text{E\_STAR}$$

$$\frac{\begin{array}{l}\vDash \Gamma \\ x : A \in \Gamma\end{array}}{\Gamma \vDash x : A} \quad \text{E\_VAR}$$

$$\frac{\begin{array}{l}\Gamma, x : A \vDash B : \star \\ \Gamma \vDash A : \star\end{array}}{\Gamma \vDash \Pi^\rho x{:}A \to B : \star} \quad \text{E\_PI}$$

$$\frac{\begin{array}{l}\Gamma, x : A \vDash a : B \\ \Gamma \vDash A : \star \\ (\rho = +) \vee (x \notin \mathsf{fv}\ a)\end{array}}{\Gamma \vDash \lambda^\rho x.a : (\Pi^\rho x{:}A \to B)} \quad \text{E\_ABS}$$

$$\frac{\begin{array}{l}\Gamma \vDash b : \Pi^+ x{:}A \to B \\ \Gamma \vDash a : A\end{array}}{\Gamma \vDash b\ \ a^+ : B\{a/x\}} \quad \text{E\_APP}$$

$$\frac{\begin{array}{l}\Gamma \vDash b : \Pi^+ x{:}A \to B \\ \Gamma \vDash a : A \\ \mathsf{Path}\ \ b = F@R, Rs\end{array}}{\Gamma \vDash b\ \ a^R : B\{a/x\}} \quad \text{E\_TAPP}$$

$$\frac{\begin{array}{l}\Gamma \vDash b : \Pi^- x{:}A \to B \\ \Gamma \vDash a : A\end{array}}{\Gamma \vDash b\ \square^- : B\{a/x\}} \quad \text{E\_IAPP}$$

$$\frac{\begin{array}{l}\Gamma \vDash a : A \\ \Gamma; \widetilde{\Gamma} \vDash A \equiv B : \star/\mathbf{Rep} \\ \Gamma \vDash B : \star\end{array}}{\Gamma \vDash a : B} \quad \text{E\_CONV}$$

$$\frac{\begin{array}{l}\Gamma, c : \phi \vDash B : \star \\ \Gamma \vDash \phi \ \ \mathsf{ok}\end{array}}{\Gamma \vDash \forall c{:}\phi.B : \star} \quad \text{E\_CPI}$$

$$\frac{\begin{array}{l}\Gamma, c : \phi \vDash a : B \\ \Gamma \vDash \phi \ \ \mathsf{ok}\end{array}}{\Gamma \vDash \Lambda c.a : \forall c{:}\phi.B} \quad \text{E\_CABS}$$

$$\frac{\begin{array}{l}\Gamma \vDash a_1 : \forall c{:}(a \sim_{A/R} b).B_1 \\ \Gamma; \widetilde{\Gamma} \vDash a \equiv b : A/R\end{array}}{\Gamma \vDash a_1[\bullet] : B_1\{\bullet/c\}} \quad \text{E\_CAPP}$$

$$\frac{\begin{array}{l}\vDash \Gamma \\ F : A@Rs \in \Sigma_0 \\ \varnothing \vDash A : \star\end{array}}{\Gamma \vDash F : A} \quad \text{E\_CONST}$$

$$\frac{\begin{array}{l}\vDash \Gamma \\ F : p \sim a : A/R_1 @ Rs \in \Sigma_0\end{array}}{\Gamma \vDash F : A} \quad \text{E\_Fam}$$

$$\frac{\begin{array}{l}\Gamma \vDash a : A \\ \Gamma \vDash F : A_1 \\ \Gamma \vDash b_1 : B \\ \Gamma \vDash b_2 : C \\ \Gamma \vDash \mathsf{case}_R\ a : A\ \mathsf{of}\ F : A_1 \Rightarrow B \mid C\end{array}}{\Gamma \vDash \mathsf{case}_R\ a\ \mathsf{of}\ F \to b_1 \|_{\_} \to b_2 : C} \quad \text{E\_Case}$$

$\boxed{\Gamma; \Delta \vDash \phi_1 \equiv \phi_2}$ prop equality

$$\frac{\begin{array}{c}\Gamma; \Delta \vDash A_1 \equiv A_2 : A/R \\ \Gamma; \Delta \vDash B_1 \equiv B_2 : A/R\end{array}}{\Gamma; \Delta \vDash A_1 \sim_{A/R} B_1 \equiv A_2 \sim_{A/R} B_2} \quad \text{E\_PropCong}$$

$$\frac{\begin{array}{l}\Gamma; \Delta \vDash A \equiv B : \star/R_0 \\ \Gamma \vDash A_1 \sim_{A/R} A_2\ \ \mathsf{ok} \\ \Gamma \vDash A_1 \sim_{B/R} A_2\ \ \mathsf{ok}\end{array}}{\Gamma; \Delta \vDash A_1 \sim_{A/R} A_2 \equiv A_1 \sim_{B/R} A_2} \quad \text{E\_IsoConv}$$

$$\frac{\Gamma; \Delta \vDash \forall c\!:\!(a_1 \sim_{A/R_1} a_2).B_1 \equiv \forall c\!:\!(b_1 \sim_{B/R_2} b_2).B_2 : \star/R'}{\Gamma; \Delta \vDash a_1 \sim_{A/R_1} a_2 \equiv b_1 \sim_{B/R_2} b_2} \quad \text{E\_CPiFst}$$

$\boxed{\Gamma; \Delta \vDash a \equiv b : A/R}$ definitional equality

$$\frac{\begin{array}{l}\vDash \Gamma \\ c : (a \sim_{A/R} b) \in \Gamma \\ c \in \Delta\end{array}}{\Gamma; \Delta \vDash a \equiv b : A/R} \quad \text{E\_Assn}$$

$$\frac{\Gamma \vDash a : A}{\Gamma; \Delta \vDash a \equiv a : A/R} \quad \text{E\_Refl}$$

$$\frac{\Gamma; \Delta \vDash b \equiv a : A/R}{\Gamma; \Delta \vDash a \equiv b : A/R} \quad \text{E\_Sym}$$

$$\frac{\begin{array}{l}\Gamma; \Delta \vDash a \equiv a_1 : A/R \\ \Gamma; \Delta \vDash a_1 \equiv b : A/R\end{array}}{\Gamma; \Delta \vDash a \equiv b : A/R} \quad \text{E\_Trans}$$

$$\frac{\begin{array}{l}\Gamma; \Delta \vDash a \equiv b : A/R_1 \\ R_1 \leq R_2\end{array}}{\Gamma; \Delta \vDash a \equiv b : A/R_2} \quad \text{E\_Sub}$$

$$\frac{\begin{array}{l}\Gamma \vDash a_1 : B \\ \Gamma \vDash a_2 : B \\ \vDash a_1 > a_2/R\end{array}}{\Gamma; \Delta \vDash a_1 \equiv a_2 : B/R} \quad \text{E\_Beta}$$

$$\frac{\begin{array}{l}\Gamma; \Delta \vDash A_1 \equiv A_2 : \star/R' \\ \Gamma, x : A_1; \Delta \vDash B_1 \equiv B_2 : \star/R' \\ \Gamma \vDash A_1 : \star \\ \Gamma \vDash \Pi^\rho x\!:\!A_1 \to B_1 : \star \\ \Gamma \vDash \Pi^\rho x\!:\!A_2 \to B_2 : \star\end{array}}{\Gamma; \Delta \vDash (\Pi^\rho x\!:\!A_1 \to B_1) \equiv (\Pi^\rho x\!:\!A_2 \to B_2) : \star/R'} \quad \text{E\_PiCong}$$

$$\frac{\begin{array}{l}\Gamma, x : A_1; \Delta \vDash b_1 \equiv b_2 : B/R' \\ \Gamma \vDash A_1 : \star \\ (\rho = +) \vee (x \notin \mathsf{fv}\ b_1) \\ (\rho = +) \vee (x \notin \mathsf{fv}\ b_2)\end{array}}{\Gamma; \Delta \vDash (\lambda^\rho x.b_1) \equiv (\lambda^\rho x.b_2) : (\Pi^\rho x\!:\!A_1 \to B)/R'} \quad \text{E\_AbsCong}$$

$$\frac{\begin{array}{l}\Gamma; \Delta \vDash a_1 \equiv b_1 : (\Pi^+ x\!:\!A \to B)/R' \\ \Gamma; \Delta \vDash a_2 \equiv b_2 : A/\mathbf{Nom}\end{array}}{\Gamma; \Delta \vDash a_1\ \ {a_2}^+ \equiv b_1\ \ {b_2}^+ : (B\{a_2/x\})/R'} \quad \text{E\_AppCong}$$

$$\frac{\begin{array}{l}\Gamma; \Delta \vDash a_1 \equiv b_1 : (\Pi^+ x\!:\!A \to B)/R' \\ \Gamma; \Delta \vDash a_2 \equiv b_2 : A/\mathbf{param}\, R\, R' \\ \mathsf{Path}\ \ a_1 = F@R, Rs \\ \mathsf{Path}\ \ b_1 = F'@R, Rs'\end{array}}{\Gamma; \Delta \vDash a_1\ \ {a_2}^R \equiv b_1\ \ {b_2}^R : (B\{a_2/x\})/R'} \quad \text{E\_TAppCong}$$

$$\frac{\begin{array}{l}\Gamma; \Delta \vDash a_1 \equiv b_1 : (\Pi^- x\!:\!A \to B)/R' \\ \Gamma \vDash a : A\end{array}}{\Gamma; \Delta \vDash a_1\ \square^- \equiv b_1\ \square^- : (B\{a/x\})/R'} \quad \text{E\_IAppCong}$$

$$\frac{\Gamma; \Delta \vDash \Pi^\rho x\!:\!A_1 \to B_1 \equiv \Pi^\rho x\!:\!A_2 \to B_2 : \star/R'}{\Gamma; \Delta \vDash A_1 \equiv A_2 : \star/R'} \quad \text{E\_PiFst}$$

$$\frac{\begin{array}{l}\Gamma; \Delta \vDash \Pi^\rho x\!:\!A_1 \to B_1 \equiv \Pi^\rho x\!:\!A_2 \to B_2 : \star/R' \\ \Gamma; \Delta \vDash a_1 \equiv a_2 : A_1/R'\end{array}}{\Gamma; \Delta \vDash B_1\{a_1/x\} \equiv B_2\{a_2/x\} : \star/R'} \quad \text{E\_PiSnd}$$

$$\frac{\begin{array}{l}\Gamma; \Delta \vDash a_1 \sim_{A_1/R} b_1 \equiv a_2 \sim_{A_2/R} b_2 \\ \Gamma, c : a_1 \sim_{A_1/R} b_1; \Delta \vDash A \equiv B : \star/R' \\ \Gamma \vDash a_1 \sim_{A_1/R} b_1\ \ \mathsf{ok} \\ \Gamma \vDash \forall c\!:\!a_1 \sim_{A_1/R} b_1.A : \star \\ \Gamma \vDash \forall c\!:\!a_2 \sim_{A_2/R} b_2.B : \star\end{array}}{\Gamma; \Delta \vDash \forall c\!:\!a_1 \sim_{A_1/R} b_1.A \equiv \forall c\!:\!a_2 \sim_{A_2/R} b_2.B : \star/R'} \quad \text{E\_CPiCong}$$

$$\frac{\begin{array}{l}\Gamma, c : \phi_1; \Delta \vDash a \equiv b : B/R \\ \Gamma \vDash \phi_1\ \ \mathsf{ok}\end{array}}{\Gamma; \Delta \vDash (\Lambda c.a) \equiv (\Lambda c.b) : \forall c\!:\!\phi_1.B/R} \quad \text{E\_CAbsCong}$$

$$\frac{\begin{array}{l}\Gamma; \Delta \vDash a_1 \equiv b_1 : (\forall c\!:\!(a \sim_{A/R} b).B)/R' \\ \Gamma; \widetilde{\Gamma} \vDash a \equiv b : A/\mathbf{param}\, R\, R'\end{array}}{\Gamma; \Delta \vDash a_1[\bullet] \equiv b_1[\bullet] : (B\{\bullet/c\})/R'} \quad \text{E\_CAppCong}$$

$$\frac{\begin{array}{l}\Gamma; \Delta \vDash \forall c\!:\!(a_1 \sim_{A/R} a_2).B_1 \equiv \forall c\!:\!(a_1' \sim_{A'/R'} a_2').B_2 : \star/R_0 \\ \Gamma; \widetilde{\Gamma} \vDash a_1 \equiv a_2 : A/\mathbf{param}\, R\, R_0 \\ \Gamma; \widetilde{\Gamma} \vDash a_1' \equiv a_2' : A'/\mathbf{param}\, R'\, R_0\end{array}}{\Gamma; \Delta \vDash B_1\{\bullet/c\} \equiv B_2\{\bullet/c\} : \star/R_0} \quad \text{E\_CPiSnd}$$

$$\frac{\begin{array}{l}\Gamma; \Delta \vDash a \equiv b : A/R \\ \Gamma; \Delta \vDash a \sim_{A/R} b \equiv a' \sim_{A'/R'} b'\end{array}}{\Gamma; \Delta \vDash a' \equiv b' : A'/R'} \quad \text{E\_Cast}$$

$$\frac{\begin{array}{l}\Gamma; \Delta \vDash a \equiv b : A/R \\ \Gamma; \widetilde{\Gamma} \vDash A \equiv B : \star/\mathbf{Rep} \\ \Gamma \vDash B : \star\end{array}}{\Gamma; \Delta \vDash a \equiv b : B/R} \quad \text{E\_EqConv}$$

$$\frac{\Gamma;\Delta \vDash a \sim_{A/R_1} b \equiv a' \sim_{A'/R_1} b'}{\Gamma;\Delta \vDash A \equiv A' : \star/\mathbf{Rep}} \quad \text{E\_IsoSnd}$$

$$\frac{\begin{array}{l}\Gamma;\Delta \vDash a \equiv a' : A/R \\ \Gamma;\Delta \vDash b_1 \equiv b_1' : B/R_0 \\ \Gamma;\Delta \vDash b_2 \equiv b_2' : B/R_0\end{array}}{\Gamma;\Delta \vDash \mathsf{case}_R\ a\ \mathsf{of}\ F \to b_1 \|_{\_} \to b_2 \equiv \mathsf{case}_R\ a'\ \mathsf{of}\ F \to b_1' \|_{\_} \to b_2' : B/R_0} \quad \text{E\_PatCong}$$

$$\frac{\begin{array}{l}\mathsf{ValuePath}\ a = F \\ \mathsf{ValuePath}\ a' = F \\ \Gamma \vDash a : \Pi^+ x\!:\!A \to B \\ \Gamma \vDash b : A \\ \Gamma \vDash a' : \Pi^+ x\!:\!A \to B \\ \Gamma \vDash b' : A \\ \Gamma;\Delta \vDash a\ b^{R_1} \equiv a'\ b'^{R_1} : B\{b/x\}/R' \\ \Gamma;\widetilde{\Gamma} \vDash B\{b/x\} \equiv B\{b'/x\} : \star/R'\end{array}}{\Gamma;\Delta \vDash a \equiv a' : \Pi^+ x\!:\!A \to B/R'} \quad \text{E\_LeftRel}$$

$$\frac{\begin{array}{l}\mathsf{ValuePath}\ a = F \\ \mathsf{ValuePath}\ a' = F \\ \Gamma \vDash a : \Pi^- x\!:\!A \to B \\ \Gamma \vDash b : A \\ \Gamma \vDash a' : \Pi^- x\!:\!A \to B \\ \Gamma \vDash b' : A \\ \Gamma;\Delta \vDash a\ \square^- \equiv a'\ \square^- : B\{b/x\}/R' \\ \Gamma;\widetilde{\Gamma} \vDash B\{b/x\} \equiv B\{b'/x\} : \star/R_0\end{array}}{\Gamma;\Delta \vDash a \equiv a' : \Pi^- x\!:\!A \to B/R'} \quad \text{E\_LeftIrrel}$$

$$\frac{\begin{array}{l}\mathsf{ValuePath}\ a = F \\ \mathsf{ValuePath}\ a' = F \\ \Gamma \vDash a : \Pi^+ x\!:\!A \to B \\ \Gamma \vDash b : A \\ \Gamma \vDash a' : \Pi^+ x\!:\!A \to B \\ \Gamma \vDash b' : A \\ \Gamma;\Delta \vDash a\ b^+ \equiv a'\ b'^+ : B\{b/x\}/R' \\ \Gamma;\widetilde{\Gamma} \vDash B\{b/x\} \equiv B\{b'/x\} : \star/R_0\end{array}}{\Gamma;\Delta \vDash b \equiv b' : A/\mathbf{param}\,R_1\ R'} \quad \text{E\_Right}$$

$$\frac{\begin{array}{l}\mathsf{ValuePath}\ a = F \\ \mathsf{ValuePath}\ a' = F \\ \Gamma \vDash a : \forall c\!:\!(a_1 \sim_{A/R_1} a_2).B \\ \Gamma \vDash a' : \forall c\!:\!(a_1 \sim_{A/R_1} a_2).B \\ \Gamma;\widetilde{\Gamma} \vDash a_1 \equiv a_2 : A/R' \\ \Gamma;\Delta \vDash a[\bullet] \equiv a'[\bullet] : B\{\bullet/c\}/R'\end{array}}{\Gamma;\Delta \vDash a \equiv a' : \forall c\!:\!(a_1 \sim_{A/R_1} a_2).B/R'} \quad \text{E\_CLeft}$$

$\boxed{\vDash \Gamma}$ context wellformedness

$$\frac{}{\vDash \varnothing} \quad \text{E\_Empty}$$

$$\frac{\begin{array}{l}\vDash \Gamma \\ \Gamma \vDash A : \star \\ x \notin \widetilde{\Gamma}\end{array}}{\vDash \Gamma, x : A} \quad \text{E\_ConsTm}$$

$$\frac{\begin{array}{l}\vDash \Gamma \\ \Gamma \vDash \phi \ \mathsf{ok} \\ c \notin \widetilde{\Gamma}\end{array}}{\vDash \Gamma, c : \phi} \quad \text{E\_ConsCo}$$

$\boxed{\vDash \Sigma}$ signature wellformedness

$$\frac{}{\vDash \varnothing} \quad \text{Sig\_Empty}$$

$$\frac{\begin{array}{l}\vDash \Sigma \\ \varnothing \vDash A : \star \\ F \notin \mathsf{dom}\,\Sigma\end{array}}{\vDash \Sigma \cup \{F : A @ Rs\}} \quad \text{Sig\_ConsConst}$$

$$\frac{\begin{array}{l}\vDash \Sigma \\ F \notin \mathsf{dom}\,\Sigma \\ \varnothing \vDash A : \star \\ \Omega; \Gamma \vDash p :_F B \Rightarrow A \\ \Gamma \vDash a : B \\ \Omega \vDash a : R\end{array}}{\vDash \Sigma \cup \{F : \ p \sim a : A/R @ \mathbf{range}\,\Omega\}} \quad \text{Sig\_ConsAx}$$

$\boxed{\Gamma \vdash \phi \ \mathsf{ok}}$ prop wellformedness

$\boxed{\Gamma \vdash a : A/R}$ typing

$\boxed{\Gamma; \Delta \vdash \gamma : \phi_1 \sim \phi_2}$ coercion between props

$\boxed{\Gamma; \Delta \vdash \gamma : A \sim_R B}$ coercion between types

$\boxed{\vdash \Gamma}$ context wellformedness

$\boxed{\Gamma \vdash a \rightsquigarrow b/R}$ single-step, weak head reduction to values for annotated language

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Definition rules:        149 good    0 bad
Definition rule clauses: 419 good    0 bad
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