$tmvar$, x, y, f, m, n	variables
$covar$, c	coercion variables
$datacon$, K	
$const$, T	
$tyfam$, F	
$index$, i	indices

$role,\ R$	$::=$			Role
	$\mid$	$\mathbf{Nom}$		
	$\mid$	$\mathbf{Rep}$		
	$\mid$	$R_1 \cap R_2$	S	
$relflag,\ \rho$	$::=$			relevance flag
	$\mid$	$+$		
	$\mid$	$-$		
$constraint,\ \phi$	$::=$			props
	$\mid$	$a \sim_{A/R} b$		
	$\mid$	(ϕ)	S	
	$\mid$	$\phi\{b/x\}$	S	
	$\mid$	$\lvert\phi\rvert$	S	
$tm,\ a,\ b,\ v,\ w,\ A,\ B$	$::=$			types and kinds
	$\mid$	$\star$		
	$\mid$	x		
	$\mid$	$\lambda^{\rho} x\!:\!A/R.b$	bind x in b	
	$\mid$	$\lambda^{R,\rho} x.b$	bind x in b	
	$\mid$	$a\ b^{R,\rho}$		
	$\mid$	F		
	$\mid$	T		
	$\mid$	$\Pi^{\rho} x\!:\!A/R \to B$	bind x in B	
	$\mid$	$a \triangleright_R \gamma$		
	$\mid$	$\forall c\!:\!\phi.B$	bind c in B	
	$\mid$	$\Lambda c\!:\!\phi.b$	bind c in b	
	$\mid$	$\Lambda c.b$	bind c in b	
	$\mid$	$a[\gamma]$		
	$\mid$	$\square$		
	$\mid$	K		
	$\mid$	$\mathbf{match}\ a\ \mathbf{with}\ brs$		
	$\mid$	$\mathbf{sub}\ R\ a$		
	$\mid$	$a\{b/x\}$	S	
	$\mid$	a	S	
	$\mid$	$a\{\gamma/c\}$	S	
	$\mid$	a	S	
	$\mid$	(a)	S	
	$\mid$	a	S	parsing precedence is hard
	$\mid$	$\lvert a\rvert R$	S	
	$\mid$	$\mathbf{Int}$	S	
	$\mid$	$\mathbf{Bool}$	S	
	$\mid$	$\mathbf{Nat}$	S	
	$\mid$	$\mathbf{Vec}$	S	
	$\mid$	0	S	
	$\mid$	S	S	
	$\mid$	$\mathbf{True}$	S	

	$\vert$	$\mathbf{Fix}$	S	
	$\vert$	$a \to b$	S	
	$\vert$	$\phi \Rightarrow A$	S	
	$\vert$	$a b^{R,+}$	S	
	$\vert$	$\lambda^{R} x . a$	S	
	$\vert$	$\lambda x : A . a$	S	
	$\vert$	$\forall\, x : A / R \to B$	S	
brs	$::=$			case branches
	$\vert$	$\mathbf{none}$		
	$\vert$	$K \Rightarrow a ; brs$		
	$\vert$	$brs\{a / x\}$	S	
	$\vert$	$brs\{\gamma / c\}$	S	
	$\vert$	(brs)	S	
$co,\ \gamma$	$::=$			explicit coercions
	$\vert$	$\bullet$		
	$\vert$	c		
	$\vert$	$\mathbf{red}\, a\, b$		
	$\vert$	$\mathbf{refl}\, a$		
	$\vert$	$(a \mid = \mid_{\gamma}\ b)$		
	$\vert$	$\mathbf{sym}\, \gamma$		
	$\vert$	$\gamma_1 ; \gamma_2$		
	$\vert$	$\mathbf{sub}\, \gamma$		
	$\vert$	$\Pi^{R,\rho} x : \gamma_1 . \gamma_2$	bind x in γ_2	
	$\vert$	$\lambda^{R,\rho} x : \gamma_1 . \gamma_2$	bind x in γ_2	
	$\vert$	$\gamma_1\ {\gamma_2}^{R,\rho}$		
	$\vert$	$\mathbf{piFst}\, \gamma$		
	$\vert$	$\mathbf{cpiFst}\, \gamma$		
	$\vert$	$\mathbf{isoSnd}\, \gamma$		
	$\vert$	$\gamma_1 @ \gamma_2$		
	$\vert$	$\forall c : \gamma_1 . \gamma_3$	bind c in γ_3	
	$\vert$	$\lambda c : \gamma_1 . \gamma_3 @ \gamma_4$	bind c in γ_3	
	$\vert$	$\gamma(\gamma_1, \gamma_2)$		
	$\vert$	$\gamma @ (\gamma_1 \sim \gamma_2)$		
	$\vert$	$\gamma_1 \triangleright_{R} \gamma_2$		
	$\vert$	$\gamma_1 \sim_{A} \gamma_2$		
	$\vert$	$\mathbf{conv}\ \phi_1 \sim_{\gamma} \phi_2$		
	$\vert$	$\mathbf{eta}\, a$		
	$\vert$	$\mathbf{left}\, \gamma\, \gamma'$		
	$\vert$	$\mathbf{right}\, \gamma\, \gamma'$		
	$\vert$	(γ)	S	
	$\vert$	γ	S	
	$\vert$	$\gamma\{a / x\}$	S	
$sort$	$::=$			binding classifier
	$\vert$	$\mathbf{Tm}\, A\, R$		

	$\mid$	$\mathbf{Co}\,\phi$		
sig_sort	::=			signature classifier
	$\mid$	$\mathbf{Cs}\,A$		
	$\mid$	$\mathbf{Ax}\,a\,A\,R$		
context, Γ	::=			contexts
	$\mid$	$\varnothing$		
	$\mid$	$\Gamma, x : A/R$		
	$\mid$	$\Gamma, c : \phi$		
	$\mid$	$\Gamma\{b/x\}$	M	
	$\mid$	$\Gamma\{\gamma/c\}$	M	
	$\mid$	Γ, Γ'	M	
	$\mid$	$\lvert\Gamma\rvert$	M	
	$\mid$	(Γ)	M	
	$\mid$	Γ	M	
sig, Σ	::=			signatures
	$\mid$	$\varnothing$		
	$\mid$	$\Sigma \cup \{T : A/R\}$		
	$\mid$	$\Sigma \cup \{F \sim a : A/R\}$		
	$\mid$	Σ_0	M	
	$\mid$	Σ_1	M	
	$\mid$	$\lvert\Sigma\rvert$	M	
available_props, Δ	::=			
	$\mid$	$\varnothing$		
	$\mid$	Δ, c		
	$\mid$	$\widetilde{\Gamma}$	M	
	$\mid$	(Δ)	M	
role_context, Ω	::=			role$_c$*ontexts*
	$\mid$	$\varnothing$		
	$\mid$	$\Omega, x : R$		
	$\mid$	(Ω)	M	
	$\mid$	Ω	M	
terminals	::=			
	$\mid$	$\leftrightarrow$		
	$\mid$	$\Leftrightarrow$		
	$\mid$	$\longrightarrow$		
	$\mid$	min		
	$\mid$	$\equiv$		
	$\mid$	$\forall$		
	$\mid$	$\in$		
	$\mid$	$\notin$		
	$\mid$	$\Leftarrow$		

$|$ $\Rightarrow$
$|$ $\Rightarrow^*$
$|$ $\rightarrow$
$|$ Λ
$|$
$|$ $\Box$
$|$ $\vdash$
$|$ $\dashv$
$|$ $\models$
$|$ $\models$
$|$ $\neq$
$|$ $\triangleright$
$|$ ok
$|$ $\_$
$|$ $\leadsto$
$|$ $\leadsto^*$
$|$ $\leadsto$
$|$ $\varnothing$
$|$ $\circ$
$|$ fv
$|$ dom
$|$ $\sim$
$|$ $\asymp$
$|$ $|$
$|$ $\bullet$
$|$ $\mathbf{fst}$
$|$ $\mathbf{snd}$
$|$ $|\Rightarrow|$
$|$ $\vdash_=$
$|$ $\mathbf{refl_2}$
$|$ $++$

$\mathit{formula},\ \psi$ ::=
$|$ $\mathit{judgement}$
$|$ $x : A/R \in \Gamma$
$|$ $x : R \in \Omega$
$|$ $c : \phi \in \Gamma$
$|$ $T : A/R \in \Sigma$
$|$ $F \sim a : A/R \in \Sigma$
$|$ $K : T\,\Gamma \in \Sigma$
$|$ $x \in \Delta$
$|$ $c \in \Delta$
$|$ $c\,\mathbf{not\,relevant} \in \gamma$
$|$ $x \notin \mathsf{fv}\,a$
$|$ $x \notin \mathsf{dom}\,\Gamma$
$|$ $\mathit{rctx\_uniq}\Omega$

	$\mid$	$c \notin \mathsf{dom}\,\Gamma$	
	$\mid$	$T \notin \mathsf{dom}\,\Sigma$	
	$\mid$	$F \notin \mathsf{dom}\,\Sigma$	
	$\mid$	$a = b$	
	$\mid$	$\phi_1 = \phi_2$	
	$\mid$	$\Gamma_1 = \Gamma_2$	
	$\mid$	$\gamma_1 = \gamma_2$	
	$\mid$	$\neg\psi$	
	$\mid$	$\psi_1 \wedge \psi_2$	
	$\mid$	$\psi_1 \vee \psi_2$	
	$\mid$	$\psi_1 \Rightarrow \psi_2$	
	$\mid$	(ψ)	
	$\mid$	ψ	
	$\mid$	$c : (a : A \sim b : B) \in \Gamma$	
	$\mid$		suppress lc hypothesis generated by Ott
JSubRole	$::=$		
	$\mid$	$R_1 \leq R_2$	Subroling judgement
JValue	$::=$		
	$\mid$	$\mathbf{CoercedValue}\,R\,A$	Values with at most one coercion at the top
	$\mid$	$\mathsf{Value}_R\ A$	values
	$\mid$	$\mathbf{ValueType}\,R\,A$	Types with head forms (erased language)
Jconsistent	$::=$		
	$\mid$	$\mathbf{consistent}\,a\,b\,R$	(erased) types do not differ in their heads
Jerased	$::=$		
	$\mid$	$\Omega \vDash \mathit{erased\_tm}\ a\ R$	
JChk	$::=$		
	$\mid$	$(\rho = +) \vee (x \notin \mathsf{fv}\ A)$	irrelevant argument check
Jpar	$::=$		
	$\mid$	$\Omega \vDash a \Rightarrow_R b$	parallel reduction (implicit language)
	$\mid$	$\Omega \vdash a \Rightarrow^*_R b$	multistep parallel reduction
	$\mid$	$\Omega \vdash a \Leftrightarrow_R b$	parallel reduction to a common term
Jbeta	$::=$		
	$\mid$	$\vDash a > b/R$	primitive reductions on erased terms
	$\mid$	$\vDash a \leadsto b/R$	single-step head reduction for implicit language
	$\mid$	$\vDash a \leadsto^* b/R$	multistep reduction
Jett	$::=$		
	$\mid$	$\Gamma \vDash \phi\ \ \mathsf{ok}$	Prop wellformedness
	$\mid$	$\Gamma \vDash a : A/R$	typing
	$\mid$	$\Gamma; \Delta \vDash \phi_1 \equiv \phi_2$	prop equality

	|	$\Gamma ; \Delta \vDash a \equiv b : A / R$	definitional equality
	|	$\vDash \Gamma$	context wellformedness
Jsig	::=		
	|	$\vDash \Sigma$	signature wellformedness
Jann	::=		
	|	$\Gamma \vdash \phi\ \mathsf{ok}$	prop wellformedness
	|	$\Gamma \vdash a : A / R$	typing
	|	$\Gamma ; \Delta \vdash \gamma : \phi_1 \sim \phi_2$	coercion between props
	|	$\Gamma ; \Delta \vdash \gamma : A \sim_R B$	coercion between types
	|	$\vdash \Gamma$	context wellformedness
	|	$\vdash \Sigma$	signature wellformedness
Jred	::=		
	|	$\Gamma \vdash a \leadsto b / R$	single-step, weak head reduction to values for annotated lang
judgement	::=		
	|	*JSubRole*	
	|	*JValue*	
	|	*Jconsistent*	
	|	*Jerased*	
	|	*JChk*	
	|	*Jpar*	
	|	*Jbeta*	
	|	*Jett*	
	|	*Jsig*	
	|	*Jann*	
	|	*Jred*	
user_syntax	::=		
	|	*tmvar*	
	|	*covar*	
	|	*datacon*	
	|	*const*	
	|	*tyfam*	
	|	*index*	
	|	*role*	
	|	*relflag*	
	|	*constraint*	
	|	*tm*	
	|	*brs*	
	|	*co*	
	|	*sort*	
	|	*sig_sort*	
	|	*context*	
	|	*sig*	

| *available_props*
| *role_context*
| *terminals*
| *formula*

$\boxed{R_1 \leq R_2}$ Subroling judgement

$$\frac{}{\mathbf{Nom} \leq \mathbf{Rep}} \quad \text{NomRep}$$

$$\frac{}{R \leq R} \quad \text{Refl}$$

$$\frac{\begin{array}{c} R_1 \leq R_2 \\ R_2 \leq R_3 \end{array}}{R_1 \leq R_3} \quad \text{Trans}$$

$\boxed{\mathbf{CoercedValue}\, R\, A}$ Values with at most one coercion at the top

$$\frac{\mathsf{Value}_R\ a}{\mathbf{CoercedValue}\, R\, a} \quad \text{CV}$$

$$\frac{\mathsf{Value}_R\ a}{\mathbf{CoercedValue}\, R\, (a \triangleright_{R_1} \bullet)} \quad \text{CC}$$

$$\frac{\begin{array}{l} \mathbf{CoercedValue}\, R\, (a \triangleright_{R_1} \bullet) \\ \neg(R_1 \leq R_2) \end{array}}{\mathbf{CoercedValue}\, R\, ((a \triangleright_{R_1} \bullet) \triangleright_{R_2} \bullet)} \quad \text{CCV}$$

$\boxed{\mathsf{Value}_R\ A}$ values

$$\frac{}{\mathsf{Value}_R\ \star} \quad \text{Value\_Star}$$

$$\frac{}{\mathsf{Value}_R\ \Pi^\rho x\!:\!A/R_1 \to B} \quad \text{Value\_Pi}$$

$$\frac{}{\mathsf{Value}_R\ \forall c\!:\!\phi.B} \quad \text{Value\_CPi}$$

$$\frac{}{\mathsf{Value}_R\ \lambda^+ x\!:\!A/R_1.a} \quad \text{Value\_AbsRel}$$

$$\frac{}{\mathsf{Value}_R\ \lambda^{R_1,+} x.a} \quad \text{Value\_UAbsRel}$$

$$\frac{\mathbf{CoercedValue}\, R\, a}{\mathsf{Value}_R\ \lambda^{R_1,-} x.a} \quad \text{Value\_UAbsIrrel}$$

$$\frac{}{\mathsf{Value}_R\ \Lambda c\!:\!\phi.a} \quad \text{Value\_CAbs}$$

$$\frac{}{\mathsf{Value}_R\ \Lambda c.a} \quad \text{Value\_UCAbs}$$

$$\frac{\begin{array}{l} F \sim a : A/R_1 \in \Sigma_0 \\ \neg(R_1 \leq R) \end{array}}{\mathsf{Value}_R\ F} \quad \text{Value\_Ax}$$

$\boxed{\mathbf{ValueType}\, R\, A}$ Types with head forms (erased language)

$$\frac{}{\mathbf{ValueType}\, R\, \star} \quad \text{value\_type\_Star}$$

$$\frac{}{\textbf{ValueType}\, R\, \Pi^\rho x\!:\!A/R_1 \to B}\ \textsc{value\_type\_Pi}$$

$$\frac{}{\textbf{ValueType}\, R\, \forall c\!:\!\phi.B}\ \textsc{value\_type\_CPi}$$

$$\frac{\begin{array}{l} F \sim a : A/R_1 \in \Sigma_0 \\ \neg(R_1 \leq R) \end{array}}{\textbf{ValueType}\, R\, F}\ \textsc{value\_type\_Ax}$$

$\boxed{\textbf{consistent}\ a\ b\ R}$ (erased) types do not differ in their heads

$$\frac{}{\textbf{consistent}\ \star\ \star\ R}\ \textsc{consistent\_a\_Star}$$

$$\frac{}{\textbf{consistent}\ (\Pi^\rho x_1\!:\!A_1/R \to B_1)\ (\Pi^\rho x_2\!:\!A_2/R \to B_2)\ R'}\ \textsc{consistent\_a\_Pi}$$

$$\frac{}{\textbf{consistent}\ (\forall c_1\!:\!\phi_1.A_1)\ (\forall c_2\!:\!\phi_2.A_2)\ R}\ \textsc{consistent\_a\_CPi}$$

$$\frac{}{\textbf{consistent}\ F\ F\ R'}\ \textsc{consistent\_a\_Fam}$$

$$\frac{\neg\textbf{ValueType}\, R\, b}{\textbf{consistent}\ a\ b\ R}\ \textsc{consistent\_a\_Step\_R}$$

$$\frac{\neg\textbf{ValueType}\, R\, a}{\textbf{consistent}\ a\ b\ R}\ \textsc{consistent\_a\_Step\_L}$$

$\boxed{\Omega \vDash \mathit{erased\_tm}\ a\ R}$

$$\frac{\mathit{rctx\_uniq}\Omega}{\Omega \vDash \mathit{erased\_tm}\ \square\ R}\ \textsc{erased\_a\_Bullet}$$

$$\frac{\mathit{rctx\_uniq}\Omega}{\Omega \vDash \mathit{erased\_tm}\ \star\ R}\ \textsc{erased\_a\_Star}$$

$$\frac{\begin{array}{l} \mathit{rctx\_uniq}\Omega \\ x : R \in \Omega \\ R \leq R_1 \end{array}}{\Omega \vDash \mathit{erased\_tm}\ x\ R_1}\ \textsc{erased\_a\_Var}$$

$$\frac{\Omega, x : R_1 \vDash \mathit{erased\_tm}\ a\ R}{\Omega \vDash \mathit{erased\_tm}\ (\lambda^{R_1,\rho} x.a)\ R}\ \textsc{erased\_a\_Abs}$$

$$\frac{\begin{array}{l} \Omega \vDash \mathit{erased\_tm}\ a\ R \\ \Omega \vDash \mathit{erased\_tm}\ b\ R_1 \end{array}}{\Omega \vDash \mathit{erased\_tm}\ (a\ \ b^{R_1,\rho})\ R}\ \textsc{erased\_a\_App}$$

$$\frac{\begin{array}{l} \Omega \vDash \mathit{erased\_tm}\ A\ R_1 \\ \Omega, x : R_1 \vDash \mathit{erased\_tm}\ B\ R \end{array}}{\Omega \vDash \mathit{erased\_tm}\ (\Pi^\rho x\!:\!A/R_1 \to B)\ R}\ \textsc{erased\_a\_Pi}$$

$$\frac{\begin{array}{l} \Omega \vDash \mathit{erased\_tm}\ a\ R_1 \\ \Omega \vDash \mathit{erased\_tm}\ b\ R_1 \\ \Omega \vDash \mathit{erased\_tm}\ A\ R_1 \\ \Omega \vDash \mathit{erased\_tm}\ B\ R \end{array}}{\Omega \vDash \mathit{erased\_tm}\ (\forall c\!:\!a \sim_{A/R_1}\ b.B)\ R}\ \textsc{erased\_a\_CPi}$$

$$\frac{\Omega \vDash \mathit{erased\_tm}\ b\ R}{\Omega \vDash \mathit{erased\_tm}\ (\Lambda c.b)\ R}\ \textsc{erased\_a\_CAbs}$$

$$\frac{\Omega \vDash erased\_tm\ a\ R}{\Omega \vDash erased\_tm\ (a[\bullet])\ R} \quad \text{ERASED\_A\_CAPP}$$

$$\frac{\begin{array}{c} rctx\_uniq\Omega \\ F \sim a : A/R \in \Sigma_0 \end{array}}{\Omega \vDash erased\_tm\ F\ R_1} \quad \text{ERASED\_A\_FAM}$$

$$\frac{rctx\_uniq\Omega}{\Omega \vDash erased\_tm\ T\ R} \quad \text{ERASED\_A\_CONST}$$

$$\frac{\Omega \vDash erased\_tm\ a\ R}{\Omega \vDash erased\_tm\ (a \triangleright_{R_1} \bullet)\ R} \quad \text{ERASED\_A\_CONV}$$

$\boxed{(\rho = +) \vee (x \notin \mathsf{fv}\ A)}$ irrelevant argument check

$$\frac{}{(+ = +) \vee (x \notin \mathsf{fv}\ A)} \quad \text{RHO\_REL}$$

$$\frac{x \notin \mathsf{fv} A}{(- = +) \vee (x \notin \mathsf{fv}\ A)} \quad \text{RHO\_IRRREL}$$

$\boxed{\Omega \vDash a \Rightarrow_R b}$ parallel reduction (implicit language)

$$\frac{\Omega \vDash erased\_tm\ a\ R}{\Omega \vDash a \Rightarrow_R a} \quad \text{PAR\_REFL}$$

$$\frac{\begin{array}{c} \Omega \vDash a \Rightarrow_R (\lambda^{R_1,\rho} x.a') \\ \Omega \vDash b \Rightarrow_{R_1} b' \end{array}}{\Omega \vDash a\ b^{R_1,\rho} \Rightarrow_R a'\{b'/x\}} \quad \text{PAR\_BETA}$$

$$\frac{\begin{array}{c} \Omega \vDash a \Rightarrow_R a' \\ \Omega \vDash b \Rightarrow_{R_1} b' \end{array}}{\Omega \vDash a\ b^{R_1,\rho} \Rightarrow_R a'\ b'^{R_1,\rho}} \quad \text{PAR\_APP}$$

$$\frac{\Omega \vDash a \Rightarrow_R (\Lambda c.a')}{\Omega \vDash a[\bullet] \Rightarrow_R a'\{\bullet/c\}} \quad \text{PAR\_CBETA}$$

$$\frac{\Omega \vDash a \Rightarrow_R a'}{\Omega \vDash a[\bullet] \Rightarrow_R a'[\bullet]} \quad \text{PAR\_CAPP}$$

$$\frac{\Omega, x : R_1 \vDash a \Rightarrow_R a'}{\Omega \vDash \lambda^{R_1,\rho} x.a \Rightarrow_R \lambda^{R_1,\rho} x.a'} \quad \text{PAR\_ABS}$$

$$\frac{\begin{array}{c} \Omega \vDash A \Rightarrow_{R_1} A' \\ \Omega, x : R_1 \vDash B \Rightarrow_R B' \end{array}}{\Omega \vDash \Pi^\rho x\!:\!A/R_1 \to B \Rightarrow_R \Pi^\rho x\!:\!A'/R_1 \to B'} \quad \text{PAR\_PI}$$

$$\frac{\Omega \vDash a \Rightarrow_R a'}{\Omega \vDash \Lambda c.a \Rightarrow_R \Lambda c.a'} \quad \text{PAR\_CABS}$$

$$\frac{\begin{array}{c} \Omega \vDash A \Rightarrow_{R_1} A' \\ \Omega \vDash a \Rightarrow_{R_1} a' \\ \Omega \vDash b \Rightarrow_{R_1} b' \\ \Omega \vDash B \Rightarrow_R B' \end{array}}{\Omega \vDash \forall c\!:\!a \sim_{A/R_1} b.B \Rightarrow_R \forall c\!:\!a' \sim_{A'/R_1} b'.B'} \quad \text{PAR\_CPI}$$

$$\frac{\begin{array}{l}F \sim a : A/R_1 \in \Sigma_0 \\ R_1 \leq R \\ rctx\_uniq\Omega\end{array}}{\Omega \vDash F \Rightarrow_R a} \quad \text{PAR\_AXIOM}$$

$$\frac{\Omega \vDash a_1 \Rightarrow_{R_1} a_2}{\Omega \vDash a_1 \triangleright_R \bullet \Rightarrow_{R_1} a_2 \triangleright_R \bullet} \quad \text{PAR\_CONG}$$

$$\frac{\Omega \vDash a_1 \Rightarrow_{R_1} (a_2 \triangleright_R \bullet)}{\Omega \vDash (a_1 \triangleright_R \bullet) \Rightarrow_{R_1} (a_2 \triangleright_R \bullet)} \quad \text{PAR\_COMBINE}$$

$$\frac{\begin{array}{c}\Omega \vDash a_1 \Rightarrow_{R_1} (a_2 \triangleright_R \bullet) \\ \Omega \vDash b_1 \Rightarrow_{R_2} b_2\end{array}}{\Omega \vDash a_1 {b_1}^{R_2,+} \Rightarrow_{R_1} (a_2 (b_2 \triangleright_R \bullet)^{R_2,+}) \triangleright_R \bullet} \quad \text{PAR\_PUSH}$$

$$\frac{\begin{array}{c}\Omega \vDash a_1 \Rightarrow_{R_1} (a_2 \triangleright_R \bullet) \\ \Omega \vDash b_1 \Rightarrow_{R_2} (b_2 \triangleright_R \bullet)\end{array}}{\Omega \vDash a_1 {b_1}^{R_2,+} \Rightarrow_{R_1} (a_2 (b_2 \triangleright_R \bullet)^{R_2,+}) \triangleright_R \bullet} \quad \text{PAR\_PUSHCOMBINE}$$

$$\frac{\Omega \vDash a_1 \Rightarrow_{R_1} (a_2 \triangleright_R \bullet)}{\Omega \vDash a_1[\bullet] \Rightarrow_{R_1} (a_2[\bullet]) \triangleright_R \bullet} \quad \text{PAR\_CPUSH}$$

$\boxed{\Omega \vdash a \Rightarrow^*_R b}$ multistep parallel reduction

$$\frac{}{\Omega \vdash a \Rightarrow^*_R a} \quad \text{MP\_REFL}$$

$$\frac{\begin{array}{c}\Omega \vDash a \Rightarrow_R b \\ \Omega \vdash b \Rightarrow^*_R a'\end{array}}{\Omega \vdash a \Rightarrow^*_R a'} \quad \text{MP\_STEP}$$

$\boxed{\Omega \vdash a \Leftrightarrow_R b}$ parallel reduction to a common term

$$\frac{\begin{array}{c}\Omega \vdash a_1 \Rightarrow^*_R b \\ \Omega \vdash a_2 \Rightarrow^*_R b\end{array}}{\Omega \vdash a_1 \Leftrightarrow_R a_2} \quad \text{JOIN}$$

$\boxed{\vDash a > b/R}$ primitive reductions on erased terms

$$\frac{\mathsf{Value}_{R_1}\ (\lambda^{R,\rho} x.v)}{\vDash (\lambda^{R,\rho} x.v)\ b^{R,\rho} > v\{b/x\}/R_1} \quad \text{BETA\_APPABS}$$

$$\frac{}{\vDash (\Lambda c.a')[\bullet] > a'\{\bullet/c\}/R} \quad \text{BETA\_CAPPCABS}$$

$$\frac{F \sim a : A/R \in \Sigma_0}{\vDash F > a/R} \quad \text{BETA\_AXIOM}$$

$\boxed{\vDash a \rightsquigarrow b/R}$ single-step head reduction for implicit language

$$\frac{\vDash a \rightsquigarrow a'/R_1}{\vDash \lambda^{R,-} x.a \rightsquigarrow \lambda^{R,-} x.a'/R_1} \quad \text{E\_ABSTERM}$$

$$\frac{\vDash a \rightsquigarrow a'/R_1}{\vDash a\ b^{R,\rho} \rightsquigarrow a'\ b^{R,\rho}/R_1} \quad \text{E\_APPLEFT}$$

$$\frac{\vDash a \rightsquigarrow a'/R}{\vDash a[\bullet] \rightsquigarrow a'[\bullet]/R} \quad \text{E\_CAPPLEFT}$$

$$\frac{\mathsf{Value}_{R_1}\ (\lambda^{R,\rho} x.v)}{\vDash (\lambda^{R,\rho} x.v)\ a^{R,\rho} \rightsquigarrow v\{a/x\}/R_1}\quad \text{E\_AppAbs}$$

$$\frac{}{\vDash (\Lambda c.b)[\bullet] \rightsquigarrow b\{\bullet/c\}/R}\quad \text{E\_CAppCAbs}$$

$$\frac{\begin{array}{l} F \sim a : A/R \in \Sigma_0 \\ R \leq R_1 \end{array}}{\vDash F \rightsquigarrow a/R_1}\quad \text{E\_Axiom}$$

$$\frac{\vDash a \rightsquigarrow a'/R_1}{\vDash a \rhd_R \bullet \rightsquigarrow a' \rhd_R \bullet/R_1}\quad \text{E\_Cong}$$

$$\frac{\begin{array}{l} \textbf{CoercedValue}\, R\, (v \rhd_{R_1} \bullet) \\ R_1 \leq R_2 \end{array}}{\vDash (v \rhd_{R_1} \bullet) \rhd_{R_2} \bullet \rightsquigarrow v \rhd_{R_2} \bullet/R}\quad \text{E\_Combine}$$

$$\frac{\textbf{CoercedValue}\, R_2\, (v_1 \rhd_R \bullet)}{\vDash (v_1 \rhd_R \bullet)\ b^{R_1,\rho} \rightsquigarrow (v_1\ (b \rhd_R \bullet)^{R_1,\rho}) \rhd_R \bullet/R_2}\quad \text{E\_Push}$$

$$\frac{\textbf{CoercedValue}\, R_1\, (v_1 \rhd_R \bullet)}{\vDash (v_1 \rhd_R \bullet)[\bullet] \rightsquigarrow (v_1[\bullet]) \rhd_R \bullet/R_1}\quad \text{E\_CPush}$$

$\boxed{\vDash a \rightsquigarrow^* b/R}$ multistep reduction

$$\frac{}{\vDash a \rightsquigarrow^* a/R}\quad \text{Equal}$$

$$\frac{\begin{array}{l} \vDash a \rightsquigarrow b/R \\ \vDash b \rightsquigarrow^* a'/R \end{array}}{\vDash a \rightsquigarrow^* a'/R}\quad \text{Step}$$

$\boxed{\Gamma \vDash \phi\ \mathsf{ok}}$ Prop wellformedness

$$\frac{\begin{array}{l} \Gamma \vDash a : A/R \\ \Gamma \vDash b : A/R \\ \Gamma \vDash A : \star/R \end{array}}{\Gamma \vDash a \sim_{A/R} b\ \ \mathsf{ok}}\quad \text{E\_Wff}$$

$\boxed{\Gamma \vDash a : A/R}$ typing

$$\frac{\begin{array}{l} R_1 \leq R_2 \\ \Gamma \vDash a : A/R_1 \end{array}}{\Gamma \vDash a : A/R_2}\quad \text{E\_SubRole}$$

$$\frac{\vDash \Gamma}{\Gamma \vDash \star : \star/R}\quad \text{E\_Star}$$

$$\frac{\begin{array}{l} \vDash \Gamma \\ x : A/R \in \Gamma \end{array}}{\Gamma \vDash x : A/R}\quad \text{E\_Var}$$

$$\frac{\begin{array}{l} \Gamma, x : A/R \vDash B : \star/R' \\ \Gamma \vDash A : \star/R \end{array}}{\Gamma \vDash \Pi^\rho x\!:\!A/R \to B : \star/R'}\quad \text{E\_Pi}$$

$$\frac{\begin{array}{l}\Gamma, x : A/R \vDash a : B/R' \\ \Gamma \vDash A : \star/R \\ (\rho = +) \vee (x \notin \mathsf{fv}\ a)\end{array}}{\Gamma \vDash \lambda^{R,\rho} x.a : (\Pi^\rho x\!:\!A/R \to B)/R'} \quad \text{E\_Abs}$$

$$\frac{\begin{array}{l}\Gamma \vDash b : \Pi^+ x\!:\!A/R \to B/R' \\ \Gamma \vDash a : A/R\end{array}}{\Gamma \vDash b\ a^{R,+} : B\{a/x\}/R'} \quad \text{E\_App}$$

$$\frac{\begin{array}{l}\Gamma \vDash b : \Pi^- x\!:\!A/R \to B/R' \\ \Gamma \vDash a : A/R\end{array}}{\Gamma \vDash b\ \Box^{R,-} : B\{a/x\}/R'} \quad \text{E\_IApp}$$

$$\frac{\begin{array}{l}\Gamma \vDash a : A/R \\ \Gamma; \widetilde{\Gamma} \vDash A \equiv B : \star/R \\ \Gamma \vDash B : \star/R\end{array}}{\Gamma \vDash a : B/R} \quad \text{E\_Conv}$$

$$\frac{\begin{array}{l}\Gamma, c : \phi \vDash B : \star/R \\ \Gamma \vDash \phi\ \mathsf{ok}\end{array}}{\Gamma \vDash \forall c\!:\!\phi.B : \star/R} \quad \text{E\_CPi}$$

$$\frac{\begin{array}{l}\Gamma, c : \phi \vDash a : B/R \\ \Gamma \vDash \phi\ \mathsf{ok}\end{array}}{\Gamma \vDash \Lambda c.a : \forall c\!:\!\phi.B/R} \quad \text{E\_CAbs}$$

$$\frac{\begin{array}{l}\Gamma \vDash a_1 : \forall c\!:\!(a \sim_{A/R} b).B_1/R' \\ \Gamma; \widetilde{\Gamma} \vDash a \equiv b : A/R\end{array}}{\Gamma \vDash a_1[\bullet] : B_1\{\bullet/c\}/R'} \quad \text{E\_CApp}$$

$$\frac{\begin{array}{l}\vDash \Gamma \\ F \sim a : A/R \in \Sigma_0 \\ \varnothing \vDash A : \star/R_1\end{array}}{\Gamma \vDash F : A/R_1} \quad \text{E\_Fam}$$

$$\frac{\begin{array}{l}\Gamma \vDash a : A_1/R_1 \\ \Gamma; \widetilde{\Gamma} \vDash A_1 \equiv A_2 : \star/R_2 \\ \Gamma \vDash A_2 : \star/R_1\end{array}}{\Gamma \vDash a \rhd_{R_2} \bullet : A_2/R_1} \quad \text{E\_TyCast}$$

$\boxed{\Gamma; \Delta \vDash \phi_1 \equiv \phi_2}$ prop equality

$$\frac{\begin{array}{c}\Gamma; \Delta \vDash A_1 \equiv A_2 : A/R \\ \Gamma; \Delta \vDash B_1 \equiv B_2 : A/R\end{array}}{\Gamma; \Delta \vDash A_1 \sim_{A/R} B_1 \equiv A_2 \sim_{A/R} B_2} \quad \text{E\_PropCong}$$

$$\frac{\begin{array}{c}\Gamma; \Delta \vDash A \equiv B : \star/R \\ \Gamma \vDash A_1 \sim_{A/R} A_2\ \mathsf{ok} \\ \Gamma \vDash A_1 \sim_{B/R} A_2\ \mathsf{ok}\end{array}}{\Gamma; \Delta \vDash A_1 \sim_{A/R} A_2 \equiv A_1 \sim_{B/R} A_2} \quad \text{E\_IsoConv}$$

$$\frac{\Gamma; \Delta \vDash \forall c\!:\!(a_1 \sim_{A/R} a_2).B_1 \equiv \forall c\!:\!(b_1 \sim_{B/R} b_2).B_2 : \star/R'}{\Gamma; \Delta \vDash a_1 \sim_{A/R} a_2 \equiv b_1 \sim_{B/R} b_2} \quad \text{E\_CPiFst}$$

$\boxed{\Gamma; \Delta \vDash a \equiv b : A/R}$ definitional equality

$$\frac{\begin{array}{l}\vDash \Gamma \\ c : (a \sim_{A/R} b) \in \Gamma \\ c \in \Delta\end{array}}{\Gamma; \Delta \vDash a \equiv b : A/R} \quad \text{E\_Assn}$$

$$\frac{\Gamma \vDash a : A/R}{\Gamma; \Delta \vDash a \equiv a : A/R} \quad \text{E\_Refl}$$

$$\frac{\Gamma; \Delta \vDash b \equiv a : A/R}{\Gamma; \Delta \vDash a \equiv b : A/R} \quad \text{E\_Sym}$$

$$\frac{\begin{array}{l}\Gamma; \Delta \vDash a \equiv a_1 : A/R \\ \Gamma; \Delta \vDash a_1 \equiv b : A/R\end{array}}{\Gamma; \Delta \vDash a \equiv b : A/R} \quad \text{E\_Trans}$$

$$\frac{\begin{array}{l}\Gamma; \Delta \vDash a \equiv b : A/R_1 \\ R_1 \leq R_2\end{array}}{\Gamma; \Delta \vDash a \equiv b : A/R_2} \quad \text{E\_Sub}$$

$$\frac{\begin{array}{l}\Gamma \vDash a_1 : B/R \\ \Gamma \vDash a_2 : B/R \\ \vDash a_1 > a_2/R\end{array}}{\Gamma; \Delta \vDash a_1 \equiv a_2 : B/R} \quad \text{E\_Beta}$$

$$\frac{\begin{array}{l}\Gamma; \Delta \vDash A_1 \equiv A_2 : \star/R \\ \Gamma, x : A_1/R; \Delta \vDash B_1 \equiv B_2 : \star/R' \\ \Gamma \vDash A_1 : \star/R \\ \Gamma \vDash \Pi^\rho x\!:\!A_1/R \to B_1 : \star/R' \\ \Gamma \vDash \Pi^\rho x\!:\!A_2/R \to B_2 : \star/R'\end{array}}{\Gamma; \Delta \vDash (\Pi^\rho x\!:\!A_1/R \to B_1) \equiv (\Pi^\rho x\!:\!A_2/R \to B_2) : \star/R'} \quad \text{E\_PiCong}$$

$$\frac{\begin{array}{l}\Gamma, x : A_1/R; \Delta \vDash b_1 \equiv b_2 : B/R' \\ \Gamma \vDash A_1 : \star/R \\ (\rho = +) \lor (x \notin \mathsf{fv}\ b_1) \\ (\rho = +) \lor (x \notin \mathsf{fv}\ b_2)\end{array}}{\Gamma; \Delta \vDash (\lambda^{R,\rho} x.b_1) \equiv (\lambda^{R,\rho} x.b_2) : (\Pi^\rho x\!:\!A_1/R \to B)/R'} \quad \text{E\_AbsCong}$$

$$\frac{\begin{array}{l}\Gamma; \Delta \vDash a_1 \equiv b_1 : (\Pi^+ x\!:\!A/R \to B)/R' \\ \Gamma; \Delta \vDash a_2 \equiv b_2 : A/R\end{array}}{\Gamma; \Delta \vDash a_1\ \ {a_2}^{R,+} \equiv b_1\ \ {b_2}^{R,+} : (B\{a_2/x\})/R'} \quad \text{E\_AppCong}$$

$$\frac{\begin{array}{l}\Gamma; \Delta \vDash a_1 \equiv b_1 : (\Pi^- x\!:\!A/R \to B)/R' \\ \Gamma \vDash a : A/R\end{array}}{\Gamma; \Delta \vDash a_1\ \square^{R,-} \equiv b_1\ \square^{R,-} : (B\{a/x\})/R'} \quad \text{E\_IAppCong}$$

$$\frac{\Gamma; \Delta \vDash \Pi^\rho x\!:\!A_1/R \to B_1 \equiv \Pi^\rho x\!:\!A_2/R \to B_2 : \star/R'}{\Gamma; \Delta \vDash A_1 \equiv A_2 : \star/R} \quad \text{E\_PiFst}$$

$$\frac{\begin{array}{l}\Gamma; \Delta \vDash \Pi^\rho x\!:\!A_1/R \to B_1 \equiv \Pi^\rho x\!:\!A_2/R \to B_2 : \star/R' \\ \Gamma; \Delta \vDash a_1 \equiv a_2 : A_1/R\end{array}}{\Gamma; \Delta \vDash B_1\{a_1/x\} \equiv B_2\{a_2/x\} : \star/R'} \quad \text{E\_PiSnd}$$

$$\frac{\begin{array}{l}\Gamma;\Delta \vDash a_1 \sim_{A_1/R} b_1 \equiv a_2 \sim_{A_2/R} b_2\\ \Gamma, c : a_1 \sim_{A_1/R} b_1;\Delta \vDash A \equiv B : \star/R'\\ \Gamma \vDash a_1 \sim_{A_1/R} b_1 \ \mathsf{ok}\\ \Gamma \vDash \forall c{:}a_1 \sim_{A_1/R} b_1.A : \star/R'\\ \Gamma \vDash \forall c{:}a_2 \sim_{A_2/R} b_2.B : \star/R'\end{array}}{\Gamma;\Delta \vDash \forall c{:}a_1 \sim_{A_1/R} b_1.A \equiv \forall c{:}a_2 \sim_{A_2/R} b_2.B : \star/R'} \quad \text{E\_CPiCong}$$

$$\frac{\begin{array}{l}\Gamma, c : \phi_1;\Delta \vDash a \equiv b : B/R\\ \Gamma \vDash \phi_1 \ \mathsf{ok}\end{array}}{\Gamma;\Delta \vDash (\Lambda c.a) \equiv (\Lambda c.b) : \forall c{:}\phi_1.B/R} \quad \text{E\_CAbsCong}$$

$$\frac{\begin{array}{l}\Gamma;\Delta \vDash a_1 \equiv b_1 : (\forall c{:}(a \sim_{A/R} b).B)/R'\\ \Gamma;\widetilde{\Gamma} \vDash a \equiv b : A/R\end{array}}{\Gamma;\Delta \vDash a_1[\bullet] \equiv b_1[\bullet] : (B\{\bullet/c\})/R'} \quad \text{E\_CAppCong}$$

$$\frac{\begin{array}{l}\Gamma;\Delta \vDash \forall c{:}(a_1 \sim_{A/R} a_2).B_1 \equiv \forall c{:}(a_1' \sim_{A'/R'} a_2').B_2 : \star/R_0\\ \Gamma;\widetilde{\Gamma} \vDash a_1 \equiv a_2 : A/R\\ \Gamma;\widetilde{\Gamma} \vDash a_1' \equiv a_2' : A'/R'\end{array}}{\Gamma;\Delta \vDash B_1\{\bullet/c\} \equiv B_2\{\bullet/c\} : \star/R_0} \quad \text{E\_CPiSnd}$$

$$\frac{\begin{array}{l}\Gamma;\Delta \vDash a \equiv b : A/R\\ \Gamma;\Delta \vDash a \sim_{A/R} b \equiv a' \sim_{A'/R} b'\end{array}}{\Gamma;\Delta \vDash a' \equiv b' : A'/R} \quad \text{E\_Cast}$$

$$\frac{\begin{array}{l}\Gamma;\Delta \vDash a \equiv b : A/R_1\\ \Gamma;\widetilde{\Gamma} \vDash A \equiv B : \star/R_2\\ R_1 \leq R_2\end{array}}{\Gamma;\Delta \vDash a \equiv b : B/R_2} \quad \text{E\_EqConv}$$

$$\frac{\Gamma;\Delta \vDash a \sim_{A/R} b \equiv a' \sim_{A'/R} b'}{\Gamma;\Delta \vDash A \equiv A' : \star/R} \quad \text{E\_IsoSnd}$$

$$\frac{\begin{array}{l}\Gamma;\Delta \vDash a_1 \equiv a_2 : A/R_1\\ \Gamma;\Delta \vDash A \equiv B : \star/R_2\\ \Gamma \vDash B : \star/R_1\end{array}}{\Gamma;\Delta \vDash a_1 \rhd_{R_2} \bullet \equiv a_2 \rhd_{R_2} \bullet : B/R_1} \quad \text{E\_CastCong}$$

$\boxed{\vDash \Gamma}$ context wellformedness

$$\frac{}{\vDash \varnothing} \quad \text{E\_Empty}$$

$$\frac{\begin{array}{l}\vDash \Gamma\\ \Gamma \vDash A : \star/R\\ x \notin \mathsf{dom}\,\Gamma\end{array}}{\vDash \Gamma, x : A/R} \quad \text{E\_ConsTm}$$

$$\frac{\begin{array}{l}\vDash \Gamma\\ \Gamma \vDash \phi \ \mathsf{ok}\\ c \notin \mathsf{dom}\,\Gamma\end{array}}{\vDash \Gamma, c : \phi} \quad \text{E\_ConsCo}$$

$\boxed{\vDash \Sigma}$ signature wellformedness

$$\frac{}{\vDash \varnothing} \quad \text{Sig\_Empty}$$

$$\frac{\begin{array}{l}\vDash \Sigma \\ \varnothing \vDash A : \star / R \\ \varnothing \vDash a : A / R' \\ F \notin \mathsf{dom}\,\Sigma \\ R' \leq R\end{array}}{\vDash \Sigma \cup \{F \sim a : A/R'\}} \quad \textsc{Sig\_ConsAx}$$

$\boxed{\Gamma \vdash \phi \ \mathsf{ok}}$ prop wellformedness

$$\frac{\begin{array}{l}\Gamma \vdash a : A/R \\ \Gamma \vdash b : B/R \\ |A|R = |B|R\end{array}}{\Gamma \vdash a \sim_{A/R} b \ \mathsf{ok}} \quad \textsc{An\_Wff}$$

$\boxed{\Gamma \vdash a : A/R}$ typing

$$\frac{\vdash \Gamma}{\Gamma \vdash \star : \star / R} \quad \textsc{An\_Star}$$

$$\frac{\begin{array}{l}\vdash \Gamma \\ x : A/R \in \Gamma\end{array}}{\Gamma \vdash x : A/R} \quad \textsc{An\_Var}$$

$$\frac{\begin{array}{l}\Gamma, x : A/R \vdash B : \star / R' \\ \Gamma \vdash A : \star / R\end{array}}{\Gamma \vdash \Pi^{\rho} x\!:\!A/R \to B : \star / R'} \quad \textsc{An\_Pi}$$

$$\frac{\begin{array}{l}\Gamma \vdash A : \star / R \\ \Gamma, x : A/R \vdash a : B/R' \\ (\rho = +) \vee (x \notin \mathsf{fv}\ |a|R') \\ R \leq R'\end{array}}{\Gamma \vdash \lambda^{\rho} x\!:\!A/R.a : (\Pi^{\rho} x\!:\!A/R \to B)/R'} \quad \textsc{An\_Abs}$$

$$\frac{\begin{array}{l}\Gamma \vdash b : (\Pi^{\rho} x\!:\!A/R \to B)/R' \\ \Gamma \vdash a : A/R\end{array}}{\Gamma \vdash b\ a^{R,\rho} : (B\{a/x\})/R'} \quad \textsc{An\_App}$$

$$\frac{\begin{array}{l}\Gamma \vdash a : A/R \\ \Gamma; \widetilde{\Gamma} \vdash \gamma : A \sim_R B \\ \Gamma \vdash B : \star / R\end{array}}{\Gamma \vdash a \rhd_R \gamma : B/R} \quad \textsc{An\_Conv}$$

$$\frac{\begin{array}{l}\Gamma \vdash \phi \ \mathsf{ok} \\ \Gamma, c : \phi \vdash B : \star / R\end{array}}{\Gamma \vdash \forall c\!:\!\phi.B : \star / R} \quad \textsc{An\_CPi}$$

$$\frac{\begin{array}{l}\Gamma \vdash \phi \ \mathsf{ok} \\ \Gamma, c : \phi \vdash a : B/R\end{array}}{\Gamma \vdash \Lambda c\!:\!\phi.a : (\forall c\!:\!\phi.B)/R} \quad \textsc{An\_CAbs}$$

$$\frac{\begin{array}{l}\Gamma \vdash a_1 : (\forall c\!:\!a \sim_{A_1/R} b.B)/R' \\ \Gamma; \widetilde{\Gamma} \vdash \gamma : a \sim_R b\end{array}}{\Gamma \vdash a_1[\gamma] : B\{\gamma/c\}/R'} \quad \textsc{An\_CApp}$$

$$\frac{\begin{array}{l}\vdash \Gamma\\ F \sim a : A/R \,\in\, \Sigma_1\\ \varnothing \vdash A : \star/R\end{array}}{\Gamma \vdash F : A/R}\quad \textsc{An\_Fam}$$

$$\frac{\begin{array}{l}R_1 \leq R_2\\ \Gamma \vdash a : A/R_1\end{array}}{\Gamma \vdash \mathbf{sub}\, R_1\; a : A/R_2}\quad \textsc{An\_SubRole}$$

$\boxed{\Gamma; \Delta \vdash \gamma : \phi_1 \sim \phi_2}$ coercion between props

$$\frac{\begin{array}{l}\Gamma; \Delta \vdash \gamma_1 : A_1 \sim_R A_2\\ \Gamma; \Delta \vdash \gamma_2 : B_1 \sim_R B_2\\ \Gamma \vdash A_1 \sim_{A/R} B_1 \;\mathsf{ok}\\ \Gamma \vdash A_2 \sim_{A/R} B_2 \;\mathsf{ok}\end{array}}{\Gamma; \Delta \vdash (\gamma_1 \sim_A \gamma_2) : (A_1 \sim_{A/R} B_1) \sim (A_2 \sim_{A/R} B_2)}\quad \textsc{An\_PropCong}$$

$$\frac{\Gamma; \Delta \vdash \gamma : \forall c{:}\phi_1.A_2 \sim_R \forall c{:}\phi_2.B_2}{\Gamma; \Delta \vdash \mathbf{cpiFst}\, \gamma : \phi_1 \sim \phi_2}\quad \textsc{An\_CPiFst}$$

$$\frac{\Gamma; \Delta \vdash \gamma : \phi_1 \sim \phi_2}{\Gamma; \Delta \vdash \mathbf{sym}\, \gamma : \phi_2 \sim \phi_1}\quad \textsc{An\_IsoSym}$$

$$\frac{\begin{array}{l}\Gamma; \Delta \vdash \gamma : A \sim_R B\\ \Gamma \vdash a_1 \sim_{A/R} a_2 \;\mathsf{ok}\\ \Gamma \vdash a_1' \sim_{B/R} a_2' \;\mathsf{ok}\\ |a_1|R = |a_1'|R\\ |a_2|R = |a_2'|R\end{array}}{\Gamma; \Delta \vdash \mathbf{conv}\; (a_1 \sim_{A/R} a_2) \sim_\gamma (a_1' \sim_{B/R} a_2') : (a_1 \sim_{A/R} a_2) \sim (a_1' \sim_{B/R} a_2')}\quad \textsc{An\_IsoConv}$$

$\boxed{\Gamma; \Delta \vdash \gamma : A \sim_R B}$ coercion between types

$$\frac{\begin{array}{l}\vdash \Gamma\\ c : a \sim_{A/R} b \,\in\, \Gamma\\ c \,\in\, \Delta\end{array}}{\Gamma; \Delta \vdash c : a \sim_R b}\quad \textsc{An\_Assn}$$

$$\frac{\Gamma \vdash a : A/R}{\Gamma; \Delta \vdash \mathbf{refl}\, a : a \sim_R a}\quad \textsc{An\_Refl}$$

$$\frac{\begin{array}{l}\Gamma \vdash a : A/R\\ \Gamma \vdash b : B/R\\ |a|R = |b|R\\ \Gamma; \widetilde{\Gamma} \vdash \gamma : A \sim_R B\end{array}}{\Gamma; \Delta \vdash (a \mid=\mid_\gamma b) : a \sim_R b}\quad \textsc{An\_EraseEq}$$

$$\frac{\begin{array}{l}\Gamma \vdash b : B/R\\ \Gamma \vdash a : A/R\\ \Gamma; \widetilde{\Gamma} \vdash \gamma_1 : B \sim_R A\\ \Gamma; \Delta \vdash \gamma : b \sim_R a\end{array}}{\Gamma; \Delta \vdash \mathbf{sym}\, \gamma : a \sim_R b}\quad \textsc{An\_Sym}$$

$$\frac{\begin{array}{l}\Gamma;\Delta \vdash \gamma_1 : a \sim_R a_1\\ \Gamma;\Delta \vdash \gamma_2 : a_1 \sim_R b\\ \Gamma \vdash a : A/R\\ \Gamma \vdash a_1 : A_1/R\\ \Gamma;\widetilde{\Gamma} \vdash \gamma_3 : A \sim_R A_1\end{array}}{\Gamma;\Delta \vdash (\gamma_1;\gamma_2) : a \sim_R b} \quad \textsc{An\_Trans}$$

$$\frac{\begin{array}{l}\Gamma \vdash a_1 : B_0/R\\ \Gamma \vdash a_2 : B_1/R\\ |B_0|R = |B_1|R\\ \models |a_1|R > |a_2|R/R\end{array}}{\Gamma;\Delta \vdash \mathbf{red}\, a_1\ a_2 : a_1 \sim_R a_2} \quad \textsc{An\_Beta}$$

$$\frac{\begin{array}{l}\Gamma;\Delta \vdash \gamma_1 : A_1 \sim_{R'} A_2\\ \Gamma, x : A_1/R;\Delta \vdash \gamma_2 : B_1 \sim_{R'} B_2\\ B_3 = B_2\{x \triangleright_{R'} \mathbf{sym}\,\gamma_1/x\}\\ \Gamma \vdash \Pi^\rho x\!:\!A_1/R \to B_1 : \star/R'\\ \Gamma \vdash \Pi^\rho x\!:\!A_1/R \to B_2 : \star/R'\\ \Gamma \vdash \Pi^\rho x\!:\!A_2/R \to B_3 : \star/R'\\ R \leq R'\end{array}}{\Gamma;\Delta \vdash \Pi^{R,\rho} x\!:\!\gamma_1.\gamma_2 : (\Pi^\rho x\!:\!A_1/R \to B_1) \sim_{R'} (\Pi^\rho x\!:\!A_2/R \to B_3)} \quad \textsc{An\_PiCong}$$

$$\frac{\begin{array}{l}\Gamma;\Delta \vdash \gamma_1 : A_1 \sim_R A_2\\ \Gamma, x : A_1/R;\Delta \vdash \gamma_2 : b_1 \sim_{R'} b_2\\ b_3 = b_2\{x \triangleright_{R'} \mathbf{sym}\,\gamma_1/x\}\\ \Gamma \vdash A_1 : \star/R\\ \Gamma \vdash A_2 : \star/R\\ (\rho = +) \vee (x \notin \mathsf{fv}\ |b_1|R')\\ (\rho = +) \vee (x \notin \mathsf{fv}\ |b_3|R')\\ \Gamma \vdash (\lambda^\rho x\!:\!A_1/R.b_2) : B/R'\\ R \leq R'\end{array}}{\Gamma;\Delta \vdash (\lambda^{R,\rho} x\!:\!\gamma_1.\gamma_2) : (\lambda^\rho x\!:\!A_1/R.b_1) \sim_{R'} (\lambda^\rho x\!:\!A_2/R.b_3)} \quad \textsc{An\_AbsCong}$$

$$\frac{\begin{array}{l}\Gamma;\Delta \vdash \gamma_1 : a_1 \sim_{R'} b_1\\ \Gamma;\Delta \vdash \gamma_2 : a_2 \sim_R b_2\\ \Gamma \vdash a_1\ {a_2}^{R,\rho} : A/R'\\ \Gamma \vdash b_1\ {b_2}^{R,\rho} : B/R'\\ \Gamma;\widetilde{\Gamma} \vdash \gamma_3 : A \sim_{R'} B\end{array}}{\Gamma;\Delta \vdash \gamma_1\ \gamma_2^{R,\rho} : a_1\ {a_2}^{R,\rho} \sim_{R'} b_1\ {b_2}^{R,\rho}} \quad \textsc{An\_AppCong}$$

$$\frac{\Gamma;\Delta \vdash \gamma : \Pi^\rho x\!:\!A_1/R \to B_1 \sim_{R'} \Pi^\rho x\!:\!A_2/R \to B_2}{\Gamma;\Delta \vdash \mathbf{piFst}\,\gamma : A_1 \sim_R A_2} \quad \textsc{An\_PiFst}$$

$$\frac{\begin{array}{l}\Gamma;\Delta \vdash \gamma_1 : \Pi^\rho x\!:\!A_1/R \to B_1 \sim_{R'} \Pi^\rho x\!:\!A_2/R \to B_2\\ \Gamma;\Delta \vdash \gamma_2 : a_1 \sim_R a_2\\ \Gamma \vdash a_1 : A_1/R\\ \Gamma \vdash a_2 : A_2/R\end{array}}{\Gamma;\Delta \vdash \gamma_1@\gamma_2 : B_1\{a_1/x\} \sim_{R'} B_2\{a_2/x\}} \quad \textsc{An\_PiSnd}$$

$$\frac{\begin{array}{l}\Gamma; \Delta \vdash \gamma_1 : a_1 \sim_{A_1/R} b_1 \sim a_2 \sim_{A_2/R} b_2 \\ \Gamma, c : a_1 \sim_{A_1/R} b_1; \Delta \vdash \gamma_3 : B_1 \sim_{R'} B_2 \\ B_3 = B_2\{c \triangleright_{R'} \mathbf{sym}\,\gamma_1/c\} \\ \Gamma \vdash \forall c{:}a_1 \sim_{A_1/R} b_1.B_1 : \star/R' \\ \Gamma \vdash \forall c{:}a_2 \sim_{A_2/R} b_2.B_3 : \star/R' \\ \Gamma \vdash \forall c{:}a_1 \sim_{A_1/R} b_1.B_2 : \star/R'\end{array}}{\Gamma; \Delta \vdash (\forall c{:}\gamma_1.\gamma_3) : (\forall c{:}a_1 \sim_{A_1/R} b_1.B_1) \sim_R (\forall c{:}a_2 \sim_{A_2/R} b_2.B_3)} \quad \textsc{An\_CPiCong}$$

$$\frac{\begin{array}{l}\Gamma; \Delta \vdash \gamma_1 : b_0 \sim_{A_1/R} b_1 \sim b_2 \sim_{A_2/R} b_3 \\ \Gamma, c : b_0 \sim_{A_1/R} b_1; \Delta \vdash \gamma_3 : a_1 \sim_{R'} a_2 \\ a_3 = a_2\{c \triangleright_{R'} \mathbf{sym}\,\gamma_1/c\} \\ \Gamma \vdash (\Lambda c{:}b_0 \sim_{A_1/R} b_1.a_1) : \forall c{:}b_0 \sim_{A_1/R} b_1.B_1/R' \\ \Gamma \vdash (\Lambda c{:}b_0 \sim_{A_1/R} b_1.a_2) : B/R' \\ \Gamma \vdash (\Lambda c{:}b_2 \sim_{A_2/R} b_3.a_3) : \forall c{:}b_2 \sim_{A_2/R} b_3.B_2/R' \\ \Gamma; \widetilde{\Gamma} \vdash \gamma_4 : \forall c{:}b_0 \sim_{A_1/R} b_1.B_1 \sim_{R'} \forall c{:}\phi_2.B_2\end{array}}{\Gamma; \Delta \vdash (\lambda c{:}\gamma_1.\gamma_3@\gamma_4) : (\Lambda c{:}b_0 \sim_{A_1/R} b_1.a_1) \sim_{R'} (\Lambda c{:}b_2 \sim_{A_2/R} b_3.a_3)} \quad \textsc{An\_CAbsCong}$$

$$\frac{\begin{array}{l}\Gamma; \Delta \vdash \gamma_1 : a_1 \sim_R b_1 \\ \Gamma; \widetilde{\Gamma} \vdash \gamma_2 : a_2 \sim_{R'} b_2 \\ \Gamma; \widetilde{\Gamma} \vdash \gamma_3 : a_3 \sim_{R'} b_3 \\ \Gamma \vdash a_1[\gamma_2] : A/R \\ \Gamma \vdash b_1[\gamma_3] : B/R \\ \Gamma; \widetilde{\Gamma} \vdash \gamma_4 : A \sim_R B\end{array}}{\Gamma; \Delta \vdash \gamma_1(\gamma_2, \gamma_3) : a_1[\gamma_2] \sim_R b_1[\gamma_3]} \quad \textsc{An\_CAppCong}$$

$$\frac{\begin{array}{l}\Gamma; \Delta \vdash \gamma_1 : (\forall c_1{:}a \sim_{A/R} a'.B_1) \sim_{R_0} (\forall c_2{:}b \sim_{B/R'} b'.B_2) \\ \Gamma; \widetilde{\Gamma} \vdash \gamma_2 : a \sim_R a' \\ \Gamma; \widetilde{\Gamma} \vdash \gamma_3 : b \sim_{R'} b'\end{array}}{\Gamma; \Delta \vdash \gamma_1@(\gamma_2 \sim \gamma_3) : B_1\{\gamma_2/c_1\} \sim_{R_0} B_2\{\gamma_3/c_2\}} \quad \textsc{An\_CPiSnd}$$

$$\frac{\begin{array}{l}\Gamma; \Delta \vdash \gamma_1 : a \sim_{R_1} a' \\ \Gamma; \Delta \vdash \gamma_2 : a \sim_{A/R_1} a' \sim b \sim_{B/R_1} b'\end{array}}{\Gamma; \Delta \vdash \gamma_1 \triangleright_{R_1} \gamma_2 : b \sim_{R_1} b'} \quad \textsc{An\_Cast}$$

$$\frac{\Gamma; \Delta \vdash \gamma : (a \sim_{A/R} a') \sim (b \sim_{B/R} b')}{\Gamma; \Delta \vdash \mathbf{isoSnd}\,\gamma : A \sim_R B} \quad \textsc{An\_IsoSnd}$$

$$\frac{\begin{array}{l}\Gamma; \Delta \vdash \gamma : a \sim_{R_1} b \\ R_1 \leq R_2\end{array}}{\Gamma; \Delta \vdash \mathbf{sub}\,\gamma : a \sim_{R_2} b} \quad \textsc{An\_Sub}$$

$\boxed{\vdash \Gamma}$ context wellformedness

$$\frac{}{\vdash \varnothing} \quad \textsc{An\_Empty}$$

$$\frac{\begin{array}{l}\vdash \Gamma \\ \Gamma \vdash A : \star/R \\ x \notin \mathsf{dom}\,\Gamma\end{array}}{\vdash \Gamma, x : A/R} \quad \textsc{An\_ConsTm}$$

$$\frac{\begin{array}{l}\vdash \Gamma \\ \Gamma \vdash \phi \;\mathsf{ok} \\ c \notin \mathsf{dom}\,\Gamma\end{array}}{\vdash \Gamma, c : \phi} \quad \textsc{An\_ConsCo}$$

$\boxed{\vdash \Sigma}$ signature wellformedness

$$\frac{}{\vdash \varnothing} \quad \textsc{An\_Sig\_Empty}$$

$$\frac{\begin{array}{l}\vdash \Sigma \\ \varnothing \vdash A : \star/R \\ \varnothing \vdash a : A/R \\ F \notin \mathsf{dom}\,\Sigma\end{array}}{\vdash \Sigma \cup \{F \sim a : A/R\}} \quad \textsc{An\_Sig\_ConsAx}$$

$\boxed{\Gamma \vdash a \leadsto b/R}$ single-step, weak head reduction to values for annotated language

$$\frac{\Gamma \vdash a \leadsto a'/R_1}{\Gamma \vdash a\ b^{R,\rho} \leadsto a'\ b^{R,\rho}/R_1} \quad \textsc{An\_AppLeft}$$

$$\frac{\mathsf{Value}_R\ (\lambda^\rho x\!:\!A/R.w)}{\Gamma \vdash (\lambda^\rho x\!:\!A/R.w)\ a^{R,\rho} \leadsto w\{a/x\}/R} \quad \textsc{An\_AppAbs}$$

$$\frac{\Gamma \vdash a \leadsto a'/R}{\Gamma \vdash a[\gamma] \leadsto a'[\gamma]/R} \quad \textsc{An\_CAppLeft}$$

$$\frac{}{\Gamma \vdash (\Lambda c\!:\!\phi.b)[\gamma] \leadsto b\{\gamma/c\}/R} \quad \textsc{An\_CAppCAbs}$$

$$\frac{\begin{array}{l}\Gamma \vdash A : \star/R \\ \Gamma, x : A/R \vdash b \leadsto b'/R_1\end{array}}{\Gamma \vdash (\lambda^- x\!:\!A/R.b) \leadsto (\lambda^- x\!:\!A/R.b')/R_1} \quad \textsc{An\_AbsTerm}$$

$$\frac{F \sim a : A/R \in \Sigma_1}{\Gamma \vdash F \leadsto a/R} \quad \textsc{An\_Axiom}$$

$$\frac{\Gamma \vdash a \leadsto a'/R}{\Gamma \vdash a \triangleright_{R_1} \gamma \leadsto a' \triangleright_{R_1} \gamma/R} \quad \textsc{An\_ConvTerm}$$

$$\frac{\mathsf{Value}_R\ v}{\Gamma \vdash (v \triangleright_{R_2} \gamma_1) \triangleright_{R_2} \gamma_2 \leadsto v \triangleright_{R_2} (\gamma_1;\gamma_2)/R} \quad \textsc{An\_Combine}$$

$$\frac{\begin{array}{l}\mathsf{Value}_R\ v \\ \Gamma;\widetilde{\Gamma} \vdash \gamma : \Pi^\rho x_1\!:\!A_1/R \to B_1 \sim_{R'} \Pi^\rho x_2\!:\!A_2/R \to B_2 \\ b' = b \triangleright_{R'} \mathbf{sym}\,(\mathbf{piFst}\,\gamma) \\ \gamma' = \gamma @ (b' \mid=\mid_{(\mathbf{piFst}\,\gamma)}\ b)\end{array}}{\Gamma \vdash (v \triangleright_{R'} \gamma)\ b^{R,\rho} \leadsto ((v\ b'^{R,\rho}) \triangleright_{R'} \gamma')/R} \quad \textsc{An\_Push}$$

$$\frac{\begin{array}{l}\mathsf{Value}_R\ v \\ \Gamma;\widetilde{\Gamma} \vdash \gamma : \forall c_1\!:\!a_1 \sim_{B_1/R} b_1.A_1 \sim_{R'} \forall c_2\!:\!a_2 \sim_{B_2/R} b_2.A_2 \\ \gamma_1' = \gamma_1 \triangleright_{R'} \mathbf{sym}\,(\mathbf{cpiFst}\,\gamma) \\ \gamma' = \gamma @ (\gamma_1' \sim \gamma_1)\end{array}}{\Gamma \vdash (v \triangleright_{R'} \gamma)[\gamma_1] \leadsto ((v[\gamma_1']) \triangleright_{R'} \gamma')/R} \quad \textsc{An\_CPush}$$

```
Definition rules:        163 good    0 bad
Definition rule clauses: 479 good    0 bad
```