$tmvar$, x, y, f, m, n	variables
$covar$, c	coercion variables
$datacon$, K	
$const$, T, F	
$index$, i	indices

relflag, ρ	$::=$			relevance flag
	$\mid$	$+$		
	$\mid$	$-$		
	$\mid$	$\mathit{app\_rho}\nu$		
	$\mid$	(ρ)		
appflag, ν	$::=$			applicative flag
	$\mid$	R		
	$\mid$	ρ		
role, R	$::=$			Role
	$\mid$	$\mathbf{Nom}$		
	$\mid$	$\mathbf{Rep}$		
	$\mid$	$R_1 \cap R_2$	S	
	$\mid$	$\mathbf{param}\, R_1\, R_2$	S	
	$\mid$	$\mathit{app\_role}\nu$	S	
	$\mid$	(R)	S	
constraint, ϕ	$::=$			props
	$\mid$	$a \sim_{A/R} b$		
	$\mid$	(ϕ)	S	
	$\mid$	$\phi\{b/x\}$	S	
	$\mid$	$\lvert\phi\rvert$	S	
	$\mid$	$a \sim_R b$	S	
tm, a, b, p, v, w, A, B, C	$::=$			types and kinds
	$\mid$	$\star$		
	$\mid$	x		
	$\mid$	$\lambda^\rho x\!:\!A.b$	bind x in b	
	$\mid$	$\lambda^\rho x.b$	bind x in b	
	$\mid$	$a\ b^\nu$		
	$\mid$	$\Pi^\rho x\!:\!A \to B$	bind x in B	
	$\mid$	$\Lambda c\!:\!\phi.b$	bind c in b	
	$\mid$	$\Lambda c.b$	bind c in b	
	$\mid$	$a[\gamma]$		
	$\mid$	$\forall c\!:\!\phi.B$	bind c in B	
	$\mid$	$a \triangleright_R \gamma$		
	$\mid$	F		
	$\mid$	$\square$		
	$\mid$	$\mathsf{case}_R\ a\ \mathsf{of}\ F \to b_1 \Vert_{\_} \to b_2$		
	$\mid$	K		
	$\mid$	$\mathbf{match}\, a\, \mathbf{with}\, \mathit{brs}$		
	$\mid$	$\mathbf{sub}\, R\, a$		
	$\mid$	$a\{b/x\}$	S	
	$\mid$	$a\{\gamma/c\}$	S	
	$\mid$	a	S	
	$\mid$	a	S	

	|	(a)	S	
	|	a	S	parsing precedence is hard
	|	$\|a\|_R$	S	
	|	**Int**	S	
	|	**Bool**	S	
	|	**Nat**	S	
	|	**Vec**	S	
	|	0	S	
	|	S	S	
	|	**True**	S	
	|	**Fix**	S	
	|	**Age**	S	
	|	$a \to b$	S	
	|	$\phi \Rightarrow A$	S	
	|	$a\ b$	S	
	|	$\lambda x.a$	S	
	|	$\lambda x\!:\!A.a$	S	
	|	$\forall\, x : A \to B$	S	
	|	**if** ϕ **then** a **else** b	S	
brs	::=			case branches
	|	**none**		
	|	$K \Rightarrow a; brs$		
	|	$brs\{a/x\}$	S	
	|	$brs\{\gamma/c\}$	S	
	|	(brs)	S	
$co,\ \gamma$	::=			explicit coercions
	|	$\bullet$		
	|	c		
	|	**red** $a\ b$		
	|	**refl** a		
	|	$(a \mid=\mid_{\gamma}\ b)$		
	|	**sym** γ		
	|	$\gamma_1;\gamma_2$		
	|	**sub** γ		
	|	$\Pi^{R,\rho} x\!:\!\gamma_1.\gamma_2$	bind x in γ_2	
	|	$\lambda^{R,\rho} x\!:\!\gamma_1.\gamma_2$	bind x in γ_2	
	|	$\gamma_1\ {\gamma_2}^{R,\rho}$		
	|	**piFst** γ		
	|	**cpiFst** γ		
	|	**isoSnd** γ		
	|	$\gamma_1 @ \gamma_2$		
	|	$\forall c\!:\!\gamma_1.\gamma_3$	bind c in γ_3	
	|	$\lambda c\!:\!\gamma_1.\gamma_3 @ \gamma_4$	bind c in γ_3	
	|	$\gamma(\gamma_1,\gamma_2)$		

	|	$\gamma @ (\gamma_1 \sim \gamma_2)$		
	|	$\gamma_1 \triangleright_R \gamma_2$		
	|	$\gamma_1 \sim_A \gamma_2$		
	|	$\mathbf{conv}\ \phi_1 \sim_\gamma \phi_2$		
	|	$\mathbf{eta}\, a$		
	|	$\mathbf{left}\, \gamma\, \gamma'$		
	|	$\mathbf{right}\, \gamma\, \gamma'$		
	|	(γ)	S	
	|	γ	S	
	|	$\gamma\{a/x\}$	S	
role_context, Ω	::=			role$_c$*ontexts*
	|	$\varnothing$		
	|	$x : R$		
	|	$\Omega, x : R$		
	|	Ω, Ω'	M	
	|	Γ_{Nom}		
	|	(Ω)	M	
	|	Ω	M	
roles, Rs	::=			
	|	$\mathbf{nilR}$		
	|	R, Rs		
sig_sort	::=			signature classifier
	|	$: A @ Rs$		
	|	$[p] \sim a : A/R @ Rs$		
sort	::=			binding classifier
	|	$\mathbf{Tm}\, A$		
	|	$\mathbf{Co}\, \phi$		
context, Γ	::=			contexts
	|	$\varnothing$		
	|	$\Gamma, x : A$		
	|	$\Gamma, c : \phi$		
	|	$\Gamma\{b/x\}$	M	
	|	$\Gamma\{\gamma/c\}$	M	
	|	Γ, Γ'	M	
	|	$\|\Gamma\|$	M	
	|	(Γ)	M	
	|	Γ	M	
sig, Σ	::=			signatures
	|	$\varnothing$		
	|	$\Sigma \cup \{F\, \mathit{sig\_sort}\}$		
	|	Σ_0	M	

	$\mid$	Σ_1	M
	$\mid$	$\lvert\Sigma\rvert$	M
$available\_props,\ \Delta$	::=		
	$\mid$	$\varnothing$	
	$\mid$	Δ, c	
	$\mid$	$\widetilde{\Gamma}$	M
	$\mid$	(Δ)	M
$terminals$	::=		
	$\mid$	$\leftrightarrow$	
	$\mid$	$\Leftrightarrow$	
	$\mid$	$\longrightarrow$	
	$\mid$	min	
	$\mid$	$\equiv$	
	$\mid$	$\forall$	
	$\mid$	$\in$	
	$\mid$	$\notin$	
	$\mid$	$\Leftarrow$	
	$\mid$	$\Rightarrow$	
	$\mid$	$\Rightarrow^*$	
	$\mid$	$\rightarrow$	
	$\mid$	Λ	
	$\mid$		
	$\mid$	$\Box$	
	$\mid$	$\vdash$	
	$\mid$	$\dashv$	
	$\mid$	$\models$	
	$\mid$	$\models$	
	$\mid$	$\neq$	
	$\mid$	$\triangleright$	
	$\mid$	ok	
	$\mid$	$\_$	
	$\mid$	$\leadsto$	
	$\mid$	$\leadsto^*$	
	$\mid$	$\leadsto$	
	$\mid$	$\varnothing$	
	$\mid$	$\circ$	
	$\mid$	fv	
	$\mid$	dom	
	$\mid$	$\sim$	
	$\mid$	$\asymp$	
	$\mid$	$\mid$	
	$\mid$	$\bullet$	
	$\mid$	$\mathbf{fst}$	
	$\mid$	$\mathbf{snd}$	

	|	**as**	
	|	$\mid\Rightarrow\mid$	
	|	$\vdash_{=}$	
	|	$\mathbf{refl_2}$	
	|	$++$	
$\mathit{formula},\ \psi$	::=		
	|	$\mathit{judgement}$	
	|	$x : A\ \in\ \Gamma$	
	|	$x : R\ \in\ \Omega$	
	|	$c : \phi\ \in\ \Gamma$	
	|	$F\ \mathit{sig\_sort}\ \in\ \Sigma$	
	|	$x\ \in\ \Delta$	
	|	$c\ \in\ \Delta$	
	|	$c\,\textbf{not relevant}\ \in\ \gamma$	
	|	$x\ \notin\ \mathsf{fv}\,a$	
	|	$x\ \notin\ \mathsf{dom}\,\Gamma$	
	|	$\mathit{uniq}\Gamma$	
	|	$\mathit{uniq}(\Omega)$	
	|	$c\ \notin\ \mathsf{dom}\,\Gamma$	
	|	$T\ \notin\ \mathsf{dom}\,\Sigma$	
	|	$F\ \notin\ \mathsf{dom}\,\Sigma$	
	|	$R_1\ =\ R_2$	
	|	$a\ =\ b$	
	|	$\phi_1\ =\ \phi_2$	
	|	$\Gamma_1\ =\ \Gamma_2$	
	|	$\gamma_1\ =\ \gamma_2$	
	|	$\neg\psi$	
	|	$\psi_1 \wedge \psi_2$	
	|	$\psi_1 \vee \psi_2$	
	|	$\psi_1 \Rightarrow \psi_2$	
	|	(ψ)	
	|	ψ	
	|	$c : (a : A \sim b : B)\ \in\ \Gamma$	
	|		suppress lc hypothesis generated by Ott
	|	$\{y/x\}B = B_1$	
	|	$\{c_1/c_2\}B = B_1$	
$\mathit{JSubRole}$	::=		
	|	$R_1 \leq R_2$	Subroling judgement
JPath	::=		
	|	$\mathsf{Path}_R\ a = F@\mathit{Rs}$	Type headed by constant (partial function)
JPat	::=		
	|	$\Gamma \vDash a : A\,\mathsf{pat}/R$	Pattern judgment

JCaseSyntax	::=		
	|	$\Gamma \vDash \mathsf{case}\ a : A\ \mathsf{of}\ b : B \Rightarrow C \vert C'$	Case Syntax judgment
JIrrelVarCheck	::=		
	|	$\mathsf{IrrelevantVar} a\ \cap\ \mathsf{fv} b = \emptyset$	Irrelevant Variable Check
JPatCtx	::=		
	|	$\vDash p : A\ \mathsf{patctx} = \Omega \mid \Gamma$	Contexts associated to a pattern
JMatchSubst	::=		
	|	$\mathsf{match}_R\ a_1\ \mathsf{with}\ a_2 \to b_1 = b_2$	match and substitute
JMatchApply	::=		
	|	$\mathsf{match}\ a\ \mathsf{applyto}\ b = b'$	match and apply arguments
JValue	::=		
	|	$\mathsf{Value}_R\ A$	values
JValueType	::=		
	|	$\mathsf{ValueType}_R\ A$	Types with head forms (erased language)
Jconsistent	::=		
	|	$\mathsf{consistent}_R\ a\ b$	(erased) types do not differ in their heads
Jroleing	::=		
	|	$\Omega \vDash a : R$	Roleing judgment
JChk	::=		
	|	$(\rho = +) \vee (x \notin \mathsf{fv}\ A)$	irrelevant argument check
Jpar	::=		
	|	$\Omega \vDash a \Rightarrow_R b$	parallel reduction (implicit language)
	|	$\Omega \vDash a \Rightarrow_R^* b$	multistep parallel reduction
	|	$\Omega \vDash a \Leftrightarrow_R b$	parallel reduction to a common term
Jbeta	::=		
	|	$\vDash a > b/R$	primitive reductions on erased terms
	|	$\vDash a \rightsquigarrow b/R$	single-step head reduction for implicit language
	|	$\vDash a \rightsquigarrow^* b/R$	multistep reduction
Jett	::=		
	|	$\Gamma \vDash \phi\ \mathsf{ok}$	Prop wellformedness
	|	$\Gamma \vDash a : A$	typing
	|	$\Gamma; \Delta \vDash \phi_1 \equiv \phi_2$	prop equality
	|	$\Gamma; \Delta \vDash a \equiv b : A/R$	definitional equality
	|	$\vDash \Gamma$	context wellformedness

Jsig	::=		
	|	$\models \Sigma$	signature wellformedness
judgement	::=		
	|	*JSubRole*	
	|	*JPath*	
	|	*JPat*	
	|	*JCaseSyntax*	
	|	*JIrrelVarCheck*	
	|	*JPatCtx*	
	|	*JMatchSubst*	
	|	*JMatchApply*	
	|	*JValue*	
	|	*JValueType*	
	|	*Jconsistent*	
	|	*Jroleing*	
	|	*JChk*	
	|	*Jpar*	
	|	*Jbeta*	
	|	*Jett*	
	|	*Jsig*	
user_syntax	::=		
	|	*tmvar*	
	|	*covar*	
	|	*datacon*	
	|	*const*	
	|	*index*	
	|	*relflag*	
	|	*appflag*	
	|	*role*	
	|	*constraint*	
	|	*tm*	
	|	*brs*	
	|	*co*	
	|	*role_context*	
	|	*roles*	
	|	*sig_sort*	
	|	*sort*	
	|	*context*	
	|	*sig*	
	|	*available_props*	
	|	*terminals*	
	|	*formula*	

$\boxed{R_1 \leq R_2}$ Subroling judgement

$$\frac{}{\mathbf{Nom} \leq R} \quad \textsc{NomBot}$$

$$\frac{}{R \leq \mathbf{Rep}} \quad \textsc{RepTop}$$

$$\frac{}{R \leq R} \quad \textsc{Refl}$$

$$\frac{\begin{array}{l} R_1 \leq R_2 \\ R_2 \leq R_3 \end{array}}{R_1 \leq R_3} \quad \textsc{Trans}$$

$\boxed{\mathsf{Path}_R\ a = F@Rs}$ Type headed by constant (partial function)

$$\frac{F\ : A@Rs\ \in \Sigma_0}{\mathsf{Path}_R\ F = F@Rs} \quad \textsc{Path\_AbsConst}$$

$$\frac{\begin{array}{l} F\ [p] \sim a : A/R_1@Rs\ \in \Sigma_0 \\ \neg(R_1 \leq R) \end{array}}{\mathsf{Path}_R\ F = F@Rs} \quad \textsc{Path\_Const}$$

$$\frac{\begin{array}{l} \mathsf{Path}_R\ a = F@R_1, Rs \\ app\_role\nu = R_1 \end{array}}{\mathsf{Path}_R\ (a\ b'^{\nu}) = F@Rs} \quad \textsc{Path\_App}$$

$$\frac{\mathsf{Path}_R\ a = F@Rs}{\mathsf{Path}_R\ (a[\bullet]) = F@Rs} \quad \textsc{Path\_CApp}$$

$\boxed{\Gamma \vDash a : A\ \mathsf{pat}/R}$ Pattern judgment

$$\frac{F\ : A@Rs\ \in \Sigma_0}{\varnothing \vDash F : A\ \mathsf{pat}/R} \quad \textsc{Pat\_AbsConst}$$

$$\frac{\begin{array}{l} F\ [p] \sim a : A/R_1@Rs\ \in \Sigma_0 \\ \neg(R_1 \leq R) \end{array}}{\varnothing \vDash F : A\ \mathsf{pat}/R} \quad \textsc{Pat\_Const}$$

$$\frac{\begin{array}{l} \Gamma \vDash a : \Pi^{\rho} y\!:\!A_1 \to B_1\ \mathsf{pat}/R \\ \{y/x\}B = B_1 \end{array}}{\Gamma, x : A_1 \vDash (a\ x^{\rho}) : B\ \mathsf{pat}/R} \quad \textsc{Pat\_App}$$

$$\frac{\begin{array}{l} \Gamma \vDash a : \forall c_1\!:\!\phi.B_1\ \mathsf{pat}/R \\ \{c_1/c\}B = B_1 \end{array}}{\Gamma, c : \phi \vDash (a[c]) : B\ \mathsf{pat}/R} \quad \textsc{Pat\_CApp}$$

$\boxed{\Gamma \vDash \mathsf{case}\ a : A\ \mathsf{of}\ b : B \Rightarrow C | C'}$ Case Syntax judgment

$$\frac{uniq\Gamma}{\Gamma \vDash \mathsf{case}\ a : A\ \mathsf{of}\ b : A \Rightarrow \forall c\!:\!(a \sim_{A/R} b).C | C} \quad \textsc{CaseSyntax\_Base}$$

$$\frac{\Gamma, x : A \vDash \mathsf{case}\ a : A_1\ \mathsf{of}\ b\ x^{+} : B \Rightarrow C | C'}{\Gamma \vDash \mathsf{case}\ a : A_1\ \mathsf{of}\ b : \Pi^{+} x\!:\!A \to B \Rightarrow \Pi^{+} x\!:\!A \to C | C'} \quad \textsc{CaseSyntax\_PiRel}$$

$$\frac{\Gamma, x : A \vDash \mathsf{case}\ a : A_1\ \mathsf{of}\ b\ x^{-} : B \Rightarrow C | C'}{\Gamma \vDash \mathsf{case}\ a : A_1\ \mathsf{of}\ b : \Pi^{-} x\!:\!A \to B \Rightarrow \Pi^{-} x\!:\!A \to C | C'} \quad \textsc{CaseSyntax\_PiIrrel}$$

$$\frac{\Gamma, c : \phi \vDash \mathsf{case}\ a : A\ \mathsf{of}\ b[c] : B \Rightarrow C | C'}{\Gamma \vDash \mathsf{case}\ a : A\ \mathsf{of}\ b : \forall c\!:\!\phi.B \Rightarrow \forall c\!:\!\phi.C | C'} \quad \textsc{CaseSyntax\_CPi}$$

$\boxed{\mathsf{IrrelevantVar} a\ \cap\ \mathsf{fv} b = \emptyset}$ Irrelevant Variable Check

$$\frac{}{\mathsf{IrrelevantVar} F\ \cap\ \mathsf{fv} b = \emptyset} \quad \textsc{IrrelVarCheck\_Const}$$

$$\frac{\mathsf{IrrelevantVar}\, a \ \cap \ \mathsf{fv}\, b = \emptyset}{\mathsf{IrrelevantVar}(a\ x^{+}) \ \cap \ \mathsf{fv}\, b = \emptyset} \quad \textsc{IrrelVarCheck\_App}$$

$$\frac{\begin{array}{l}\mathsf{IrrelevantVar}\, a \ \cap \ \mathsf{fv}\, b = \emptyset \\ x \ \notin \ \mathsf{fv}\, b\end{array}}{\mathsf{IrrelevantVar}(a\ x^{-}) \ \cap \ \mathsf{fv}\, b = \emptyset} \quad \textsc{IrrelVarCheck\_IApp}$$

$$\frac{\mathsf{IrrelevantVar}\, a \ \cap \ \mathsf{fv}\, b = \emptyset}{\mathsf{IrrelevantVar}(a[c]) \ \cap \ \mathsf{fv}\, b = \emptyset} \quad \textsc{IrrelVarCheck\_CApp}$$

$\boxed{\vDash p : A \ \mathsf{patctx} = \Omega \mid \Gamma}$ Contexts associated to a pattern

$$\frac{}{\vDash F : A \ \mathsf{patctx} = \varnothing \mid \varnothing} \quad \textsc{PatCtx\_Const}$$

$$\frac{\vDash p : \Pi^{+} x\!:\!A' \to A \ \mathsf{patctx} = \Omega \mid \Gamma}{\vDash p\ x : A \ \mathsf{patctx} = \Omega, x : R \mid \Gamma, x : A'} \quad \textsc{PatCtx\_PiRel}$$

$$\frac{\vDash p : \Pi^{-} x\!:\!A' \to A \ \mathsf{patctx} = \Omega \mid \Gamma}{\vDash p\ \square : A \ \mathsf{patctx} = \Omega \mid \Gamma, x : A'} \quad \textsc{PatCtx\_PiIrr}$$

$$\frac{\vDash p : \forall c\!:\!\phi. A \ \mathsf{patctx} = \Omega \mid \Gamma}{\vDash p[\bullet] : A \ \mathsf{patctx} = \Omega \mid \Gamma, c : \phi} \quad \textsc{PatCtx\_CPi}$$

$\boxed{\mathsf{match}_R \ a_1 \ \mathsf{with} \ a_2 \to b_1 = b_2}$ match and substitute

$$\frac{F \ : A @ Rs \ \in \ \Sigma_0}{\mathsf{match}_R \ F \ \mathsf{with} \ F \to b = b} \quad \textsc{MatchSubst\_AbsConst}$$

$$\frac{\begin{array}{l} F \ [p] \sim a : A / R_1 @ Rs \ \in \ \Sigma_0 \\ \neg(R_1 \leq R) \end{array}}{\mathsf{match}_R \ F \ \mathsf{with} \ F \to b = b} \quad \textsc{MatchSubst\_Const}$$

$$\frac{\mathsf{match}_R \ a_1 \ \mathsf{with} \ a_2 \to b_1 = b_2}{\mathsf{match}_R \ (a_1 \ a^{R'}) \ \mathsf{with} \ (a_2 \ x^{+}) \to b_1 = (b_2\{a/x\})} \quad \textsc{MatchSubst\_AppRelR}$$

$$\frac{\mathsf{match}_R \ a_1 \ \mathsf{with} \ a_2 \to b_1 = b_2}{\mathsf{match}_R \ (a_1 \ a^{+}) \ \mathsf{with} \ (a_2 \ x^{+}) \to b_1 = (b_2\{a/x\})} \quad \textsc{MatchSubst\_AppRel}$$

$$\frac{\mathsf{match}_R \ a_1 \ \mathsf{with} \ a_2 \to b_1 = b_2}{\mathsf{match}_R \ (a_1 \ \square^{-}) \ \mathsf{with} \ (a_2 \ x^{-}) \to b_1 = (b_2\{\square/x\})} \quad \textsc{MatchSubst\_AppIrrel}$$

$$\frac{\mathsf{match}_R \ a_1 \ \mathsf{with} \ a_2 \to b_1 = b_2}{\mathsf{match}_R \ (a_1[\bullet]) \ \mathsf{with} \ (a_2[c]) \to b_1 = (b_2\{\bullet/c\})} \quad \textsc{MatchSubst\_CApp}$$

$\boxed{\mathsf{match} \ a \ \mathsf{applyto} \ b = b'}$ match and apply arguments

$$\frac{F \ : A @ Rs \ \in \ \Sigma_0}{\mathsf{match} \ F \ \mathsf{applyto} \ b = b} \quad \textsc{MatchApply\_Const}$$

$$\frac{\mathsf{match} \ a \ \mathsf{applyto} \ b = b'}{\mathsf{match} \ a \ a'^{\nu} \ \mathsf{applyto} \ b = b' \ a'^{(app\_rho\nu)}} \quad \textsc{MatchApply\_App}$$

$$\frac{\mathsf{match} \ a \ \mathsf{applyto} \ b = b'}{\mathsf{match} \ a[\gamma] \ \mathsf{applyto} \ b = b'[\gamma]} \quad \textsc{MatchApply\_CApp}$$

$\boxed{\mathsf{Value}_R \ A}$ values

$$\frac{}{\mathsf{Value}_R \ \star} \quad \textsc{Value\_Star}$$

$$\frac{}{\mathsf{Value}_R\ \Pi^\rho x\!:\!A \to B}\quad \textsc{Value\_Pi}$$

$$\frac{}{\mathsf{Value}_R\ \forall c\!:\!\phi.B}\quad \textsc{Value\_CPi}$$

$$\frac{}{\mathsf{Value}_R\ \lambda^+ x\!:\!A.a}\quad \textsc{Value\_AbsRel}$$

$$\frac{}{\mathsf{Value}_R\ \lambda^+ x.a}\quad \textsc{Value\_UAbsRel}$$

$$\frac{\mathsf{Value}_R\ a}{\mathsf{Value}_R\ \lambda^- x.a}\quad \textsc{Value\_UAbsIrrel}$$

$$\frac{}{\mathsf{Value}_R\ \Lambda c\!:\!\phi.a}\quad \textsc{Value\_CAbs}$$

$$\frac{}{\mathsf{Value}_R\ \Lambda c.a}\quad \textsc{Value\_UCAbs}$$

$$\frac{\mathsf{Path}_R\ a = F@Rs}{\mathsf{Value}_R\ a}\quad \textsc{Value\_Path}$$

$\boxed{\mathsf{ValueType}_R\ A}$ Types with head forms (erased language)

$$\frac{}{\mathsf{ValueType}_R\ \star}\quad \textsc{value\_type\_Star}$$

$$\frac{}{\mathsf{ValueType}_R\ \Pi^\rho x\!:\!A \to B}\quad \textsc{value\_type\_Pi}$$

$$\frac{}{\mathsf{ValueType}_R\ \forall c\!:\!\phi.B}\quad \textsc{value\_type\_CPi}$$

$$\frac{\mathsf{Path}_R\ a = F@Rs}{\mathsf{ValueType}_R\ a}\quad \textsc{value\_type\_Path}$$

$\boxed{\mathsf{consistent}_R\ a\ b}$ (erased) types do not differ in their heads

$$\frac{}{\mathsf{consistent}_R\ \star\ \star}\quad \textsc{consistent\_a\_Star}$$

$$\frac{}{\mathsf{consistent}_{R'}\ (\Pi^\rho x_1\!:\!A_1 \to B_1)\ (\Pi^\rho x_2\!:\!A_2 \to B_2)}\quad \textsc{consistent\_a\_Pi}$$

$$\frac{}{\mathsf{consistent}_R\ (\forall c_1\!:\!\phi_1.A_1)\ (\forall c_2\!:\!\phi_2.A_2)}\quad \textsc{consistent\_a\_CPi}$$

$$\frac{\begin{array}{c}\mathsf{Path}_R\ a_1 = F@Rs\\ \mathsf{Path}_R\ a_2 = F@Rs\end{array}}{\mathsf{consistent}_R\ a_1\ a_2}\quad \textsc{consistent\_a\_Path}$$

$$\frac{\neg\mathsf{ValueType}_R\ b}{\mathsf{consistent}_R\ a\ b}\quad \textsc{consistent\_a\_Step\_R}$$

$$\frac{\neg\mathsf{ValueType}_R\ a}{\mathsf{consistent}_R\ a\ b}\quad \textsc{consistent\_a\_Step\_L}$$

$\boxed{\Omega \vDash a : R}$ Roleing judgment

$$\frac{\mathit{uniq}(\Omega)}{\Omega \vDash \Box : R}\quad \textsc{role\_a\_Bullet}$$

$$\frac{\mathit{uniq}(\Omega)}{\Omega \vDash \star : R}\quad \textsc{role\_a\_Star}$$

$$\frac{\begin{array}{l} uniq(\Omega) \\ x : R \in \Omega \\ R \leq R_1 \end{array}}{\Omega \vDash x : R_1} \quad \textsc{role\_a\_Var}$$

$$\frac{\Omega, x : \mathbf{Nom} \vDash a : R}{\Omega \vDash (\lambda^\rho x.a) : R} \quad \textsc{role\_a\_Abs}$$

$$\frac{\begin{array}{l} \Omega \vDash a : R \\ \Omega \vDash b : app\_role\nu \end{array}}{\Omega \vDash (a\ b^\nu) : R} \quad \textsc{role\_a\_App}$$

$$\frac{\begin{array}{l} \Omega \vDash A : R \\ \Omega, x : \mathbf{Nom} \vDash B : R \end{array}}{\Omega \vDash (\Pi^\rho x\!:\!A \to B) : R} \quad \textsc{role\_a\_Pi}$$

$$\frac{\begin{array}{l} \Omega \vDash a : R_1 \\ \Omega \vDash b : R_1 \\ \Omega \vDash A : R_0 \\ \Omega \vDash B : R \end{array}}{\Omega \vDash (\forall c\!:\!a \sim_{A/R_1} b.B) : R} \quad \textsc{role\_a\_CPi}$$

$$\frac{\Omega \vDash b : R}{\Omega \vDash (\Lambda c.b) : R} \quad \textsc{role\_a\_CAbs}$$

$$\frac{\Omega \vDash a : R}{\Omega \vDash (a[\bullet]) : R} \quad \textsc{role\_a\_CApp}$$

$$\frac{\begin{array}{l} uniq(\Omega) \\ F\ : A@Rs \in \Sigma_0 \end{array}}{\Omega \vDash F : R} \quad \textsc{role\_a\_Const}$$

$$\frac{\begin{array}{l} uniq(\Omega) \\ F\ [p] \sim a : A/R@Rs \in \Sigma_0 \end{array}}{\Omega \vDash F : R_1} \quad \textsc{role\_a\_Fam}$$

$$\frac{\begin{array}{l} \Omega \vDash a : R \\ \Omega, \Gamma_{\mathsf{Nom}} \vDash b_1 : R_1 \\ \Omega \vDash b_2 : R_1 \end{array}}{\Omega \vDash \mathsf{case}_R\ a\ \mathsf{of}\ F \to b_1 \|_\_ \to b_2 : R_1} \quad \textsc{role\_a\_Pattern}$$

$\boxed{(\rho = +) \vee (x \notin \mathsf{fv}\ A)}$ irrelevant argument check

$$\frac{}{(+ = +) \vee (x \notin \mathsf{fv}\ A)} \quad \textsc{Rho\_Rel}$$

$$\frac{x \notin \mathsf{fv} A}{(- = +) \vee (x \notin \mathsf{fv}\ A)} \quad \textsc{Rho\_IrrRel}$$

$\boxed{\Omega \vDash a \Rightarrow_R b}$ parallel reduction (implicit language)

$$\frac{\Omega \vDash a : R}{\Omega \vDash a \Rightarrow_R a} \quad \textsc{Par\_Refl}$$

$$\frac{\begin{array}{l} \Omega \vDash a \Rightarrow_R (\lambda^\rho x.a') \\ \Omega \vDash b \Rightarrow_{app\_role\nu} b' \end{array}}{\Omega \vDash a\ b^\nu \Rightarrow_R a'\{b'/x\}} \quad \textsc{Par\_Beta}$$

$$\frac{\begin{array}{l}\Omega \vDash a \Rightarrow_R a' \\ \Omega \vDash b \Rightarrow_{app\_role\,\nu} b'\end{array}}{\Omega \vDash a\ b^{\nu} \Rightarrow_R a'\ b'^{\nu}} \quad \textsc{Par\_App}$$

$$\frac{\Omega \vDash a \Rightarrow_R (\Lambda c.a')}{\Omega \vDash a[\bullet] \Rightarrow_R a'\{\bullet/c\}} \quad \textsc{Par\_CBeta}$$

$$\frac{\Omega \vDash a \Rightarrow_R a'}{\Omega \vDash a[\bullet] \Rightarrow_R a'[\bullet]} \quad \textsc{Par\_CApp}$$

$$\frac{\Omega, x : \mathbf{Nom} \vDash a \Rightarrow_R a'}{\Omega \vDash \lambda^{\rho} x.a \Rightarrow_R \lambda^{\rho} x.a'} \quad \textsc{Par\_Abs}$$

$$\frac{\begin{array}{l}\Omega \vDash A \Rightarrow_R A' \\ \Omega, x : \mathbf{Nom} \vDash B \Rightarrow_R B'\end{array}}{\Omega \vDash \Pi^{\rho} x\!:\!A \to B \Rightarrow_R \Pi^{\rho} x\!:\!A' \to B'} \quad \textsc{Par\_Pi}$$

$$\frac{\Omega \vDash a \Rightarrow_R a'}{\Omega \vDash \Lambda c.a \Rightarrow_R \Lambda c.a'} \quad \textsc{Par\_CAbs}$$

$$\frac{\begin{array}{l}\Omega \vDash A \Rightarrow_{R_0} A' \\ \Omega \vDash a \Rightarrow_{R_1} a' \\ \Omega \vDash b \Rightarrow_{R_1} b' \\ \Omega \vDash B \Rightarrow_R B'\end{array}}{\Omega \vDash \forall c\!:\!a \sim_{A/R_1} b.B \Rightarrow_R \forall c\!:\!a' \sim_{A'/R_1} b'.B'} \quad \textsc{Par\_CPi}$$

$$\frac{\begin{array}{l}\mathsf{Path}_{R_1}\ a = F@Rs \\ F\ [p] \sim b : A/R_1@Rs \ \in\ \Sigma_0 \\ \Omega \vDash a \Rightarrow_R a' \\ \mathsf{match}_{R_2}\ a'\ \mathsf{with}\ p \to b = b' \\ R_1 \leq R \\ uniq(\Omega)\end{array}}{\Omega \vDash a \Rightarrow_R b'} \quad \textsc{Par\_Axiom}$$

$$\frac{\begin{array}{l}\Omega \vDash a \Rightarrow_R a' \\ \Omega \vDash b_1 \Rightarrow_{R_0} b_1' \\ \Omega \vDash b_2 \Rightarrow_{R_0} b_2'\end{array}}{\Omega \vDash (\mathsf{case}_R\ a\ \mathsf{of}\ F \to b_1 \|_{\_} \to b_2) \Rightarrow_{R_0} (\mathsf{case}_R\ a'\ \mathsf{of}\ F \to b_1' \|_{\_} \to b_2')} \quad \textsc{Par\_Pattern}$$

$$\frac{\begin{array}{l}\Omega \vDash a \Rightarrow_R a' \\ \Omega \vDash b_1 \Rightarrow_{R_0} b_1' \\ \mathsf{Path}_{R_1}\ a' = F@Rs \\ \mathsf{match}\ a'\ \mathsf{applyto}\ b_1' = b\end{array}}{\Omega \vDash (\mathsf{case}_R\ a\ \mathsf{of}\ F \to b_1 \|_{\_} \to b_2) \Rightarrow_{R_0} b[\bullet]} \quad \textsc{Par\_PatternTrue}$$

$$\frac{\begin{array}{l}\Omega \vDash a \Rightarrow_R a' \\ \Omega \vDash b_2 \Rightarrow_{R_0} b_2' \\ \mathsf{Value}_R\ a' \\ \neg(\mathsf{Path}_{R_1}\ a' = F@Rs)\end{array}}{\Omega \vDash (\mathsf{case}_R\ a\ \mathsf{of}\ F \to b_1 \|_{\_} \to b_2) \Rightarrow_{R_0} b_2'} \quad \textsc{Par\_PatternFalse}$$

$\boxed{\Omega \vDash a \Rightarrow_R^* b}$ multistep parallel reduction

$$\frac{}{\Omega \vDash a \Rightarrow_R^* a} \quad \textsc{MP\_Refl}$$

$$\frac{\Omega \vDash a \Rightarrow_R b \quad \Omega \vDash b \Rightarrow_R^* a'}{\Omega \vDash a \Rightarrow_R^* a'} \quad \text{MP\_STEP}$$

$\boxed{\Omega \vDash a \Leftrightarrow_R b}$ parallel reduction to a common term

$$\frac{\Omega \vDash a_1 \Rightarrow_R^* b \quad \Omega \vDash a_2 \Rightarrow_R^* b}{\Omega \vDash a_1 \Leftrightarrow_R a_2} \quad \text{JOIN}$$

$\boxed{\vDash a > b/R}$ primitive reductions on erased terms

$$\frac{\mathsf{Value}_{R_1}\ (\lambda^\rho x.v)}{\vDash (\lambda^\rho x.v)\ b^\nu > v\{b/x\}/R_1} \quad \text{BETA\_APPABS}$$

$$\frac{}{\vDash (\Lambda c.a')[\bullet] > a'\{\bullet/c\}/R} \quad \text{BETA\_CAPPCABS}$$

$$\frac{\begin{array}{l}\mathsf{Path}_{R_1}\ a = F@Rs \\ F\ [p] \sim b : A/R_1@Rs \in \Sigma_0 \\ \mathsf{match}_{R_2}\ a\ \mathsf{with}\ p \to b = b' \\ R_1 \leq R\end{array}}{\vDash a > b'/R} \quad \text{BETA\_AXIOM}$$

$$\frac{\begin{array}{l}\mathsf{Path}_R\ a = F@Rs \\ \mathsf{match}\ a\ \mathsf{applyto}\ b_1 = b_1'\end{array}}{\vDash \mathsf{case}_R\ a\ \mathsf{of}\ F \to b_1 \|\_ \to b_2 > b_1'[\bullet]/R_0} \quad \text{BETA\_PATTERNTRUE}$$

$$\frac{\begin{array}{l}\mathsf{Value}_R\ a \\ \neg(\mathsf{Path}_R\ a = F@Rs)\end{array}}{\vDash \mathsf{case}_R\ a\ \mathsf{of}\ F \to b_1 \|\_ \to b_2 > b_2/R_0} \quad \text{BETA\_PATTERNFALSE}$$

$\boxed{\vDash a \rightsquigarrow b/R}$ single-step head reduction for implicit language

$$\frac{\vDash a \rightsquigarrow a'/R_1}{\vDash \lambda^- x.a \rightsquigarrow \lambda^- x.a'/R_1} \quad \text{E\_ABSTERM}$$

$$\frac{\vDash a \rightsquigarrow a'/R_1}{\vDash a\ b^\nu \rightsquigarrow a'\ b^\nu/R_1} \quad \text{E\_APPLEFT}$$

$$\frac{\vDash a \rightsquigarrow a'/R}{\vDash a[\bullet] \rightsquigarrow a'[\bullet]/R} \quad \text{E\_CAPPLEFT}$$

$$\frac{\vDash a \rightsquigarrow a'/R}{\vDash \mathsf{case}_R\ a\ \mathsf{of}\ F \to b_1 \|\_ \to b_2 \rightsquigarrow \mathsf{case}_R\ a'\ \mathsf{of}\ F \to b_1 \|\_ \to b_2/R_0} \quad \text{E\_PATTERN}$$

$$\frac{\vDash a > b/R}{\vDash a \rightsquigarrow b/R} \quad \text{E\_PRIM}$$

$\boxed{\vDash a \rightsquigarrow^* b/R}$ multistep reduction

$$\frac{}{\vDash a \rightsquigarrow^* a/R} \quad \text{EQUAL}$$

$$\frac{\vDash a \rightsquigarrow b/R \quad \vDash b \rightsquigarrow^* a'/R}{\vDash a \rightsquigarrow^* a'/R} \quad \text{STEP}$$

$\boxed{\Gamma \vDash \phi\ \mathsf{ok}}$ Prop wellformedness

$$\frac{\Gamma \vDash a : A \quad \Gamma \vDash b : A \quad \Gamma \vDash A : \star}{\Gamma \vDash a \sim_{A/R} b \;\; \mathsf{ok}} \quad \text{E\_WFF}$$

$\boxed{\Gamma \vDash a : A}$ typing

$$\frac{\vDash \Gamma}{\Gamma \vDash \star : \star} \quad \text{E\_STAR}$$

$$\frac{\vDash \Gamma \quad x : A \in \Gamma}{\Gamma \vDash x : A} \quad \text{E\_VAR}$$

$$\frac{\Gamma, x : A \vDash B : \star \quad \Gamma \vDash A : \star}{\Gamma \vDash \Pi^{\rho} x\!:\!A \to B : \star} \quad \text{E\_PI}$$

$$\frac{\Gamma, x : A \vDash a : B \quad \Gamma \vDash A : \star \quad (\rho = +) \vee (x \notin \mathsf{fv}\ a)}{\Gamma \vDash \lambda^{\rho} x.a : (\Pi^{\rho} x\!:\!A \to B)} \quad \text{E\_ABS}$$

$$\frac{\Gamma \vDash b : \Pi^{+} x\!:\!A \to B \quad \Gamma \vDash a : A}{\Gamma \vDash b\ a^{+} : B\{a/x\}} \quad \text{E\_APP}$$

$$\frac{\Gamma \vDash b : \Pi^{+} x\!:\!A \to B \quad \Gamma \vDash a : A \quad \mathsf{Path}_{R'}\ b = F@R, Rs}{\Gamma \vDash b\ a^{R} : B\{a/x\}} \quad \text{E\_TAPP}$$

$$\frac{\Gamma \vDash b : \Pi^{-} x\!:\!A \to B \quad \Gamma \vDash a : A}{\Gamma \vDash b\ \Box^{-} : B\{a/x\}} \quad \text{E\_IAPP}$$

$$\frac{\Gamma \vDash a : A \quad \Gamma; \widetilde{\Gamma} \vDash A \equiv B : \star/\mathbf{Rep} \quad \Gamma \vDash B : \star}{\Gamma \vDash a : B} \quad \text{E\_CONV}$$

$$\frac{\Gamma, c : \phi \vDash B : \star \quad \Gamma \vDash \phi \;\; \mathsf{ok}}{\Gamma \vDash \forall c\!:\!\phi.B : \star} \quad \text{E\_CPI}$$

$$\frac{\Gamma, c : \phi \vDash a : B \quad \Gamma \vDash \phi \;\; \mathsf{ok}}{\Gamma \vDash \Lambda c.a : \forall c\!:\!\phi.B} \quad \text{E\_CABS}$$

$$\frac{\Gamma \vDash a_1 : \forall c\!:\!(a \sim_{A/R} b).B_1 \quad \Gamma; \widetilde{\Gamma} \vDash a \equiv b : A/R}{\Gamma \vDash a_1[\bullet] : B_1\{\bullet/c\}} \quad \text{E\_CAPP}$$

$$\frac{\vDash \Gamma \quad F : A@Rs \in \Sigma_0 \quad \varnothing \vDash A : \star}{\Gamma \vDash F : A} \quad \text{E\_CONST}$$

$$\frac{\begin{array}{l}\vDash \Gamma \\ F\ [p] \sim a : A/R_1@Rs \in \Sigma_0 \\ \varnothing \vDash A : \star\end{array}}{\Gamma \vDash F : A} \quad \text{E\_FAM}$$

$$\frac{\begin{array}{l}\Gamma \vDash a : A \\ \Gamma \vDash F : A_1 \\ \Gamma \vDash b_1 : B \\ \Gamma \vDash b_2 : C \\ \Gamma \vDash \mathsf{case}\ a : A\ \mathsf{of}\ F : A_1 \Rightarrow B | C\end{array}}{\Gamma \vDash \mathsf{case}_R\ a\ \mathsf{of}\ F \to b_1 \|\_ \to b_2 : C} \quad \text{E\_CASE}$$

$\boxed{\Gamma; \Delta \vDash \phi_1 \equiv \phi_2}$ prop equality

$$\frac{\begin{array}{c}\Gamma; \Delta \vDash A_1 \equiv A_2 : A/R \\ \Gamma; \Delta \vDash B_1 \equiv B_2 : A/R\end{array}}{\Gamma; \Delta \vDash A_1 \sim_{A/R} B_1 \equiv A_2 \sim_{A/R} B_2} \quad \text{E\_PROPCONG}$$

$$\frac{\begin{array}{c}\Gamma; \Delta \vDash A \equiv B : \star/R_0 \\ \Gamma \vDash A_1 \sim_{A/R} A_2\ \mathsf{ok} \\ \Gamma \vDash A_1 \sim_{B/R} A_2\ \mathsf{ok}\end{array}}{\Gamma; \Delta \vDash A_1 \sim_{A/R} A_2 \equiv A_1 \sim_{B/R} A_2} \quad \text{E\_ISOCONV}$$

$$\frac{\Gamma; \Delta \vDash \forall c{:}(a_1 \sim_{A/R_1} a_2).B_1 \equiv \forall c{:}(b_1 \sim_{B/R_2} b_2).B_2 : \star/R'}{\Gamma; \Delta \vDash a_1 \sim_{A/R_1} a_2 \equiv b_1 \sim_{B/R_2} b_2} \quad \text{E\_CPIFST}$$

$\boxed{\Gamma; \Delta \vDash a \equiv b : A/R}$ definitional equality

$$\frac{\begin{array}{l}\vDash \Gamma \\ c : (a \sim_{A/R} b) \in \Gamma \\ c \in \Delta\end{array}}{\Gamma; \Delta \vDash a \equiv b : A/R} \quad \text{E\_ASSN}$$

$$\frac{\Gamma \vDash a : A}{\Gamma; \Delta \vDash a \equiv a : A/\mathbf{Nom}} \quad \text{E\_REFL}$$

$$\frac{\Gamma; \Delta \vDash b \equiv a : A/R}{\Gamma; \Delta \vDash a \equiv b : A/R} \quad \text{E\_SYM}$$

$$\frac{\begin{array}{c}\Gamma; \Delta \vDash a \equiv a_1 : A/R \\ \Gamma; \Delta \vDash a_1 \equiv b : A/R\end{array}}{\Gamma; \Delta \vDash a \equiv b : A/R} \quad \text{E\_TRANS}$$

$$\frac{\begin{array}{l}\Gamma; \Delta \vDash a \equiv b : A/R_1 \\ R_1 \leq R_2\end{array}}{\Gamma; \Delta \vDash a \equiv b : A/R_2} \quad \text{E\_SUB}$$

$$\frac{\begin{array}{l}\Gamma \vDash a_1 : B \\ \Gamma \vDash a_2 : B \\ \vDash a_1 > a_2/R\end{array}}{\Gamma; \Delta \vDash a_1 \equiv a_2 : B/R} \quad \text{E\_BETA}$$

$$\frac{\begin{array}{l}\Gamma; \Delta \vDash A_1 \equiv A_2 : \star/R' \\ \Gamma, x : A_1; \Delta \vDash B_1 \equiv B_2 : \star/R' \\ \Gamma \vDash A_1 : \star \\ \Gamma \vDash \Pi^\rho x{:}A_1 \to B_1 : \star \\ \Gamma \vDash \Pi^\rho x{:}A_2 \to B_2 : \star\end{array}}{\Gamma; \Delta \vDash (\Pi^\rho x{:}A_1 \to B_1) \equiv (\Pi^\rho x{:}A_2 \to B_2) : \star/R'} \quad \text{E\_PICONG}$$

$$\frac{\begin{array}{l}\Gamma, x : A_1; \Delta \vDash b_1 \equiv b_2 : B/R' \\ \Gamma \vDash A_1 : \star \\ (\rho = +) \vee (x \notin \mathsf{fv}\ b_1) \\ (\rho = +) \vee (x \notin \mathsf{fv}\ b_2)\end{array}}{\Gamma; \Delta \vDash (\lambda^\rho x.b_1) \equiv (\lambda^\rho x.b_2) : (\Pi^\rho x\!:\!A_1 \to B)/R'} \quad \textsc{E\_AbsCong}$$

$$\frac{\begin{array}{l}\Gamma; \Delta \vDash a_1 \equiv b_1 : (\Pi^+ x\!:\!A \to B)/R' \\ \Gamma; \Delta \vDash a_2 \equiv b_2 : A/\mathbf{Nom}\end{array}}{\Gamma; \Delta \vDash a_1\ {a_2}^+ \equiv b_1\ {b_2}^+ : (B\{a_2/x\})/R'} \quad \textsc{E\_AppCong}$$

$$\frac{\begin{array}{l}\Gamma; \Delta \vDash a_1 \equiv b_1 : (\Pi^+ x\!:\!A \to B)/R' \\ \mathsf{Path}_{R'}\ a_1 = F@R, Rs \\ \Gamma; \Delta \vDash a_2 \equiv b_2 : A/\mathbf{param}\, R\, R'\end{array}}{\Gamma; \Delta \vDash a_1\ {a_2}^R \equiv b_1\ {b_2}^R : (B\{a_2/x\})/R'} \quad \textsc{E\_TAppCong}$$

$$\frac{\begin{array}{l}\Gamma; \Delta \vDash a_1 \equiv b_1 : (\Pi^- x\!:\!A \to B)/R' \\ \Gamma \vDash a : A\end{array}}{\Gamma; \Delta \vDash a_1\ \Box^- \equiv b_1\ \Box^- : (B\{a/x\})/R'} \quad \textsc{E\_IAppCong}$$

$$\frac{\Gamma; \Delta \vDash \Pi^\rho x\!:\!A_1 \to B_1 \equiv \Pi^\rho x\!:\!A_2 \to B_2 : \star/R'}{\Gamma; \Delta \vDash A_1 \equiv A_2 : \star/R'} \quad \textsc{E\_PiFst}$$

$$\frac{\begin{array}{l}\Gamma; \Delta \vDash \Pi^\rho x\!:\!A_1 \to B_1 \equiv \Pi^\rho x\!:\!A_2 \to B_2 : \star/R' \\ \Gamma; \Delta \vDash a_1 \equiv a_2 : A_1/R'\end{array}}{\Gamma; \Delta \vDash B_1\{a_1/x\} \equiv B_2\{a_2/x\} : \star/R'} \quad \textsc{E\_PiSnd}$$

$$\frac{\begin{array}{l}\Gamma; \Delta \vDash a_1 \sim_{A_1/R} b_1 \equiv a_2 \sim_{A_2/R} b_2 \\ \Gamma, c : a_1 \sim_{A_1/R} b_1; \Delta \vDash A \equiv B : \star/R' \\ \Gamma \vDash a_1 \sim_{A_1/R} b_1\ \mathsf{ok} \\ \Gamma \vDash \forall c\!:\!a_1 \sim_{A_1/R} b_1.A : \star \\ \Gamma \vDash \forall c\!:\!a_2 \sim_{A_2/R} b_2.B : \star\end{array}}{\Gamma; \Delta \vDash \forall c\!:\!a_1 \sim_{A_1/R} b_1.A \equiv \forall c\!:\!a_2 \sim_{A_2/R} b_2.B : \star/R'} \quad \textsc{E\_CPiCong}$$

$$\frac{\begin{array}{l}\Gamma, c : \phi_1; \Delta \vDash a \equiv b : B/R \\ \Gamma \vDash \phi_1\ \mathsf{ok}\end{array}}{\Gamma; \Delta \vDash (\Lambda c.a) \equiv (\Lambda c.b) : \forall c\!:\!\phi_1.B/R} \quad \textsc{E\_CAbsCong}$$

$$\frac{\begin{array}{l}\Gamma; \Delta \vDash a_1 \equiv b_1 : (\forall c\!:\!(a \sim_{A/R} b).B)/R' \\ \Gamma; \widetilde{\Gamma} \vDash a \equiv b : A/\mathbf{param}\, R\, R'\end{array}}{\Gamma; \Delta \vDash a_1[\bullet] \equiv b_1[\bullet] : (B\{\bullet/c\})/R'} \quad \textsc{E\_CAppCong}$$

$$\frac{\begin{array}{l}\Gamma; \Delta \vDash \forall c\!:\!(a_1 \sim_{A/R} a_2).B_1 \equiv \forall c\!:\!(a_1' \sim_{A'/R'} a_2').B_2 : \star/R_0 \\ \Gamma; \widetilde{\Gamma} \vDash a_1 \equiv a_2 : A/\mathbf{param}\, R\, R_0 \\ \Gamma; \widetilde{\Gamma} \vDash a_1' \equiv a_2' : A'/\mathbf{param}\, R'\, R_0\end{array}}{\Gamma; \Delta \vDash B_1\{\bullet/c\} \equiv B_2\{\bullet/c\} : \star/R_0} \quad \textsc{E\_CPiSnd}$$

$$\frac{\begin{array}{l}\Gamma; \Delta \vDash a \equiv b : A/R \\ \Gamma; \Delta \vDash a \sim_{A/R} b \equiv a' \sim_{A'/R'} b'\end{array}}{\Gamma; \Delta \vDash a' \equiv b' : A'/R'} \quad \textsc{E\_Cast}$$

$$\frac{\begin{array}{l}\Gamma; \Delta \vDash a \equiv b : A/R \\ \Gamma; \widetilde{\Gamma} \vDash A \equiv B : \star/\mathbf{Rep} \\ \Gamma \vDash B : \star\end{array}}{\Gamma; \Delta \vDash a \equiv b : B/R} \quad \textsc{E\_EqConv}$$

$$\frac{\Gamma;\Delta \vDash a \sim_{A/R_1} b \equiv a' \sim_{A'/R_1} b'}{\Gamma;\Delta \vDash A \equiv A' : \star/\mathbf{Rep}} \quad \text{E\_IsoSnd}$$

$$\frac{\begin{array}{c}\Gamma;\Delta \vDash a \equiv a' : A/R \\ \Gamma;\Delta \vDash b_1 \equiv b_1' : B/R_0 \\ \Gamma;\Delta \vDash b_2 \equiv b_2' : B/R_0\end{array}}{\Gamma;\Delta \vDash \mathsf{case}_R\ a\ \mathsf{of}\ F \to b_1 \|\_ \to b_2 \equiv \mathsf{case}_R\ a'\ \mathsf{of}\ F \to b_1' \|\_ \to b_2' : B/R_0} \quad \text{E\_PatCong}$$

$$\frac{\begin{array}{l}\mathsf{Path}_{R'}\ a = F@R, Rs \\ \mathsf{Path}_{R'}\ a' = F@R, Rs \\ \Gamma \vDash a : \Pi^+ x\!:\!A \to B \\ \Gamma \vDash b : A \\ \Gamma \vDash a' : \Pi^+ x\!:\!A \to B \\ \Gamma \vDash b' : A \\ \Gamma;\Delta \vDash a\ b^{R_1} \equiv a'\ b'^{R_1} : B\{b/x\}/R' \\ \Gamma;\widetilde{\Gamma} \vDash B\{b/x\} \equiv B\{b'/x\} : \star/R'\end{array}}{\Gamma;\Delta \vDash a \equiv a' : \Pi^+ x\!:\!A \to B/R'} \quad \text{E\_LeftRel}$$

$$\frac{\begin{array}{l}\mathsf{Path}_{R'}\ a = F@R, Rs \\ \mathsf{Path}_{R'}\ a' = F@R, Rs \\ \Gamma \vDash a : \Pi^- x\!:\!A \to B \\ \Gamma \vDash b : A \\ \Gamma \vDash a' : \Pi^- x\!:\!A \to B \\ \Gamma \vDash b' : A \\ \Gamma;\Delta \vDash a\ \square^- \equiv a'\ \square^- : B\{b/x\}/R' \\ \Gamma;\widetilde{\Gamma} \vDash B\{b/x\} \equiv B\{b'/x\} : \star/R_0\end{array}}{\Gamma;\Delta \vDash a \equiv a' : \Pi^- x\!:\!A \to B/R'} \quad \text{E\_LeftIrrel}$$

$$\frac{\begin{array}{l}\mathsf{Path}_{R'}\ a = F@R, Rs \\ \mathsf{Path}_{R'}\ a' = F@R, Rs \\ \Gamma \vDash a : \Pi^+ x\!:\!A \to B \\ \Gamma \vDash b : A \\ \Gamma \vDash a' : \Pi^+ x\!:\!A \to B \\ \Gamma \vDash b' : A \\ \Gamma;\Delta \vDash a\ b^+ \equiv a'\ b'^+ : B\{b/x\}/R' \\ \Gamma;\widetilde{\Gamma} \vDash B\{b/x\} \equiv B\{b'/x\} : \star/R_0\end{array}}{\Gamma;\Delta \vDash b \equiv b' : A/\mathbf{param}\ R_1\ R'} \quad \text{E\_Right}$$

$$\frac{\begin{array}{l}\mathsf{Path}_{R'}\ a = F@R, Rs \\ \mathsf{Path}_{R'}\ a' = F@R, Rs \\ \Gamma \vDash a : \forall c\!:\!(a_1 \sim_{A/R_1} a_2).B \\ \Gamma \vDash a' : \forall c\!:\!(a_1 \sim_{A/R_1} a_2).B \\ \Gamma;\widetilde{\Gamma} \vDash a_1 \equiv a_2 : A/R' \\ \Gamma;\Delta \vDash a[\bullet] \equiv a'[\bullet] : B\{\bullet/c\}/R'\end{array}}{\Gamma;\Delta \vDash a \equiv a' : \forall c\!:\!(a_1 \sim_{A/R_1} a_2).B/R'} \quad \text{E\_CLeft}$$

$\boxed{\vDash \Gamma}$ context wellformedness

$$\frac{}{\vDash \varnothing} \quad \text{E\_Empty}$$

$$\frac{\begin{array}{l}\vDash \Gamma \\ \Gamma \vDash A : \star \\ x \notin \mathsf{dom}\,\Gamma\end{array}}{\vDash \Gamma, x : A} \quad \text{E\_ConsTm}$$

$$\frac{\begin{array}{l}\vDash \Gamma \\ \Gamma \vDash \phi \ \ \mathsf{ok} \\ c \notin \mathsf{dom}\,\Gamma\end{array}}{\vDash \Gamma, c : \phi} \quad \textsc{E\_ConsCo}$$

$\boxed{\vDash \Sigma}$ signature wellformedness

$$\frac{}{\vDash \varnothing} \quad \textsc{Sig\_Empty}$$

$$\frac{\begin{array}{l}\vDash \Sigma \\ \varnothing \vDash A : \star \\ F \notin \mathsf{dom}\,\Sigma\end{array}}{\vDash \Sigma \cup \{F : A @ Rs\}} \quad \textsc{Sig\_ConsConst}$$

$$\frac{\begin{array}{l}\vDash \Sigma \\ F \notin \mathsf{dom}\,\Sigma \\ \vDash p : A \ \mathsf{patctx} = \Omega \mid \Gamma \\ \Gamma \vDash a : A \\ \Omega \vDash a : \mathbf{Rep}\end{array}}{\vDash \Sigma \cup \{F\,[p] \sim a : A/R @ Rs\}} \quad \textsc{Sig\_ConsAx}$$

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Definition rules:        143 good    0 bad
Definition rule clauses: 403 good    0 bad
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