$tmvar$, x, y, f, m, n	variables
$covar$, c	coercion variables
$datacon$, K	
$const$, T, F	
$index$, i	indices

role, R	$::=$			Role
	$\|$	**Nom**		
	$\|$	**Rep**		
	$\|$	$R_1 \cap R_2$	S	
relflag, ρ	$::=$			relevance flag
	$\|$	$+$		
	$\|$	$-$		
constraint, ϕ	$::=$			props
	$\|$	$a \sim_{A/R} b$		
	$\|$	(ϕ)	S	
	$\|$	$\phi\{b/x\}$	S	
	$\|$	$\|\phi\|$	S	
	$\|$	$a \sim_R b$	S	
tm, a, b, v, w, A, B	$::=$			types and kinds
	$\|$	$\star$		
	$\|$	x		
	$\|$	$\lambda^\rho x : A/R.b$	bind x in b	
	$\|$	$\lambda^{R,\rho} x.b$	bind x in b	
	$\|$	$a\ b^{R,\rho}$		
	$\|$	F		
	$\|$	$\Pi^\rho x : A/R \to B$	bind x in B	
	$\|$	$a \rhd_R \gamma$		
	$\|$	$\forall c : \phi.B$	bind c in B	
	$\|$	$\Lambda c : \phi.b$	bind c in b	
	$\|$	$\Lambda c.b$	bind c in b	
	$\|$	$a[\gamma]$		
	$\|$	$\square$		
	$\|$	**ifPath** $R\ a'\ a\ b_1\ b_2$		
	$\|$	K		
	$\|$	**match** a **with** brs		
	$\|$	**sub** $R\ a$		
	$\|$	$a\{b/x\}$	S	
	$\|$	a	S	
	$\|$	$a\{\gamma/c\}$	S	
	$\|$	a	S	
	$\|$	(a)	S	
	$\|$	a	S	parsing precedence is hard
	$\|$	$\|a\|_R$	S	
	$\|$	**Int**	S	
	$\|$	**Bool**	S	
	$\|$	**Nat**	S	
	$\|$	**Vec**	S	
	$\|$	0	S	
	$\|$	S	S	

	$\mid$	**True**	S	
	$\mid$	**Fix**	S	
	$\mid$	**Age**	S	
	$\mid$	$a \to b$	S	
	$\mid$	$\phi \Rightarrow A$	S	
	$\mid$	$a\ b$	S	
	$\mid$	$\lambda x.a$	S	
	$\mid$	$\lambda x\!:\!A.a$	S	
	$\mid$	$\forall\, x : A/R \to B$	S	
	$\mid$	**if** ϕ **then** a **else** b	S	
brs	$::=$			case branches
	$\mid$	**none**		
	$\mid$	$K \Rightarrow a; brs$		
	$\mid$	$brs\{a/x\}$	S	
	$\mid$	$brs\{\gamma/c\}$	S	
	$\mid$	(brs)	S	
$co,\ \gamma$	$::=$			explicit coercions
	$\mid$	$\bullet$		
	$\mid$	c		
	$\mid$	**red** $a\ b$		
	$\mid$	**refl** a		
	$\mid$	$(a \mid=\mid_{\gamma}\ b)$		
	$\mid$	**sym** γ		
	$\mid$	$\gamma_1; \gamma_2$		
	$\mid$	**sub** γ		
	$\mid$	$\Pi^{R,\rho} x\!:\!\gamma_1.\gamma_2$	bind x in γ_2	
	$\mid$	$\lambda^{R,\rho} x\!:\!\gamma_1.\gamma_2$	bind x in γ_2	
	$\mid$	$\gamma_1\ \gamma_2^{R,\rho}$		
	$\mid$	**piFst** γ		
	$\mid$	**cpiFst** γ		
	$\mid$	**isoSnd** γ		
	$\mid$	$\gamma_1 @ \gamma_2$		
	$\mid$	$\forall c\!:\!\gamma_1.\gamma_3$	bind c in γ_3	
	$\mid$	$\lambda c\!:\!\gamma_1.\gamma_3 @ \gamma_4$	bind c in γ_3	
	$\mid$	$\gamma(\gamma_1, \gamma_2)$		
	$\mid$	$\gamma @ (\gamma_1 \sim \gamma_2)$		
	$\mid$	$\gamma_1 \triangleright_R \gamma_2$		
	$\mid$	$\gamma_1 \sim_A \gamma_2$		
	$\mid$	**conv** $\phi_1 \sim_{\gamma} \phi_2$		
	$\mid$	**eta** a		
	$\mid$	**left** $\gamma\,\gamma'$		
	$\mid$	**right** $\gamma\,\gamma'$		
	$\mid$	(γ)	S	
	$\mid$	γ	S	

	$\vert$	$\gamma\{a/x\}$	S	
role_context, Ω	$::=$			role$_c$*ontexts*
	$\vert$	$\varnothing$		
	$\vert$	$\Omega, x : R$		
	$\vert$	(Ω)	M	
	$\vert$	Ω	M	
sig_sort	$::=$			signature classifier
	$\vert$	$: A/R$		
	$\vert$	$\sim a : A/R$		
sort	$::=$			binding classifier
	$\vert$	$\mathbf{Tm}\, A\, R$		
	$\vert$	$\mathbf{Co}\, \phi$		
context, Γ	$::=$			contexts
	$\vert$	$\varnothing$		
	$\vert$	$\Gamma, x : A/R$		
	$\vert$	$\Gamma, c : \phi$		
	$\vert$	$\Gamma\{b/x\}$	M	
	$\vert$	$\Gamma\{\gamma/c\}$	M	
	$\vert$	Γ, Γ'	M	
	$\vert$	$\vert\Gamma\vert$	M	
	$\vert$	(Γ)	M	
	$\vert$	Γ	M	
sig, Σ	$::=$			signatures
	$\vert$	$\varnothing$		
	$\vert$	$\Sigma \cup \{F\, sig\_sort\}$		
	$\vert$	Σ_0	M	
	$\vert$	Σ_1	M	
	$\vert$	$\vert\Sigma\vert$	M	
available_props, Δ	$::=$			
	$\vert$	$\varnothing$		
	$\vert$	Δ, c		
	$\vert$	$\widetilde{\Gamma}$	M	
	$\vert$	(Δ)	M	
terminals	$::=$			
	$\vert$	$\leftrightarrow$		
	$\vert$	$\Leftrightarrow$		
	$\vert$	$\longrightarrow$		
	$\vert$	min		
	$\vert$	$\equiv$		
	$\vert$	$\forall$		

| $\in$
| $\notin$
| $\Leftarrow$
| $\Rightarrow$
| $\Rightarrow^*$
| $\rightarrow$
| Λ
|
| $\square$
| $\vdash$
| $\dashv$
| $\models$
| $\models$
| $\neq$
| $\triangleright$
| ok
| $\_$
| $\leadsto$
| $\leadsto^*$
| $\leadsto$
| $\varnothing$
| $\circ$
| fv
| dom
| $\sim$
| $\asymp$
| $|$
| $\bullet$
| $\mathbf{fst}$
| $\mathbf{snd}$
| $| \Rightarrow |$
| $\vdash_{=}$
| $\mathbf{refl_2}$
| $++$

$formula,\ \psi$ ::=
| $judgement$
| $x : A/R \in \Gamma$
| $x : R \in \Omega$
| $c : \phi \in \Gamma$
| $F\ sig\_sort \in \Sigma$
| $K : T\,\Gamma \in \Sigma$
| $x \in \Delta$
| $c \in \Delta$
| $c\ \mathbf{not\ relevant} \in \gamma$
| $x \notin \mathsf{fv}\,a$

	$\mid$	$x \notin \mathsf{dom}\,\Gamma$	
	$\mid$	$\mathit{uniq}(\Omega)$	
	$\mid$	$c \notin \mathsf{dom}\,\Gamma$	
	$\mid$	$T \notin \mathsf{dom}\,\Sigma$	
	$\mid$	$F \notin \mathsf{dom}\,\Sigma$	
	$\mid$	$a = b$	
	$\mid$	$\phi_1 = \phi_2$	
	$\mid$	$\Gamma_1 = \Gamma_2$	
	$\mid$	$\gamma_1 = \gamma_2$	
	$\mid$	$\neg\psi$	
	$\mid$	$\psi_1 \wedge \psi_2$	
	$\mid$	$\psi_1 \vee \psi_2$	
	$\mid$	$\psi_1 \Rightarrow \psi_2$	
	$\mid$	(ψ)	
	$\mid$	ψ	
	$\mid$	$c : (a : A \sim b : B) \in \Gamma$	
	$\mid$		suppress lc hypothesis generated by Ott
JSubRole	$::=$		
	$\mid$	$R_1 \leq R_2$	Subroling judgement
JPath	$::=$		
	$\mid$	$\mathsf{Path}_R\ a = F$	Type headed by constant (partial function)
JValue	$::=$		
	$\mid$	$\mathsf{Value}_R\ A$	values
JValueType	$::=$		
	$\mid$	$\mathsf{ValueType}_R\ A$	Types with head forms (erased language)
Jconsistent	$::=$		
	$\mid$	$\mathsf{consistent}_R\ ab$	(erased) types do not differ in their heads
Jerased	$::=$		
	$\mid$	$\Omega \vDash a : R$	
JChk	$::=$		
	$\mid$	$(\rho = +) \vee (x \notin \mathsf{fv}\ A)$	irrelevant argument check
Jpar	$::=$		
	$\mid$	$\Omega \vDash a \Rightarrow_R b$	parallel reduction (implicit language)
	$\mid$	$\Omega \vdash a \Rightarrow^*_R b$	multistep parallel reduction
	$\mid$	$\Omega \vdash a \Leftrightarrow_R b$	parallel reduction to a common term
Jbeta	$::=$		
	$\mid$	$\vDash a > b/R$	primitive reductions on erased terms
	$\mid$	$\vDash a \rightsquigarrow b/R$	single-step head reduction for implicit language

	$\vert$	$\models a \rightsquigarrow^{*} b / R$	multistep reduction
Jett	$::=$		
	$\vert$	$\Gamma \models \phi\ \mathsf{ok}$	Prop wellformedness
	$\vert$	$\Gamma \models a : A / R$	typing
	$\vert$	$\Gamma; \Delta \models \phi_1 \equiv \phi_2$	prop equality
	$\vert$	$\Gamma; \Delta \models a \equiv b : A / R$	definitional equality
	$\vert$	$\models \Gamma$	context wellformedness
Jsig	$::=$		
	$\vert$	$\models \Sigma$	signature wellformedness
Jann	$::=$		
	$\vert$	$\Gamma \vdash \phi\ \mathsf{ok}$	prop wellformedness
	$\vert$	$\Gamma \vdash a : A / R$	typing
	$\vert$	$\Gamma; \Delta \vdash \gamma : \phi_1 \sim \phi_2$	coercion between props
	$\vert$	$\Gamma; \Delta \vdash \gamma : A \sim_R B$	coercion between types
	$\vert$	$\vdash \Gamma$	context wellformedness
	$\vert$	$\vdash \Sigma$	signature wellformedness
Jred	$::=$		
	$\vert$	$\Gamma \vdash a \rightsquigarrow b / R$	single-step, weak head reduction to values for annotated lang
judgement	$::=$		
	$\vert$	*JSubRole*	
	$\vert$	*JPath*	
	$\vert$	*JValue*	
	$\vert$	*JValueType*	
	$\vert$	*Jconsistent*	
	$\vert$	*Jerased*	
	$\vert$	*JChk*	
	$\vert$	*Jpar*	
	$\vert$	*Jbeta*	
	$\vert$	*Jett*	
	$\vert$	*Jsig*	
	$\vert$	*Jann*	
	$\vert$	*Jred*	
user_syntax	$::=$		
	$\vert$	*tmvar*	
	$\vert$	*covar*	
	$\vert$	*datacon*	
	$\vert$	*const*	
	$\vert$	*index*	
	$\vert$	*role*	
	$\vert$	*relflag*	
	$\vert$	*constraint*	

| *tm*
| *brs*
| *co*
| *role_context*
| *sig_sort*
| *sort*
| *context*
| *sig*
| *available_props*
| *terminals*
| *formula*

$\boxed{R_1 \leq R_2}$ Subroling judgement

$$\frac{}{\mathbf{Nom} \leq R} \quad \textsc{NomBot}$$

$$\frac{}{R \leq \mathbf{Rep}} \quad \textsc{RepTop}$$

$$\frac{}{R \leq R} \quad \textsc{Refl}$$

$$\frac{R_1 \leq R_2 \qquad R_2 \leq R_3}{R_1 \leq R_3} \quad \textsc{Trans}$$

$\boxed{\mathsf{Path}_R\ a = F}$ Type headed by constant (partial function)

$$\frac{F \sim a : A/R_1 \in \Sigma_0 \qquad \neg(R_1 \leq R)}{\mathsf{Path}_R\ F = F} \quad \textsc{Path\_Const}$$

$$\frac{\mathsf{Path}_R\ a = F}{\mathsf{Path}_R\ (a\ b'^{R_1,\rho}) = F} \quad \textsc{Path\_App}$$

$$\frac{\mathsf{Path}_R\ a = F}{\mathsf{Path}_R\ (a[\bullet]) = F} \quad \textsc{Path\_CApp}$$

$\boxed{\mathsf{Value}_R\ A}$ values

$$\frac{}{\mathsf{Value}_R\ \star} \quad \textsc{Value\_Star}$$

$$\frac{}{\mathsf{Value}_R\ \Pi^\rho x\!:\!A/R_1 \to B} \quad \textsc{Value\_Pi}$$

$$\frac{}{\mathsf{Value}_R\ \forall c\!:\!\phi.B} \quad \textsc{Value\_CPi}$$

$$\frac{}{\mathsf{Value}_R\ \lambda^+ x\!:\!A/R_1.a} \quad \textsc{Value\_AbsRel}$$

$$\frac{}{\mathsf{Value}_R\ \lambda^{R_1,+} x.a} \quad \textsc{Value\_UAbsRel}$$

$$\frac{\mathsf{Value}_R\ a}{\mathsf{Value}_R\ \lambda^{R_1,-} x.a} \quad \textsc{Value\_UAbsIrrel}$$

$$\frac{}{\mathsf{Value}_R\ \Lambda c\!:\!\phi.a} \quad \textsc{Value\_CAbs}$$

$$\frac{}{\mathsf{Value}_R\ \Lambda c.a} \quad \textsc{Value\_UCAbs}$$

$$\frac{\mathsf{Path}_R\ a = F}{\mathsf{Value}_R\ a} \quad \textsc{Value\_Path}$$

$\boxed{\mathsf{ValueType}_R\ A}$ Types with head forms (erased language)

$$\frac{}{\mathsf{ValueType}_R\ \star} \quad \textsc{value\_type\_Star}$$

$$\frac{}{\mathsf{ValueType}_R\ \Pi^\rho x\!:\!A/R_1 \to B} \quad \textsc{value\_type\_Pi}$$

$$\frac{}{\mathsf{ValueType}_R\ \forall c\!:\!\phi.B} \quad \textsc{value\_type\_CPi}$$

$$\frac{\begin{array}{l}\mathsf{Path}_R\ A = F\\ \mathsf{Value}_R\ A\end{array}}{\mathsf{ValueType}_R\ A} \quad \textsc{value\_type\_Path}$$

$\boxed{\mathsf{consistent}_R\ ab}$ (erased) types do not differ in their heads

$$\frac{}{\mathsf{consistent}_R\ \star\star} \quad \textsc{consistent\_a\_Star}$$

$$\frac{}{\mathsf{consistent}_{R'}\ (\Pi^\rho x_1\!:\!A_1/R \to B_1)(\Pi^\rho x_2\!:\!A_2/R \to B_2)} \quad \textsc{consistent\_a\_Pi}$$

$$\frac{}{\mathsf{consistent}_R\ (\forall c_1\!:\!\phi_1.A_1)(\forall c_2\!:\!\phi_2.A_2)} \quad \textsc{consistent\_a\_CPi}$$

$$\frac{\begin{array}{l}\mathsf{Path}_R\ a_1 = F\\ \mathsf{Path}_R\ a_2 = F\end{array}}{\mathsf{consistent}_R\ a_1 a_2} \quad \textsc{consistent\_a\_Path}$$

$$\frac{\neg\mathsf{ValueType}_R\ b}{\mathsf{consistent}_R\ ab} \quad \textsc{consistent\_a\_Step\_R}$$

$$\frac{\neg\mathsf{ValueType}_R\ a}{\mathsf{consistent}_R\ ab} \quad \textsc{consistent\_a\_Step\_L}$$

$\boxed{\Omega \vDash a : R}$

$$\frac{uniq(\Omega)}{\Omega \vDash \square : R} \quad \textsc{erased\_a\_Bullet}$$

$$\frac{uniq(\Omega)}{\Omega \vDash \star : R} \quad \textsc{erased\_a\_Star}$$

$$\frac{\begin{array}{l}uniq(\Omega)\\ x : R \in \Omega\\ R \leq R_1\end{array}}{\Omega \vDash x : R_1} \quad \textsc{erased\_a\_Var}$$

$$\frac{\Omega, x : R_1 \vDash a : R}{\Omega \vDash (\lambda^{R_1,\rho} x.a) : R} \quad \textsc{erased\_a\_Abs}$$

$$\frac{\begin{array}{l}\Omega \vDash a : R\\ \Omega \vDash b : R_1\end{array}}{\Omega \vDash (a\ \ b^{R_1,\rho}) : R} \quad \textsc{erased\_a\_App}$$

$$\frac{\begin{array}{l}\Omega \vDash A : R\\ \Omega, x : R_1 \vDash B : R\end{array}}{\Omega \vDash (\Pi^\rho x\!:\!A/R_1 \to B) : R} \quad \textsc{erased\_a\_Pi}$$

$$\frac{\begin{array}{c}\Omega \vDash a : R_1 \\ \Omega \vDash b : R_1 \\ \Omega \vDash A : R_0 \\ \Omega \vDash B : R\end{array}}{\Omega \vDash (\forall c\!:\!a \sim_{A/R_1} b.B) : R} \quad \textsc{erased\_a\_CPi}$$

$$\frac{\Omega \vDash b : R}{\Omega \vDash (\Lambda c.b) : R} \quad \textsc{erased\_a\_CAbs}$$

$$\frac{\Omega \vDash a : R}{\Omega \vDash (a[\bullet]) : R} \quad \textsc{erased\_a\_CApp}$$

$$\frac{\begin{array}{l}uniq(\Omega) \\ F \sim a : A/R \in \Sigma_0\end{array}}{\Omega \vDash F : R_1} \quad \textsc{erased\_a\_Fam}$$

$$\frac{\begin{array}{l}F \sim a_0 : A/R' \in \Sigma_0 \\ \Omega \vDash a : R \\ \Omega \vDash b_1 : R_1 \\ \Omega \vDash b_2 : R_1\end{array}}{\Omega \vDash (\textbf{ifPath}\, R\, F\, a\, b_1\, b_2) : R_1} \quad \textsc{erased\_a\_Pattern}$$

$\boxed{(\rho = +) \vee (x \notin \mathsf{fv}\ A)}$ irrelevant argument check

$$\frac{}{(+ = +) \vee (x \notin \mathsf{fv}\ A)} \quad \textsc{Rho\_Rel}$$

$$\frac{x \notin \mathsf{fv} A}{(- = +) \vee (x \notin \mathsf{fv}\ A)} \quad \textsc{Rho\_IrrRel}$$

$\boxed{\Omega \vDash a \Rightarrow_R b}$ parallel reduction (implicit language)

$$\frac{\Omega \vDash a : R}{\Omega \vDash a \Rightarrow_R a} \quad \textsc{Par\_Refl}$$

$$\frac{\begin{array}{c}\Omega \vDash a \Rightarrow_R (\lambda^{R_1,\rho} x.a') \\ \Omega \vDash b \Rightarrow_{R_1} b'\end{array}}{\Omega \vDash a\ b^{R_1,\rho} \Rightarrow_R a'\{b'/x\}} \quad \textsc{Par\_Beta}$$

$$\frac{\begin{array}{c}\Omega \vDash a \Rightarrow_R a' \\ \Omega \vDash b \Rightarrow_{R_1} b'\end{array}}{\Omega \vDash a\ b^{R_1,\rho} \Rightarrow_R a'\ b'^{R_1,\rho}} \quad \textsc{Par\_App}$$

$$\frac{\Omega \vDash a \Rightarrow_R (\Lambda c.a')}{\Omega \vDash a[\bullet] \Rightarrow_R a'\{\bullet/c\}} \quad \textsc{Par\_CBeta}$$

$$\frac{\Omega \vDash a \Rightarrow_R a'}{\Omega \vDash a[\bullet] \Rightarrow_R a'[\bullet]} \quad \textsc{Par\_CApp}$$

$$\frac{\Omega, x : R_1 \vDash a \Rightarrow_R a'}{\Omega \vDash \lambda^{R_1,\rho} x.a \Rightarrow_R \lambda^{R_1,\rho} x.a'} \quad \textsc{Par\_Abs}$$

$$\frac{\begin{array}{c}\Omega \vDash A \Rightarrow_R A' \\ \Omega, x : R_1 \vDash B \Rightarrow_R B'\end{array}}{\Omega \vDash \Pi^\rho x\!:\!A/R_1 \to B \Rightarrow_R \Pi^\rho x\!:\!A'/R_1 \to B'} \quad \textsc{Par\_Pi}$$

$$\frac{\Omega \vDash a \Rightarrow_R a'}{\Omega \vDash \Lambda c.a \Rightarrow_R \Lambda c.a'} \quad \textsc{Par\_CAbs}$$

$$\frac{\begin{array}{c}\Omega \vDash A \Rightarrow_{R_0} A' \\ \Omega \vDash a \Rightarrow_{R_1} a' \\ \Omega \vDash b \Rightarrow_{R_1} b' \\ \Omega \vDash B \Rightarrow_R B'\end{array}}{\Omega \vDash \forall c\!:\!a \sim_{A/R_1} b.B \Rightarrow_R \forall c\!:\!a' \sim_{A'/R_1} b'.B'} \quad \textsc{Par\_CPi}$$

$$\frac{\begin{array}{l}F \sim a : A/R_1 \in \Sigma_0 \\ R_1 \leq R \\ \mathit{uniq}(\Omega)\end{array}}{\Omega \vDash F \Rightarrow_R a} \quad \textsc{Par\_Axiom}$$

$$\frac{\begin{array}{l}F \sim a_0 : A/R' \in \Sigma_0 \\ \Omega \vDash a \Rightarrow_R a' \\ \Omega \vDash b_1 \Rightarrow_{R_0} b_1' \\ \Omega \vDash b_2 \Rightarrow_{R_0} b_2'\end{array}}{\Omega \vDash \textbf{ifPath}\, R\, F\ a\ b_1\ b_2 \Rightarrow_{R_0} \textbf{ifPath}\, R\, F\ a'\ b_1'\ b_2'} \quad \textsc{Par\_Pattern}$$

$$\frac{\begin{array}{l}\Omega \vDash a \Rightarrow_R a' \\ \Omega \vDash b_1 \Rightarrow_{R_0} b_1' \\ \Omega \vDash b_2 \Rightarrow_{R_0} b_2' \\ \mathsf{Path}_R\ a' = F\end{array}}{\Omega \vDash \textbf{ifPath}\, R\, F\ a\ b_1\ b_2 \Rightarrow_{R_0} b_1'} \quad \textsc{Par\_PatternTrue}$$

$$\frac{\begin{array}{l}F \sim a_0 : A/R' \in \Sigma_0 \\ \Omega \vDash a \Rightarrow_R a' \\ \Omega \vDash b_1 \Rightarrow_{R_0} b_1' \\ \Omega \vDash b_2 \Rightarrow_{R_0} b_2' \\ \mathsf{Value}_R\ a' \\ \neg(\mathsf{Path}_R\ a' = F)\end{array}}{\Omega \vDash \textbf{ifPath}\, R\, F\ a\ b_1\ b_2 \Rightarrow_{R_0} b_2'} \quad \textsc{Par\_PatternFalse}$$

$\boxed{\Omega \vdash a \Rightarrow^*_R b}$ multistep parallel reduction

$$\frac{}{\Omega \vdash a \Rightarrow^*_R a} \quad \textsc{MP\_Refl}$$

$$\frac{\begin{array}{c}\Omega \vDash a \Rightarrow_R b \\ \Omega \vdash b \Rightarrow^*_R a'\end{array}}{\Omega \vdash a \Rightarrow^*_R a'} \quad \textsc{MP\_Step}$$

$\boxed{\Omega \vdash a \Leftrightarrow_R b}$ parallel reduction to a common term

$$\frac{\begin{array}{c}\Omega \vdash a_1 \Rightarrow^*_R b \\ \Omega \vdash a_2 \Rightarrow^*_R b\end{array}}{\Omega \vdash a_1 \Leftrightarrow_R a_2} \quad \textsc{join}$$

$\boxed{\vDash a > b/R}$ primitive reductions on erased terms

$$\frac{\mathsf{Value}_{R_1}\ (\lambda^{R,\rho} x.v)}{\vDash (\lambda^{R,\rho} x.v)\ b^{R,\rho} > v\{b/x\}/R_1} \quad \textsc{Beta\_AppAbs}$$

$$\frac{}{\vDash (\Lambda c.a')[\bullet] > a'\{\bullet/c\}/R} \quad \textsc{Beta\_CAppCAbs}$$

$$\frac{\begin{array}{l}F \sim a : A/R \in \Sigma_0 \\ R \leq R_1\end{array}}{\vDash F > a/R_1} \quad \textsc{Beta\_Axiom}$$

$$\frac{\mathsf{Path}_R\ a = F}{\vDash \textbf{ifPath}\, R\, F\ a\ b_1\ b_2 > b_1/R_0}\quad \textsc{Beta\_PatternTrue}$$

$$\frac{\begin{array}{l} F\ \sim a_0 : A/R' \in \Sigma_0 \\ \mathsf{Value}_R\ a \\ \neg(\mathsf{Path}_R\ a = F) \end{array}}{\vDash \textbf{ifPath}\, R\, F\ a\ b_1\ b_2 > b_2/R_0}\quad \textsc{Beta\_PatternFalse}$$

$\boxed{\vDash a \rightsquigarrow b/R}$ single-step head reduction for implicit language

$$\frac{\vDash a \rightsquigarrow a'/R_1}{\vDash \lambda^{R,-}x.a \rightsquigarrow \lambda^{R,-}x.a'/R_1}\quad \textsc{E\_AbsTerm}$$

$$\frac{\vDash a \rightsquigarrow a'/R_1}{\vDash a\ b^{R,\rho} \rightsquigarrow a'\ b^{R,\rho}/R_1}\quad \textsc{E\_AppLeft}$$

$$\frac{\vDash a \rightsquigarrow a'/R}{\vDash a[\bullet] \rightsquigarrow a'[\bullet]/R}\quad \textsc{E\_CAppLeft}$$

$$\frac{\vDash a \rightsquigarrow a'/R}{\vDash \textbf{ifPath}\, R\, F\ a\ b_1\ b_2 \rightsquigarrow \textbf{ifPath}\, R\, F\ a'\ b_1\ b_2/R_0}\quad \textsc{E\_Pattern}$$

$$\frac{\vDash a > b/R}{\vDash a \rightsquigarrow b/R}\quad \textsc{E\_Prim}$$

$\boxed{\vDash a \rightsquigarrow^* b/R}$ multistep reduction

$$\frac{}{\vDash a \rightsquigarrow^* a/R}\quad \textsc{Equal}$$

$$\frac{\begin{array}{l} \vDash a \rightsquigarrow b/R \\ \vDash b \rightsquigarrow^* a'/R \end{array}}{\vDash a \rightsquigarrow^* a'/R}\quad \textsc{Step}$$

$\boxed{\Gamma \vDash \phi\ \mathsf{ok}}$ Prop wellformedness

$$\frac{\begin{array}{l} \Gamma \vDash a : A/R \\ \Gamma \vDash b : A/R \\ \Gamma \vDash A : \star/R_0 \end{array}}{\Gamma \vDash a \sim_{A/R} b\ \ \mathsf{ok}}\quad \textsc{E\_Wff}$$

$\boxed{\Gamma \vDash a : A/R}$ typing

$$\frac{\begin{array}{l} R_1 \leq R_2 \\ \Gamma \vDash a : A/R_1 \end{array}}{\Gamma \vDash a : A/R_2}\quad \textsc{E\_SubRole}$$

$$\frac{\vDash \Gamma}{\Gamma \vDash \star : \star/R}\quad \textsc{E\_Star}$$

$$\frac{\begin{array}{l} \vDash \Gamma \\ x : A/R \in \Gamma \end{array}}{\Gamma \vDash x : A/R}\quad \textsc{E\_Var}$$

$$\frac{\begin{array}{l} \Gamma, x : A/R \vDash B : \star/R' \\ \Gamma \vDash A : \star/R' \end{array}}{\Gamma \vDash \Pi^\rho x\!:\!A/R \to B : \star/R'}\quad \textsc{E\_Pi}$$

$$\frac{\begin{array}{l}\Gamma, x : A/R \vDash a : B/R' \\ \Gamma \vDash A : \star/R_0 \\ (\rho = +) \vee (x \notin \mathsf{fv}\ a)\end{array}}{\Gamma \vDash \lambda^{R,\rho} x.a : (\Pi^\rho x\!:\!A/R \to B)/R'} \quad \text{E\_Abs}$$

$$\frac{\begin{array}{l}\Gamma \vDash b : \Pi^+ x\!:\!A/R \to B/R' \\ \Gamma \vDash a : A/R\end{array}}{\Gamma \vDash b\ a^{R,+} : B\{a/x\}/R'} \quad \text{E\_App}$$

$$\frac{\begin{array}{l}\Gamma \vDash b : \Pi^- x\!:\!A/R \to B/R' \\ \Gamma \vDash a : A/R\end{array}}{\Gamma \vDash b\ \Box^{R,-} : B\{a/x\}/R'} \quad \text{E\_IApp}$$

$$\frac{\begin{array}{l}\Gamma \vDash a : A/R \\ \Gamma; \widetilde{\Gamma} \vDash A \equiv B : \star/\mathbf{Rep} \\ \Gamma \vDash B : \star/R_0\end{array}}{\Gamma \vDash a : B/R} \quad \text{E\_Conv}$$

$$\frac{\begin{array}{l}\Gamma, c : \phi \vDash B : \star/R \\ \Gamma \vDash \phi\ \mathsf{ok}\end{array}}{\Gamma \vDash \forall c\!:\!\phi.B : \star/R} \quad \text{E\_CPi}$$

$$\frac{\begin{array}{l}\Gamma, c : \phi \vDash a : B/R \\ \Gamma \vDash \phi\ \mathsf{ok}\end{array}}{\Gamma \vDash \Lambda c.a : \forall c\!:\!\phi.B/R} \quad \text{E\_CAbs}$$

$$\frac{\begin{array}{l}\Gamma \vDash a_1 : \forall c\!:\!(a \sim_{A/R} b).B_1/R' \\ \Gamma; \widetilde{\Gamma} \vDash a \equiv b : A/R\end{array}}{\Gamma \vDash a_1[\bullet] : B_1\{\bullet/c\}/R'} \quad \text{E\_CApp}$$

$$\frac{\begin{array}{l}\vDash \Gamma \\ F\ \sim a : A/R\ \in\ \Sigma_0 \\ \varnothing \vDash A : \star/R_0\end{array}}{\Gamma \vDash F : A/R_1} \quad \text{E\_Fam}$$

$$\frac{\begin{array}{l}F\ \sim a_0 : A_0/R'\ \in\ \Sigma_0 \\ \Gamma \vDash a : A/R \\ \Gamma \vDash b_1 : B/R_0 \\ \Gamma \vDash b_2 : B/R_0\end{array}}{\Gamma \vDash \mathbf{ifPath}\, R\, F\ a\ b_1\ b_2 : B/R_0} \quad \text{E\_Pat}$$

$\boxed{\Gamma; \Delta \vDash \phi_1 \equiv \phi_2}$ prop equality

$$\frac{\begin{array}{c}\Gamma; \Delta \vDash A_1 \equiv A_2 : A/R \\ \Gamma; \Delta \vDash B_1 \equiv B_2 : A/R\end{array}}{\Gamma; \Delta \vDash A_1 \sim_{A/R} B_1 \equiv A_2 \sim_{A/R} B_2} \quad \text{E\_PropCong}$$

$$\frac{\begin{array}{c}\Gamma; \Delta \vDash A \equiv B : \star/R_0 \\ \Gamma \vDash A_1 \sim_{A/R} A_2\ \mathsf{ok} \\ \Gamma \vDash A_1 \sim_{B/R} A_2\ \mathsf{ok}\end{array}}{\Gamma; \Delta \vDash A_1 \sim_{A/R} A_2 \equiv A_1 \sim_{B/R} A_2} \quad \text{E\_IsoConv}$$

$$\frac{\Gamma; \Delta \vDash \forall c\!:\!(a_1 \sim_{A/R_1} a_2).B_1 \equiv \forall c\!:\!(b_1 \sim_{B/R_2} b_2).B_2 : \star/R'}{\Gamma; \Delta \vDash a_1 \sim_{A/R_1} a_2 \equiv b_1 \sim_{B/R_2} b_2} \quad \text{E\_CPiFst}$$

$\boxed{\Gamma; \Delta \vDash a \equiv b : A/R}$ definitional equality

$$\frac{\begin{array}{l}\vDash \Gamma \\ c : (a \sim_{A/R} b) \in \Gamma \\ c \in \Delta\end{array}}{\Gamma; \Delta \vDash a \equiv b : A/R} \quad \text{E\_ASSN}$$

$$\frac{\Gamma \vDash a : A/R}{\Gamma; \Delta \vDash a \equiv a : A/R} \quad \text{E\_REFL}$$

$$\frac{\Gamma; \Delta \vDash b \equiv a : A/R}{\Gamma; \Delta \vDash a \equiv b : A/R} \quad \text{E\_SYM}$$

$$\frac{\begin{array}{l}\Gamma; \Delta \vDash a \equiv a_1 : A/R \\ \Gamma; \Delta \vDash a_1 \equiv b : A/R\end{array}}{\Gamma; \Delta \vDash a \equiv b : A/R} \quad \text{E\_TRANS}$$

$$\frac{\begin{array}{l}\Gamma; \Delta \vDash a \equiv b : A/R_1 \\ R_1 \leq R_2\end{array}}{\Gamma; \Delta \vDash a \equiv b : A/R_2} \quad \text{E\_SUB}$$

$$\frac{\begin{array}{l}\Gamma \vDash a_1 : B/R \\ \Gamma \vDash a_2 : B/R \\ \vDash a_1 > a_2/R\end{array}}{\Gamma; \Delta \vDash a_1 \equiv a_2 : B/R} \quad \text{E\_BETA}$$

$$\frac{\begin{array}{l}\Gamma; \Delta \vDash A_1 \equiv A_2 : \star/R' \\ \Gamma, x : A_1/R; \Delta \vDash B_1 \equiv B_2 : \star/R' \\ \Gamma \vDash A_1 : \star/R' \\ \Gamma \vDash \Pi^\rho x\!:\!A_1/R \to B_1 : \star/R' \\ \Gamma \vDash \Pi^\rho x\!:\!A_2/R \to B_2 : \star/R'\end{array}}{\Gamma; \Delta \vDash (\Pi^\rho x\!:\!A_1/R \to B_1) \equiv (\Pi^\rho x\!:\!A_2/R \to B_2) : \star/R'} \quad \text{E\_PICONG}$$

$$\frac{\begin{array}{l}\Gamma, x : A_1/R; \Delta \vDash b_1 \equiv b_2 : B/R' \\ \Gamma \vDash A_1 : \star/R_0 \\ (\rho = +) \lor (x \notin \mathsf{fv}\ b_1) \\ (\rho = +) \lor (x \notin \mathsf{fv}\ b_2)\end{array}}{\Gamma; \Delta \vDash (\lambda^{R,\rho} x.b_1) \equiv (\lambda^{R,\rho} x.b_2) : (\Pi^\rho x\!:\!A_1/R \to B)/R'} \quad \text{E\_ABSCONG}$$

$$\frac{\begin{array}{l}\Gamma; \Delta \vDash a_1 \equiv b_1 : (\Pi^+ x\!:\!A/R \to B)/R' \\ \Gamma; \Delta \vDash a_2 \equiv b_2 : A/R\end{array}}{\Gamma; \Delta \vDash a_1\ {a_2}^{R,+} \equiv b_1\ {b_2}^{R,+} : (B\{a_2/x\})/R'} \quad \text{E\_APPCONG}$$

$$\frac{\begin{array}{l}\Gamma; \Delta \vDash a_1 \equiv b_1 : (\Pi^- x\!:\!A/R \to B)/R' \\ \Gamma \vDash a : A/R\end{array}}{\Gamma; \Delta \vDash a_1\ \Box^{R,-} \equiv b_1\ \Box^{R,-} : (B\{a/x\})/R'} \quad \text{E\_IAPPCONG}$$

$$\frac{\Gamma; \Delta \vDash \Pi^\rho x\!:\!A_1/R \to B_1 \equiv \Pi^\rho x\!:\!A_2/R \to B_2 : \star/R'}{\Gamma; \Delta \vDash A_1 \equiv A_2 : \star/R'} \quad \text{E\_PIFST}$$

$$\frac{\begin{array}{l}\Gamma; \Delta \vDash \Pi^\rho x\!:\!A_1/R \to B_1 \equiv \Pi^\rho x\!:\!A_2/R \to B_2 : \star/R' \\ \Gamma; \Delta \vDash a_1 \equiv a_2 : A_1/R\end{array}}{\Gamma; \Delta \vDash B_1\{a_1/x\} \equiv B_2\{a_2/x\} : \star/R'} \quad \text{E\_PISND}$$

$$\frac{\begin{array}{l}\Gamma;\Delta \vDash a_1 \sim_{A_1/R} b_1 \equiv a_2 \sim_{A_2/R} b_2\\ \Gamma, c : a_1 \sim_{A_1/R} b_1; \Delta \vDash A \equiv B : \star/R'\\ \Gamma \vDash a_1 \sim_{A_1/R} b_1 \;\; \mathsf{ok}\\ \Gamma \vDash \forall c{:}\, a_1 \sim_{A_1/R} b_1 . A : \star/R'\\ \Gamma \vDash \forall c{:}\, a_2 \sim_{A_2/R} b_2 . B : \star/R'\end{array}}{\Gamma;\Delta \vDash \forall c{:}\, a_1 \sim_{A_1/R} b_1 . A \equiv \forall c{:}\, a_2 \sim_{A_2/R} b_2 . B : \star/R'} \quad \text{E\_CPiCong}$$

$$\frac{\begin{array}{l}\Gamma, c : \phi_1; \Delta \vDash a \equiv b : B/R\\ \Gamma \vDash \phi_1 \;\; \mathsf{ok}\end{array}}{\Gamma;\Delta \vDash (\Lambda c.a) \equiv (\Lambda c.b) : \forall c{:}\,\phi_1 . B/R} \quad \text{E\_CAbsCong}$$

$$\frac{\begin{array}{l}\Gamma;\Delta \vDash a_1 \equiv b_1 : (\forall c{:}(a \sim_{A/R} b).B)/R'\\ \Gamma;\widetilde{\Gamma} \vDash a \equiv b : A/R\end{array}}{\Gamma;\Delta \vDash a_1[\bullet] \equiv b_1[\bullet] : (B\{\bullet/c\})/R'} \quad \text{E\_CAppCong}$$

$$\frac{\begin{array}{l}\Gamma;\Delta \vDash \forall c{:}(a_1 \sim_{A/R} a_2).B_1 \equiv \forall c{:}(a_1' \sim_{A'/R'} a_2').B_2 : \star/R_0\\ \Gamma;\widetilde{\Gamma} \vDash a_1 \equiv a_2 : A/R\\ \Gamma;\widetilde{\Gamma} \vDash a_1' \equiv a_2' : A'/R'\end{array}}{\Gamma;\Delta \vDash B_1\{\bullet/c\} \equiv B_2\{\bullet/c\} : \star/R_0} \quad \text{E\_CPiSnd}$$

$$\frac{\begin{array}{l}\Gamma;\Delta \vDash a \equiv b : A/R\\ \Gamma;\Delta \vDash a \sim_{A/R} b \equiv a' \sim_{A'/R'} b'\end{array}}{\Gamma;\Delta \vDash a' \equiv b' : A'/R'} \quad \text{E\_Cast}$$

$$\frac{\begin{array}{l}\Gamma;\Delta \vDash a \equiv b : A/R\\ \Gamma;\widetilde{\Gamma} \vDash A \equiv B : \star/\mathbf{Rep}\\ \Gamma \vDash B : \star/R_0\end{array}}{\Gamma;\Delta \vDash a \equiv b : B/R} \quad \text{E\_EqConv}$$

$$\frac{\Gamma;\Delta \vDash a \sim_{A/R_1} b \equiv a' \sim_{A'/R_1} b'}{\Gamma;\Delta \vDash A \equiv A' : \star/R_1} \quad \text{E\_IsoSnd}$$

$$\frac{\begin{array}{l}F \sim a_0 : A_0/R' \in \Sigma_0\\ \Gamma;\Delta \vDash a \equiv a' : A/R\\ \Gamma;\Delta \vDash b_1 \equiv b_1' : B/R_0\\ \Gamma;\Delta \vDash b_2 \equiv b_2' : B/R_0\end{array}}{\Gamma;\Delta \vDash \mathbf{ifPath}\, R\, F\, a\, b_1\, b_2 \equiv \mathbf{ifPath}\, R\, F\, a'\, b_1'\, b_2' : B/R_0} \quad \text{E\_PatCong}$$

$$\frac{\begin{array}{l}\mathsf{Path}_{R'}\;\, a = F\\ \mathsf{Path}_{R'}\;\, a' = F\\ \Gamma \vDash a : \Pi^{+} x{:}\, A/R_1 \to B/R'\\ \Gamma \vDash b : A/R_1\\ \Gamma \vDash a' : \Pi^{+} x{:}\, A/R_1 \to B/R'\\ \Gamma \vDash b' : A/R_1\\ \Gamma;\Delta \vDash a\;\, b^{R_1,+} \equiv a'\;\, b'^{R_1,+} : B\{b/x\}/R'\\ \Gamma;\widetilde{\Gamma} \vDash B\{b/x\} \equiv B\{b'/x\} : \star/R_0\end{array}}{\Gamma;\Delta \vDash a \equiv a' : \Pi^{+} x{:}\, A/R_1 \to B/R'} \quad \text{E\_LeftRel}$$

$$\frac{\begin{array}{l}\mathsf{Path}_{R'}\ a = F\\ \mathsf{Path}_{R'}\ a' = F\\ \Gamma \vDash a : \Pi^{-} x\!:\!A/R_1 \to B/R'\\ \Gamma \vDash b : A/R_1\\ \Gamma \vDash a' : \Pi^{-} x\!:\!A/R_1 \to B/R'\\ \Gamma \vDash b' : A/R_1\\ \Gamma;\Delta \vDash a\ \square^{R_1,-} \equiv a'\ \square^{R_1,-} : B\{b/x\}/R'\\ \Gamma;\widetilde{\Gamma} \vDash B\{b/x\} \equiv B\{b'/x\} : \star/R_0\end{array}}{\Gamma;\Delta \vDash a \equiv a' : \Pi^{-} x\!:\!A/R_1 \to B/R'} \quad \text{E\_LeftIrrel}$$

$$\frac{\begin{array}{l}\mathsf{Path}_{R'}\ a = F\\ \mathsf{Path}_{R'}\ a' = F\\ \Gamma \vDash a : \Pi^{+} x\!:\!A/R_1 \to B/R'\\ \Gamma \vDash b : A/R_1\\ \Gamma \vDash a' : \Pi^{+} x\!:\!A/R_1 \to B/R'\\ \Gamma \vDash b' : A/R_1\\ \Gamma;\Delta \vDash a\ b^{R_1,+} \equiv a'\ b'^{R_1,+} : B\{b/x\}/R'\\ \Gamma;\widetilde{\Gamma} \vDash B\{b/x\} \equiv B\{b'/x\} : \star/R_0\end{array}}{\Gamma;\Delta \vDash b \equiv b' : A/R_1} \quad \text{E\_Right}$$

$$\frac{\begin{array}{l}\mathsf{Path}_{R'}\ a = F\\ \mathsf{Path}_{R'}\ a' = F\\ \Gamma \vDash a : \forall c\!:\!(a_1 \sim_{A/R_1} a_2).B/R'\\ \Gamma \vDash a' : \forall c\!:\!(a_1 \sim_{A/R_1} a_2).B/R'\\ \Gamma;\widetilde{\Gamma} \vDash a_1 \equiv a_2 : A/R_1\\ \Gamma;\Delta \vDash a[\bullet] \equiv a'[\bullet] : B\{\bullet/c\}/R'\end{array}}{\Gamma;\Delta \vDash a \equiv a' : \forall c\!:\!(a_1 \sim_{A/R_1} a_2).B/R'} \quad \text{E\_CLeft}$$

$\boxed{\vDash \Gamma}$ context wellformedness

$$\frac{}{\vDash \varnothing} \quad \text{E\_Empty}$$

$$\frac{\begin{array}{l}\vDash \Gamma\\ \Gamma \vDash A : \star/R'\\ x \notin \mathsf{dom}\,\Gamma\end{array}}{\vDash \Gamma, x : A/R} \quad \text{E\_ConsTm}$$

$$\frac{\begin{array}{l}\vDash \Gamma\\ \Gamma \vDash \phi\ \mathsf{ok}\\ c \notin \mathsf{dom}\,\Gamma\end{array}}{\vDash \Gamma, c : \phi} \quad \text{E\_ConsCo}$$

$\boxed{\vDash \Sigma}$ signature wellformedness

$$\frac{}{\vDash \varnothing} \quad \text{Sig\_Empty}$$

$$\frac{\begin{array}{l}\vDash \Sigma\\ \varnothing \vDash a : A/R'\\ F \notin \mathsf{dom}\,\Sigma\end{array}}{\vDash \Sigma \cup \{F \sim a : A/R'\}} \quad \text{Sig\_ConsAx}$$

$\boxed{\Gamma \vdash \phi\ \mathsf{ok}}$ prop wellformedness

$$\frac{\Gamma \vdash a : A/R \quad \Gamma \vdash b : B/R \quad |A|_R = |B|_R}{\Gamma \vdash a \sim_{A/R} b \ \mathsf{ok}} \quad \textsc{An\_Wff}$$

$\boxed{\Gamma \vdash a : A/R}$ typing

$$\frac{\vdash \Gamma}{\Gamma \vdash \star : \star/R} \quad \textsc{An\_Star}$$

$$\frac{\vdash \Gamma \quad x : A/R \in \Gamma}{\Gamma \vdash x : A/R} \quad \textsc{An\_Var}$$

$$\frac{\Gamma, x : A/R \vdash B : \star/R' \quad \Gamma \vdash A : \star/R}{\Gamma \vdash \Pi^{\rho} x\!:\!A/R \to B : \star/R'} \quad \textsc{An\_Pi}$$

$$\frac{\Gamma \vdash A : \star/R \quad \Gamma, x : A/R \vdash a : B/R' \quad (\rho = +) \vee (x \notin \mathsf{fv}\ |a|_{R'}) \quad R \leq R'}{\Gamma \vdash \lambda^{\rho} x\!:\!A/R.a : (\Pi^{\rho} x\!:\!A/R \to B)/R'} \quad \textsc{An\_Abs}$$

$$\frac{\Gamma \vdash b : (\Pi^{\rho} x\!:\!A/R \to B)/R' \quad \Gamma \vdash a : A/R}{\Gamma \vdash b\ a^{R,\rho} : (B\{a/x\})/R'} \quad \textsc{An\_App}$$

$$\frac{\Gamma \vdash a : A/R \quad \Gamma; \widetilde{\Gamma} \vdash \gamma : A \sim_R B \quad \Gamma \vdash B : \star/R}{\Gamma \vdash a \triangleright_R \gamma : B/R} \quad \textsc{An\_Conv}$$

$$\frac{\Gamma \vdash \phi \ \mathsf{ok} \quad \Gamma, c : \phi \vdash B : \star/R}{\Gamma \vdash \forall c\!:\!\phi.B : \star/R} \quad \textsc{An\_CPi}$$

$$\frac{\Gamma \vdash \phi \ \mathsf{ok} \quad \Gamma, c : \phi \vdash a : B/R}{\Gamma \vdash \Lambda c\!:\!\phi.a : (\forall c\!:\!\phi.B)/R} \quad \textsc{An\_CAbs}$$

$$\frac{\Gamma \vdash a_1 : (\forall c\!:\!a \sim_{A_1/R} b.B)/R' \quad \Gamma; \widetilde{\Gamma} \vdash \gamma : a \sim_R b}{\Gamma \vdash a_1[\gamma] : B\{\gamma/c\}/R'} \quad \textsc{An\_CApp}$$

$$\frac{\vdash \Gamma \quad F \sim a : A/R \in \Sigma_1 \quad \varnothing \vdash A : \star/R_0}{\Gamma \vdash F : A/R_1} \quad \textsc{An\_Fam}$$

$$\frac{R_1 \leq R_2 \quad \Gamma \vdash a : A/R_1}{\Gamma \vdash \mathbf{sub}\, R_1\, a : A/R_2} \quad \textsc{An\_SubRole}$$

$\boxed{\Gamma; \Delta \vdash \gamma : \phi_1 \sim \phi_2}$ coercion between props

$$\frac{\begin{array}{l}\Gamma;\Delta\vdash\gamma_1 : A_1 \sim_R A_2\\ \Gamma;\Delta\vdash\gamma_2 : B_1 \sim_R B_2\\ \Gamma\vdash A_1 \sim_{A/R} B_1\ \ \mathsf{ok}\\ \Gamma\vdash A_2 \sim_{A/R} B_2\ \ \mathsf{ok}\end{array}}{\Gamma;\Delta\vdash(\gamma_1 \sim_A \gamma_2) : (A_1 \sim_{A/R} B_1) \sim (A_2 \sim_{A/R} B_2)}\quad\textsc{An\_PropCong}$$

$$\frac{\Gamma;\Delta\vdash\gamma : \forall c{:}\phi_1.A_2 \sim_R \forall c{:}\phi_2.B_2}{\Gamma;\Delta\vdash\mathbf{cpiFst}\,\gamma : \phi_1 \sim \phi_2}\quad\textsc{An\_CPiFst}$$

$$\frac{\Gamma;\Delta\vdash\gamma : \phi_1 \sim \phi_2}{\Gamma;\Delta\vdash\mathbf{sym}\,\gamma : \phi_2 \sim \phi_1}\quad\textsc{An\_IsoSym}$$

$$\frac{\begin{array}{l}\Gamma;\Delta\vdash\gamma : A \sim_R B\\ \Gamma\vdash a_1 \sim_{A/R} a_2\ \ \mathsf{ok}\\ \Gamma\vdash a_1' \sim_{B/R} a_2'\ \ \mathsf{ok}\\ |a_1|_R = |a_1'|_R\\ |a_2|_R = |a_2'|_R\end{array}}{\Gamma;\Delta\vdash\mathbf{conv}\ (a_1 \sim_{A/R} a_2) \sim_\gamma (a_1' \sim_{B/R} a_2') : (a_1 \sim_{A/R} a_2) \sim (a_1' \sim_{B/R} a_2')}\quad\textsc{An\_IsoConv}$$

$\boxed{\Gamma;\Delta\vdash\gamma : A \sim_R B}$ coercion between types

$$\frac{\begin{array}{l}\vdash\Gamma\\ c : a \sim_{A/R} b\ \in\ \Gamma\\ c\ \in\ \Delta\end{array}}{\Gamma;\Delta\vdash c : a \sim_R b}\quad\textsc{An\_Assn}$$

$$\frac{\Gamma\vdash a : A/R}{\Gamma;\Delta\vdash\mathbf{refl}\,a : a \sim_R a}\quad\textsc{An\_Refl}$$

$$\frac{\begin{array}{l}\Gamma\vdash a : A/R\\ \Gamma\vdash b : B/R\\ |a|_R = |b|_R\\ \Gamma;\widetilde{\Gamma}\vdash\gamma : A \sim_R B\end{array}}{\Gamma;\Delta\vdash(a \mathrel{|{=}|_\gamma} b) : a \sim_R b}\quad\textsc{An\_EraseEq}$$

$$\frac{\begin{array}{l}\Gamma\vdash b : B/R\\ \Gamma\vdash a : A/R\\ \Gamma;\widetilde{\Gamma}\vdash\gamma_1 : B \sim_R A\\ \Gamma;\Delta\vdash\gamma : b \sim_R a\end{array}}{\Gamma;\Delta\vdash\mathbf{sym}\,\gamma : a \sim_R b}\quad\textsc{An\_Sym}$$

$$\frac{\begin{array}{l}\Gamma;\Delta\vdash\gamma_1 : a \sim_R a_1\\ \Gamma;\Delta\vdash\gamma_2 : a_1 \sim_R b\\ \Gamma\vdash a : A/R\\ \Gamma\vdash a_1 : A_1/R\\ \Gamma;\widetilde{\Gamma}\vdash\gamma_3 : A \sim_R A_1\end{array}}{\Gamma;\Delta\vdash(\gamma_1;\gamma_2) : a \sim_R b}\quad\textsc{An\_Trans}$$

$$\frac{\begin{array}{l}\Gamma\vdash a_1 : B_0/R\\ \Gamma\vdash a_2 : B_1/R\\ |B_0|_R = |B_1|_R\\ \vDash |a_1|_R > |a_2|_R/R\end{array}}{\Gamma;\Delta\vdash\mathbf{red}\,a_1\ a_2 : a_1 \sim_R a_2}\quad\textsc{An\_Beta}$$

$$\frac{\begin{array}{l}\Gamma;\Delta \vdash \gamma_1 : A_1 \sim_{R'} A_2 \\ \Gamma, x : A_1/R;\Delta \vdash \gamma_2 : B_1 \sim_{R'} B_2 \\ B_3 = B_2\{x \triangleright_{R'} \mathbf{sym}\, \gamma_1/x\} \\ \Gamma \vdash \Pi^{\rho} x\!:\!A_1/R \to B_1 : \star/R' \\ \Gamma \vdash \Pi^{\rho} x\!:\!A_1/R \to B_2 : \star/R' \\ \Gamma \vdash \Pi^{\rho} x\!:\!A_2/R \to B_3 : \star/R' \\ R \leq R'\end{array}}{\Gamma;\Delta \vdash \Pi^{R,\rho} x\!:\!\gamma_1.\gamma_2 : (\Pi^{\rho} x\!:\!A_1/R \to B_1) \sim_{R'} (\Pi^{\rho} x\!:\!A_2/R \to B_3)} \quad \textsc{An\_PiCong}$$

$$\frac{\begin{array}{l}\Gamma;\Delta \vdash \gamma_1 : A_1 \sim_{R} A_2 \\ \Gamma, x : A_1/R;\Delta \vdash \gamma_2 : b_1 \sim_{R'} b_2 \\ b_3 = b_2\{x \triangleright_{R'} \mathbf{sym}\, \gamma_1/x\} \\ \Gamma \vdash A_1 : \star/R \\ \Gamma \vdash A_2 : \star/R \\ (\rho = +) \vee (x \notin \mathsf{fv}\, |b_1|_{R'}) \\ (\rho = +) \vee (x \notin \mathsf{fv}\, |b_3|_{R'}) \\ \Gamma \vdash (\lambda^{\rho} x\!:\!A_1/R.b_2) : B/R' \\ R \leq R'\end{array}}{\Gamma;\Delta \vdash (\lambda^{R,\rho} x\!:\!\gamma_1.\gamma_2) : (\lambda^{\rho} x\!:\!A_1/R.b_1) \sim_{R'} (\lambda^{\rho} x\!:\!A_2/R.b_3)} \quad \textsc{An\_AbsCong}$$

$$\frac{\begin{array}{l}\Gamma;\Delta \vdash \gamma_1 : a_1 \sim_{R'} b_1 \\ \Gamma;\Delta \vdash \gamma_2 : a_2 \sim_{R} b_2 \\ \Gamma \vdash a_1\ {a_2}^{R,\rho} : A/R' \\ \Gamma \vdash b_1\ {b_2}^{R,\rho} : B/R' \\ \Gamma;\widetilde{\Gamma} \vdash \gamma_3 : A \sim_{R'} B\end{array}}{\Gamma;\Delta \vdash \gamma_1\ \gamma_2^{R,\rho} : a_1\ {a_2}^{R,\rho} \sim_{R'} b_1\ {b_2}^{R,\rho}} \quad \textsc{An\_AppCong}$$

$$\frac{\Gamma;\Delta \vdash \gamma : \Pi^{\rho} x\!:\!A_1/R \to B_1 \sim_{R'} \Pi^{\rho} x\!:\!A_2/R \to B_2}{\Gamma;\Delta \vdash \mathbf{piFst}\, \gamma : A_1 \sim_{R} A_2} \quad \textsc{An\_PiFst}$$

$$\frac{\begin{array}{l}\Gamma;\Delta \vdash \gamma_1 : \Pi^{\rho} x\!:\!A_1/R \to B_1 \sim_{R'} \Pi^{\rho} x\!:\!A_2/R \to B_2 \\ \Gamma;\Delta \vdash \gamma_2 : a_1 \sim_{R} a_2 \\ \Gamma \vdash a_1 : A_1/R \\ \Gamma \vdash a_2 : A_2/R\end{array}}{\Gamma;\Delta \vdash \gamma_1 @ \gamma_2 : B_1\{a_1/x\} \sim_{R'} B_2\{a_2/x\}} \quad \textsc{An\_PiSnd}$$

$$\frac{\begin{array}{l}\Gamma;\Delta \vdash \gamma_1 : a_1 \sim_{A_1/R} b_1 \sim a_2 \sim_{A_2/R} b_2 \\ \Gamma, c : a_1 \sim_{A_1/R} b_1;\Delta \vdash \gamma_3 : B_1 \sim_{R'} B_2 \\ B_3 = B_2\{c \triangleright_{R'} \mathbf{sym}\, \gamma_1/c\} \\ \Gamma \vdash \forall c\!:\!a_1 \sim_{A_1/R} b_1.B_1 : \star/R' \\ \Gamma \vdash \forall c\!:\!a_2 \sim_{A_2/R} b_2.B_3 : \star/R' \\ \Gamma \vdash \forall c\!:\!a_1 \sim_{A_1/R} b_1.B_2 : \star/R'\end{array}}{\Gamma;\Delta \vdash (\forall c\!:\!\gamma_1.\gamma_3) : (\forall c\!:\!a_1 \sim_{A_1/R} b_1.B_1) \sim_{R} (\forall c\!:\!a_2 \sim_{A_2/R} b_2.B_3)} \quad \textsc{An\_CPiCong}$$

$$\frac{\begin{array}{l}\Gamma;\Delta \vdash \gamma_1 : b_0 \sim_{A_1/R} b_1 \sim b_2 \sim_{A_2/R} b_3 \\ \Gamma, c : b_0 \sim_{A_1/R} b_1;\Delta \vdash \gamma_3 : a_1 \sim_{R'} a_2 \\ a_3 = a_2\{c \triangleright_{R'} \mathbf{sym}\, \gamma_1/c\} \\ \Gamma \vdash (\Lambda c\!:\!b_0 \sim_{A_1/R} b_1.a_1) : \forall c\!:\!b_0 \sim_{A_1/R} b_1.B_1/R' \\ \Gamma \vdash (\Lambda c\!:\!b_0 \sim_{A_1/R} b_1.a_2) : B/R' \\ \Gamma \vdash (\Lambda c\!:\!b_2 \sim_{A_2/R} b_3.a_3) : \forall c\!:\!b_2 \sim_{A_2/R} b_3.B_2/R' \\ \Gamma;\widetilde{\Gamma} \vdash \gamma_4 : \forall c\!:\!b_0 \sim_{A_1/R} b_1.B_1 \sim_{R'} \forall c\!:\!\phi_2.B_2\end{array}}{\Gamma;\Delta \vdash (\lambda c\!:\!\gamma_1.\gamma_3 @ \gamma_4) : (\Lambda c\!:\!b_0 \sim_{A_1/R} b_1.a_1) \sim_{R'} (\Lambda c\!:\!b_2 \sim_{A_2/R} b_3.a_3)} \quad \textsc{An\_CAbsCong}$$

$$\frac{\begin{array}{l}\Gamma;\Delta\vdash\gamma_1 : a_1 \sim_R b_1\\ \Gamma;\widetilde{\Gamma}\vdash\gamma_2 : a_2 \sim_{R'} b_2\\ \Gamma;\widetilde{\Gamma}\vdash\gamma_3 : a_3 \sim_{R'} b_3\\ \Gamma\vdash a_1[\gamma_2] : A/R\\ \Gamma\vdash b_1[\gamma_3] : B/R\\ \Gamma;\widetilde{\Gamma}\vdash\gamma_4 : A \sim_R B\end{array}}{\Gamma;\Delta\vdash\gamma_1(\gamma_2,\gamma_3) : a_1[\gamma_2] \sim_R b_1[\gamma_3]}\quad \textsc{An\_CAppCong}$$

$$\frac{\begin{array}{l}\Gamma;\Delta\vdash\gamma_1 : (\forall c_1\!:\!a \sim_{A/R} a'.B_1) \sim_{R_0} (\forall c_2\!:\!b \sim_{B/R'} b'.B_2)\\ \Gamma;\widetilde{\Gamma}\vdash\gamma_2 : a \sim_R a'\\ \Gamma;\widetilde{\Gamma}\vdash\gamma_3 : b \sim_{R'} b'\end{array}}{\Gamma;\Delta\vdash\gamma_1@(\gamma_2\sim\gamma_3) : B_1\{\gamma_2/c_1\} \sim_{R_0} B_2\{\gamma_3/c_2\}}\quad \textsc{An\_CPiSnd}$$

$$\frac{\begin{array}{l}\Gamma;\Delta\vdash\gamma_1 : a \sim_{R_1} a'\\ \Gamma;\Delta\vdash\gamma_2 : a \sim_{A/R_1} a' \sim b \sim_{B/R_1} b'\end{array}}{\Gamma;\Delta\vdash\gamma_1 \rhd_{R_1} \gamma_2 : b \sim_{R_1} b'}\quad \textsc{An\_Cast}$$

$$\frac{\Gamma;\Delta\vdash\gamma : (a \sim_{A/R} a') \sim (b \sim_{B/R} b')}{\Gamma;\Delta\vdash\mathbf{isoSnd}\,\gamma : A \sim_R B}\quad \textsc{An\_IsoSnd}$$

$$\frac{\begin{array}{l}\Gamma;\Delta\vdash\gamma : a \sim_{R_1} b\\ R_1 \leq R_2\end{array}}{\Gamma;\Delta\vdash\mathbf{sub}\,\gamma : a \sim_{R_2} b}\quad \textsc{An\_Sub}$$

$\boxed{\vdash\Gamma}$ context wellformedness

$$\frac{}{\vdash\varnothing}\quad \textsc{An\_Empty}$$

$$\frac{\begin{array}{l}\vdash\Gamma\\ \Gamma\vdash A : \star/R\\ x \notin \mathsf{dom}\,\Gamma\end{array}}{\vdash\Gamma, x : A/R}\quad \textsc{An\_ConsTm}$$

$$\frac{\begin{array}{l}\vdash\Gamma\\ \Gamma\vdash\phi\ \mathsf{ok}\\ c \notin \mathsf{dom}\,\Gamma\end{array}}{\vdash\Gamma, c : \phi}\quad \textsc{An\_ConsCo}$$

$\boxed{\vdash\Sigma}$ signature wellformedness

$$\frac{}{\vdash\varnothing}\quad \textsc{An\_Sig\_Empty}$$

$$\frac{\begin{array}{l}\vdash\Sigma\\ \varnothing\vdash A : \star/R\\ \varnothing\vdash a : A/R\\ F \notin \mathsf{dom}\,\Sigma\end{array}}{\vdash\Sigma\cup\{F \sim a : A/R\}}\quad \textsc{An\_Sig\_ConsAx}$$

$\boxed{\Gamma\vdash a \rightsquigarrow b/R}$ single-step, weak head reduction to values for annotated language

$$\frac{\Gamma\vdash a \rightsquigarrow a'/R_1}{\Gamma\vdash a\ b^{R,\rho} \rightsquigarrow a'\ b^{R,\rho}/R_1}\quad \textsc{An\_AppLeft}$$

$$\frac{\mathsf{Value}_R\ (\lambda^\rho x\!:\!A/R.w)}{\Gamma\vdash(\lambda^\rho x\!:\!A/R.w)\ a^{R,\rho} \rightsquigarrow w\{a/x\}/R}\quad \textsc{An\_AppAbs}$$

$$\frac{\Gamma \vdash a \leadsto a'/R}{\Gamma \vdash a[\gamma] \leadsto a'[\gamma]/R} \quad \textsc{An\_CAppLeft}$$

$$\frac{}{\Gamma \vdash (\Lambda c\!:\!\phi.b)[\gamma] \leadsto b\{\gamma/c\}/R} \quad \textsc{An\_CAppCAbs}$$

$$\frac{\begin{array}{l}\Gamma \vdash A : \star/R \\ \Gamma, x : A/R \vdash b \leadsto b'/R_1\end{array}}{\Gamma \vdash (\lambda^- x\!:\!A/R.b) \leadsto (\lambda^- x\!:\!A/R.b')/R_1} \quad \textsc{An\_AbsTerm}$$

$$\frac{F \sim a : A/R \in \Sigma_1}{\Gamma \vdash F \leadsto a/R} \quad \textsc{An\_Axiom}$$

$$\frac{\Gamma \vdash a \leadsto a'/R}{\Gamma \vdash a \triangleright_{R_1} \gamma \leadsto a' \triangleright_{R_1} \gamma/R} \quad \textsc{An\_ConvTerm}$$

$$\frac{\mathsf{Value}_R\ v}{\Gamma \vdash (v \triangleright_{R_2} \gamma_1) \triangleright_{R_2} \gamma_2 \leadsto v \triangleright_{R_2} (\gamma_1; \gamma_2)/R} \quad \textsc{An\_Combine}$$

$$\frac{\begin{array}{l}\mathsf{Value}_R\ v \\ \Gamma; \widetilde{\Gamma} \vdash \gamma : \Pi^\rho x_1\!:\!A_1/R \to B_1 \sim_{R'} \Pi^\rho x_2\!:\!A_2/R \to B_2 \\ b' = b \triangleright_{R'} \mathbf{sym}\,(\mathbf{piFst}\,\gamma) \\ \gamma' = \gamma @ (b' \mid=\mid_{(\mathbf{piFst}\,\gamma)} b)\end{array}}{\Gamma \vdash (v \triangleright_{R'} \gamma)\ b^{R,\rho} \leadsto ((v\ b'^{R,\rho}) \triangleright_{R'} \gamma')/R} \quad \textsc{An\_Push}$$

$$\frac{\begin{array}{l}\mathsf{Value}_R\ v \\ \Gamma; \widetilde{\Gamma} \vdash \gamma : \forall c_1\!:\!a_1 \sim_{B_1/R} b_1.A_1 \sim_{R'} \forall c_2\!:\!a_2 \sim_{B_2/R} b_2.A_2 \\ \gamma_1' = \gamma_1 \triangleright_{R'} \mathbf{sym}\,(\mathbf{cpiFst}\,\gamma) \\ \gamma' = \gamma @ (\gamma_1' \sim \gamma_1)\end{array}}{\Gamma \vdash (v \triangleright_{R'} \gamma)[\gamma_1] \leadsto ((v[\gamma_1']) \triangleright_{R'} \gamma')/R} \quad \textsc{An\_CPush}$$

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Definition rules:        162 good    0 bad
Definition rule clauses: 517 good    0 bad
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