$tmvar$, x, y, f, m, n	variables
$covar$, c	coercion variables
$datacon$, K	
$const$, T, F	
$index$, i	indices

relflag, ρ	$::=$			relevance flag
	$\mid$	$+$		
	$\mid$	$-$		
ν	$::=$			
	$\mid$	R		
	$\mid$	ρ		
role, R	$::=$			Role
	$\mid$	$\mathbf{Nom}$		
	$\mid$	$\mathbf{Rep}$		
	$\mid$	$R_1 \cap R_2$	S	
	$\mid$	$\mathbf{param}\, R_1\, R_2$	S	
	$\mid$	$app\_role \nu$	S	
	$\mid$	(R)	S	
constraint, ϕ	$::=$			props
	$\mid$	$a \sim_{A/R} b$		
	$\mid$	(ϕ)	S	
	$\mid$	$\phi\{b/x\}$	S	
	$\mid$	$\lvert\phi\rvert$	S	
	$\mid$	$a \sim_R b$	S	
tm, a, b, v, w, A, B	$::=$			types and kinds
	$\mid$	$\star$		
	$\mid$	x		
	$\mid$	$\lambda^\rho x\!:\!A.b$	bind x in b	
	$\mid$	$\lambda^\rho x.b$	bind x in b	
	$\mid$	$a\ b^\nu$		
	$\mid$	F		
	$\mid$	$\Pi^\rho x\!:\!A \to B$	bind x in B	
	$\mid$	$a \rhd_R \gamma$		
	$\mid$	$\forall c\!:\!\phi.B$	bind c in B	
	$\mid$	$\Lambda c\!:\!\phi.b$	bind c in b	
	$\mid$	$\Lambda c.b$	bind c in b	
	$\mid$	$a[\gamma]$		
	$\mid$	$\square$		
	$\mid$	$case_R\ a\ of\ a' \to b_1 \Vert_{\_} \to b_2$		
	$\mid$	$caseRa_1 of a_2 \to b_1 \Vert_{\_} \to b_2$		
	$\mid$	K		
	$\mid$	$\mathbf{match}\, a\, \mathbf{with}\, brs$		
	$\mid$	$\mathbf{sub}\, R\, a$		
	$\mid$	$a\{b/x\}$	S	
	$\mid$	a	S	
	$\mid$	$a\{\gamma/c\}$	S	
	$\mid$	a	S	
	$\mid$	(a)	S	

	|	a	S	parsing precedence is hard
	|	$\|a\|_R$	S	
	|	**Int**	S	
	|	**Bool**	S	
	|	**Nat**	S	
	|	**Vec**	S	
	|	0	S	
	|	S	S	
	|	**True**	S	
	|	**Fix**	S	
	|	**Age**	S	
	|	$a \to b$	S	
	|	$a/R \to b$	S	
	|	$\phi \Rightarrow A$	S	
	|	$a\ b$	S	
	|	$\lambda x.a$	S	
	|	$\lambda x\!:\!A.a$	S	
	|	$\forall\, x : A \to B$	S	
	|	**if** ϕ **then** a **else** b	S	
brs	::=			case branches
	|	**none**		
	|	$K \Rightarrow a; brs$		
	|	$brs\{a/x\}$	S	
	|	$brs\{\gamma/c\}$	S	
	|	(brs)	S	
$co,\ \gamma$	::=			explicit coercions
	|	$\bullet$		
	|	c		
	|	**red** $a\ b$		
	|	**refl** a		
	|	$(a \mid=\mid_\gamma b)$		
	|	**sym** γ		
	|	$\gamma_1; \gamma_2$		
	|	**sub** γ		
	|	$\Pi^{R,\rho} x\!:\!\gamma_1.\gamma_2$	bind x in γ_2	
	|	$\lambda^{R,\rho} x\!:\!\gamma_1.\gamma_2$	bind x in γ_2	
	|	$\gamma_1\ \gamma_2^{R,\rho}$		
	|	**piFst** γ		
	|	**cpiFst** γ		
	|	**isoSnd** γ		
	|	$\gamma_1 @ \gamma_2$		
	|	$\forall c\!:\!\gamma_1.\gamma_3$	bind c in γ_3	
	|	$\lambda c\!:\!\gamma_1.\gamma_3 @ \gamma_4$	bind c in γ_3	
	|	$\gamma(\gamma_1, \gamma_2)$		

	|	$\gamma@(\gamma_1 \sim \gamma_2)$		
	|	$\gamma_1 \triangleright_R \gamma_2$		
	|	$\gamma_1 \sim_A \gamma_2$		
	|	$\mathbf{conv}\ \phi_1 \sim_\gamma \phi_2$		
	|	$\mathbf{eta}\, a$		
	|	$\mathbf{left}\, \gamma\, \gamma'$		
	|	$\mathbf{right}\, \gamma\, \gamma'$		
	|	(γ)	S	
	|	γ	S	
	|	$\gamma\{a/x\}$	S	
$\mathit{role\_context},\ \Omega$	::=			role$_c$*ontexts*
	|	$\varnothing$		
	|	$\Omega, x : R$		
	|	(Ω)	M	
	|	Ω	M	
$\mathit{roles},\ \mathit{Rs}$	::=			
	|	$\mathbf{nilR}$		
	|	R, Rs		
$\mathit{sig\_sort}$	::=			signature classifier
	|	$: A@\mathit{Rs}$		
	|	$\sim a : A/R@\mathit{Rs}$		
sort	::=			binding classifier
	|	$\mathbf{Tm}\, A$		
	|	$\mathbf{Co}\, \phi$		
$\mathit{context},\ \Gamma$	::=			contexts
	|	$\varnothing$		
	|	$\Gamma, x : A$		
	|	$\Gamma, c : \phi$		
	|	$\Gamma\{b/x\}$	M	
	|	$\Gamma\{\gamma/c\}$	M	
	|	Γ, Γ'	M	
	|	$\|\Gamma\|$	M	
	|	(Γ)	M	
	|	Γ	M	
$\mathit{sig},\ \Sigma$	::=			signatures
	|	$\varnothing$		
	|	$\Sigma \cup \{F\, \mathit{sig\_sort}\}$		
	|	Σ_0	M	
	|	Σ_1	M	
	|	$\|\Sigma\|$	M	

$available\_props,\ \Delta$	::=		
	|	$\varnothing$	
	|	Δ, c	
	|	$\widetilde{\Gamma}$	M
	|	(Δ)	M

$terminals$	::=	
	|	$\leftrightarrow$
	|	$\Leftrightarrow$
	|	$\longrightarrow$
	|	min
	|	$\equiv$
	|	$\forall$
	|	$\in$
	|	$\notin$
	|	$\Leftarrow$
	|	$\Rightarrow$
	|	$\Rightarrow^{*}$
	|	$\rightarrow$
	|	Λ
	|	
	|	$\Box$
	|	$\vdash$
	|	$\dashv$
	|	$\models$
	|	$\models$
	|	$\neq$
	|	$\triangleright$
	|	$\ \mathsf{ok}$
	|	$\_$
	|	$\leadsto$
	|	$\leadsto^{*}$
	|	$\leadsto$
	|	$\varnothing$
	|	$\circ$
	|	fv
	|	dom
	|	$\sim$
	|	$\asymp$
	|	$\mid$
	|	$\bullet$
	|	$\mathbf{fst}$
	|	$\mathbf{snd}$
	|	$\mid\Rightarrow\mid$
	|	$\vdash_{=}$
	|	$\mathbf{refl_2}$

	$\mid$	$++$	
formula, ψ	$::=$		
	$\mid$	*judgement*	
	$\mid$	$x : A \in \Gamma$	
	$\mid$	$x : R \in \Omega$	
	$\mid$	$c : \phi \in \Gamma$	
	$\mid$	$F\ sig\_sort \in \Sigma$	
	$\mid$	$x \in \Delta$	
	$\mid$	$c \in \Delta$	
	$\mid$	$c\ \mathbf{not\ relevant} \in \gamma$	
	$\mid$	$x \notin \mathsf{fv}\, a$	
	$\mid$	$x \notin \mathsf{dom}\, \Gamma$	
	$\mid$	$uniq(\Omega)$	
	$\mid$	$c \notin \mathsf{dom}\, \Gamma$	
	$\mid$	$T \notin \mathsf{dom}\, \Sigma$	
	$\mid$	$F \notin \mathsf{dom}\, \Sigma$	
	$\mid$	$R_1 = R_2$	
	$\mid$	$a = b$	
	$\mid$	$\phi_1 = \phi_2$	
	$\mid$	$\Gamma_1 = \Gamma_2$	
	$\mid$	$\gamma_1 = \gamma_2$	
	$\mid$	$\neg\psi$	
	$\mid$	$\psi_1 \wedge \psi_2$	
	$\mid$	$\psi_1 \vee \psi_2$	
	$\mid$	$\psi_1 \Rightarrow \psi_2$	
	$\mid$	(ψ)	
	$\mid$	ψ	
	$\mid$	$c : (a : A \sim b : B) \in \Gamma$	
	$\mid$		suppress lc hypothesis generated by Ott
	$\mid$	$\{y/x\}B = B_1$	
	$\mid$	$\{c_1/c_2\}B = B_1$	
JSubRole	$::=$		
	$\mid$	$R_1 \leq R_2$	Subroling judgement
JPath	$::=$		
	$\mid$	$\mathsf{Path}_R\ a = F@Rs$	Type headed by constant (partial function)
JPat	$::=$		
	$\mid$	$\Gamma \vDash a : A\ \mathbf{pat}\ R@Rs$	Pattern judgment
JValue	$::=$		
	$\mid$	$\mathsf{Value}_R\ A$	values
JValueType	$::=$		
	$\mid$	$\mathsf{ValueType}_R\ A$	Types with head forms (erased language)

Jconsistent	::=		
	|	$\mathsf{consistent}_R\ ab$	(erased) types do not differ in their heads
Jroleing	::=		
	|	$\Omega \vDash a : R$	
JChk	::=		
	|	$(\rho = +) \vee (x \notin \mathsf{fv}\ A)$	irrelevant argument check
Jpar	::=		
	|	$\Omega \vDash a \Rightarrow_R b$	parallel reduction (implicit language)
	|	$\Omega \vdash a \Rightarrow^*_R b$	multistep parallel reduction
	|	$\Omega \vdash a \Leftrightarrow_R b$	parallel reduction to a common term
Jbeta	::=		
	|	$\vDash a > b/R$	primitive reductions on erased terms
	|	$\vDash a \rightsquigarrow b/R$	single-step head reduction for implicit language
	|	$\vDash a \rightsquigarrow^* b/R$	multistep reduction
Jett	::=		
	|	$\Gamma \vDash \phi\ \mathsf{ok}$	Prop wellformedness
	|	$\Gamma \vDash a : A$	typing
	|	$\Gamma; \Delta \vDash \phi_1 \equiv \phi_2$	prop equality
	|	$\Gamma; \Delta \vDash a \equiv b : A/R$	definitional equality
	|	$\vDash \Gamma$	context wellformedness
Jsig	::=		
	|	$\vDash \Sigma$	signature wellformedness
judgement	::=		
	|	*JSubRole*	
	|	*JPath*	
	|	*JPat*	
	|	*JValue*	
	|	*JValueType*	
	|	*Jconsistent*	
	|	*Jroleing*	
	|	*JChk*	
	|	*Jpar*	
	|	*Jbeta*	
	|	*Jett*	
	|	*Jsig*	
user_syntax	::=		
	|	*tmvar*	
	|	*covar*	
	|	*datacon*	

| *const*
| *index*
| *relflag*
| ν
| *role*
| *constraint*
| *tm*
| *brs*
| *co*
| *role_context*
| *roles*
| *sig_sort*
| *sort*
| *context*
| *sig*
| *available_props*
| *terminals*
| *formula*

$\boxed{R_1 \leq R_2}$ Subroling judgement

$$\frac{}{\mathbf{Nom} \leq R} \quad \text{NomBot}$$

$$\frac{}{R \leq \mathbf{Rep}} \quad \text{RepTop}$$

$$\frac{}{R \leq R} \quad \text{Refl}$$

$$\frac{R_1 \leq R_2 \quad R_2 \leq R_3}{R_1 \leq R_3} \quad \text{Trans}$$

$\boxed{\mathsf{Path}_R\ a = F@Rs}$ Type headed by constant (partial function)

$$\frac{F\ : A@Rs\ \in\ \Sigma_0}{\mathsf{Path}_R\ F = F@Rs} \quad \text{Path\_AbsConst}$$

$$\frac{F\ \sim a : A/R_1@Rs\ \in\ \Sigma_0 \quad \neg(R_1 \leq R)}{\mathsf{Path}_R\ F = F@R, Rs} \quad \text{Path\_Const}$$

$$\frac{\mathsf{Path}_R\ a = F@R_1, Rs \quad \mathit{app\_role}\nu = R_1}{\mathsf{Path}_R\ (a\ b'^{\nu}) = F@Rs} \quad \text{Path\_App}$$

$$\frac{\mathsf{Path}_R\ a = F@Rs}{\mathsf{Path}_R\ (a[\bullet]) = F@Rs} \quad \text{Path\_CApp}$$

$\boxed{\Gamma \vDash a : A\, \mathbf{pat}\, R@Rs}$ Pattern judgment

$$\frac{F\ : A@Rs\ \in\ \Sigma_0}{\varnothing \vDash F : A\, \mathbf{pat}\, R@Rs} \quad \text{Pat\_AbsConst}$$

$$\frac{F\ \sim a : A/R_1@Rs\ \in\ \Sigma_0 \quad \neg(R_1 \leq R)}{\varnothing \vDash F : A\, \mathbf{pat}\, R@Rs} \quad \text{Pat\_Const}$$

$$\frac{\begin{array}{l}\Gamma \vDash a : \Pi^\rho y\!:\!A_1 \to B_1 \ \mathbf{pat}\ R@R_1, Rs \\ \{y/x\}B = B_1 \\ \mathit{app\_role}\nu = R_1\end{array}}{\Gamma, x : A_1 \vDash (a\ x^\nu) : B\ \mathbf{pat}\ R@Rs} \quad \textsc{Pat\_App}$$

$$\frac{\begin{array}{l}\Gamma \vDash a : \forall c_1\!:\!\phi.B_1\ \mathbf{pat}\ R@Rs \\ \{c_1/c\}B = B_1\end{array}}{\Gamma, c : \phi \vDash (a[\bullet]) : B\ \mathbf{pat}\ R@Rs} \quad \textsc{Pat\_CApp}$$

$\boxed{\mathsf{Value}_R\ A}$ values

$$\frac{}{\mathsf{Value}_R\ \star} \quad \textsc{Value\_Star}$$

$$\frac{}{\mathsf{Value}_R\ \Pi^\rho x\!:\!A \to B} \quad \textsc{Value\_Pi}$$

$$\frac{}{\mathsf{Value}_R\ \forall c\!:\!\phi.B} \quad \textsc{Value\_CPi}$$

$$\frac{}{\mathsf{Value}_R\ \lambda^+ x\!:\!A.a} \quad \textsc{Value\_AbsRel}$$

$$\frac{}{\mathsf{Value}_R\ \lambda^+ x.a} \quad \textsc{Value\_UAbsRel}$$

$$\frac{\mathsf{Value}_R\ a}{\mathsf{Value}_R\ \lambda^- x.a} \quad \textsc{Value\_UAbsIrrel}$$

$$\frac{}{\mathsf{Value}_R\ \Lambda c\!:\!\phi.a} \quad \textsc{Value\_CAbs}$$

$$\frac{}{\mathsf{Value}_R\ \Lambda c.a} \quad \textsc{Value\_UCAbs}$$

$$\frac{\mathsf{Path}_R\ a = F@Rs}{\mathsf{Value}_R\ a} \quad \textsc{Value\_Path}$$

$\boxed{\mathsf{ValueType}_R\ A}$ Types with head forms (erased language)

$$\frac{}{\mathsf{ValueType}_R\ \star} \quad \textsc{value\_type\_Star}$$

$$\frac{}{\mathsf{ValueType}_R\ \Pi^\rho x\!:\!A \to B} \quad \textsc{value\_type\_Pi}$$

$$\frac{}{\mathsf{ValueType}_R\ \forall c\!:\!\phi.B} \quad \textsc{value\_type\_CPi}$$

$$\frac{\mathsf{Path}_R\ a = F@Rs}{\mathsf{ValueType}_R\ a} \quad \textsc{value\_type\_Path}$$

$\boxed{\mathsf{consistent}_R\ ab}$ (erased) types do not differ in their heads

$$\frac{}{\mathsf{consistent}_R\ \star\star} \quad \textsc{consistent\_a\_Star}$$

$$\frac{}{\mathsf{consistent}_{R'}\ (\Pi^\rho x_1\!:\!A_1 \to B_1)(\Pi^\rho x_2\!:\!A_2 \to B_2)} \quad \textsc{consistent\_a\_Pi}$$

$$\frac{}{\mathsf{consistent}_R\ (\forall c_1\!:\!\phi_1.A_1)(\forall c_2\!:\!\phi_2.A_2)} \quad \textsc{consistent\_a\_CPi}$$

$$\frac{\begin{array}{l}\mathsf{Path}_R\ a_1 = F@Rs \\ \mathsf{Path}_R\ a_2 = F@Rs\end{array}}{\mathsf{consistent}_R\ a_1 a_2} \quad \textsc{consistent\_a\_Path}$$

$$\frac{\neg\mathsf{ValueType}_R\ b}{\mathsf{consistent}_R\ ab}\ \textsc{consistent\_a\_Step\_R}$$

$$\frac{\neg\mathsf{ValueType}_R\ a}{\mathsf{consistent}_R\ ab}\ \textsc{consistent\_a\_Step\_L}$$

$\boxed{\Omega \vDash a : R}$

$$\frac{uniq(\Omega)}{\Omega \vDash \Box : R}\ \textsc{role\_a\_Bullet}$$

$$\frac{uniq(\Omega)}{\Omega \vDash \star : R}\ \textsc{role\_a\_Star}$$

$$\frac{\begin{array}{l} uniq(\Omega) \\ x : R\ \in\ \Omega \\ R \leq R_1 \end{array}}{\Omega \vDash x : R_1}\ \textsc{role\_a\_Var}$$

$$\frac{\Omega, x : \mathbf{Nom} \vDash a : R}{\Omega \vDash (\lambda^\rho x.a) : R}\ \textsc{role\_a\_Abs}$$

$$\frac{\begin{array}{l} \Omega \vDash a : R \\ \Omega \vDash b : app\_role\nu \end{array}}{\Omega \vDash (a\ b^\nu) : R}\ \textsc{role\_a\_App}$$

$$\frac{\begin{array}{l} \Omega \vDash A : R \\ \Omega, x : \mathbf{Nom} \vDash B : R \end{array}}{\Omega \vDash (\Pi^\rho x\!:\!A \to B) : R}\ \textsc{role\_a\_Pi}$$

$$\frac{\begin{array}{l} \Omega \vDash a : R_1 \\ \Omega \vDash b : R_1 \\ \Omega \vDash A : R_0 \\ \Omega \vDash B : R \end{array}}{\Omega \vDash (\forall c\!:\!a \sim_{A/R_1} b.B) : R}\ \textsc{role\_a\_CPi}$$

$$\frac{\Omega \vDash b : R}{\Omega \vDash (\Lambda c.b) : R}\ \textsc{role\_a\_CAbs}$$

$$\frac{\Omega \vDash a : R}{\Omega \vDash (a[\bullet]) : R}\ \textsc{role\_a\_CApp}$$

$$\frac{\begin{array}{l} uniq(\Omega) \\ F\ : A@Rs\ \in\ \Sigma_0 \end{array}}{\Omega \vDash F : R}\ \textsc{role\_a\_Const}$$

$$\frac{\begin{array}{l} uniq(\Omega) \\ F\ \sim a : A/R@Rs\ \in\ \Sigma_0 \end{array}}{\Omega \vDash F : R_1}\ \textsc{role\_a\_Fam}$$

$$\frac{\begin{array}{l} F\ sig\_sort\ \in\ \Sigma_0 \\ \Omega \vDash a : R \\ \Omega \vDash b_1 : R_1 \\ \Omega \vDash b_2 : R_1 \end{array}}{\Omega \vDash (case_R\ a\ of\ F \to b_1 \|\_ \to b_2) : R_1}\ \textsc{role\_a\_Pattern}$$

$\boxed{(\rho = +) \vee (x \notin \mathsf{fv}\ A)}$ irrelevant argument check

$$\frac{}{(+ = +) \vee (x \notin \mathsf{fv}\ A)} \quad \textsc{Rho\_Rel}$$

$$\frac{x \ \notin \ \mathsf{fv} A}{(- = +) \vee (x \notin \mathsf{fv}\ A)} \quad \textsc{Rho\_IrrRel}$$

$\boxed{\Omega \vDash a \Rightarrow_R b}$ parallel reduction (implicit language)

$$\frac{\Omega \vDash a : R}{\Omega \vDash a \Rightarrow_R a} \quad \textsc{Par\_Refl}$$

$$\frac{\begin{array}{c}\Omega \vDash a \Rightarrow_R (\lambda^\rho x.a') \\ \Omega \vDash b \Rightarrow_{\mathit{app\_role}\nu}\ b'\end{array}}{\Omega \vDash a\ b^\nu \Rightarrow_R a'\{b'/x\}} \quad \textsc{Par\_Beta}$$

$$\frac{\begin{array}{c}\Omega \vDash a \Rightarrow_R a' \\ \Omega \vDash b \Rightarrow_{\mathit{app\_role}\nu}\ b'\end{array}}{\Omega \vDash a\ b^\nu \Rightarrow_R a'\ b'^\nu} \quad \textsc{Par\_App}$$

$$\frac{\Omega \vDash a \Rightarrow_R (\Lambda c.a')}{\Omega \vDash a[\bullet] \Rightarrow_R a'\{\bullet/c\}} \quad \textsc{Par\_CBeta}$$

$$\frac{\Omega \vDash a \Rightarrow_R a'}{\Omega \vDash a[\bullet] \Rightarrow_R a'[\bullet]} \quad \textsc{Par\_CApp}$$

$$\frac{\Omega, x : \mathbf{Nom} \vDash a \Rightarrow_R a'}{\Omega \vDash \lambda^\rho x.a \Rightarrow_R \lambda^\rho x.a'} \quad \textsc{Par\_Abs}$$

$$\frac{\begin{array}{l}\Omega \vDash A \Rightarrow_R A' \\ \Omega, x : \mathbf{Nom} \vDash B \Rightarrow_R B'\end{array}}{\Omega \vDash \Pi^\rho x\!:\!A \to B \Rightarrow_R \Pi^\rho x\!:\!A' \to B'} \quad \textsc{Par\_Pi}$$

$$\frac{\Omega \vDash a \Rightarrow_R a'}{\Omega \vDash \Lambda c.a \Rightarrow_R \Lambda c.a'} \quad \textsc{Par\_CAbs}$$

$$\frac{\begin{array}{l}\Omega \vDash A \Rightarrow_{R_0} A' \\ \Omega \vDash a \Rightarrow_{R_1} a' \\ \Omega \vDash b \Rightarrow_{R_1} b' \\ \Omega \vDash B \Rightarrow_R B'\end{array}}{\Omega \vDash \forall c\!:\!a \sim_{A/R_1} b.B \Rightarrow_R \forall c\!:\!a' \sim_{A'/R_1} b'.B'} \quad \textsc{Par\_CPi}$$

$$\frac{\begin{array}{l}F\ \sim a : A/R_1 @ Rs\ \in\ \Sigma_0 \\ R_1 \leq R \\ \mathit{uniq}(\Omega)\end{array}}{\Omega \vDash F \Rightarrow_R a} \quad \textsc{Par\_Axiom}$$

$$\frac{\begin{array}{l}F\ \mathit{sig\_sort}\ \in\ \Sigma_0 \\ \Omega \vDash a \Rightarrow_R a' \\ \Omega \vDash b_1 \Rightarrow_{R_0} b_1' \\ \Omega \vDash b_2 \Rightarrow_{R_0} b_2'\end{array}}{\Omega \vDash \mathit{case}_R\ a\ \mathit{of}\ F \to b_1 \|_{\_} \to b_2 \Rightarrow_{R_0} \mathit{case}_R\ a'\ \mathit{of}\ F \to b_1' \|_{\_} \to b_2'} \quad \textsc{Par\_Pattern}$$

$$\frac{\begin{array}{l}\Omega \vDash a \Rightarrow_R a' \\ \Omega \vDash b_1 \Rightarrow_{R_0} b_1' \\ \Omega \vDash b_2 \Rightarrow_{R_0} b_2' \\ \mathsf{Path}_R\ a' = F @ Rs\end{array}}{\Omega \vDash \mathit{case}_R\ a\ \mathit{of}\ F \to b_1 \|_{\_} \to b_2 \Rightarrow_{R_0} b_1'} \quad \textsc{Par\_PatternTrue}$$

$$\frac{\begin{array}{l} F\ sig\_sort\ \in\ \Sigma_0 \\ \Omega \vDash a \Rightarrow_R a' \\ \Omega \vDash b_1 \Rightarrow_{R_0} b_1' \\ \Omega \vDash b_2 \Rightarrow_{R_0} b_2' \\ \mathsf{Value}_R\ a' \\ \neg(\mathsf{Path}_R\ a' = F@Rs) \end{array}}{\Omega \vDash case_R\ a\ of\ F \to b_1 \|\_ \to b_2 \Rightarrow_{R_0} b_2'} \quad \textsc{Par\_PatternFalse}$$

$\boxed{\Omega \vdash a \Rightarrow^*_R b}$ multistep parallel reduction

$$\frac{}{\Omega \vdash a \Rightarrow^*_R a} \quad \textsc{MP\_Refl}$$

$$\frac{\begin{array}{l} \Omega \vDash a \Rightarrow_R b \\ \Omega \vdash b \Rightarrow^*_R a' \end{array}}{\Omega \vdash a \Rightarrow^*_R a'} \quad \textsc{MP\_Step}$$

$\boxed{\Omega \vdash a \Leftrightarrow_R b}$ parallel reduction to a common term

$$\frac{\begin{array}{l} \Omega \vdash a_1 \Rightarrow^*_R b \\ \Omega \vdash a_2 \Rightarrow^*_R b \end{array}}{\Omega \vdash a_1 \Leftrightarrow_R a_2} \quad \textsc{join}$$

$\boxed{\vDash a > b/R}$ primitive reductions on erased terms

$$\frac{\mathsf{Value}_{R_1}\ (\lambda^\rho x.v)}{\vDash (\lambda^\rho x.v)\ b^\nu > v\{b/x\}/R_1} \quad \textsc{Beta\_AppAbs}$$

$$\frac{}{\vDash (\Lambda c.a')[\bullet] > a'\{\bullet/c\}/R} \quad \textsc{Beta\_CAppCAbs}$$

$$\frac{\begin{array}{l} F\ \sim a : A/R@Rs\ \in\ \Sigma_0 \\ R \leq R_1 \end{array}}{\vDash F > a/R_1} \quad \textsc{Beta\_Axiom}$$

$$\frac{\mathsf{Path}_R\ a = F@Rs}{\vDash case_R\ a\ of\ F \to b_1 \|\_ \to b_2 > b_1/R_0} \quad \textsc{Beta\_PatternTrue}$$

$$\frac{\begin{array}{l} F\ sig\_sort\ \in\ \Sigma_0 \\ \mathsf{Value}_R\ a \\ \neg(\mathsf{Path}_R\ a = F@Rs) \end{array}}{\vDash case_R\ a\ of\ F \to b_1 \|\_ \to b_2 > b_2/R_0} \quad \textsc{Beta\_PatternFalse}$$

$\boxed{\vDash a \leadsto b/R}$ single-step head reduction for implicit language

$$\frac{\vDash a \leadsto a'/R_1}{\vDash \lambda^- x.a \leadsto \lambda^- x.a'/R_1} \quad \textsc{E\_AbsTerm}$$

$$\frac{\vDash a \leadsto a'/R_1}{\vDash a\ b^\nu \leadsto a'\ b^\nu/R_1} \quad \textsc{E\_AppLeft}$$

$$\frac{\vDash a \leadsto a'/R}{\vDash a[\bullet] \leadsto a'[\bullet]/R} \quad \textsc{E\_CAppLeft}$$

$$\frac{\vDash a \leadsto a'/R}{\vDash case_R\ a\ of\ F \to b_1 \|\_ \to b_2 \leadsto case_R\ a'\ of\ F \to b_1 \|\_ \to b_2/R_0} \quad \textsc{E\_Pattern}$$

$$\frac{\vDash a > b/R}{\vDash a \leadsto b/R} \quad \textsc{E\_Prim}$$

$\boxed{\vDash a \rightsquigarrow^* b/R}$ multistep reduction

$$\frac{}{\vDash a \rightsquigarrow^* a/R} \quad \textsc{Equal}$$

$$\frac{\begin{array}{c}\vDash a \rightsquigarrow b/R \\ \vDash b \rightsquigarrow^* a'/R\end{array}}{\vDash a \rightsquigarrow^* a'/R} \quad \textsc{Step}$$

$\boxed{\Gamma \vDash \phi \ \mathsf{ok}}$ Prop wellformedness

$$\frac{\begin{array}{c}\Gamma \vDash a : A \\ \Gamma \vDash b : A \\ \Gamma \vDash A : \star\end{array}}{\Gamma \vDash a \sim_{A/R} b \ \ \mathsf{ok}} \quad \text{E\_}\textsc{Wff}$$

$\boxed{\Gamma \vDash a : A}$ typing

$$\frac{\vDash \Gamma}{\Gamma \vDash \star : \star} \quad \text{E\_}\textsc{Star}$$

$$\frac{\begin{array}{c}\vDash \Gamma \\ x : A \in \Gamma\end{array}}{\Gamma \vDash x : A} \quad \text{E\_}\textsc{Var}$$

$$\frac{\begin{array}{c}\Gamma, x : A \vDash B : \star \\ \Gamma \vDash A : \star\end{array}}{\Gamma \vDash \Pi^\rho x \!:\! A \to B : \star} \quad \text{E\_}\textsc{Pi}$$

$$\frac{\begin{array}{l}\Gamma, x : A \vDash a : B \\ \Gamma \vDash A : \star \\ (\rho = +) \lor (x \notin \mathsf{fv}\ a)\end{array}}{\Gamma \vDash \lambda^\rho x.a : (\Pi^\rho x \!:\! A \to B)} \quad \text{E\_}\textsc{Abs}$$

$$\frac{\begin{array}{l}\Gamma \vDash b : \Pi^+ x \!:\! A \to B \\ \Gamma \vDash a : A\end{array}}{\Gamma \vDash b\ \ a^+ : B\{a/x\}} \quad \text{E\_}\textsc{App}$$

$$\frac{\begin{array}{l}\Gamma \vDash b : \Pi^+ x \!:\! A \to B \\ \Gamma \vDash a : A \\ \mathsf{Path}_R\ a = F @ Rs\end{array}}{\Gamma \vDash b\ \ a^R : B\{a/x\}} \quad \text{E\_T}\textsc{App}$$

$$\frac{\begin{array}{l}\Gamma \vDash b : \Pi^- x \!:\! A \to B \\ \Gamma \vDash a : A\end{array}}{\Gamma \vDash b\ \square^- : B\{a/x\}} \quad \text{E\_I}\textsc{App}$$

$$\frac{\begin{array}{l}\Gamma \vDash a : A \\ \Gamma; \widetilde{\Gamma} \vDash A \equiv B : \star/\mathbf{Rep} \\ \Gamma \vDash B : \star\end{array}}{\Gamma \vDash a : B} \quad \text{E\_}\textsc{Conv}$$

$$\frac{\begin{array}{l}\Gamma, c : \phi \vDash B : \star \\ \Gamma \vDash \phi \ \ \mathsf{ok}\end{array}}{\Gamma \vDash \forall c \!:\! \phi.B : \star} \quad \text{E\_CP}\textsc{i}$$

$$\frac{\begin{array}{l}\Gamma, c : \phi \vDash a : B \\ \Gamma \vDash \phi \ \ \mathsf{ok}\end{array}}{\Gamma \vDash \Lambda c.a : \forall c \!:\! \phi.B} \quad \text{E\_CA}\textsc{bs}$$

$$\frac{\Gamma \vDash a_1 : \forall c{:}(a \sim_{A/R} b).B_1 \qquad \Gamma; \widetilde{\Gamma} \vDash a \equiv b : A/R}{\Gamma \vDash a_1[\bullet] : B_1\{\bullet/c\}} \quad \text{E\_CApp}$$

$$\frac{\vDash \Gamma \qquad F : A@Rs \in \Sigma_0 \qquad \varnothing \vDash A : \star}{\Gamma \vDash F : A} \quad \text{E\_Const}$$

$$\frac{\vDash \Gamma \qquad F \sim a : A/R_1@Rs \in \Sigma_0 \qquad \varnothing \vDash A : \star}{\Gamma \vDash F : A} \quad \text{E\_Fam}$$

$$\frac{F\ sig\_sort \in \Sigma_0 \qquad \Gamma \vDash a : A \qquad \Gamma \vDash b_1 : B \qquad \Gamma \vDash b_2 : B}{\Gamma \vDash case_R\ a\ of\ F \to b_1 \|_{\_} \to b_2 : B} \quad \text{E\_Pat}$$

$$\frac{\Gamma \vDash a_1 : A \qquad \Gamma' \vDash a_2 : A\ \mathbf{pat}\ R@Rs \qquad \Gamma, (\Gamma', c : \phi_1) \vDash b_1 : B \qquad \Gamma \vDash b_2 : B \qquad \phi_1 = (a_1 \sim_{A/R} a_2)}{\Gamma \vDash caseRa_1of\,a_2 \to b_1 \|_{\_} \to b_2 : B} \quad \text{E\_case}$$

$\boxed{\Gamma; \Delta \vDash \phi_1 \equiv \phi_2}$ prop equality

$$\frac{\Gamma; \Delta \vDash A_1 \equiv A_2 : A/R \qquad \Gamma; \Delta \vDash B_1 \equiv B_2 : A/R}{\Gamma; \Delta \vDash A_1 \sim_{A/R} B_1 \equiv A_2 \sim_{A/R} B_2} \quad \text{E\_PropCong}$$

$$\frac{\Gamma; \Delta \vDash A \equiv B : \star/R_0 \qquad \Gamma \vDash A_1 \sim_{A/R} A_2\ \mathsf{ok} \qquad \Gamma \vDash A_1 \sim_{B/R} A_2\ \mathsf{ok}}{\Gamma; \Delta \vDash A_1 \sim_{A/R} A_2 \equiv A_1 \sim_{B/R} A_2} \quad \text{E\_IsoConv}$$

$$\frac{\Gamma; \Delta \vDash \forall c{:}(a_1 \sim_{A/R_1} a_2).B_1 \equiv \forall c{:}(b_1 \sim_{B/R_2} b_2).B_2 : \star/R'}{\Gamma; \Delta \vDash a_1 \sim_{A/R_1} a_2 \equiv b_1 \sim_{B/R_2} b_2} \quad \text{E\_CPiFst}$$

$\boxed{\Gamma; \Delta \vDash a \equiv b : A/R}$ definitional equality

$$\frac{\vDash \Gamma \qquad c : (a \sim_{A/R} b) \in \Gamma \qquad c \in \Delta}{\Gamma; \Delta \vDash a \equiv b : A/R} \quad \text{E\_Assn}$$

$$\frac{\Gamma \vDash a : A}{\Gamma; \Delta \vDash a \equiv a : A/\mathbf{Nom}} \quad \text{E\_Refl}$$

$$\frac{\Gamma; \Delta \vDash b \equiv a : A/R}{\Gamma; \Delta \vDash a \equiv b : A/R} \quad \text{E\_Sym}$$

$$\frac{\Gamma; \Delta \vDash a \equiv a_1 : A/R \qquad \Gamma; \Delta \vDash a_1 \equiv b : A/R}{\Gamma; \Delta \vDash a \equiv b : A/R} \quad \text{E\_Trans}$$

$$\frac{\begin{array}{l}\Gamma;\Delta \vDash a \equiv b : A/R_1\\ R_1 \leq R_2\end{array}}{\Gamma;\Delta \vDash a \equiv b : A/R_2}\ \textsc{E\_Sub}$$

$$\frac{\begin{array}{l}\Gamma \vDash a_1 : B\\ \Gamma \vDash a_2 : B\\ \vDash a_1 > a_2/R\end{array}}{\Gamma;\Delta \vDash a_1 \equiv a_2 : B/R}\ \textsc{E\_Beta}$$

$$\frac{\begin{array}{l}\Gamma;\Delta \vDash A_1 \equiv A_2 : \star/R'\\ \Gamma, x : A_1;\Delta \vDash B_1 \equiv B_2 : \star/R'\\ \Gamma \vDash A_1 : \star\\ \Gamma \vDash \Pi^\rho x\!:\!A_1 \to B_1 : \star\\ \Gamma \vDash \Pi^\rho x\!:\!A_2 \to B_2 : \star\end{array}}{\Gamma;\Delta \vDash (\Pi^\rho x\!:\!A_1 \to B_1) \equiv (\Pi^\rho x\!:\!A_2 \to B_2) : \star/R'}\ \textsc{E\_PiCong}$$

$$\frac{\begin{array}{l}\Gamma, x : A_1;\Delta \vDash b_1 \equiv b_2 : B/R'\\ \Gamma \vDash A_1 : \star\\ (\rho = +) \vee (x \notin \mathsf{fv}\ b_1)\\ (\rho = +) \vee (x \notin \mathsf{fv}\ b_2)\end{array}}{\Gamma;\Delta \vDash (\lambda^\rho x.b_1) \equiv (\lambda^\rho x.b_2) : (\Pi^\rho x\!:\!A_1 \to B)/R'}\ \textsc{E\_AbsCong}$$

$$\frac{\begin{array}{l}\Gamma;\Delta \vDash a_1 \equiv b_1 : (\Pi^+ x\!:\!A \to B)/R'\\ \Gamma;\Delta \vDash a_2 \equiv b_2 : A/R'\end{array}}{\Gamma;\Delta \vDash a_1\ a_2{}^+ \equiv b_1\ b_2{}^+ : (B\{a_2/x\})/R'}\ \textsc{E\_AppCong}$$

$$\frac{\begin{array}{l}\Gamma;\Delta \vDash a_1 \equiv b_1 : (\Pi^- x\!:\!A \to B)/R'\\ \Gamma \vDash a : A\end{array}}{\Gamma;\Delta \vDash a_1\ \Box^- \equiv b_1\ \Box^- : (B\{a/x\})/R'}\ \textsc{E\_IAppCong}$$

$$\frac{\Gamma;\Delta \vDash \Pi^\rho x\!:\!A_1 \to B_1 \equiv \Pi^\rho x\!:\!A_2 \to B_2 : \star/R'}{\Gamma;\Delta \vDash A_1 \equiv A_2 : \star/R'}\ \textsc{E\_PiFst}$$

$$\frac{\begin{array}{l}\Gamma;\Delta \vDash \Pi^\rho x\!:\!A_1 \to B_1 \equiv \Pi^\rho x\!:\!A_2 \to B_2 : \star/R'\\ \Gamma;\Delta \vDash a_1 \equiv a_2 : A_1/R'\end{array}}{\Gamma;\Delta \vDash B_1\{a_1/x\} \equiv B_2\{a_2/x\} : \star/R'}\ \textsc{E\_PiSnd}$$

$$\frac{\begin{array}{l}\Gamma;\Delta \vDash a_1 \sim_{A_1/R} b_1 \equiv a_2 \sim_{A_2/R} b_2\\ \Gamma, c : a_1 \sim_{A_1/R} b_1;\Delta \vDash A \equiv B : \star/R'\\ \Gamma \vDash a_1 \sim_{A_1/R} b_1\ \ \mathsf{ok}\\ \Gamma \vDash \forall c\!:\!a_1 \sim_{A_1/R} b_1.A : \star\\ \Gamma \vDash \forall c\!:\!a_2 \sim_{A_2/R} b_2.B : \star\end{array}}{\Gamma;\Delta \vDash \forall c\!:\!a_1 \sim_{A_1/R} b_1.A \equiv \forall c\!:\!a_2 \sim_{A_2/R} b_2.B : \star/R'}\ \textsc{E\_CPiCong}$$

$$\frac{\begin{array}{l}\Gamma, c : \phi_1;\Delta \vDash a \equiv b : B/R\\ \Gamma \vDash \phi_1\ \ \mathsf{ok}\end{array}}{\Gamma;\Delta \vDash (\Lambda c.a) \equiv (\Lambda c.b) : \forall c\!:\!\phi_1.B/R}\ \textsc{E\_CAbsCong}$$

$$\frac{\begin{array}{l}\Gamma;\Delta \vDash a_1 \equiv b_1 : (\forall c\!:\!(a \sim_{A/R} b).B)/R'\\ \Gamma;\widetilde{\Gamma} \vDash a \equiv b : A/\mathbf{param}\,R\,R'\end{array}}{\Gamma;\Delta \vDash a_1[\bullet] \equiv b_1[\bullet] : (B\{\bullet/c\})/R'}\ \textsc{E\_CAppCong}$$

$$\frac{\begin{array}{l}\Gamma;\Delta \vDash \forall c\!:\!(a_1 \sim_{A/R} a_2).B_1 \equiv \forall c\!:\!(a_1' \sim_{A'/R'} a_2').B_2 : \star/R_0\\ \Gamma;\widetilde{\Gamma} \vDash a_1 \equiv a_2 : A/\mathbf{param}\,R\,R_0\\ \Gamma;\widetilde{\Gamma} \vDash a_1' \equiv a_2' : A'/\mathbf{param}\,R'\,R_0\end{array}}{\Gamma;\Delta \vDash B_1\{\bullet/c\} \equiv B_2\{\bullet/c\} : \star/R_0}\ \textsc{E\_CPiSnd}$$

$$\frac{\begin{array}{l}\Gamma;\Delta\vDash a\equiv b:A/R\\ \Gamma;\Delta\vDash a\sim_{A/R}\ b\equiv a'\sim_{A'/R'}\ b'\end{array}}{\Gamma;\Delta\vDash a'\equiv b':A'/R'}\quad\textsc{E\_Cast}$$

$$\frac{\begin{array}{l}\Gamma;\Delta\vDash a\equiv b:A/R\\ \Gamma;\widetilde{\Gamma}\vDash A\equiv B:\star/\mathbf{Rep}\\ \Gamma\vDash B:\star\end{array}}{\Gamma;\Delta\vDash a\equiv b:B/R}\quad\textsc{E\_EqConv}$$

$$\frac{\Gamma;\Delta\vDash a\sim_{A/R_1}\ b\equiv a'\sim_{A'/R_1}\ b'}{\Gamma;\Delta\vDash A\equiv A':\star/\mathbf{Rep}}\quad\textsc{E\_IsoSnd}$$

$$\frac{\begin{array}{l}F\ sig\_sort\ \in\ \Sigma_0\\ \Gamma;\Delta\vDash a\equiv a':A/R\\ \Gamma;\Delta\vDash b_1\equiv b_1':B/R_0\\ \Gamma;\Delta\vDash b_2\equiv b_2':B/R_0\end{array}}{\Gamma;\Delta\vDash case_R\ a\ of\ F\to b_1\|\_\to b_2\equiv case_R\ a'\ of\ F\to b_1'\|\_\to b_2':B/R_0}\quad\textsc{E\_PatCong}$$

$$\frac{\begin{array}{l}\mathsf{Path}_{R'}\ a=F@R,Rs\\ \mathsf{Path}_{R'}\ a'=F@R,Rs\\ \Gamma\vDash a:\Pi^+x\!:\!A\to B\\ \Gamma\vDash b:A\\ \Gamma\vDash a':\Pi^+x\!:\!A\to B\\ \Gamma\vDash b':A\\ \Gamma;\Delta\vDash a\ b^{R_1}\equiv a'\ b'^{R_1}:B\{b/x\}/R'\\ \Gamma;\widetilde{\Gamma}\vDash B\{b/x\}\equiv B\{b'/x\}:\star/R'\end{array}}{\Gamma;\Delta\vDash a\equiv a':\Pi^+x\!:\!A\to B/R'}\quad\textsc{E\_LeftRel}$$

$$\frac{\begin{array}{l}\mathsf{Path}_{R'}\ a=F@R,Rs\\ \mathsf{Path}_{R'}\ a'=F@R,Rs\\ \Gamma\vDash a:\Pi^-x\!:\!A\to B\\ \Gamma\vDash b:A\\ \Gamma\vDash a':\Pi^-x\!:\!A\to B\\ \Gamma\vDash b':A\\ \Gamma;\Delta\vDash a\ \Box^-\equiv a'\ \Box^-:B\{b/x\}/R'\\ \Gamma;\widetilde{\Gamma}\vDash B\{b/x\}\equiv B\{b'/x\}:\star/R_0\end{array}}{\Gamma;\Delta\vDash a\equiv a':\Pi^-x\!:\!A\to B/R'}\quad\textsc{E\_LeftIrrel}$$

$$\frac{\begin{array}{l}\mathsf{Path}_{R'}\ a=F@R,Rs\\ \mathsf{Path}_{R'}\ a'=F@R,Rs\\ \Gamma\vDash a:\Pi^+x\!:\!A\to B\\ \Gamma\vDash b:A\\ \Gamma\vDash a':\Pi^+x\!:\!A\to B\\ \Gamma\vDash b':A\\ \Gamma;\Delta\vDash a\ b^+\equiv a'\ b'^+:B\{b/x\}/R'\\ \Gamma;\widetilde{\Gamma}\vDash B\{b/x\}\equiv B\{b'/x\}:\star/R_0\end{array}}{\Gamma;\Delta\vDash b\equiv b':A/\mathbf{param}\,R_1\ R'}\quad\textsc{E\_Right}$$

$$\frac{\begin{array}{l}\mathsf{Path}_{R'}\ a=F@R,Rs\\ \mathsf{Path}_{R'}\ a'=F@R,Rs\\ \Gamma\vDash a:\forall c\!:\!(a_1\sim_{A/R_1}\ a_2).B\\ \Gamma\vDash a':\forall c\!:\!(a_1\sim_{A/R_1}\ a_2).B\\ \Gamma;\widetilde{\Gamma}\vDash a_1\equiv a_2:A/R'\\ \Gamma;\Delta\vDash a[\bullet]\equiv a'[\bullet]:B\{\bullet/c\}/R'\end{array}}{\Gamma;\Delta\vDash a\equiv a':\forall c\!:\!(a_1\sim_{A/R_1}\ a_2).B/R'}\quad\textsc{E\_CLeft}$$

$\boxed{\vDash \Gamma}$ context wellformedness

$$\frac{}{\vDash \varnothing} \quad \textsc{E\_Empty}$$

$$\frac{\begin{array}{l} \vDash \Gamma \\ \Gamma \vDash A : \star \\ x \notin \mathsf{dom}\,\Gamma \end{array}}{\vDash \Gamma, x : A} \quad \textsc{E\_ConsTm}$$

$$\frac{\begin{array}{l} \vDash \Gamma \\ \Gamma \vDash \phi \;\; \mathsf{ok} \\ c \notin \mathsf{dom}\,\Gamma \end{array}}{\vDash \Gamma, c : \phi} \quad \textsc{E\_ConsCo}$$

$\boxed{\vDash \Sigma}$ signature wellformedness

$$\frac{}{\vDash \varnothing} \quad \textsc{Sig\_Empty}$$

$$\frac{\begin{array}{l} \vDash \Sigma \\ \varnothing \vDash A : \star \\ F \notin \mathsf{dom}\,\Sigma \end{array}}{\vDash \Sigma \cup \{F : A @ Rs\}} \quad \textsc{Sig\_ConsConst}$$

$$\frac{\begin{array}{l} \vDash \Sigma \\ \varnothing \vDash a : A \\ F \notin \mathsf{dom}\,\Sigma \end{array}}{\vDash \Sigma \cup \{F \sim a : A / R @ Rs\}} \quad \textsc{Sig\_ConsAx}$$

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Definition rules:        122 good    0 bad
Definition rule clauses: 361 good    0 bad
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