

$tmvar, x, y, f, m, n$	variables
$covar, c$	coercion variables
$datacon, K$	
$const, T$	
$tyfam, F$	
$index, i$	indices

		Fix	S	
		$a \rightarrow b$	S	
		$\phi \Rightarrow A$	S	
		$ab^{R,+}$	S	
		$\lambda^R x. a$	S	
		$\lambda x : A. a$	S	
		$\forall x : A/R \rightarrow B$	S	
brs	::=			case branches
		none		
		$K \Rightarrow a; brs$		
		$brs\{a/x\}$	S	
		$brs\{\gamma/c\}$	S	
		(brs)	S	
co, γ	::=			explicit coercions
		•		
		c		
		red $a\ b$		
		refl a		
		$(a \models_\gamma b)$		
		sym γ		
		$\gamma_1; \gamma_2$		
		sub γ		
		$\Pi^{R,\rho} x : \gamma_1. \gamma_2$	bind x in γ_2	
		$\lambda^{R,\rho} x : \gamma_1. \gamma_2$	bind x in γ_2	
		$\gamma_1 \gamma_2^{R,\rho}$		
		piFst γ		
		cpiFst γ		
		isoSnd γ		
		$\gamma_1 @ \gamma_2$		
		$\forall c : \gamma_1. \gamma_3$	bind c in γ_3	
		$\lambda c : \gamma_1. \gamma_3 @ \gamma_4$	bind c in γ_3	
		$\gamma(\gamma_1, \gamma_2)$		
		$\gamma @ (\gamma_1 \sim \gamma_2)$		
		$\gamma_1 \triangleright_R \gamma_2$		
		$\gamma_1 \sim_A \gamma_2$		
		conv $\phi_1 \sim_\gamma \phi_2$		
		eta a		
		left $\gamma \gamma'$		
		right $\gamma \gamma'$		
		(γ)	S	
		γ	S	
		$\gamma\{a/x\}$	S	
$sort$::=			binding classifier
		Tm $A\ R$		

		Co ϕ	
<i>sig_sort</i>	::=		signature classifier
		Cs A	
		Ax $a A R$	
<i>context</i> , Γ	::=		contexts
		\emptyset	
		$\Gamma, x : A/R$	
		$\Gamma, c : \phi$	
		$\Gamma\{b/x\}$	M
		$\Gamma\{\gamma/c\}$	M
		Γ, Γ'	M
		$ \Gamma $	M
		(Γ)	M
		Γ	M
<i>sig</i> , Σ	::=		signatures
		\emptyset	
		$\Sigma \cup \{T : A/R\}$	
		$\Sigma \cup \{F \sim a : A/R\}$	
		Σ_0	M
		Σ_1	M
		$ \Sigma $	M
<i>available_props</i> , Δ	::=		
		\emptyset	
		Δ, c	
		$\tilde{\Gamma}$	M
		(Δ)	M
<i>role_context</i> , Ω	::=		<i>role_contexts</i>
		\emptyset	
		$\Omega, x : R$	
		(Ω)	M
		Ω	M
<i>terminals</i>	::=		
		\leftrightarrow	
		\Leftrightarrow	
		\longrightarrow	
		min	
		\equiv	
		\forall	
		\in	
		\notin	
		\Leftarrow	

	\Rightarrow
	\Rightarrow^*
	\rightarrow
	Λ
	\square
	\vdash
	\vdash
	\models
	\models
	\neq
	\triangleright
	ok
	-
	\rightsquigarrow
	\rightsquigarrow^*
	\rightsquigarrow
	\emptyset
	\circ
	fv
	dom
	\sim
	\succ
	•
	fst
	snd
	$ \Rightarrow $
	$\vdash_{=}$
	refl₂
	++
<i>formula, ψ</i>	$::=$
	<i>judgement</i>
	$x : A/R \in \Gamma$
	$x : R \in \Omega$
	$c : \phi \in \Gamma$
	$T : A/R \in \Sigma$
	$F \sim a : A/R \in \Sigma$
	$K : T\Gamma \in \Sigma$
	$x \in \Delta$
	$c \in \Delta$
	$c \text{ not relevant} \in \gamma$
	$x \notin \text{fva}$
	$x \notin \text{dom } \Gamma$
	$\text{rctx_uniq}\Omega$

	$ \begin{array}{ l} c \notin \text{dom } \Gamma \\ T \notin \text{dom } \Sigma \\ F \notin \text{dom } \Sigma \\ a = b \\ \phi_1 = \phi_2 \\ \Gamma_1 = \Gamma_2 \\ \gamma_1 = \gamma_2 \\ \neg \psi \\ \psi_1 \wedge \psi_2 \\ \psi_1 \vee \psi_2 \\ \psi_1 \Rightarrow \psi_2 \\ (\psi) \\ \psi \\ c : (a : A \sim b : B) \in \Gamma \end{array} $	suppress lc hypothesis generated by Ott
<i>JSubRole</i>	$ \begin{array}{ l} R_1 \leq R_2 \end{array} $	Subroling judgement
<i>JValue</i>	$ \begin{array}{ l} \mathbf{CoercedValue } R A \\ \mathbf{Value}_R A \\ \mathbf{ValueType } R A \end{array} $	Values with at most one coercion at the top values Types with head forms (erased language)
<i>Jconsistent</i>	$ \begin{array}{ l} \mathbf{consistent } a b \end{array} $	(erased) types do not differ in their heads
<i>Jerased</i>	$ \begin{array}{ l} \Omega \models \text{erased_tm } a R \end{array} $	
<i>JChk</i>	$ \begin{array}{ l} (\rho = +) \vee (x \notin \text{fv } A) \end{array} $	irrelevant argument check
<i>Jpar</i>	$ \begin{array}{ l} \Omega \models a \Rightarrow_R b \\ \Omega \vdash a \Rightarrow_R^* b \\ \Omega \vdash a \Leftrightarrow_R b \end{array} $	parallel reduction (implicit language) multistep parallel reduction parallel reduction to a common term
<i>Jbeta</i>	$ \begin{array}{ l} \models a > b/R \\ \models a \rightsquigarrow b/R \\ \models a \rightsquigarrow^* b/R \end{array} $	primitive reductions on erased terms single-step head reduction for implicit language multistep reduction
<i>Jett</i>	$ \begin{array}{ l} \Gamma \models \phi \text{ ok} \\ \Gamma \models a : A/R \\ \Gamma; \Delta \models \phi_1 \equiv \phi_2 \end{array} $	Prop wellformedness typing prop equality

		$\Gamma; \Delta \models a \equiv b : A/R$	definitional equality
		$\models \Gamma$	context wellformedness
$Jsig$	$::=$		
		$\models \Sigma$	signature wellformedness
$Jann$	$::=$		
		$\Gamma \vdash \phi \text{ ok}$	prop wellformedness
		$\Gamma \vdash a : A/R$	typing
		$\Gamma; \Delta \vdash \gamma : \phi_1 \sim \phi_2$	coercion between props
		$\Gamma; \Delta \vdash \gamma : A \sim_R B$	coercion between types
		$\vdash \Gamma$	context wellformedness
		$\vdash \Sigma$	signature wellformedness
$Jred$	$::=$		
		$\Gamma \vdash a \rightsquigarrow b/R$	single-step, weak head reduction to values for annotated lang
$judgement$	$::=$		
		$JSubRole$	
		$JValue$	
		$Jconsistent$	
		$Jerased$	
		$JChk$	
		$Jpar$	
		$Jbeta$	
		$Jett$	
		$Jsig$	
		$Jann$	
		$Jred$	
$user_syntax$	$::=$		
		$tmvar$	
		$covar$	
		$datacon$	
		$const$	
		$tyfam$	
		$index$	
		$role$	
		$relflag$	
		$constraint$	
		tm	
		brs	
		co	
		$sort$	
		sig_sort	
		$context$	
		sig	

| *available_props*
 | *role_context*
 | *terminals*
 | *formula*

$R_1 \leq R_2$ Subroling judgement

$$\frac{}{\mathbf{Nom} \leq \mathbf{Rep}} \text{NOMREP}$$

$$\frac{}{\overline{R \leq R}} \text{REFL}$$

$$\frac{R_1 \leq R_2 \quad R_2 \leq R_3}{R_1 \leq R_3} \text{TRANS}$$

$\mathbf{CoercedValue} \ R \ A$ Values with at most one coercion at the top

$$\frac{\mathbf{Value}_R \ a}{\mathbf{CoercedValue} \ R \ a} \text{CV}$$

$$\frac{\mathbf{Value}_R \ a \quad \neg(R_1 \leq R)}{\mathbf{CoercedValue} \ R \ (a \triangleright_{R_1} \gamma)} \text{CC}$$

$\mathbf{Value}_R \ A$ values

$$\frac{}{\mathbf{Value}_R \ \star} \text{VALUE_STAR}$$

$$\frac{}{\mathbf{Value}_R \ \Pi^\rho x : A / R_1 \rightarrow B} \text{VALUE_PI}$$

$$\frac{}{\mathbf{Value}_R \ \forall c : \phi. B} \text{VALUE_CPI}$$

$$\frac{}{\mathbf{Value}_R \ \lambda^+ x : A / R_1. a} \text{VALUE_ABSR}$$

$$\frac{}{\mathbf{Value}_R \ \lambda^{R_1, +} x. a} \text{VALUE_UABSR}$$

$$\frac{\mathbf{CoercedValue} \ R \ a}{\mathbf{Value}_R \ \lambda^{R_1, -} x. a} \text{VALUE_UABSI}$$

$$\frac{\mathbf{CoercedValue} \ R \ a}{\mathbf{Value}_R \ \lambda^- x : A / R_1. a} \text{VALUE_ABSI}$$

$$\frac{}{\mathbf{Value}_R \ \Lambda c : \phi. a} \text{VALUE_CABS}$$

$$\frac{}{\mathbf{Value}_R \ \Lambda c. a} \text{VALUE_UCABS}$$

$$\frac{F \sim a : A / R_1 \in \Sigma_0 \quad \neg(R_1 \leq R)}{\mathbf{Value}_R \ F} \text{VALUE_AX}$$

$\mathbf{ValueType} \ R \ A$ Types with head forms (erased language)

$$\frac{}{\mathbf{ValueType} \ R \ \star} \text{VALUE_TYPE_STAR}$$

$$\frac{}{\mathbf{ValueType} \ R \ \Pi^\rho x : A/R_1 \rightarrow B} \text{VALUE_TYPE_PI}$$

$$\frac{}{\mathbf{ValueType} \ R \ \forall c : \phi. B} \text{VALUE_TYPE_CPI}$$

$$\frac{\begin{array}{c} F \sim a : A/R_1 \in \Sigma_0 \\ \neg(R_1 \leq R) \end{array}}{\mathbf{ValueType} \ R \ F} \text{VALUE_TYPE_AX}$$

consistent $a \ b$ (erased) types do not differ in their heads

$$\frac{}{\mathbf{consistent} \ \star \ \star} \text{CONSISTENT_A_STAR}$$

$$\frac{}{\mathbf{consistent} \ (\Pi^\rho x_1 : A_1/R \rightarrow B_1) \ (\Pi^\rho x_2 : A_2/R \rightarrow B_2)} \text{CONSISTENT_A_PI}$$

$$\frac{}{\mathbf{consistent} \ (\forall c_1 : \phi_1. A_1) \ (\forall c_2 : \phi_2. A_2)} \text{CONSISTENT_A_CPI}$$

$$\frac{\neg \mathbf{ValueType} \ R \ b}{\mathbf{consistent} \ a \ b} \text{CONSISTENT_A_STEP_R}$$

$$\frac{\neg \mathbf{ValueType} \ R \ a}{\mathbf{consistent} \ a \ b} \text{CONSISTENT_A_STEP_L}$$

$\Omega \models \text{erased_tm} \ a \ R$

$$\frac{rctx_uniq \ \Omega}{\Omega \models \text{erased_tm} \ \Box \ R} \text{ERASED_A_BULLET}$$

$$\frac{rctx_uniq \ \Omega}{\Omega \models \text{erased_tm} \ \star \ R} \text{ERASED_A_STAR}$$

$$\frac{\begin{array}{c} rctx_uniq \ \Omega \\ x : R \in \Omega \\ R \leq R_1 \end{array}}{\Omega \models \text{erased_tm} \ x \ R_1} \text{ERASED_A_VAR}$$

$$\frac{\Omega, x : R_1 \models \text{erased_tm} \ a \ R}{\Omega \models \text{erased_tm} \ (\lambda^{R_1, \rho} x. a) \ R} \text{ERASED_A_ABS}$$

$$\frac{\begin{array}{c} \Omega \models \text{erased_tm} \ a \ R \\ \Omega \models \text{erased_tm} \ b \ R_1 \end{array}}{\Omega \models \text{erased_tm} \ (a \ b^{R_1, \rho}) \ R} \text{ERASED_A_APP}$$

$$\frac{\begin{array}{c} \Omega \models \text{erased_tm} \ A \ R_1 \\ \Omega, x : R_1 \models \text{erased_tm} \ B \ R \end{array}}{\Omega \models \text{erased_tm} \ (\Pi^\rho x : A/R_1 \rightarrow B) \ R} \text{ERASED_A_PI}$$

$$\frac{\begin{array}{c} \Omega \models \text{erased_tm} \ a \ R_1 \\ \Omega \models \text{erased_tm} \ b \ R_1 \\ \Omega \models \text{erased_tm} \ A \ R_1 \\ \Omega \models \text{erased_tm} \ B \ R \end{array}}{\Omega \models \text{erased_tm} \ (\forall c : a \sim_{A/R_1} b. B) \ R} \text{ERASED_A_CPI}$$

$$\frac{\Omega \models \text{erased_tm} \ b \ R}{\Omega \models \text{erased_tm} \ (\Lambda c. b) \ R} \text{ERASED_A_CABS}$$

$$\frac{\Omega \models \text{erased_tm} \ a \ R}{\Omega \models \text{erased_tm} \ (a[\bullet]) \ R} \text{ERASED_A_CAPP}$$

$$\frac{\begin{array}{c} rctx_uniq\Omega \\ F \sim a : A/R \in \Sigma_0 \\ R \leq R_1 \end{array}}{\Omega \models erased_tm F R_1} \quad \text{ERASED_A_FAM}$$

$$\frac{rctx_uniq\Omega}{\Omega \models erased_tm T R} \quad \text{ERASED_A_CONST}$$

$$\frac{\Omega \models erased_tm a R}{\Omega \models erased_tm (a \triangleright_{R_1} \bullet) R} \quad \text{ERASED_A_CONV}$$

$$\boxed{(\rho = +) \vee (x \notin \text{fv } A)} \quad \text{irrelevant argument check}$$

$$\overline{(+ = +) \vee (x \notin \text{fv } A)} \quad \text{RHO_REL}$$

$$\frac{x \notin \text{fv } A}{(- = +) \vee (x \notin \text{fv } A)} \quad \text{RHO_IRRREL}$$

$$\boxed{\Omega \models a \Rightarrow_R b} \quad \text{parallel reduction (implicit language)}$$

$$\frac{\Omega \models erased_tm a R}{\Omega \models a \Rightarrow_R a} \quad \text{PAR_REFL}$$

$$\frac{\begin{array}{c} \Omega \models a \Rightarrow_R (\lambda^{R_1, \rho} x. a') \\ \Omega \models b \Rightarrow_{R_1} b' \end{array}}{\Omega \models a \ b^{R_1, \rho} \Rightarrow_R a' \{b'/x\}} \quad \text{PAR_BETA}$$

$$\frac{\begin{array}{c} \Omega \models a \Rightarrow_R a' \\ \Omega \models b \Rightarrow_{R_1} b' \end{array}}{\Omega \models a \ b^{R_1, \rho} \Rightarrow_R a' \ b'^{R_1, \rho}} \quad \text{PAR_APP}$$

$$\frac{\Omega \models a \Rightarrow_R (\Lambda c. a')}{\Omega \models a[\bullet] \Rightarrow_R a' \{\bullet/c\}} \quad \text{PAR_CBETA}$$

$$\frac{\Omega \models a \Rightarrow_R a'}{\Omega \models a[\bullet] \Rightarrow_R a'[\bullet]} \quad \text{PAR_CAPP}$$

$$\frac{\Omega, x : R_1 \models a \Rightarrow_R a'}{\Omega \models \lambda^{R_1, \rho} x. a \Rightarrow_R \lambda^{R_1, \rho} x. a'} \quad \text{PAR_ABS}$$

$$\frac{\begin{array}{c} \Omega \models A \Rightarrow_{R_1} A' \\ \Omega, x : R_1 \models B \Rightarrow_R B' \end{array}}{\Omega \models \Pi^{\rho} x : A/R_1 \rightarrow B \Rightarrow_R \Pi^{\rho} x : A'/R_1 \rightarrow B'} \quad \text{PAR_PI}$$

$$\frac{\Omega \models a \Rightarrow_R a'}{\Omega \models \Lambda c. a \Rightarrow_R \Lambda c. a'} \quad \text{PAR_CABS}$$

$$\frac{\begin{array}{c} \Omega \models A \Rightarrow_{R_1} A' \\ \Omega \models a \Rightarrow_{R_1} a' \\ \Omega \models b \Rightarrow_{R_1} b' \\ \Omega \models B \Rightarrow_R B' \end{array}}{\Omega \models \forall c : a \sim_{A/R_1} b. B \Rightarrow_R \forall c : a' \sim_{A'/R_1} b'. B'} \quad \text{PAR_CPI}$$

$$\frac{\begin{array}{c} F \sim a : A/R_1 \in \Sigma_0 \\ R_1 \leq R \\ rctx_uniq\Omega \end{array}}{\Omega \models F \Rightarrow_R a} \quad \text{PAR_AXIOM}$$

$$\begin{array}{c}
\frac{\Omega \models a_1 \Rightarrow_{R_1} a_2}{\Omega \models a_1 \triangleright_R \bullet \Rightarrow_{R_1} a_2 \triangleright_R \bullet} \text{PAR_CONG} \\
\frac{\Omega \models a_1 \Rightarrow_{R_1} (a_2 \triangleright_R \bullet)}{\Omega \models (a_1 \triangleright_R \bullet) \Rightarrow_{R_1} (a_2 \triangleright_R \bullet)} \text{PAR_COMBINE} \\
\frac{\Omega \models a_1 \Rightarrow_{R_1} (a_2 \triangleright_R \bullet) \quad \Omega \models b_1 \Rightarrow_{R_2} b_2}{\Omega \models a_1 b_1^{R_2,+} \Rightarrow_{R_1} (a_2(b_2 \triangleright_R \bullet)^{R_2,+}) \triangleright_R \bullet} \text{PAR_PUSH} \\
\frac{\Omega \models a_1 \Rightarrow_{R_1} (a_2 \triangleright_R \bullet) \quad \Omega \models b_1 \Rightarrow_{R_2} (b_2 \triangleright_R \bullet)}{\Omega \models a_1 b_1^{R_2,+} \Rightarrow_{R_1} (a_2(b_2 \triangleright_R \bullet)^{R_2,+}) \triangleright_R \bullet} \text{PAR_PUSHCOMBINE} \\
\frac{\Omega \models a_1 \Rightarrow_{R_1} (a_2 \triangleright_R \bullet)}{\Omega \models a_1[\bullet] \Rightarrow_{R_1} (a_2[\bullet]) \triangleright_R \bullet} \text{PAR_CPUSH}
\end{array}$$

$\boxed{\Omega \vdash a \Rightarrow_R^* b}$ multistep parallel reduction

$$\begin{array}{c}
\overline{\Omega \vdash a \Rightarrow_R^* a} \text{MP_REFL} \\
\frac{\Omega \models a \Rightarrow_R b \quad \Omega \vdash b \Rightarrow_R^* a'}{\Omega \vdash a \Rightarrow_R^* a'} \text{MP_STEP}
\end{array}$$

$\boxed{\Omega \vdash a \Leftrightarrow_R b}$ parallel reduction to a common term

$$\frac{\Omega \vdash a_1 \Rightarrow_R^* b \quad \Omega \vdash a_2 \Rightarrow_R^* b}{\Omega \vdash a_1 \Leftrightarrow_R a_2} \text{JOIN}$$

$\boxed{\models a > b/R}$ primitive reductions on erased terms

$$\begin{array}{c}
\frac{\text{Value}_{R_1}(\lambda^{R,\rho} x.v)}{\models (\lambda^{R,\rho} x.v) \ b^{R,\rho} > v\{b/x\}/R_1} \text{BETA_APPABS} \\
\overline{\models (\Lambda c.a')[\bullet] > a'\{\bullet/c\}/R} \text{BETA_CAPPCABS} \\
\frac{F \sim a : A/R \in \Sigma_0}{\models F > a/R} \text{BETA_AXIOM}
\end{array}$$

$\boxed{\models a \rightsquigarrow b/R}$ single-step head reduction for implicit language

$$\begin{array}{c}
\frac{\models a \rightsquigarrow a'/R_1}{\models \lambda^{R,-} x.a \rightsquigarrow \lambda^{R,-} x.a'/R_1} \text{E_ABSTERM} \\
\frac{\models a \rightsquigarrow a'/R_1}{\models a \ b^{R,\rho} \rightsquigarrow a' \ b^{R,\rho}/R_1} \text{E_APPLEFT} \\
\frac{\models a \rightsquigarrow a'/R}{\models a[\bullet] \rightsquigarrow a'[\bullet]/R} \text{E_CAPPLEFT} \\
\frac{\text{Value}_{R_1}(\lambda^{R,\rho} x.v)}{\models (\lambda^{R,\rho} x.v) \ a^{R,\rho} \rightsquigarrow v\{a/x\}/R_1} \text{E_APPABS} \\
\overline{\models (\Lambda c.b)[\bullet] \rightsquigarrow b\{\bullet/c\}/R} \text{E_CAPPCABS}
\end{array}$$

$$\begin{array}{c}
\frac{F \sim a : A/R \in \Sigma_0}{R \leq R_1} \quad \text{E_AXIOM} \\
\frac{\vdash a \rightsquigarrow a'/R_1 \quad \neg(R \leq R_1)}{\vdash a \triangleright_R \bullet \rightsquigarrow a' \triangleright_R \bullet / R_1} \quad \text{E_CONG} \\
\frac{}{\vdash (a \triangleright_R \bullet) \triangleright_R \bullet \rightsquigarrow a \triangleright_R \bullet / R_1} \quad \text{E_COMBINE} \\
\frac{}{\vdash (v_1 \triangleright_R \bullet) v_2^{R_1, +} \rightsquigarrow (v_1 (v_2 \triangleright_R \bullet)^{R_1, +}) \triangleright_R \bullet / R_2} \quad \text{E_PUSH} \\
\frac{}{\vdash (v_1 \triangleright_R \bullet)[\bullet] \rightsquigarrow (v_1[\bullet]) \triangleright_R \bullet / R_1} \quad \text{E_CPUSH}
\end{array}$$

$$\boxed{\vdash a \rightsquigarrow^* b/R} \quad \text{multistep reduction}$$

$$\begin{array}{c}
\frac{}{\vdash a \rightsquigarrow^* a/R} \quad \text{EQUAL} \\
\frac{\vdash a \rightsquigarrow b/R \quad \vdash b \rightsquigarrow^* a'/R}{\vdash a \rightsquigarrow^* a'/R} \quad \text{STEP}
\end{array}$$

$$\boxed{\Gamma \models \phi \text{ ok}} \quad \text{Prop wellformedness}$$

$$\frac{\Gamma \models a : A/R \quad \Gamma \models b : A/R \quad \Gamma \models A : \star/R}{\Gamma \models a \sim_{A/R} b \text{ ok}} \quad \text{E_WFF}$$

$$\boxed{\Gamma \models a : A/R} \quad \text{typing}$$

$$\begin{array}{c}
\frac{R_1 \leq R_2 \quad \Gamma \models a : A/R_1}{\Gamma \models a : A/R_2} \quad \text{E_SUBROLE} \\
\frac{\vdash \Gamma}{\Gamma \models \star : \star/R} \quad \text{E_STAR} \\
\frac{\vdash \Gamma \quad x : A/R \in \Gamma}{\Gamma \models x : A/R} \quad \text{E_VAR} \\
\frac{\Gamma, x : A/R \models B : \star/R' \quad \Gamma \models A : \star/R \quad R \leq R'}{\Gamma \models \Pi^\rho x : A/R \rightarrow B : \star/R'} \quad \text{E_PI} \\
\frac{\Gamma, x : A/R \models a : B/R' \quad \Gamma \models A : \star/R \quad (\rho = +) \vee (x \notin \text{fv } a) \quad R \leq R'}{\Gamma \models \lambda^{R, \rho} x. a : (\Pi^\rho x : A/R \rightarrow B)/R'} \quad \text{E_ABS} \\
\frac{\Gamma \models b : \Pi^+ x : A/R \rightarrow B/R' \quad \Gamma \models a : A/R}{\Gamma \models b \ a^{R, +} : B\{a/x\}/R'} \quad \text{E_APP}
\end{array}$$

$$\frac{\Gamma \models b : \Pi^- x : A/R \rightarrow B/R' \quad \Gamma \models a : A/R}{\Gamma \models b \square^{R,-} : B\{a/x\}/R'} \quad \text{E_IAPP}$$

$$\frac{\Gamma \models a : A/R \quad \Gamma; \tilde{\Gamma} \models A \equiv B : \star/R \quad \Gamma \models B : \star/R}{\Gamma \models a : B/R} \quad \text{E_CONV}$$

$$\frac{\Gamma, c : \phi \models B : \star/R \quad \Gamma \models \phi \text{ ok}}{\Gamma \models \forall c : \phi. B : \star/R} \quad \text{E_CPI}$$

$$\frac{\Gamma, c : \phi \models a : B/R \quad \Gamma \models \phi \text{ ok}}{\Gamma \models \Lambda c. a : \forall c : \phi. B/R} \quad \text{E_CABS}$$

$$\frac{\Gamma \models a_1 : \forall c : (a \sim_{A/R} b). B_1/R' \quad \Gamma; \tilde{\Gamma} \models a \equiv b : A/R}{\Gamma \models a_1[\bullet] : B_1\{\bullet/c\}/R'} \quad \text{E_CAPP}$$

$$\frac{\models \Gamma \quad F \sim a : A/R \in \Sigma_0 \quad \emptyset \models A : \star/R}{\Gamma \models F : A/R} \quad \text{E_FAM}$$

$$\frac{\Gamma \models a : A_1/R_1 \quad \Gamma; \tilde{\Gamma} \models A_1 \equiv A_2 : \star/R_2 \quad \neg(R_2 \leq R_1) \quad \Gamma \models A_2 : \star/R_1}{\Gamma \models a \triangleright_{R_2} \bullet : A_2/R_1} \quad \text{E_TYCAST}$$

$$\boxed{\Gamma; \Delta \models \phi_1 \equiv \phi_2} \quad \text{prop equality}$$

$$\frac{\Gamma; \Delta \models A_1 \equiv A_2 : A/R \quad \Gamma; \Delta \models B_1 \equiv B_2 : A/R}{\Gamma; \Delta \models A_1 \sim_{A/R} B_1 \equiv A_2 \sim_{A/R} B_2} \quad \text{E_PROPCONG}$$

$$\frac{\Gamma; \Delta \models A \equiv B : \star/R \quad \Gamma \models A_1 \sim_{A/R} A_2 \text{ ok} \quad \Gamma \models A_1 \sim_{B/R} A_2 \text{ ok}}{\Gamma; \Delta \models A_1 \sim_{A/R} A_2 \equiv A_1 \sim_{B/R} A_2} \quad \text{E_ISOCONV}$$

$$\frac{\Gamma; \Delta \models \forall c : \phi_1. B_1 \equiv \forall c : \phi_2. B_2 : \star/R}{\Gamma; \Delta \models \phi_1 \equiv \phi_2} \quad \text{E_CPIFST}$$

$$\boxed{\Gamma; \Delta \models a \equiv b : A/R} \quad \text{definitional equality}$$

$$\frac{\models \Gamma \quad c : (a \sim_{A/R} b) \in \Gamma \quad c \in \Delta}{\Gamma; \Delta \models a \equiv b : A/R} \quad \text{E_ASSN}$$

$$\frac{\Gamma \models a : A/R}{\Gamma; \Delta \models a \equiv a : A/R} \quad \text{E_REFL}$$

$$\begin{array}{c}
\frac{\Gamma; \Delta \models b \equiv a : A/R}{\Gamma; \Delta \models a \equiv b : A/R} \quad \text{E_SYM} \\
\\
\frac{\Gamma; \Delta \models a \equiv a_1 : A/R \quad \Gamma; \Delta \models a_1 \equiv b : A/R}{\Gamma; \Delta \models a \equiv b : A/R} \quad \text{E_TRANS} \\
\\
\frac{\Gamma; \Delta \models a \equiv b : A/R_1 \quad R_1 \leq R_2}{\Gamma; \Delta \models a \equiv b : A/R_2} \quad \text{E_SUB} \\
\\
\frac{\Gamma \models a_1 : B/R \quad \Gamma \models a_2 : B/R \quad \models a_1 > a_2/R}{\Gamma; \Delta \models a_1 \equiv a_2 : B/R} \quad \text{E_BETA} \\
\\
\frac{\Gamma; \Delta \models A_1 \equiv A_2 : \star/R \quad \Gamma, x : A_1/R; \Delta \models B_1 \equiv B_2 : \star/R' \quad \Gamma \models A_1 : \star/R \quad \Gamma \models \Pi^\rho x : A_1/R \rightarrow B_1 : \star/R' \quad \Gamma \models \Pi^\rho x : A_2/R \rightarrow B_2 : \star/R' \quad R \leq R'}{\Gamma; \Delta \models (\Pi^\rho x : A_1/R \rightarrow B_1) \equiv (\Pi^\rho x : A_2/R \rightarrow B_2) : \star/R'} \quad \text{E_PICONG} \\
\\
\frac{\Gamma, x : A_1/R; \Delta \models b_1 \equiv b_2 : B/R' \quad \Gamma \models A_1 : \star/R \quad R \leq R' \quad (\rho = +) \vee (x \notin \text{fv } b_1) \quad (\rho = +) \vee (x \notin \text{fv } b_2)}{\Gamma; \Delta \models (\lambda^{R, \rho} x. b_1) \equiv (\lambda^{R, \rho} x. b_2) : (\Pi^\rho x : A_1/R \rightarrow B)/R'} \quad \text{E_ABSCONG} \\
\\
\frac{\Gamma; \Delta \models a_1 \equiv b_1 : (\Pi^+ x : A/R \rightarrow B)/R' \quad \Gamma; \Delta \models a_2 \equiv b_2 : A/R}{\Gamma; \Delta \models a_1 \ a_2^{R, +} \equiv b_1 \ b_2^{R, +} : (B\{a_2/x\})/R'} \quad \text{E_APPCONG} \\
\\
\frac{\Gamma; \Delta \models a_1 \equiv b_1 : (\Pi^- x : A/R \rightarrow B)/R' \quad \Gamma \models a : A/R}{\Gamma; \Delta \models a_1 \ \Box^{R, -} \equiv b_1 \ \Box^{R, -} : (B\{a/x\})/R'} \quad \text{E_IAPPCONG} \\
\\
\frac{\Gamma; \Delta \models \Pi^\rho x : A_1/R \rightarrow B_1 \equiv \Pi^\rho x : A_2/R \rightarrow B_2 : \star/R'}{\Gamma; \Delta \models A_1 \equiv A_2 : \star/R} \quad \text{E_PIFST} \\
\\
\frac{\Gamma; \Delta \models \Pi^\rho x : A_1/R \rightarrow B_1 \equiv \Pi^\rho x : A_2/R \rightarrow B_2 : \star/R' \quad \Gamma; \Delta \models a_1 \equiv a_2 : A_1/R}{\Gamma; \Delta \models B_1\{a_1/x\} \equiv B_2\{a_2/x\} : \star/R'} \quad \text{E_PISND} \\
\\
\frac{\Gamma; \Delta \models a_1 \sim_{A_1/R} b_1 \equiv a_2 \sim_{A_2/R} b_2 \quad \Gamma, c : a_1 \sim_{A_1/R} b_1; \Delta \models A \equiv B : \star/R' \quad \Gamma \models a_1 \sim_{A_1/R} b_1 \text{ ok} \quad \Gamma \models \forall c : a_1 \sim_{A_1/R} b_1. A : \star/R' \quad \Gamma \models \forall c : a_2 \sim_{A_2/R} b_2. B : \star/R'}{\Gamma; \Delta \models \forall c : a_1 \sim_{A_1/R} b_1. A \equiv \forall c : a_2 \sim_{A_2/R} b_2. B : \star/R'} \quad \text{E_CPICONG} \\
\\
\frac{\Gamma, c : \phi_1; \Delta \models a \equiv b : B/R \quad \Gamma \models \phi_1 \text{ ok}}{\Gamma; \Delta \models (\Lambda c. a) \equiv (\Lambda c. b) : \forall c : \phi_1. B/R} \quad \text{E_CABSCONG}
\end{array}$$

$$\frac{\begin{array}{l} \Gamma; \Delta \models a_1 \equiv b_1 : (\forall c : (a \sim_{A/R} b). B) / R' \\ \Gamma; \tilde{\Gamma} \models a \equiv b : A/R \end{array}}{\Gamma; \Delta \models a_1[\bullet] \equiv b_1[\bullet] : (B\{\bullet/c\})/R'} \quad \text{E_CAPP_CONG}$$

$$\frac{\begin{array}{l} \Gamma; \Delta \models \forall c : (a_1 \sim_{A/R} a_2). B_1 \equiv \forall c : (a'_1 \sim_{A'/R'} a'_2). B_2 : \star / R_0 \\ \Gamma; \tilde{\Gamma} \models a_1 \equiv a_2 : A/R \\ \Gamma; \tilde{\Gamma} \models a'_1 \equiv a'_2 : A'/R' \end{array}}{\Gamma; \Delta \models B_1\{\bullet/c\} \equiv B_2\{\bullet/c\} : \star / R_0} \quad \text{E_CPI_SND}$$

$$\frac{\begin{array}{l} \Gamma; \Delta \models a \equiv b : A/R \\ \Gamma; \Delta \models a \sim_{A/R} b \equiv a' \sim_{A'/R'} b' \end{array}}{\Gamma; \Delta \models a' \equiv b' : A'/R'} \quad \text{E_CAST}$$

$$\frac{\begin{array}{l} \Gamma; \Delta \models a \equiv b : A/R_1 \\ \Gamma; \tilde{\Gamma} \models A \equiv B : \star / R_2 \\ R_1 \leq R_2 \end{array}}{\Gamma; \Delta \models a \equiv b : B/R_2} \quad \text{E_EQ_CONV}$$

$$\frac{\Gamma; \Delta \models a \sim_{A/R} b \equiv a' \sim_{A'/R} b'}{\Gamma; \Delta \models A \equiv A' : \star / R} \quad \text{E_ISO_SND}$$

$$\frac{\begin{array}{l} \Gamma; \Delta \models a_1 \equiv a_2 : A/R_1 \\ \Gamma; \Delta \models A \equiv B : \star / R_2 \\ \neg(R_2 \leq R_1) \\ \Gamma \models B : \star / R_1 \end{array}}{\Gamma; \Delta \models a_1 \triangleright_{R_2} \bullet \equiv a_2 \triangleright_{R_2} \bullet : B/R_1} \quad \text{E_CAST_CONG}$$

$\boxed{\models \Gamma}$ context wellformedness

$$\frac{}{\models \emptyset} \quad \text{E_EMPTY}$$

$$\frac{\begin{array}{l} \models \Gamma \\ \Gamma \models A : \star / R \\ x \notin \text{dom } \Gamma \end{array}}{\models \Gamma, x : A/R} \quad \text{E_CONST_M}$$

$$\frac{\begin{array}{l} \models \Gamma \\ \Gamma \models \phi \text{ ok} \\ c \notin \text{dom } \Gamma \end{array}}{\models \Gamma, c : \phi} \quad \text{E_CONS_CO}$$

$\boxed{\models \Sigma}$ signature wellformedness

$$\frac{}{\models \emptyset} \quad \text{SIG_EMPTY}$$

$$\frac{\begin{array}{l} \models \Sigma \\ \emptyset \models A : \star / R \\ \emptyset \models a : A/R' \\ F \notin \text{dom } \Sigma \\ R' \leq R \end{array}}{\models \Sigma \cup \{F \sim a : A/R'\}} \quad \text{SIG_CONS_AX}$$

$\boxed{\Gamma \vdash \phi \text{ ok}}$ prop wellformedness

$$\frac{\begin{array}{c} \Gamma \vdash a : A/R \\ \Gamma \vdash b : B/R \\ |A|R = |B|R \end{array}}{\Gamma \vdash a \sim_{A/R} b \text{ ok}} \text{ AN_WFF}$$

$\boxed{\Gamma \vdash a : A/R}$ typing

$$\frac{\vdash \Gamma}{\Gamma \vdash \star : \star/R} \text{ AN_STAR}$$

$$\frac{\begin{array}{c} \vdash \Gamma \\ x : A/R \in \Gamma \end{array}}{\Gamma \vdash x : A/R} \text{ AN_VAR}$$

$$\frac{\begin{array}{c} \Gamma, x : A/R \vdash B : \star/R' \\ \Gamma \vdash A : \star/R \end{array}}{\Gamma \vdash \Pi^{\rho} x : A/R \rightarrow B : \star/R'} \text{ AN_PI}$$

$$\frac{\begin{array}{c} \Gamma \vdash A : \star/R \\ \Gamma, x : A/R \vdash a : B/R' \\ (\rho = +) \vee (x \notin \text{fv } |a|R') \\ R \leq R' \end{array}}{\Gamma \vdash \lambda^{\rho} x : A/R. a : (\Pi^{\rho} x : A/R \rightarrow B)/R'} \text{ AN_ABS}$$

$$\frac{\begin{array}{c} \Gamma \vdash b : (\Pi^{\rho} x : A/R \rightarrow B)/R' \\ \Gamma \vdash a : A/R \end{array}}{\Gamma \vdash b \ a^{R,\rho} : (B\{a/x\})/R'} \text{ AN_APP}$$

$$\frac{\begin{array}{c} \Gamma \vdash a : A/R \\ \Gamma; \tilde{\Gamma} \vdash \gamma : A \sim_R B \\ \Gamma \vdash B : \star/R \end{array}}{\Gamma \vdash a \triangleright_R \gamma : B/R} \text{ AN_CONV}$$

$$\frac{\begin{array}{c} \Gamma \vdash \phi \text{ ok} \\ \Gamma, c : \phi \vdash B : \star/R \end{array}}{\Gamma \vdash \forall c : \phi. B : \star/R} \text{ AN_CPI}$$

$$\frac{\begin{array}{c} \Gamma \vdash \phi \text{ ok} \\ \Gamma, c : \phi \vdash a : B/R \end{array}}{\Gamma \vdash \Lambda c : \phi. a : (\forall c : \phi. B)/R} \text{ AN_CABS}$$

$$\frac{\begin{array}{c} \Gamma \vdash a_1 : (\forall c : a \sim_{A_1/R} b. B)/R' \\ \Gamma; \tilde{\Gamma} \vdash \gamma : a \sim_R b \end{array}}{\Gamma \vdash a_1[\gamma] : B\{\gamma/c\}/R'} \text{ AN_CAPP}$$

$$\frac{\begin{array}{c} \vdash \Gamma \\ F \sim a : A/R \in \Sigma_1 \\ \emptyset \vdash A : \star/R \end{array}}{\Gamma \vdash F : A/R} \text{ AN_FAM}$$

$$\frac{\begin{array}{c} R_1 \leq R_2 \\ \Gamma \vdash a : A/R_1 \end{array}}{\Gamma \vdash a : A/R_2} \text{ AN_SUBROLE}$$

$\boxed{\Gamma; \Delta \vdash \gamma : \phi_1 \sim \phi_2}$ coercion between props

$$\begin{array}{c}
\frac{\begin{array}{l}
\Gamma; \Delta \vdash \gamma_1 : A_1 \sim_R A_2 \\
\Gamma; \Delta \vdash \gamma_2 : B_1 \sim_R B_2 \\
\Gamma \vdash A_1 \sim_{A/R} B_1 \text{ ok} \\
\Gamma \vdash A_2 \sim_{A/R} B_2 \text{ ok}
\end{array}}{\Gamma; \Delta \vdash (\gamma_1 \sim_A \gamma_2) : (A_1 \sim_{A/R} B_1) \sim (A_2 \sim_{A/R} B_2)} \text{AN_PROP_CONG} \\
\\
\frac{\Gamma; \Delta \vdash \gamma : \forall c : \phi_1. A_2 \sim_R \forall c : \phi_2. B_2}{\Gamma; \Delta \vdash \mathbf{cpiFst} \gamma : \phi_1 \sim \phi_2} \text{AN_CPIFST} \\
\\
\frac{\Gamma; \Delta \vdash \gamma : \phi_1 \sim \phi_2}{\Gamma; \Delta \vdash \mathbf{sym} \gamma : \phi_2 \sim \phi_1} \text{AN_ISO_SYM} \\
\\
\frac{\begin{array}{l}
\Gamma; \Delta \vdash \gamma : A \sim_R B \\
\Gamma \vdash a_1 \sim_{A/R} a_2 \text{ ok} \\
\Gamma \vdash a'_1 \sim_{B/R} a'_2 \text{ ok} \\
|a_1|_R = |a'_1|_R \\
|a_2|_R = |a'_2|_R
\end{array}}{\Gamma; \Delta \vdash \mathbf{conv} (a_1 \sim_{A/R} a_2) \sim_\gamma (a'_1 \sim_{B/R} a'_2) : (a_1 \sim_{A/R} a_2) \sim (a'_1 \sim_{B/R} a'_2)} \text{AN_ISO_CONV} \\
\\
\boxed{\Gamma; \Delta \vdash \gamma : A \sim_R B} \quad \text{coercion between types} \\
\\
\frac{\begin{array}{l}
\vdash \Gamma \\
c : a \sim_{A/R} b \in \Gamma \\
c \in \Delta
\end{array}}{\Gamma; \Delta \vdash c : a \sim_R b} \text{AN_ASSN} \\
\\
\frac{\Gamma \vdash a : A/R}{\Gamma; \Delta \vdash \mathbf{refl} a : a \sim_R a} \text{AN_REFL} \\
\\
\frac{\begin{array}{l}
\Gamma \vdash a : A/R \\
\Gamma \vdash b : B/R \\
|a|_R = |b|_R \\
\Gamma; \tilde{\Gamma} \vdash \gamma : A \sim_R B
\end{array}}{\Gamma; \Delta \vdash (a \models_\gamma b) : a \sim_R b} \text{AN_ERASEEQ} \\
\\
\frac{\begin{array}{l}
\Gamma \vdash b : B/R \\
\Gamma \vdash a : A/R \\
\Gamma; \tilde{\Gamma} \vdash \gamma_1 : B \sim_R A \\
\Gamma; \Delta \vdash \gamma : b \sim_R a
\end{array}}{\Gamma; \Delta \vdash \mathbf{sym} \gamma : a \sim_R b} \text{AN_SYM} \\
\\
\frac{\begin{array}{l}
\Gamma; \Delta \vdash \gamma_1 : a \sim_R a_1 \\
\Gamma; \Delta \vdash \gamma_2 : a_1 \sim_R b \\
\Gamma \vdash a : A/R \\
\Gamma \vdash a_1 : A_1/R \\
\Gamma; \tilde{\Gamma} \vdash \gamma_3 : A \sim_R A_1
\end{array}}{\Gamma; \Delta \vdash (\gamma_1; \gamma_2) : a \sim_R b} \text{AN_TRANS} \\
\\
\frac{\begin{array}{l}
\Gamma \vdash a_1 : B_0/R \\
\Gamma \vdash a_2 : B_1/R \\
|B_0|_R = |B_1|_R \\
\models |a_1|_R > |a_2|_R/R
\end{array}}{\Gamma; \Delta \vdash \mathbf{red} a_1 a_2 : a_1 \sim_R a_2} \text{AN_BETA}
\end{array}$$

$$\begin{array}{c}
\Gamma; \Delta \vdash \gamma_1 : A_1 \sim_{R'} A_2 \\
\Gamma, x : A_1/R; \Delta \vdash \gamma_2 : B_1 \sim_{R'} B_2 \\
B_3 = B_2\{x \triangleright_{R'} \mathbf{sym} \gamma_1/x\} \\
\Gamma \vdash \Pi^\rho x : A_1/R \rightarrow B_1 : \star/R' \\
\Gamma \vdash \Pi^\rho x : A_1/R \rightarrow B_2 : \star/R' \\
\Gamma \vdash \Pi^\rho x : A_2/R \rightarrow B_3 : \star/R' \\
R \leq R' \\
\hline
\Gamma; \Delta \vdash \Pi^{R,\rho} x : \gamma_1.\gamma_2 : (\Pi^\rho x : A_1/R \rightarrow B_1) \sim_{R'} (\Pi^\rho x : A_2/R \rightarrow B_3) \quad \text{AN_PiCong}
\end{array}$$

$$\begin{array}{c}
\Gamma; \Delta \vdash \gamma_1 : A_1 \sim_R A_2 \\
\Gamma, x : A_1/R; \Delta \vdash \gamma_2 : b_1 \sim_{R'} b_2 \\
b_3 = b_2\{x \triangleright_{R'} \mathbf{sym} \gamma_1/x\} \\
\Gamma \vdash A_1 : \star/R \\
\Gamma \vdash A_2 : \star/R \\
(\rho = +) \vee (x \notin \mathbf{fv} |b_1|R') \\
(\rho = +) \vee (x \notin \mathbf{fv} |b_3|R') \\
\Gamma \vdash (\lambda^\rho x : A_1/R.b_2) : B/R' \\
R \leq R' \\
\hline
\Gamma; \Delta \vdash (\lambda^{R,\rho} x : \gamma_1.\gamma_2) : (\lambda^\rho x : A_1/R.b_1) \sim_{R'} (\lambda^\rho x : A_2/R.b_3) \quad \text{AN_AbsCong}
\end{array}$$

$$\begin{array}{c}
\Gamma; \Delta \vdash \gamma_1 : a_1 \sim_{R'} b_1 \\
\Gamma; \Delta \vdash \gamma_2 : a_2 \sim_R b_2 \\
\Gamma \vdash a_1 \ a_2^{R,\rho} : A/R' \\
\Gamma \vdash b_1 \ b_2^{R,\rho} : B/R' \\
\Gamma; \tilde{\Gamma} \vdash \gamma_3 : A \sim_{R'} B \\
\hline
\Gamma; \Delta \vdash \gamma_1 \ \gamma_2^{R,\rho} : a_1 \ a_2^{R,\rho} \sim_{R'} b_1 \ b_2^{R,\rho} \quad \text{AN_AppCong}
\end{array}$$

$$\begin{array}{c}
\Gamma; \Delta \vdash \gamma : \Pi^\rho x : A_1/R \rightarrow B_1 \sim_{R'} \Pi^\rho x : A_2/R \rightarrow B_2 \\
\hline
\Gamma; \Delta \vdash \mathbf{piFst} \gamma : A_1 \sim_R A_2 \quad \text{AN_PiFst}
\end{array}$$

$$\begin{array}{c}
\Gamma; \Delta \vdash \gamma_1 : \Pi^\rho x : A_1/R \rightarrow B_1 \sim_{R'} \Pi^\rho x : A_2/R \rightarrow B_2 \\
\Gamma; \Delta \vdash \gamma_2 : a_1 \sim_R a_2 \\
\Gamma \vdash a_1 : A_1/R \\
\Gamma \vdash a_2 : A_2/R \\
\hline
\Gamma; \Delta \vdash \gamma_1 @ \gamma_2 : B_1\{a_1/x\} \sim_{R'} B_2\{a_2/x\} \quad \text{AN_PiSnd}
\end{array}$$

$$\begin{array}{c}
\Gamma; \Delta \vdash \gamma_1 : a_1 \sim_{A_1/R} b_1 \sim_{A_2/R} b_2 \\
\Gamma, c : a_1 \sim_{A_1/R} b_1; \Delta \vdash \gamma_3 : B_1 \sim_{R'} B_2 \\
B_3 = B_2\{c \triangleright_{R'} \mathbf{sym} \gamma_1/c\} \\
\Gamma \vdash \forall c : a_1 \sim_{A_1/R} b_1.B_1 : \star/R' \\
\Gamma \vdash \forall c : a_2 \sim_{A_2/R} b_2.B_3 : \star/R' \\
\Gamma \vdash \forall c : a_1 \sim_{A_1/R} b_1.B_2 : \star/R' \\
\hline
\Gamma; \Delta \vdash (\forall c : \gamma_1.\gamma_3) : (\forall c : a_1 \sim_{A_1/R} b_1.B_1) \sim_R (\forall c : a_2 \sim_{A_2/R} b_2.B_3) \quad \text{AN_CPiCong}
\end{array}$$

$$\begin{array}{c}
\Gamma; \Delta \vdash \gamma_1 : b_0 \sim_{A_1/R} b_1 \sim_{A_2/R} b_3 \\
\Gamma, c : b_0 \sim_{A_1/R} b_1; \Delta \vdash \gamma_3 : a_1 \sim_{R'} a_2 \\
a_3 = a_2\{c \triangleright_{R'} \mathbf{sym} \gamma_1/c\} \\
\Gamma \vdash (\Lambda c : b_0 \sim_{A_1/R} b_1.a_1) : \forall c : b_0 \sim_{A_1/R} b_1.B_1/R' \\
\Gamma \vdash (\Lambda c : b_0 \sim_{A_1/R} b_1.a_2) : B/R' \\
\Gamma \vdash (\Lambda c : b_2 \sim_{A_2/R} b_3.a_3) : \forall c : b_2 \sim_{A_2/R} b_3.B_2/R' \\
\Gamma; \tilde{\Gamma} \vdash \gamma_4 : \forall c : b_0 \sim_{A_1/R} b_1.B_1 \sim_{R'} \forall c : \phi_2.B_2 \\
\hline
\Gamma; \Delta \vdash (\lambda c : \gamma_1.\gamma_3 @ \gamma_4) : (\Lambda c : b_0 \sim_{A_1/R} b_1.a_1) \sim_{R'} (\Lambda c : b_2 \sim_{A_2/R} b_3.a_3) \quad \text{AN_CAbsCong}
\end{array}$$

$$\begin{array}{c}
\Gamma; \Delta \vdash \gamma_1 : a_1 \sim_R b_1 \\
\Gamma; \tilde{\Gamma} \vdash \gamma_2 : a_2 \sim_{R'} b_2 \\
\Gamma; \tilde{\Gamma} \vdash \gamma_3 : a_3 \sim_{R'} b_3 \\
\Gamma \vdash a_1[\gamma_2] : A/R \\
\Gamma \vdash b_1[\gamma_3] : B/R \\
\Gamma; \tilde{\Gamma} \vdash \gamma_4 : A \sim_R B \\
\hline
\Gamma; \Delta \vdash \gamma_1(\gamma_2, \gamma_3) : a_1[\gamma_2] \sim_R b_1[\gamma_3] \quad \text{AN_CAPP_CONG} \\
\\
\Gamma; \Delta \vdash \gamma_1 : (\forall c_1 : a \sim_{A/R} a'. B_1) \sim_{R_0} (\forall c_2 : b \sim_{B/R'} b'. B_2) \\
\Gamma; \tilde{\Gamma} \vdash \gamma_2 : a \sim_R a' \\
\Gamma; \tilde{\Gamma} \vdash \gamma_3 : b \sim_{R'} b' \\
\hline
\Gamma; \Delta \vdash \gamma_1 @ (\gamma_2 \sim \gamma_3) : B_1\{\gamma_2/c_1\} \sim_{R_0} B_2\{\gamma_3/c_2\} \quad \text{AN_CPI_SND} \\
\\
\Gamma; \Delta \vdash \gamma_1 : a \sim_{R_1} a' \\
\Gamma; \Delta \vdash \gamma_2 : a \sim_{A/R_1} a' \sim b \sim_{B/R_2} b' \\
\hline
\Gamma; \Delta \vdash \gamma_1 \triangleright_{R_2} \gamma_2 : b \sim_{R_2} b' \quad \text{AN_CAST} \\
\\
\Gamma; \Delta \vdash \gamma : (a \sim_{A/R} a') \sim (b \sim_{B/R} b') \\
\hline
\Gamma; \Delta \vdash \mathbf{isoSnd} \gamma : A \sim_R B \quad \text{AN_ISO_SND} \\
\\
\Gamma; \Delta \vdash \gamma : a \sim_{R_1} b \\
R_1 \leq R_2 \\
\hline
\Gamma; \Delta \vdash \mathbf{sub} \gamma : a \sim_{R_2} b \quad \text{AN_SUB}
\end{array}$$

$\boxed{\vdash \Gamma}$ context wellformedness

$$\begin{array}{c}
\overline{\vdash \emptyset} \quad \text{AN_EMPTY} \\
\\
\vdash \Gamma \\
\Gamma \vdash A : \star/R \\
x \notin \text{dom } \Gamma \\
\hline
\vdash \Gamma, x : A/R \quad \text{AN_CONSTM} \\
\\
\vdash \Gamma \\
\Gamma \vdash \phi \text{ ok} \\
c \notin \text{dom } \Gamma \\
\hline
\vdash \Gamma, c : \phi \quad \text{AN_CONSCo}
\end{array}$$

$\boxed{\vdash \Sigma}$ signature wellformedness

$$\begin{array}{c}
\overline{\vdash \emptyset} \quad \text{AN_SIG_EMPTY} \\
\\
\vdash \Sigma \\
\emptyset \vdash A : \star/R \\
\emptyset \vdash a : A/R \\
F \notin \text{dom } \Sigma \\
\hline
\vdash \Sigma \cup \{F \sim a : A/R\} \quad \text{AN_SIG_CONSAx}
\end{array}$$

$\boxed{\Gamma \vdash a \rightsquigarrow b/R}$ single-step, weak head reduction to values for annotated language

$$\begin{array}{c}
\Gamma \vdash a \rightsquigarrow a'/R_1 \\
\hline
\Gamma \vdash a \ b^{R,\rho} \rightsquigarrow a' \ b^{R,\rho}/R_1 \quad \text{AN_APPLEFT} \\
\\
\text{Value}_R (\lambda^\rho x : A/R.w) \\
\hline
\Gamma \vdash (\lambda^\rho x : A/R.w) \ a^{R,\rho} \rightsquigarrow w\{a/x\}/R \quad \text{AN_APPABS}
\end{array}$$

$$\begin{array}{c}
\frac{\Gamma \vdash a \rightsquigarrow a'/R}{\Gamma \vdash a[\gamma] \rightsquigarrow a'[\gamma]/R} \quad \text{AN_CAPPLEFT} \\
\\
\frac{}{\Gamma \vdash (\Lambda c:\phi.b)[\gamma] \rightsquigarrow b\{\gamma/c\}/R} \quad \text{AN_CAPPCABS} \\
\\
\frac{\Gamma \vdash A : \star/R \quad \Gamma, x : A/R \vdash b \rightsquigarrow b'/R_1}{\Gamma \vdash (\lambda^- x:A/R.b) \rightsquigarrow (\lambda^- x:A/R.b')/R_1} \quad \text{AN_ABSTERM} \\
\\
\frac{F \sim a : A/R \in \Sigma_1}{\Gamma \vdash F \rightsquigarrow a/R} \quad \text{AN_AXIOM} \\
\\
\frac{\Gamma \vdash a \rightsquigarrow a'/R}{\Gamma \vdash a \triangleright_{R_1} \gamma \rightsquigarrow a' \triangleright_{R_1} \gamma/R} \quad \text{AN_CONVTERM} \\
\\
\frac{\text{Value}_R v}{\Gamma \vdash (v \triangleright_{R_2} \gamma_1) \triangleright_{R_2} \gamma_2 \rightsquigarrow v \triangleright_{R_2} (\gamma_1; \gamma_2)/R} \quad \text{AN_COMBINE} \\
\\
\frac{\begin{array}{l} \text{Value}_R v \\ \Gamma; \tilde{\Gamma} \vdash \gamma : \Pi^\rho x_1:A_1/R \rightarrow B_1 \sim_{R'} \Pi^\rho x_2:A_2/R \rightarrow B_2 \\ b' = b \triangleright_{R'} \mathbf{sym}(\mathbf{piFst} \gamma) \\ \gamma' = \gamma @ (b' \models_{(\mathbf{piFst} \gamma)} b) \end{array}}{\Gamma \vdash (v \triangleright_{R'} \gamma) \ b^{R,\rho} \rightsquigarrow ((v \ b'^{R,\rho}) \triangleright_{R'} \gamma')/R} \quad \text{AN_PUSH} \\
\\
\frac{\begin{array}{l} \text{Value}_R v \\ \Gamma; \tilde{\Gamma} \vdash \gamma : \forall c_1:a_1 \sim_{B_1/R} b_1.A_1 \sim_{R'} \forall c_2:a_2 \sim_{B_2/R} b_2.A_2 \\ \gamma'_1 = \gamma_1 \triangleright_{R'} \mathbf{sym}(\mathbf{cpiFst} \gamma) \\ \gamma' = \gamma @ (\gamma'_1 \sim \gamma_1) \end{array}}{\Gamma \vdash (v \triangleright_{R'} \gamma)[\gamma_1] \rightsquigarrow ((v[\gamma'_1]) \triangleright_{R'} \gamma')/R} \quad \text{AN_CPUSH}
\end{array}$$

Definition rules: 162 good 0 bad

Definition rule clauses: 482 good 0 bad