$tmvar$, x, y, f, m, n	variables
$covar$, c	coercion variables
$datacon$, K	
$const$, T, F	
$index$, i	indices

role, R	$::=$			Role
	$\mid$	**Nom**		
	$\mid$	**Rep**		
	$\mid$	**Phm**		
	$\mid$	$R_1 \cap R_2$	S	
relflag, ρ	$::=$			relevance flag
	$\mid$	$+$		
	$\mid$	$-$		
constraint, ϕ	$::=$			props
	$\mid$	$a \sim_{A/R} b$		
	$\mid$	(ϕ)	S	
	$\mid$	$\phi\{b/x\}$	S	
	$\mid$	$\lvert\phi\rvert$	S	
tm, a, b, v, w, A, B	$::=$			types and kinds
	$\mid$	$\star$		
	$\mid$	x		
	$\mid$	$\lambda^{\rho} x\!:\!A/R.b$	bind x in b	
	$\mid$	$\lambda^{R,\rho} x.b$	bind x in b	
	$\mid$	$a\ b^{R,\rho}$		
	$\mid$	F		
	$\mid$	$\Pi^{\rho} x\!:\!A/R \to B$	bind x in B	
	$\mid$	$a \rhd_R \gamma$		
	$\mid$	$\forall c\!:\!\phi.B$	bind c in B	
	$\mid$	$\Lambda c\!:\!\phi.b$	bind c in b	
	$\mid$	$\Lambda c.b$	bind c in b	
	$\mid$	$a[\gamma]$		
	$\mid$	$\square$		
	$\mid$	K		
	$\mid$	**match** a **with** brs		
	$\mid$	**sub** $R\ a$		
	$\mid$	$a\{b/x\}$	S	
	$\mid$	a	S	
	$\mid$	$a\{\gamma/c\}$	S	
	$\mid$	a	S	
	$\mid$	(a)	S	
	$\mid$	a	S	parsing precedence is hard
	$\mid$	$\lvert a\rvert R$	S	
	$\mid$	**Int**	S	
	$\mid$	**Bool**	S	
	$\mid$	**Nat**	S	
	$\mid$	**Vec**	S	
	$\mid$	0	S	
	$\mid$	S	S	
	$\mid$	**True**	S	

	|	**Fix**	S	
	|	**Age**	S	
	|	$a \to b$	S	
	|	$\phi \Rightarrow A$	S	
	|	$a b^{R,+}$	S	
	|	$\lambda^R x.a$	S	
	|	$\lambda x : A.a$	S	
	|	$\forall\, x : A/R \to B$	S	
brs	::=			case branches
	|	**none**		
	|	$K \Rightarrow a; brs$		
	|	$brs\{a/x\}$	S	
	|	$brs\{\gamma/c\}$	S	
	|	(brs)	S	
$co,\ \gamma$	::=			explicit coercions
	|	$\bullet$		
	|	c		
	|	**red** $a\ b$		
	|	**refl** a		
	|	$(a \mid=\mid_{\gamma}\ b)$		
	|	**sym** γ		
	|	$\gamma_1; \gamma_2$		
	|	**sub** γ		
	|	$\Pi^{R,\rho} x : \gamma_1 . \gamma_2$	bind x in γ_2	
	|	$\lambda^{R,\rho} x : \gamma_1 . \gamma_2$	bind x in γ_2	
	|	$\gamma_1\ \gamma_2^{R,\rho}$		
	|	**piFst** γ		
	|	**cpiFst** γ		
	|	**isoSnd** γ		
	|	$\gamma_1 @ \gamma_2$		
	|	$\forall c : \gamma_1 . \gamma_3$	bind c in γ_3	
	|	$\lambda c : \gamma_1 . \gamma_3 @ \gamma_4$	bind c in γ_3	
	|	$\gamma(\gamma_1, \gamma_2)$		
	|	$\gamma @ (\gamma_1 \sim \gamma_2)$		
	|	$\gamma_1 \triangleright_R \gamma_2$		
	|	$\gamma_1 \sim_A \gamma_2$		
	|	**conv** $\phi_1 \sim_{\gamma} \phi_2$		
	|	**eta** a		
	|	**left** $\gamma\,\gamma'$		
	|	**right** $\gamma\,\gamma'$		
	|	(γ)	S	
	|	γ	S	
	|	$\gamma\{a/x\}$	S	
$role\_context,\ \Omega$	::=			role$_{c}ontexts$

	$\vert$	$\varnothing$		
	$\vert$	$\Omega, x : R$		
	$\vert$	(Ω)	M	
	$\vert$	Ω	M	
sig_sort	::=			signature classifier
	$\vert$	$: A/R$		
	$\vert$	$\sim a : A/R$		
sort	::=			binding classifier
	$\vert$	$\mathbf{Tm}\, A\, R$		
	$\vert$	$\mathbf{Co}\, \phi$		
context, Γ	::=			contexts
	$\vert$	$\varnothing$		
	$\vert$	$\Gamma, x : A/R$		
	$\vert$	$\Gamma, c : \phi$		
	$\vert$	$\Gamma\{b/x\}$	M	
	$\vert$	$\Gamma\{\gamma/c\}$	M	
	$\vert$	Γ, Γ'	M	
	$\vert$	$\vert\Gamma\vert$	M	
	$\vert$	(Γ)	M	
	$\vert$	Γ	M	
sig, Σ	::=			signatures
	$\vert$	$\varnothing$		
	$\vert$	$\Sigma \cup \{F\, sig\_sort\}$		
	$\vert$	Σ_0	M	
	$\vert$	Σ_1	M	
	$\vert$	$\vert\Sigma\vert$	M	
available_props, Δ	::=			
	$\vert$	$\varnothing$		
	$\vert$	Δ, c		
	$\vert$	$\widetilde{\Gamma}$	M	
	$\vert$	(Δ)	M	
terminals	::=			
	$\vert$	$\leftrightarrow$		
	$\vert$	$\Leftrightarrow$		
	$\vert$	$\longrightarrow$		
	$\vert$	min		
	$\vert$	$\equiv$		
	$\vert$	$\forall$		
	$\vert$	$\in$		
	$\vert$	$\notin$		
	$\vert$	$\Leftarrow$		

| | $\Rightarrow$
| | $\Rightarrow^*$
| | $\rightarrow$
| | Λ
| |
| | $\Box$
| | $\vdash$
| | $\dashv$
| | $\models$
| | $\models$
| | $\neq$
| | $\triangleright$
| | ok
| | $\_$
| | $\leadsto$
| | $\leadsto^*$
| | $\leadsto$
| | $\varnothing$
| | $\circ$
| | fv
| | dom
| | $\sim$
| | $\asymp$
| | $|$
| | $\bullet$
| | $\mathbf{fst}$
| | $\mathbf{snd}$
| | $|\Rightarrow|$
| | $\vdash_=$
| | $\mathbf{refl_2}$
| | $++$

$formula,\ \psi$ ::=
| | $judgement$
| | $x : A/R\ \in\ \Gamma$
| | $x : R\ \in\ \Omega$
| | $c : \phi\ \in\ \Gamma$
| | $F\ sig\_sort\ \in\ \Sigma$
| | $K\ :\ T\,\Gamma\ \in\ \Sigma$
| | $x\ \in\ \Delta$
| | $c\ \in\ \Delta$
| | $c\,\textbf{not relevant}\ \in\ \gamma$
| | $x\ \notin\ \mathsf{fv}\,a$
| | $x\ \notin\ \mathsf{dom}\,\Gamma$
| | $uniq(\Omega)$
| | $c\ \notin\ \mathsf{dom}\,\Gamma$

	$\mid$	$T \notin \mathsf{dom}\,\Sigma$	
	$\mid$	$F \notin \mathsf{dom}\,\Sigma$	
	$\mid$	$a = b$	
	$\mid$	$\phi_1 = \phi_2$	
	$\mid$	$\Gamma_1 = \Gamma_2$	
	$\mid$	$\gamma_1 = \gamma_2$	
	$\mid$	$\neg\psi$	
	$\mid$	$\psi_1 \wedge \psi_2$	
	$\mid$	$\psi_1 \vee \psi_2$	
	$\mid$	$\psi_1 \Rightarrow \psi_2$	
	$\mid$	(ψ)	
	$\mid$	ψ	
	$\mid$	$c : (a : A \sim b : B) \in \Gamma$	
	$\mid$		suppress lc hypothesis generated by Ott
JSubRole	$::=$		
	$\mid$	$R_1 \leq R_2$	Subroling judgement
JPath	$::=$		
	$\mid$	$\mathsf{Path}_R\ a = F$	Type headed by constant (partial function)
JValue	$::=$		
	$\mid$	$\mathsf{CoercedValue}_R\ A$	Values with at most one coercion at the top
	$\mid$	$\mathsf{Value}_R\ A$	values
	$\mid$	$\mathsf{ValueType}_R\ A$	Types with head forms (erased language)
Jconsistent	$::=$		
	$\mid$	$\mathsf{consistent}_R\ ab$	(erased) types do not differ in their heads
Jerased	$::=$		
	$\mid$	$\Omega \vDash a : R$	
JChk	$::=$		
	$\mid$	$(\rho = +) \vee (x \notin \mathsf{fv}\ A)$	irrelevant argument check
Jpar	$::=$		
	$\mid$	$\Omega \vDash a \Rightarrow_R b$	parallel reduction (implicit language)
	$\mid$	$\Omega \vdash a \Rightarrow^*_R b$	multistep parallel reduction
	$\mid$	$\Omega \vdash a \Leftrightarrow_R b$	parallel reduction to a common term
Jbeta	$::=$		
	$\mid$	$\vDash a > b/R$	primitive reductions on erased terms
	$\mid$	$\vDash a \rightsquigarrow b/R$	single-step head reduction for implicit language
	$\mid$	$\vDash a \rightsquigarrow^* b/R$	multistep reduction
Jett	$::=$		
	$\mid$	$\Gamma \vDash \phi\ \mathsf{ok}$	Prop wellformedness

	|	$\Gamma \vDash a : A / R$	typing
	|	$\Gamma ; \Delta \vDash \phi_1 \equiv \phi_2$	prop equality
	|	$\Gamma ; \Delta \vDash a \equiv b : A / R$	definitional equality
	|	$\vDash \Gamma$	context wellformedness
Jsig	::=		
	|	$\vDash \Sigma$	signature wellformedness
Jann	::=		
	|	$\Gamma \vdash \phi\ \mathsf{ok}$	prop wellformedness
	|	$\Gamma \vdash a : A / R$	typing
	|	$\Gamma ; \Delta \vdash \gamma : \phi_1 \sim \phi_2$	coercion between props
	|	$\Gamma ; \Delta \vdash \gamma : A \sim_R B$	coercion between types
	|	$\vdash \Gamma$	context wellformedness
	|	$\vdash \Sigma$	signature wellformedness
Jred	::=		
	|	$\Gamma \vdash a \leadsto b / R$	single-step, weak head reduction to values for annotated lang
judgement	::=		
	|	*JSubRole*	
	|	*JPath*	
	|	*JValue*	
	|	*Jconsistent*	
	|	*Jerased*	
	|	*JChk*	
	|	*Jpar*	
	|	*Jbeta*	
	|	*Jett*	
	|	*Jsig*	
	|	*Jann*	
	|	*Jred*	
user_syntax	::=		
	|	*tmvar*	
	|	*covar*	
	|	*datacon*	
	|	*const*	
	|	*index*	
	|	*role*	
	|	*relflag*	
	|	*constraint*	
	|	*tm*	
	|	*brs*	
	|	*co*	
	|	*role_context*	
	|	*sig_sort*	

| *sort*
| *context*
| *sig*
| *available_props*
| *terminals*
| *formula*

$\boxed{R_1 \leq R_2}$ Subroling judgement

$$\frac{}{\mathbf{Nom} \leq R} \quad \text{NomBot}$$

$$\frac{}{R \leq \mathbf{Phm}} \quad \text{PhmTop}$$

$$\frac{}{R \leq R} \quad \text{Refl}$$

$$\frac{R_1 \leq R_2 \quad R_2 \leq R_3}{R_1 \leq R_3} \quad \text{Trans}$$

$\boxed{\mathsf{Path}_R\ a = F}$ Type headed by constant (partial function)

$$\frac{F \sim a : A/R_1 \in \Sigma_0 \quad \neg(R_1 \leq R)}{\mathsf{Path}_R\ F = F} \quad \text{Path\_Const}$$

$$\frac{\mathsf{Path}_R\ a = F}{\mathsf{Path}_R\ (a\ b'^{R_1,\rho}) = F} \quad \text{Path\_App}$$

$$\frac{\mathsf{Path}_R\ a = F}{\mathsf{Path}_R\ (a[\bullet]) = F} \quad \text{Path\_CApp}$$

$$\frac{\mathsf{Path}_R\ a = F}{\mathsf{Path}_R\ (a \triangleright_{R_1} \bullet) = F} \quad \text{Path\_Conv}$$

$\boxed{\mathsf{CoercedValue}_R\ A}$ Values with at most one coercion at the top

$$\frac{\mathsf{Value}_R\ a}{\mathsf{CoercedValue}_R\ a} \quad \text{CV}$$

$$\frac{\mathsf{Value}_R\ a}{\mathsf{CoercedValue}_R\ (a \triangleright_{R_1} \bullet)} \quad \text{CC}$$

$$\frac{\mathsf{CoercedValue}_R\ (a \triangleright_{R_1} \bullet) \quad \neg(R_1 \leq R_2)}{\mathsf{CoercedValue}_R\ ((a \triangleright_{R_1} \bullet) \triangleright_{R_2} \bullet)} \quad \text{CCV}$$

$\boxed{\mathsf{Value}_R\ A}$ values

$$\frac{}{\mathsf{Value}_R\ \star} \quad \text{Value\_Star}$$

$$\frac{}{\mathsf{Value}_R\ \Pi^\rho x\!:\!A/R_1 \to B} \quad \text{Value\_Pi}$$

$$\frac{}{\mathsf{Value}_R\ \forall c\!:\!\phi.B} \quad \text{Value\_CPi}$$

$$\frac{}{\mathsf{Value}_R\ \lambda^+ x\!:\!A/R_1.a} \quad \text{Value\_AbsRel}$$

$$\frac{}{\mathsf{Value}_R\ \lambda^{R_1,+}x.a}\quad\textsc{Value\_UAbsRel}$$

$$\frac{\mathsf{CoercedValue}_R\ a}{\mathsf{Value}_R\ \lambda^{R_1,-}x.a}\quad\textsc{Value\_UAbsIrrel}$$

$$\frac{}{\mathsf{Value}_R\ \Lambda c\!:\!\phi.a}\quad\textsc{Value\_CAbs}$$

$$\frac{}{\mathsf{Value}_R\ \Lambda c.a}\quad\textsc{Value\_UCAbs}$$

$$\frac{\begin{array}{l}F\ \sim a : A/R_1\ \in \Sigma_0\\ \neg(R_1 \leq R)\end{array}}{\mathsf{Value}_R\ F}\quad\textsc{Value\_Ax}$$

$$\frac{\begin{array}{l}\mathsf{Path}_R\ a = F\\ \mathsf{Value}_R\ a\end{array}}{\mathsf{Value}_R\ (a\ b'^{R_1,\rho})}\quad\textsc{Value\_App}$$

$$\frac{\begin{array}{l}\mathsf{Path}_R\ a = F\\ \mathsf{Value}_R\ a\end{array}}{\mathsf{Value}_R\ (a[\bullet])}\quad\textsc{Value\_CApp}$$

$$\frac{}{\mathsf{Value}_R\ \square}\quad\textsc{Value\_Bullet}$$

$\boxed{\mathsf{ValueType}_R\ A}$ Types with head forms (erased language)

$$\frac{}{\mathsf{ValueType}_R\ \star}\quad\textsc{value\_type\_Star}$$

$$\frac{}{\mathsf{ValueType}_R\ \Pi^\rho x\!:\!A/R_1 \to B}\quad\textsc{value\_type\_Pi}$$

$$\frac{}{\mathsf{ValueType}_R\ \forall c\!:\!\phi.B}\quad\textsc{value\_type\_CPi}$$

$$\frac{\begin{array}{l}\mathsf{Path}_R\ A = F\\ \mathsf{Value}_R\ A\end{array}}{\mathsf{ValueType}_R\ A}\quad\textsc{value\_type\_Path}$$

$\boxed{\mathsf{consistent}_R\ ab}$ (erased) types do not differ in their heads

$$\frac{}{\mathsf{consistent}_R\ \star\star}\quad\textsc{consistent\_a\_Star}$$

$$\frac{}{\mathsf{consistent}_{R'}\ (\Pi^\rho x_1\!:\!A_1/R \to B_1)(\Pi^\rho x_2\!:\!A_2/R \to B_2)}\quad\textsc{consistent\_a\_Pi}$$

$$\frac{}{\mathsf{consistent}_R\ (\forall c_1\!:\!\phi_1.A_1)(\forall c_2\!:\!\phi_2.A_2)}\quad\textsc{consistent\_a\_CPi}$$

$$\frac{\begin{array}{l}\mathsf{Path}_R\ a_1 = F\\ \mathsf{Path}_R\ a_2 = F\end{array}}{\mathsf{consistent}_R\ a_1 a_2}\quad\textsc{consistent\_a\_Path}$$

$$\frac{\neg\mathsf{ValueType}_R\ b}{\mathsf{consistent}_R\ ab}\quad\textsc{consistent\_a\_Step\_R}$$

$$\frac{\neg\mathsf{ValueType}_R\ a}{\mathsf{consistent}_R\ ab}\quad\textsc{consistent\_a\_Step\_L}$$

$$\frac{}{\mathsf{consistent}_R\ \square\square}\quad\textsc{consistent\_a\_Bullet}$$

$\boxed{\Omega \vDash a : R}$

$$\frac{uniq(\Omega)}{\Omega \vDash \Box : R} \quad \textsc{erased\_a\_Bullet}$$

$$\frac{uniq(\Omega)}{\Omega \vDash \star : R} \quad \textsc{erased\_a\_Star}$$

$$\frac{\begin{array}{l} uniq(\Omega) \\ x : R \in \Omega \\ R \leq R_1 \end{array}}{\Omega \vDash x : R_1} \quad \textsc{erased\_a\_Var}$$

$$\frac{\Omega, x : R_1 \vDash a : R}{\Omega \vDash (\lambda^{R_1,\rho} x.a) : R} \quad \textsc{erased\_a\_Abs}$$

$$\frac{\begin{array}{l} \Omega \vDash a : R \\ \Omega \vDash b : R_1 \end{array}}{\Omega \vDash (a\ b^{R_1,\rho}) : R} \quad \textsc{erased\_a\_App}$$

$$\frac{\begin{array}{l} \Omega \vDash A : R_1 \\ \Omega, x : R_1 \vDash B : R \end{array}}{\Omega \vDash (\Pi^\rho x\!:\!A/R_1 \to B) : R} \quad \textsc{erased\_a\_Pi}$$

$$\frac{\begin{array}{l} \Omega \vDash a : R_1 \\ \Omega \vDash b : R_1 \\ \Omega \vDash A : R_1 \\ \Omega \vDash B : R \end{array}}{\Omega \vDash (\forall c\!:\!a \sim_{A/R_1} b.B) : R} \quad \textsc{erased\_a\_CPi}$$

$$\frac{\Omega \vDash b : R}{\Omega \vDash (\Lambda c.b) : R} \quad \textsc{erased\_a\_CAbs}$$

$$\frac{\Omega \vDash a : R}{\Omega \vDash (a[\bullet]) : R} \quad \textsc{erased\_a\_CApp}$$

$$\frac{\begin{array}{l} uniq(\Omega) \\ F \sim a : A/R \in \Sigma_0 \end{array}}{\Omega \vDash F : R_1} \quad \textsc{erased\_a\_Fam}$$

$$\frac{\Omega \vDash a : R}{\Omega \vDash (a \rhd_{R_1} \bullet) : R} \quad \textsc{erased\_a\_Conv}$$

$\boxed{(\rho = +) \vee (x \notin \mathsf{fv}\ A)}$ irrelevant argument check

$$\frac{}{(+ = +) \vee (x \notin \mathsf{fv}\ A)} \quad \textsc{Rho\_Rel}$$

$$\frac{x \notin \mathsf{fv} A}{(- = +) \vee (x \notin \mathsf{fv}\ A)} \quad \textsc{Rho\_IrrRel}$$

$\boxed{\Omega \vDash a \Rightarrow_R b}$ parallel reduction (implicit language)

$$\frac{\Omega \vDash a : R}{\Omega \vDash a \Rightarrow_R a} \quad \textsc{Par\_Refl}$$

$$\frac{\begin{array}{l} \Omega \vDash a \Rightarrow_R (\lambda^{R_1,\rho} x.a') \\ \Omega \vDash b \Rightarrow_{R_1} b' \end{array}}{\Omega \vDash a\ b^{R_1,\rho} \Rightarrow_R a'\{b'/x\}} \quad \textsc{Par\_Beta}$$

$$\frac{\Omega \vDash a \Rightarrow_R a' \quad \Omega \vDash b \Rightarrow_{R_1} b'}{\Omega \vDash a\ b^{R_1,\rho} \Rightarrow_R a'\ b'^{R_1,\rho}} \quad \textsc{Par\_App}$$

$$\frac{\Omega \vDash a \Rightarrow_R (\Lambda c.a')}{\Omega \vDash a[\bullet] \Rightarrow_R a'\{\bullet/c\}} \quad \textsc{Par\_CBeta}$$

$$\frac{\Omega \vDash a \Rightarrow_R a'}{\Omega \vDash a[\bullet] \Rightarrow_R a'[\bullet]} \quad \textsc{Par\_CApp}$$

$$\frac{\Omega, x : R_1 \vDash a \Rightarrow_R a'}{\Omega \vDash \lambda^{R_1,\rho}x.a \Rightarrow_R \lambda^{R_1,\rho}x.a'} \quad \textsc{Par\_Abs}$$

$$\frac{\Omega \vDash A \Rightarrow_{R_1} A' \quad \Omega, x : R_1 \vDash B \Rightarrow_R B'}{\Omega \vDash \Pi^\rho x\!:\!A/R_1 \to B \Rightarrow_R \Pi^\rho x\!:\!A'/R_1 \to B'} \quad \textsc{Par\_Pi}$$

$$\frac{\Omega \vDash a \Rightarrow_R a'}{\Omega \vDash \Lambda c.a \Rightarrow_R \Lambda c.a'} \quad \textsc{Par\_CAbs}$$

$$\frac{\Omega \vDash A \Rightarrow_{R_1} A' \quad \Omega \vDash a \Rightarrow_{R_1} a' \quad \Omega \vDash b \Rightarrow_{R_1} b' \quad \Omega \vDash B \Rightarrow_R B'}{\Omega \vDash \forall c\!:\!a \sim_{A/R_1} b.B \Rightarrow_R \forall c\!:\!a' \sim_{A'/R_1} b'.B'} \quad \textsc{Par\_CPi}$$

$$\frac{F \sim a : A/R_1 \in \Sigma_0 \quad R_1 \leq R \quad \mathit{uniq}(\Omega)}{\Omega \vDash F \Rightarrow_R a} \quad \textsc{Par\_Axiom}$$

$$\frac{\Omega \vDash a_1 \Rightarrow_{R_1} a_2}{\Omega \vDash a_1 \rhd_R \bullet \Rightarrow_{R_1} a_2 \rhd_R \bullet} \quad \textsc{Par\_Cong}$$

$$\frac{\Omega \vDash a_1 \Rightarrow_{R_1} (a_2 \rhd_R \bullet)}{\Omega \vDash (a_1 \rhd_R \bullet) \Rightarrow_{R_1} (a_2 \rhd_R \bullet)} \quad \textsc{Par\_Combine}$$

$$\frac{\Omega \vDash a_1 \Rightarrow_{R_1} (a_2 \rhd_R \bullet) \quad \Omega \vDash b_1 \Rightarrow_{R_2} b_2}{\Omega \vDash a_1 b_1{}^{R_2,+} \Rightarrow_{R_1} (a_2 (b_2 \rhd_R \bullet)^{R_2,+}) \rhd_R \bullet} \quad \textsc{Par\_Push}$$

$$\frac{\Omega \vDash a_1 \Rightarrow_{R_1} (a_2 \rhd_R \bullet) \quad \Omega \vDash b_1 \Rightarrow_{R_2} (b_2 \rhd_R \bullet)}{\Omega \vDash a_1 b_1{}^{R_2,+} \Rightarrow_{R_1} (a_2 (b_2 \rhd_R \bullet)^{R_2,+}) \rhd_R \bullet} \quad \textsc{Par\_PushCombine}$$

$$\frac{\Omega \vDash a_1 \Rightarrow_{R_1} (a_2 \rhd_R \bullet)}{\Omega \vDash a_1[\bullet] \Rightarrow_{R_1} (a_2[\bullet]) \rhd_R \bullet} \quad \textsc{Par\_CPush}$$

$$\frac{\Omega \vDash a : \mathbf{Phm}}{\Omega \vDash a \Rightarrow_{\mathbf{Phm}} \Box} \quad \textsc{Par\_Bullet}$$

$\boxed{\Omega \vdash a \Rightarrow^*_R b}$ multistep parallel reduction

$$\frac{}{\Omega \vdash a \Rightarrow^*_R a} \quad \textsc{MP\_Refl}$$

$$\frac{\Omega \vDash a \Rightarrow_R b \quad \Omega \vdash b \Rightarrow^*_R a'}{\Omega \vdash a \Rightarrow^*_R a'} \quad \textsc{MP\_Step}$$

$\boxed{\Omega \vdash a \Leftrightarrow_R b}$ parallel reduction to a common term

$$\frac{\begin{array}{l}\Omega \vdash a_1 \Rightarrow^*_R b \\ \Omega \vdash a_2 \Rightarrow^*_R b\end{array}}{\Omega \vdash a_1 \Leftrightarrow_R a_2} \quad \textsc{join}$$

$\boxed{\vDash a > b / R}$ primitive reductions on erased terms

$$\frac{\mathsf{Value}_{R_1}\ (\lambda^{R,\rho} x.v)}{\vDash (\lambda^{R,\rho} x.v)\ b^{R,\rho} > v\{b/x\}/R_1} \quad \textsc{Beta\_AppAbs}$$

$$\frac{}{\vDash (\Lambda c.a')[\bullet] > a'\{\bullet/c\}/R} \quad \textsc{Beta\_CAppCAbs}$$

$$\frac{\begin{array}{l}F \sim a : A/R \in \Sigma_0 \\ R \leq R_1\end{array}}{\vDash F > a/R_1} \quad \textsc{Beta\_Axiom}$$

$$\frac{}{\vDash a > \Box/\mathbf{Phm}} \quad \textsc{Beta\_Bullet}$$

$\boxed{\vDash a \rightsquigarrow b / R}$ single-step head reduction for implicit language

$$\frac{\vDash a \rightsquigarrow a'/R_1}{\vDash \lambda^{R,-} x.a \rightsquigarrow \lambda^{R,-} x.a'/R_1} \quad \textsc{E\_AbsTerm}$$

$$\frac{\vDash a \rightsquigarrow a'/R_1}{\vDash a\ b^{R,\rho} \rightsquigarrow a'\ b^{R,\rho}/R_1} \quad \textsc{E\_AppLeft}$$

$$\frac{\vDash a \rightsquigarrow a'/R}{\vDash a[\bullet] \rightsquigarrow a'[\bullet]/R} \quad \textsc{E\_CAppLeft}$$

$$\frac{\mathsf{Value}_{R_1}\ (\lambda^{R,\rho} x.v)}{\vDash (\lambda^{R,\rho} x.v)\ a^{R,\rho} \rightsquigarrow v\{a/x\}/R_1} \quad \textsc{E\_AppAbs}$$

$$\frac{}{\vDash (\Lambda c.b)[\bullet] \rightsquigarrow b\{\bullet/c\}/R} \quad \textsc{E\_CAppCAbs}$$

$$\frac{\begin{array}{l}F \sim a : A/R \in \Sigma_0 \\ R \leq R_1\end{array}}{\vDash F \rightsquigarrow a/R_1} \quad \textsc{E\_Axiom}$$

$$\frac{\vDash a \rightsquigarrow a'/R_1}{\vDash a \triangleright_R \bullet \rightsquigarrow a' \triangleright_R \bullet/R_1} \quad \textsc{E\_Cong}$$

$$\frac{\begin{array}{l}\mathsf{CoercedValue}_R\ (v \triangleright_{R_1} \bullet) \\ R_1 \leq R_2\end{array}}{\vDash (v \triangleright_{R_1} \bullet) \triangleright_{R_2} \bullet \rightsquigarrow v \triangleright_{R_2} \bullet/R} \quad \textsc{E\_Combine}$$

$$\frac{\mathsf{CoercedValue}_{R_2}\ (v_1 \triangleright_R \bullet)}{\vDash (v_1 \triangleright_R \bullet)\ b^{R_1,\rho} \rightsquigarrow (v_1\ (b \triangleright_R \bullet)^{R_1,\rho}) \triangleright_R \bullet/R_2} \quad \textsc{E\_Push}$$

$$\frac{\mathsf{CoercedValue}_{R_1}\ (v_1 \triangleright_R \bullet)}{\vDash (v_1 \triangleright_R \bullet)[\bullet] \rightsquigarrow (v_1[\bullet]) \triangleright_R \bullet/R_1} \quad \textsc{E\_CPush}$$

$\boxed{\vDash a \rightsquigarrow^* b / R}$ multistep reduction

$$\frac{}{\vDash a \rightsquigarrow^* a/R} \quad \textsc{Equal}$$

$$\frac{\vDash a \leadsto b/R \quad \vDash b \leadsto^* a'/R}{\vDash a \leadsto^* a'/R}\ \textsc{Step}$$

$\boxed{\Gamma \vDash \phi\ \mathsf{ok}}$ Prop wellformedness

$$\frac{\Gamma \vDash a : A/R \quad \Gamma \vDash b : A/R \quad \Gamma \vDash A : \star/R}{\Gamma \vDash a \sim_{A/R} b\ \mathsf{ok}}\ \text{E\_}\textsc{Wff}$$

$\boxed{\Gamma \vDash a : A/R}$ typing

$$\frac{R_1 \leq R_2 \quad \Gamma \vDash a : A/R_1}{\Gamma \vDash a : A/R_2}\ \text{E\_}\textsc{SubRole}$$

$$\frac{\vDash \Gamma}{\Gamma \vDash \star : \star/R}\ \text{E\_}\textsc{Star}$$

$$\frac{\vDash \Gamma \quad x : A/R \in \Gamma}{\Gamma \vDash x : A/R}\ \text{E\_}\textsc{Var}$$

$$\frac{\Gamma, x : A/R \vDash B : \star/R' \quad \Gamma \vDash A : \star/R}{\Gamma \vDash \Pi^\rho x{:}A/R \to B : \star/R'}\ \text{E\_}\textsc{Pi}$$

$$\frac{\Gamma, x : A/R \vDash a : B/R' \quad \Gamma \vDash A : \star/R \quad (\rho = +) \vee (x \notin \mathsf{fv}\ a)}{\Gamma \vDash \lambda^{R,\rho}x.a : (\Pi^\rho x{:}A/R \to B)/R'}\ \text{E\_}\textsc{Abs}$$

$$\frac{\Gamma \vDash b : \Pi^+ x{:}A/R \to B/R' \quad \Gamma \vDash a : A/R}{\Gamma \vDash b\ a^{R,+} : B\{a/x\}/R'}\ \text{E\_}\textsc{App}$$

$$\frac{\Gamma \vDash b : \Pi^- x{:}A/R \to B/R' \quad \Gamma \vDash a : A/R}{\Gamma \vDash b\ \Box^{R,-} : B\{a/x\}/R'}\ \text{E\_I}\textsc{App}$$

$$\frac{\Gamma \vDash a : A/R \quad \Gamma; \widetilde{\Gamma} \vDash A \equiv B : \star/R \quad \Gamma \vDash B : \star/R}{\Gamma \vDash a : B/R}\ \text{E\_}\textsc{Conv}$$

$$\frac{\Gamma, c : \phi \vDash B : \star/R \quad \Gamma \vDash \phi\ \mathsf{ok}}{\Gamma \vDash \forall c{:}\phi.B : \star/R}\ \text{E\_CP}\textsc{i}$$

$$\frac{\Gamma, c : \phi \vDash a : B/R \quad \Gamma \vDash \phi\ \mathsf{ok}}{\Gamma \vDash \Lambda c.a : \forall c{:}\phi.B/R}\ \text{E\_CA}\textsc{bs}$$

$$\frac{\Gamma \vDash a_1 : \forall c{:}(a \sim_{A/R} b).B_1/R' \quad \Gamma; \widetilde{\Gamma} \vDash a \equiv b : A/R}{\Gamma \vDash a_1[\bullet] : B_1\{\bullet/c\}/R'}\ \text{E\_CA}\textsc{pp}$$

$$\frac{\begin{array}{l}\vDash \Gamma \\ F \sim a : A/R \in \Sigma_0 \\ \varnothing \vDash A : \star/R_1\end{array}}{\Gamma \vDash F : A/R_1} \quad \text{E\_FAM}$$

$$\frac{\begin{array}{l}\Gamma \vDash a : A_1/R_1 \\ \Gamma; \widetilde{\Gamma} \vDash A_1 \equiv A_2 : \star/R_2 \\ \Gamma \vDash A_2 : \star/R_1\end{array}}{\Gamma \vDash a \rhd_{R_2} \bullet : A_2/R_1} \quad \text{E\_TYCAST}$$

$$\frac{\Gamma \vDash A : \star/\mathbf{Phm}}{\Gamma \vDash \Box : A/\mathbf{Phm}} \quad \text{E\_BULLET}$$

$\boxed{\Gamma; \Delta \vDash \phi_1 \equiv \phi_2}$ prop equality

$$\frac{\begin{array}{c}\Gamma; \Delta \vDash A_1 \equiv A_2 : A/R \\ \Gamma; \Delta \vDash B_1 \equiv B_2 : A/R\end{array}}{\Gamma; \Delta \vDash A_1 \sim_{A/R} B_1 \equiv A_2 \sim_{A/R} B_2} \quad \text{E\_PROPCONG}$$

$$\frac{\begin{array}{c}\Gamma; \Delta \vDash A \equiv B : \star/R \\ \Gamma \vDash A_1 \sim_{A/R} A_2 \ \mathsf{ok} \\ \Gamma \vDash A_1 \sim_{B/R} A_2 \ \mathsf{ok}\end{array}}{\Gamma; \Delta \vDash A_1 \sim_{A/R} A_2 \equiv A_1 \sim_{B/R} A_2} \quad \text{E\_ISOCONV}$$

$$\frac{\Gamma; \Delta \vDash \forall c{:}(a_1 \sim_{A/R} a_2).B_1 \equiv \forall c{:}(b_1 \sim_{B/R} b_2).B_2 : \star/R'}{\Gamma; \Delta \vDash a_1 \sim_{A/R} a_2 \equiv b_1 \sim_{B/R} b_2} \quad \text{E\_CPIFST}$$

$\boxed{\Gamma; \Delta \vDash a \equiv b : A/R}$ definitional equality

$$\frac{\begin{array}{l}\vDash \Gamma \\ c : (a \sim_{A/R} b) \in \Gamma \\ c \in \Delta\end{array}}{\Gamma; \Delta \vDash a \equiv b : A/R} \quad \text{E\_ASSN}$$

$$\frac{\Gamma \vDash a : A/R}{\Gamma; \Delta \vDash a \equiv a : A/R} \quad \text{E\_REFL}$$

$$\frac{\Gamma; \Delta \vDash b \equiv a : A/R}{\Gamma; \Delta \vDash a \equiv b : A/R} \quad \text{E\_SYM}$$

$$\frac{\begin{array}{c}\Gamma; \Delta \vDash a \equiv a_1 : A/R \\ \Gamma; \Delta \vDash a_1 \equiv b : A/R\end{array}}{\Gamma; \Delta \vDash a \equiv b : A/R} \quad \text{E\_TRANS}$$

$$\frac{\begin{array}{l}\Gamma; \Delta \vDash a \equiv b : A/R_1 \\ R_1 \leq R_2\end{array}}{\Gamma; \Delta \vDash a \equiv b : A/R_2} \quad \text{E\_SUB}$$

$$\frac{\begin{array}{c}\Gamma \vDash a_1 : B/R \\ \Gamma \vDash a_2 : B/R \\ \vDash a_1 > a_2/R\end{array}}{\Gamma; \Delta \vDash a_1 \equiv a_2 : B/R} \quad \text{E\_BETA}$$

$$\frac{\begin{array}{l}\Gamma; \Delta \vDash A_1 \equiv A_2 : \star/R \\ \Gamma, x : A_1/R; \Delta \vDash B_1 \equiv B_2 : \star/R' \\ \Gamma \vDash A_1 : \star/R \\ \Gamma \vDash \Pi^\rho x\!:\!A_1/R \to B_1 : \star/R' \\ \Gamma \vDash \Pi^\rho x\!:\!A_2/R \to B_2 : \star/R'\end{array}}{\Gamma; \Delta \vDash (\Pi^\rho x\!:\!A_1/R \to B_1) \equiv (\Pi^\rho x\!:\!A_2/R \to B_2) : \star/R'} \quad \textsc{E\_PiCong}$$

$$\frac{\begin{array}{l}\Gamma, x : A_1/R; \Delta \vDash b_1 \equiv b_2 : B/R' \\ \Gamma \vDash A_1 : \star/R \\ (\rho = +) \vee (x \notin \mathsf{fv}\ b_1) \\ (\rho = +) \vee (x \notin \mathsf{fv}\ b_2)\end{array}}{\Gamma; \Delta \vDash (\lambda^{R,\rho} x.b_1) \equiv (\lambda^{R,\rho} x.b_2) : (\Pi^\rho x\!:\!A_1/R \to B)/R'} \quad \textsc{E\_AbsCong}$$

$$\frac{\begin{array}{l}\Gamma; \Delta \vDash a_1 \equiv b_1 : (\Pi^+ x\!:\!A/R \to B)/R' \\ \Gamma; \Delta \vDash a_2 \equiv b_2 : A/R\end{array}}{\Gamma; \Delta \vDash a_1\ \ {a_2}^{R,+} \equiv b_1\ \ {b_2}^{R,+} : (B\{a_2/x\})/R'} \quad \textsc{E\_AppCong}$$

$$\frac{\begin{array}{l}\Gamma; \Delta \vDash a_1 \equiv b_1 : (\Pi^- x\!:\!A/R \to B)/R' \\ \Gamma \vDash a : A/R\end{array}}{\Gamma; \Delta \vDash a_1\ \square^{R,-} \equiv b_1\ \square^{R,-} : (B\{a/x\})/R'} \quad \textsc{E\_IAppCong}$$

$$\frac{\Gamma; \Delta \vDash \Pi^\rho x\!:\!A_1/R \to B_1 \equiv \Pi^\rho x\!:\!A_2/R \to B_2 : \star/R'}{\Gamma; \Delta \vDash A_1 \equiv A_2 : \star/R} \quad \textsc{E\_PiFst}$$

$$\frac{\begin{array}{l}\Gamma; \Delta \vDash \Pi^\rho x\!:\!A_1/R \to B_1 \equiv \Pi^\rho x\!:\!A_2/R \to B_2 : \star/R' \\ \Gamma; \Delta \vDash a_1 \equiv a_2 : A_1/R\end{array}}{\Gamma; \Delta \vDash B_1\{a_1/x\} \equiv B_2\{a_2/x\} : \star/R'} \quad \textsc{E\_PiSnd}$$

$$\frac{\begin{array}{l}\Gamma; \Delta \vDash a_1 \sim_{A_1/R} b_1 \equiv a_2 \sim_{A_2/R} b_2 \\ \Gamma, c : a_1 \sim_{A_1/R} b_1; \Delta \vDash A \equiv B : \star/R' \\ \Gamma \vDash a_1 \sim_{A_1/R} b_1\ \ \mathsf{ok} \\ \Gamma \vDash \forall c\!:\!a_1 \sim_{A_1/R} b_1.A : \star/R' \\ \Gamma \vDash \forall c\!:\!a_2 \sim_{A_2/R} b_2.B : \star/R'\end{array}}{\Gamma; \Delta \vDash \forall c\!:\!a_1 \sim_{A_1/R} b_1.A \equiv \forall c\!:\!a_2 \sim_{A_2/R} b_2.B : \star/R'} \quad \textsc{E\_CPiCong}$$

$$\frac{\begin{array}{l}\Gamma, c : \phi_1; \Delta \vDash a \equiv b : B/R \\ \Gamma \vDash \phi_1\ \ \mathsf{ok}\end{array}}{\Gamma; \Delta \vDash (\Lambda c.a) \equiv (\Lambda c.b) : \forall c\!:\!\phi_1.B/R} \quad \textsc{E\_CAbsCong}$$

$$\frac{\begin{array}{l}\Gamma; \Delta \vDash a_1 \equiv b_1 : (\forall c\!:\!(a \sim_{A/R} b).B)/R' \\ \Gamma; \widetilde{\Gamma} \vDash a \equiv b : A/R\end{array}}{\Gamma; \Delta \vDash a_1[\bullet] \equiv b_1[\bullet] : (B\{\bullet/c\})/R'} \quad \textsc{E\_CAppCong}$$

$$\frac{\begin{array}{l}\Gamma; \Delta \vDash \forall c\!:\!(a_1 \sim_{A/R} a_2).B_1 \equiv \forall c\!:\!(a_1' \sim_{A'/R'} a_2').B_2 : \star/R_0 \\ \Gamma; \widetilde{\Gamma} \vDash a_1 \equiv a_2 : A/R \\ \Gamma; \widetilde{\Gamma} \vDash a_1' \equiv a_2' : A'/R'\end{array}}{\Gamma; \Delta \vDash B_1\{\bullet/c\} \equiv B_2\{\bullet/c\} : \star/R_0} \quad \textsc{E\_CPiSnd}$$

$$\frac{\begin{array}{l}\Gamma; \Delta \vDash a \equiv b : A/R \\ \Gamma; \Delta \vDash a \sim_{A/R} b \equiv a' \sim_{A'/R} b'\end{array}}{\Gamma; \Delta \vDash a' \equiv b' : A'/R} \quad \textsc{E\_Cast}$$

$$\frac{\begin{array}{l}\Gamma; \Delta \vDash a \equiv b : A/R_1 \\ \Gamma; \widetilde{\Gamma} \vDash A \equiv B : \star/R_2 \\ R_1 \leq R_2\end{array}}{\Gamma; \Delta \vDash a \equiv b : B/R_2} \quad \textsc{E\_EqConv}$$

$$\frac{\Gamma; \Delta \vDash a \sim_{A/R} b \equiv a' \sim_{A'/R} b'}{\Gamma; \Delta \vDash A \equiv A' : \star/R} \quad \textsc{E\_IsoSnd}$$

$$\frac{\begin{array}{l}\Gamma; \Delta \vDash a_1 \equiv a_2 : A/R_1 \\ \Gamma; \widetilde{\Gamma} \vDash A \equiv B : \star/R_2 \\ \Gamma \vDash B : \star/R_1\end{array}}{\Gamma; \Delta \vDash a_1 \triangleright_{R_2} \bullet \equiv a_2 \triangleright_{R_2} \bullet : B/R_1} \quad \textsc{E\_CastCong}$$

$\boxed{\vDash \Gamma}$ context wellformedness

$$\frac{}{\vDash \varnothing} \quad \textsc{E\_Empty}$$

$$\frac{\begin{array}{l}\vDash \Gamma \\ \Gamma \vDash A : \star/R \\ x \notin \mathsf{dom}\,\Gamma\end{array}}{\vDash \Gamma, x : A/R} \quad \textsc{E\_ConsTm}$$

$$\frac{\begin{array}{l}\vDash \Gamma \\ \Gamma \vDash \phi\ \ \mathsf{ok} \\ c \notin \mathsf{dom}\,\Gamma\end{array}}{\vDash \Gamma, c : \phi} \quad \textsc{E\_ConsCo}$$

$\boxed{\vDash \Sigma}$ signature wellformedness

$$\frac{}{\vDash \varnothing} \quad \textsc{Sig\_Empty}$$

$$\frac{\begin{array}{l}\vDash \Sigma \\ \varnothing \vDash A : \star/R \\ \varnothing \vDash a : A/R' \\ F \notin \mathsf{dom}\,\Sigma \\ R' \leq R\end{array}}{\vDash \Sigma \cup \{F \sim a : A/R'\}} \quad \textsc{Sig\_ConsAx}$$

$\boxed{\Gamma \vdash \phi\ \ \mathsf{ok}}$ prop wellformedness

$$\frac{\begin{array}{l}\Gamma \vdash a : A/R \\ \Gamma \vdash b : B/R \\ |A|R = |B|R\end{array}}{\Gamma \vdash a \sim_{A/R} b\ \ \mathsf{ok}} \quad \textsc{An\_Wff}$$

$\boxed{\Gamma \vdash a : A/R}$ typing

$$\frac{\vdash \Gamma}{\Gamma \vdash \star : \star/R} \quad \textsc{An\_Star}$$

$$\frac{\begin{array}{l}\vdash \Gamma \\ x : A/R \in \Gamma\end{array}}{\Gamma \vdash x : A/R} \quad \textsc{An\_Var}$$

$$\frac{\begin{array}{l}\Gamma, x : A/R \vdash B : \star/R' \\ \Gamma \vdash A : \star/R\end{array}}{\Gamma \vdash \Pi^{\rho} x\!:\!A/R \to B : \star/R'} \quad \textsc{An\_Pi}$$

$$\frac{\begin{array}{l}\Gamma \vdash A : \star/R \\ \Gamma, x : A/R \vdash a : B/R' \\ (\rho = +) \vee (x \notin \mathsf{fv}\ |a|R') \\ R \leq R'\end{array}}{\Gamma \vdash \lambda^{\rho} x\!:\!A/R.a : (\Pi^{\rho} x\!:\!A/R \to B)/R'} \quad \textsc{An\_Abs}$$

$$\frac{\begin{array}{l}\Gamma \vdash b : (\Pi^{\rho} x\!:\!A/R \to B)/R' \\ \Gamma \vdash a : A/R\end{array}}{\Gamma \vdash b\ \ a^{R,\rho} : (B\{a/x\})/R'} \quad \textsc{An\_App}$$

$$\frac{\begin{array}{l}\Gamma \vdash a : A/R \\ \Gamma; \widetilde{\Gamma} \vdash \gamma : A \sim_R B \\ \Gamma \vdash B : \star/R\end{array}}{\Gamma \vdash a \rhd_R \gamma : B/R} \quad \textsc{An\_Conv}$$

$$\frac{\begin{array}{l}\Gamma \vdash \phi\ \ \mathsf{ok} \\ \Gamma, c : \phi \vdash B : \star/R\end{array}}{\Gamma \vdash \forall c\!:\!\phi.B : \star/R} \quad \textsc{An\_CPi}$$

$$\frac{\begin{array}{l}\Gamma \vdash \phi\ \ \mathsf{ok} \\ \Gamma, c : \phi \vdash a : B/R\end{array}}{\Gamma \vdash \Lambda c\!:\!\phi.a : (\forall c\!:\!\phi.B)/R} \quad \textsc{An\_CAbs}$$

$$\frac{\begin{array}{l}\Gamma \vdash a_1 : (\forall c\!:\!a \sim_{A_1/R} b.B)/R' \\ \Gamma; \widetilde{\Gamma} \vdash \gamma : a \sim_R b\end{array}}{\Gamma \vdash a_1[\gamma] : B\{\gamma/c\}/R'} \quad \textsc{An\_CApp}$$

$$\frac{\begin{array}{l}\vdash \Gamma \\ F\ \sim a : A/R\ \in\ \Sigma_1 \\ \varnothing \vdash A : \star/R\end{array}}{\Gamma \vdash F : A/R} \quad \textsc{An\_Fam}$$

$$\frac{\begin{array}{l}R_1 \leq R_2 \\ \Gamma \vdash a : A/R_1\end{array}}{\Gamma \vdash \mathbf{sub}\, R_1\ a : A/R_2} \quad \textsc{An\_SubRole}$$

$\boxed{\Gamma; \Delta \vdash \gamma : \phi_1 \sim \phi_2}$ coercion between props

$$\frac{\begin{array}{l}\Gamma; \Delta \vdash \gamma_1 : A_1 \sim_R A_2 \\ \Gamma; \Delta \vdash \gamma_2 : B_1 \sim_R B_2 \\ \Gamma \vdash A_1 \sim_{A/R} B_1\ \ \mathsf{ok} \\ \Gamma \vdash A_2 \sim_{A/R} B_2\ \ \mathsf{ok}\end{array}}{\Gamma; \Delta \vdash (\gamma_1 \sim_A \gamma_2) : (A_1 \sim_{A/R} B_1) \sim (A_2 \sim_{A/R} B_2)} \quad \textsc{An\_PropCong}$$

$$\frac{\Gamma; \Delta \vdash \gamma : \forall c\!:\!\phi_1.A_2 \sim_R \forall c\!:\!\phi_2.B_2}{\Gamma; \Delta \vdash \mathbf{cpiFst}\, \gamma : \phi_1 \sim \phi_2} \quad \textsc{An\_CPiFst}$$

$$\frac{\Gamma; \Delta \vdash \gamma : \phi_1 \sim \phi_2}{\Gamma; \Delta \vdash \mathbf{sym}\, \gamma : \phi_2 \sim \phi_1} \quad \textsc{An\_IsoSym}$$

$$\frac{\begin{array}{l}\Gamma; \Delta \vdash \gamma : A \sim_R B \\ \Gamma \vdash a_1 \sim_{A/R} a_2\ \ \mathsf{ok} \\ \Gamma \vdash a_1' \sim_{B/R} a_2'\ \ \mathsf{ok} \\ |a_1|R = |a_1'|R \\ |a_2|R = |a_2'|R\end{array}}{\Gamma; \Delta \vdash \mathbf{conv}\ (a_1 \sim_{A/R} a_2) \sim_{\gamma} (a_1' \sim_{B/R} a_2') : (a_1 \sim_{A/R} a_2) \sim (a_1' \sim_{B/R} a_2')} \quad \textsc{An\_IsoConv}$$

$\boxed{\Gamma; \Delta \vdash \gamma : A \sim_R B}$ coercion between types

$$\frac{\begin{array}{l} \vdash \Gamma \\ c : a \sim_{A/R} b \in \Gamma \\ c \in \Delta \end{array}}{\Gamma; \Delta \vdash c : a \sim_R b} \quad \textsc{An\_Assn}$$

$$\frac{\Gamma \vdash a : A/R}{\Gamma; \Delta \vdash \mathbf{refl}\, a : a \sim_R a} \quad \textsc{An\_Refl}$$

$$\frac{\begin{array}{l} \Gamma \vdash a : A/R \\ \Gamma \vdash b : B/R \\ |a|R = |b|R \\ \Gamma; \widetilde{\Gamma} \vdash \gamma : A \sim_R B \end{array}}{\Gamma; \Delta \vdash (a \mathrel{|{=}|_{\gamma}} b) : a \sim_R b} \quad \textsc{An\_EraseEq}$$

$$\frac{\begin{array}{l} \Gamma \vdash b : B/R \\ \Gamma \vdash a : A/R \\ \Gamma; \widetilde{\Gamma} \vdash \gamma_1 : B \sim_R A \\ \Gamma; \Delta \vdash \gamma : b \sim_R a \end{array}}{\Gamma; \Delta \vdash \mathbf{sym}\, \gamma : a \sim_R b} \quad \textsc{An\_Sym}$$

$$\frac{\begin{array}{l} \Gamma; \Delta \vdash \gamma_1 : a \sim_R a_1 \\ \Gamma; \Delta \vdash \gamma_2 : a_1 \sim_R b \\ \Gamma \vdash a : A/R \\ \Gamma \vdash a_1 : A_1/R \\ \Gamma; \widetilde{\Gamma} \vdash \gamma_3 : A \sim_R A_1 \end{array}}{\Gamma; \Delta \vdash (\gamma_1; \gamma_2) : a \sim_R b} \quad \textsc{An\_Trans}$$

$$\frac{\begin{array}{l} \Gamma \vdash a_1 : B_0/R \\ \Gamma \vdash a_2 : B_1/R \\ |B_0|R = |B_1|R \\ \vDash |a_1|R > |a_2|R/R \end{array}}{\Gamma; \Delta \vdash \mathbf{red}\, a_1\, a_2 : a_1 \sim_R a_2} \quad \textsc{An\_Beta}$$

$$\frac{\begin{array}{l} \Gamma; \Delta \vdash \gamma_1 : A_1 \sim_{R'} A_2 \\ \Gamma, x : A_1/R; \Delta \vdash \gamma_2 : B_1 \sim_{R'} B_2 \\ B_3 = B_2\{x \rhd_{R'} \mathbf{sym}\, \gamma_1/x\} \\ \Gamma \vdash \Pi^\rho x\!:\!A_1/R \to B_1 : \star/R' \\ \Gamma \vdash \Pi^\rho x\!:\!A_1/R \to B_2 : \star/R' \\ \Gamma \vdash \Pi^\rho x\!:\!A_2/R \to B_3 : \star/R' \\ R \leq R' \end{array}}{\Gamma; \Delta \vdash \Pi^{R,\rho} x\!:\!\gamma_1.\gamma_2 : (\Pi^\rho x\!:\!A_1/R \to B_1) \sim_{R'} (\Pi^\rho x\!:\!A_2/R \to B_3)} \quad \textsc{An\_PiCong}$$

$$\frac{\begin{array}{l} \Gamma; \Delta \vdash \gamma_1 : A_1 \sim_R A_2 \\ \Gamma, x : A_1/R; \Delta \vdash \gamma_2 : b_1 \sim_{R'} b_2 \\ b_3 = b_2\{x \rhd_{R'} \mathbf{sym}\, \gamma_1/x\} \\ \Gamma \vdash A_1 : \star/R \\ \Gamma \vdash A_2 : \star/R \\ (\rho = +) \vee (x \notin \mathsf{fv}\ |b_1|R') \\ (\rho = +) \vee (x \notin \mathsf{fv}\ |b_3|R') \\ \Gamma \vdash (\lambda^\rho x\!:\!A_1/R.b_2) : B/R' \\ R \leq R' \end{array}}{\Gamma; \Delta \vdash (\lambda^{R,\rho} x\!:\!\gamma_1.\gamma_2) : (\lambda^\rho x\!:\!A_1/R.b_1) \sim_{R'} (\lambda^\rho x\!:\!A_2/R.b_3)} \quad \textsc{An\_AbsCong}$$

$$\frac{\begin{array}{c}\Gamma;\Delta \vdash \gamma_1 : a_1 \sim_{R'} b_1 \\ \Gamma;\Delta \vdash \gamma_2 : a_2 \sim_{R} b_2 \\ \Gamma \vdash a_1\ a_2{}^{R,\rho} : A/R' \\ \Gamma \vdash b_1\ b_2{}^{R,\rho} : B/R' \\ \Gamma;\widetilde{\Gamma} \vdash \gamma_3 : A \sim_{R'} B\end{array}}{\Gamma;\Delta \vdash \gamma_1\ \gamma_2^{R,\rho} : a_1\ a_2{}^{R,\rho} \sim_{R'} b_1\ b_2{}^{R,\rho}}\ \textsc{An\_AppCong}$$

$$\frac{\Gamma;\Delta \vdash \gamma : \Pi^\rho x\!:\!A_1/R \to B_1 \sim_{R'} \Pi^\rho x\!:\!A_2/R \to B_2}{\Gamma;\Delta \vdash \mathbf{piFst}\,\gamma : A_1 \sim_R A_2}\ \textsc{An\_PiFst}$$

$$\frac{\begin{array}{l}\Gamma;\Delta \vdash \gamma_1 : \Pi^\rho x\!:\!A_1/R \to B_1 \sim_{R'} \Pi^\rho x\!:\!A_2/R \to B_2 \\ \Gamma;\Delta \vdash \gamma_2 : a_1 \sim_R a_2 \\ \Gamma \vdash a_1 : A_1/R \\ \Gamma \vdash a_2 : A_2/R\end{array}}{\Gamma;\Delta \vdash \gamma_1 @ \gamma_2 : B_1\{a_1/x\} \sim_{R'} B_2\{a_2/x\}}\ \textsc{An\_PiSnd}$$

$$\frac{\begin{array}{l}\Gamma;\Delta \vdash \gamma_1 : a_1 \sim_{A_1/R} b_1 \sim a_2 \sim_{A_2/R} b_2 \\ \Gamma, c : a_1 \sim_{A_1/R} b_1;\Delta \vdash \gamma_3 : B_1 \sim_{R'} B_2 \\ B_3 = B_2\{c \triangleright_{R'} \mathbf{sym}\,\gamma_1/c\} \\ \Gamma \vdash \forall c\!:\!a_1 \sim_{A_1/R} b_1.B_1 : \star/R' \\ \Gamma \vdash \forall c\!:\!a_2 \sim_{A_2/R} b_2.B_3 : \star/R' \\ \Gamma \vdash \forall c\!:\!a_1 \sim_{A_1/R} b_1.B_2 : \star/R'\end{array}}{\Gamma;\Delta \vdash (\forall c\!:\!\gamma_1.\gamma_3) : (\forall c\!:\!a_1 \sim_{A_1/R} b_1.B_1) \sim_R (\forall c\!:\!a_2 \sim_{A_2/R} b_2.B_3)}\ \textsc{An\_CPiCong}$$

$$\frac{\begin{array}{l}\Gamma;\Delta \vdash \gamma_1 : b_0 \sim_{A_1/R} b_1 \sim b_2 \sim_{A_2/R} b_3 \\ \Gamma, c : b_0 \sim_{A_1/R} b_1;\Delta \vdash \gamma_3 : a_1 \sim_{R'} a_2 \\ a_3 = a_2\{c \triangleright_{R'} \mathbf{sym}\,\gamma_1/c\} \\ \Gamma \vdash (\Lambda c\!:\!b_0 \sim_{A_1/R} b_1.a_1) : \forall c\!:\!b_0 \sim_{A_1/R} b_1.B_1/R' \\ \Gamma \vdash (\Lambda c\!:\!b_0 \sim_{A_1/R} b_1.a_2) : B/R' \\ \Gamma \vdash (\Lambda c\!:\!b_2 \sim_{A_2/R} b_3.a_3) : \forall c\!:\!b_2 \sim_{A_2/R} b_3.B_2/R' \\ \Gamma;\widetilde{\Gamma} \vdash \gamma_4 : \forall c\!:\!b_0 \sim_{A_1/R} b_1.B_1 \sim_{R'} \forall c\!:\!\phi_2.B_2\end{array}}{\Gamma;\Delta \vdash (\lambda c\!:\!\gamma_1.\gamma_3 @ \gamma_4) : (\Lambda c\!:\!b_0 \sim_{A_1/R} b_1.a_1) \sim_{R'} (\Lambda c\!:\!b_2 \sim_{A_2/R} b_3.a_3)}\ \textsc{An\_CAbsCong}$$

$$\frac{\begin{array}{l}\Gamma;\Delta \vdash \gamma_1 : a_1 \sim_R b_1 \\ \Gamma;\widetilde{\Gamma} \vdash \gamma_2 : a_2 \sim_{R'} b_2 \\ \Gamma;\widetilde{\Gamma} \vdash \gamma_3 : a_3 \sim_{R'} b_3 \\ \Gamma \vdash a_1[\gamma_2] : A/R \\ \Gamma \vdash b_1[\gamma_3] : B/R \\ \Gamma;\widetilde{\Gamma} \vdash \gamma_4 : A \sim_R B\end{array}}{\Gamma;\Delta \vdash \gamma_1(\gamma_2,\gamma_3) : a_1[\gamma_2] \sim_R b_1[\gamma_3]}\ \textsc{An\_CAppCong}$$

$$\frac{\begin{array}{l}\Gamma;\Delta \vdash \gamma_1 : (\forall c_1\!:\!a \sim_{A/R} a'.B_1) \sim_{R_0} (\forall c_2\!:\!b \sim_{B/R'} b'.B_2) \\ \Gamma;\widetilde{\Gamma} \vdash \gamma_2 : a \sim_R a' \\ \Gamma;\widetilde{\Gamma} \vdash \gamma_3 : b \sim_{R'} b'\end{array}}{\Gamma;\Delta \vdash \gamma_1 @ (\gamma_2 \sim \gamma_3) : B_1\{\gamma_2/c_1\} \sim_{R_0} B_2\{\gamma_3/c_2\}}\ \textsc{An\_CPiSnd}$$

$$\frac{\begin{array}{l}\Gamma;\Delta \vdash \gamma_1 : a \sim_{R_1} a' \\ \Gamma;\Delta \vdash \gamma_2 : a \sim_{A/R_1} a' \sim b \sim_{B/R_1} b'\end{array}}{\Gamma;\Delta \vdash \gamma_1 \triangleright_{R_1} \gamma_2 : b \sim_{R_1} b'}\ \textsc{An\_Cast}$$

$$\frac{\Gamma;\Delta \vdash \gamma : (a \sim_{A/R} a') \sim (b \sim_{B/R} b')}{\Gamma;\Delta \vdash \mathbf{isoSnd}\,\gamma : A \sim_R B}\ \textsc{An\_IsoSnd}$$

$$\frac{\begin{array}{c}\Gamma; \Delta \vdash \gamma : a \sim_{R_1} b \\ R_1 \leq R_2\end{array}}{\Gamma; \Delta \vdash \mathbf{sub}\, \gamma : a \sim_{R_2} b} \quad \text{AN\_SUB}$$

$\boxed{\vdash \Gamma}$ context wellformedness

$$\frac{}{\vdash \varnothing} \quad \text{AN\_EMPTY}$$

$$\frac{\begin{array}{c}\vdash \Gamma \\ \Gamma \vdash A : \star / R \\ x \notin \mathsf{dom}\, \Gamma\end{array}}{\vdash \Gamma, x : A / R} \quad \text{AN\_CONSTM}$$

$$\frac{\begin{array}{c}\vdash \Gamma \\ \Gamma \vdash \phi \ \mathsf{ok} \\ c \notin \mathsf{dom}\, \Gamma\end{array}}{\vdash \Gamma, c : \phi} \quad \text{AN\_CONSCO}$$

$\boxed{\vdash \Sigma}$ signature wellformedness

$$\frac{}{\vdash \varnothing} \quad \text{AN\_SIG\_EMPTY}$$

$$\frac{\begin{array}{c}\vdash \Sigma \\ \varnothing \vdash A : \star / R \\ \varnothing \vdash a : A / R \\ F \notin \mathsf{dom}\, \Sigma\end{array}}{\vdash \Sigma \cup \{F \sim a : A / R\}} \quad \text{AN\_SIG\_CONSAX}$$

$\boxed{\Gamma \vdash a \rightsquigarrow b / R}$ single-step, weak head reduction to values for annotated language

$$\frac{\Gamma \vdash a \rightsquigarrow a' / R_1}{\Gamma \vdash a\ b^{R,\rho} \rightsquigarrow a'\ b^{R,\rho} / R_1} \quad \text{AN\_APPLEFT}$$

$$\frac{\mathsf{Value}_R\ (\lambda^\rho x\!:\!A/R.w)}{\Gamma \vdash (\lambda^\rho x\!:\!A/R.w)\ a^{R,\rho} \rightsquigarrow w\{a/x\} / R} \quad \text{AN\_APPABS}$$

$$\frac{\Gamma \vdash a \rightsquigarrow a' / R}{\Gamma \vdash a[\gamma] \rightsquigarrow a'[\gamma] / R} \quad \text{AN\_CAPPLEFT}$$

$$\frac{}{\Gamma \vdash (\Lambda c\!:\!\phi.b)[\gamma] \rightsquigarrow b\{\gamma/c\} / R} \quad \text{AN\_CAPPCABS}$$

$$\frac{\begin{array}{c}\Gamma \vdash A : \star / R \\ \Gamma, x : A / R \vdash b \rightsquigarrow b' / R_1\end{array}}{\Gamma \vdash (\lambda^- x\!:\!A/R.b) \rightsquigarrow (\lambda^- x\!:\!A/R.b') / R_1} \quad \text{AN\_ABSTERM}$$

$$\frac{F \sim a : A / R \in \Sigma_1}{\Gamma \vdash F \rightsquigarrow a / R} \quad \text{AN\_AXIOM}$$

$$\frac{\Gamma \vdash a \rightsquigarrow a' / R}{\Gamma \vdash a \triangleright_{R_1} \gamma \rightsquigarrow a' \triangleright_{R_1} \gamma / R} \quad \text{AN\_CONVTERM}$$

$$\frac{\mathsf{Value}_R\ v}{\Gamma \vdash (v \triangleright_{R_2} \gamma_1) \triangleright_{R_2} \gamma_2 \rightsquigarrow v \triangleright_{R_2} (\gamma_1 ; \gamma_2) / R} \quad \text{AN\_COMBINE}$$

$$\frac{\begin{array}{l}\mathsf{Value}_R\ v\\ \Gamma;\widetilde{\Gamma}\vdash\gamma:\Pi^\rho x_1\!:\!A_1/R\to B_1\sim_{R'}\Pi^\rho x_2\!:\!A_2/R\to B_2\\ b'=b\rhd_{R'}\mathbf{sym}\,(\mathbf{piFst}\,\gamma)\\ \gamma'=\gamma@(b'\ |{=}|_{(\mathbf{piFst}\,\gamma)}\ b)\end{array}}{\Gamma\vdash(v\rhd_{R'}\gamma)\ b^{R,\rho}\leadsto((v\ b'^{R,\rho})\rhd_{R'}\gamma')/R}\quad\textsc{An\_Push}$$

$$\frac{\begin{array}{l}\mathsf{Value}_R\ v\\ \Gamma;\widetilde{\Gamma}\vdash\gamma:\forall c_1\!:\!a_1\sim_{B_1/R}b_1.A_1\sim_{R'}\forall c_2\!:\!a_2\sim_{B_2/R}b_2.A_2\\ \gamma_1'=\gamma_1\rhd_{R'}\mathbf{sym}\,(\mathbf{cpiFst}\,\gamma)\\ \gamma'=\gamma@(\gamma_1'\sim\gamma_1)\end{array}}{\Gamma\vdash(v\rhd_{R'}\gamma)[\gamma_1]\leadsto((v[\gamma_1'])\rhd_{R'}\gamma')/R}\quad\textsc{An\_CPush}$$

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Definition rules:        174 good    0 bad
Definition rule clauses: 503 good    0 bad
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