$tmvar$, x, y, f, m, n	variables
$covar$, c	coercion variables
$datacon$, K	
$const$, T	
$tyfam$, F	
$index$, i	indices

$role,\ R$	$::=$			Role
	$\|$	$\mathbf{Nom}$		
	$\|$	$\mathbf{Rep}$		
	$\|$	$R_1 \cap R_2$	S	

$relflag,\ \rho$	$::=$			relevance flag
	$\|$	$+$		
	$\|$	$-$		

$constraint,\ \phi$	$::=$			props
	$\|$	$a \sim_{A/R} b$		
	$\|$	(ϕ)	S	
	$\|$	$\phi\{b/x\}$	S	
	$\|$	$\|\phi\|$	S	

$tm,\ a,\ b,\ v,\ w,\ A,\ B$	$::=$			types and kinds
	$\|$	$\star$		
	$\|$	x		
	$\|$	$\lambda^{\rho} x : A/R.b$	bind x in b	
	$\|$	$\lambda^{R,\rho} x.b$	bind x in b	
	$\|$	$a\ b^{R,\rho}$		
	$\|$	F		
	$\|$	T		
	$\|$	$\Pi^{\rho} x : A/R \to B$	bind x in B	
	$\|$	$a \triangleright_R \gamma$		
	$\|$	$\forall c : \phi.B$	bind c in B	
	$\|$	$\Lambda c : \phi.b$	bind c in b	
	$\|$	$\Lambda c.b$	bind c in b	
	$\|$	$a[\gamma]$		
	$\|$	$\square$		
	$\|$	K		
	$\|$	$\mathbf{match}\ a\ \mathbf{with}\ brs$		
	$\|$	$\mathbf{sub}\ R\ a$		
	$\|$	$a\{b/x\}$	S	
	$\|$	a	S	
	$\|$	$a\{\gamma/c\}$	S	
	$\|$	a	S	
	$\|$	(a)	S	
	$\|$	a	S	parsing precedence is hard
	$\|$	$\|a\|R$	S	
	$\|$	$\mathbf{Int}$	S	
	$\|$	$\mathbf{Bool}$	S	
	$\|$	$\mathbf{Nat}$	S	
	$\|$	$\mathbf{Vec}$	S	
	$\|$	0	S	
	$\|$	S	S	
	$\|$	$\mathbf{True}$	S	

	Fix	S
	$a \rightarrow b$	S
	$\phi \Rightarrow A$	S
	$ab^{R,+}$	S
	$\lambda^R x.a$	S
	$\lambda x : A.a$	S
	$\forall\, x : A/R \rightarrow B$	S

brs ::=   case branches
	none	
	$K \Rightarrow a; brs$	
	$brs\{a/x\}$	S
	$brs\{\gamma/c\}$	S
	(brs)	S

$co,\ \gamma$ ::=   explicit coercions
	$\bullet$	
	c	
	red $a\ b$	
	refl a	
	$(a \mathrel{|{=}|}_\gamma b)$	
	$\mathbf{sym}\,\gamma$	
	$\gamma_1; \gamma_2$	
	$\mathbf{sub}\,\gamma$	
	$\Pi^{R,\rho} x : \gamma_1.\gamma_2$	bind x in γ_2
	$\lambda^{R,\rho} x : \gamma_1.\gamma_2$	bind x in γ_2
	$\gamma_1\ \gamma_2^{R,\rho}$	
	$\mathbf{piFst}\,\gamma$	
	$\mathbf{cpiFst}\,\gamma$	
	$\mathbf{isoSnd}\,\gamma$	
	$\gamma_1 @ \gamma_2$	
	$\forall c : \gamma_1.\gamma_3$	bind c in γ_3
	$\lambda c : \gamma_1.\gamma_3 @ \gamma_4$	bind c in γ_3
	$\gamma(\gamma_1, \gamma_2)$	
	$\gamma @ (\gamma_1 \sim \gamma_2)$	
	$\gamma_1 \triangleright_R \gamma_2$	
	$\gamma_1 \sim_A \gamma_2$	
	$\mathbf{conv}\ \phi_1 \sim_\gamma \phi_2$	
	$\mathbf{eta}\,a$	
	$\mathbf{left}\,\gamma\,\gamma'$	
	$\mathbf{right}\,\gamma\,\gamma'$	
	(γ)	S
	γ	S
	$\gamma\{a/x\}$	S

$sort$ ::=   binding classifier
| | $\mathbf{Tm}\,A\,R$ | |

| **Co** ϕ

sig_sort ::=   signature classifier
| **Cs** A
| **Ax** $a\,A\,R$

context, Γ ::=   contexts
| $\varnothing$
| $\Gamma, x : A/R$
| $\Gamma, c : \phi$
| $\Gamma\{b/x\}$ M
| $\Gamma\{\gamma/c\}$ M
| Γ, Γ' M
| $|\Gamma|$ M
| (Γ) M
| Γ M

sig, Σ ::=   signatures
| $\varnothing$
| $\Sigma \cup \{T : A/R\}$
| $\Sigma \cup \{F \sim a : A/R\}$
| Σ_0 M
| Σ_1 M
| $|\Sigma|$ M

available_props, Δ ::=
| $\varnothing$
| Δ, c
| $\widetilde{\Gamma}$ M
| (Δ) M

role_context, Ω ::=   role$_c$ontexts
| $\varnothing$
| $\Omega, x : R$
| (Ω) M
| Ω M

terminals ::=
| $\leftrightarrow$
| $\Leftrightarrow$
| $\longrightarrow$
| min
| $\equiv$
| $\forall$
| $\in$
| $\notin$
| $\Leftarrow$

| $\Rightarrow$
| $\Rightarrow^*$
| $\rightarrow$
| Λ
|
| $\Box$
| $\vdash$
| $\dashv$
| $\vDash$
| $\vDash$
| $\neq$
| $\rhd$
| ok
|
| -
| $\rightsquigarrow$
| $\rightsquigarrow^*$
| $\rightsquigarrow$
| $\varnothing$
| $\circ$
| fv
| dom
| $\sim$
| $\asymp$
| $|$
| $\bullet$
| **fst**
| **snd**
| $| \Rightarrow |$
| $\vdash_=$
| $\mathbf{refl_2}$
| $++$

$formula,\ \psi \qquad ::=$

| $judgement$
| $x : A/R \in \Gamma$
| $x : R \in \Omega$
| $c : \phi \in \Gamma$
| $T : A/R \in \Sigma$
| $F \sim a : A/R \in \Sigma$
| $K : T\, \Gamma \in \Sigma$
| $x \in \Delta$
| $c \in \Delta$
| $c\,\mathbf{not\,relevant} \in \gamma$
| $x \notin \mathsf{fv}\, a$
| $x \notin \mathsf{dom}\,\Gamma$
| $rctx\_uniq\,\Omega$

	$c \notin \text{dom}\,\Gamma$	
	$T \notin \text{dom}\,\Sigma$	
	$F \notin \text{dom}\,\Sigma$	
	$a = b$	
	$\phi_1 = \phi_2$	
	$\Gamma_1 = \Gamma_2$	
	$\gamma_1 = \gamma_2$	
	$\neg\psi$	
	$\psi_1 \wedge \psi_2$	
	$\psi_1 \vee \psi_2$	
	$\psi_1 \Rightarrow \psi_2$	
	(ψ)	
	ψ	
	$c : (a : A \sim b : B) \in \Gamma$	
		suppress lc hypothesis generated by Ott

| *JSubRole* | ::= | | |
| | | $R_1 \leq R_2$ | Subroling judgement |

JValue	::=		
		CoercedValue $R\,A$	Values with at most one coercion at the top
		$\mathsf{Value}_R\ A$	values
		ValueType $R\,A$	Types with head forms (erased language)

| *Jconsistent* | ::= | | |
| | | **consistent** $a\,b\,R$ | (erased) types do not differ in their heads |

| *Jerased* | ::= | | |
| | | $\Omega \vDash erased\_tm\,a\,R$ | |

| *JChk* | ::= | | |
| | | $(\rho = +) \vee (x \notin \mathsf{fv}\,A)$ | irrelevant argument check |

Jpar	::=		
		$\Omega \vDash a \Rightarrow_R b$	parallel reduction (implicit language)
		$\Omega \vdash a \Rightarrow_R^* b$	multistep parallel reduction
		$\Omega \vdash a \Leftrightarrow_R b$	parallel reduction to a common term

Jbeta	::=		
		$\vDash a > b/R$	primitive reductions on erased terms
		$\vDash a \rightsquigarrow b/R$	single-step head reduction for implicit language
		$\vDash a \rightsquigarrow^* b/R$	multistep reduction

Jett	::=		
		$\Gamma \vDash \phi\ \mathsf{ok}$	Prop wellformedness
		$\Gamma \vDash a : A/R$	typing
		$\Gamma; \Delta \vDash \phi_1 \equiv \phi_2$	prop equality

| | $\Gamma; \Delta \vDash a \equiv b : A/R$ | definitional equality |
| | $\vDash \Gamma$ | context wellformedness |

| $Jsig$ | $::=$ | |
| | $\vDash \Sigma$ | signature wellformedness |

$Jann$	$::=$	
	$\Gamma \vdash \phi$ ok	prop wellformedness
	$\Gamma \vdash a : A/R$	typing
	$\Gamma; \Delta \vdash \gamma : \phi_1 \sim \phi_2$	coercion between props
	$\Gamma; \Delta \vdash \gamma : A \sim_R B$	coercion between types
	$\vdash \Gamma$	context wellformedness
	$\vdash \Sigma$	signature wellformedness

| $Jred$ | $::=$ | |
| | $\Gamma \vdash a \rightsquigarrow b/R$ | single-step, weak head reduction to values for annotated lang |

$judgement$	$::=$	
	$JSubRole$	
	$JValue$	
	$Jconsistent$	
	$Jerased$	
	$JChk$	
	$Jpar$	
	$Jbeta$	
	$Jett$	
	$Jsig$	
	$Jann$	
	$Jred$	

$user\_syntax$	$::=$	
	$tmvar$	
	$covar$	
	$datacon$	
	$const$	
	$tyfam$	
	$index$	
	$role$	
	$relflag$	
	$constraint$	
	tm	
	brs	
	co	
	$sort$	
	sig_sort	
	$context$	
	sig	

| *available_props*
| *role_context*
| *terminals*
| *formula*

$\boxed{R_1 \leq R_2}$ Subroling judgement

$$\frac{}{\mathbf{Nom} \leq \mathbf{Rep}} \quad \text{NomRep}$$

$$\frac{}{R \leq R} \quad \text{Refl}$$

$$\frac{\begin{array}{c} R_1 \leq R_2 \\ R_2 \leq R_3 \end{array}}{R_1 \leq R_3} \quad \text{Trans}$$

$\boxed{\mathbf{CoercedValue}\, R\, A}$ Values with at most one coercion at the top

$$\frac{\mathsf{Value}_R\, a}{\mathbf{CoercedValue}\, R\, a} \quad \text{CV}$$

$$\frac{\begin{array}{c} \mathsf{Value}_R\, a \\ \neg(R_1 \leq R) \end{array}}{\mathbf{CoercedValue}\, R\, (a \triangleright_{R_1} \gamma)} \quad \text{CC}$$

$\boxed{\mathsf{Value}_R\, A}$ values

$$\frac{}{\mathsf{Value}_R\, \star} \quad \text{Value\_Star}$$

$$\frac{}{\mathsf{Value}_R\, \Pi^\rho x : A/R_1 \to B} \quad \text{Value\_Pi}$$

$$\frac{}{\mathsf{Value}_R\, \forall c : \phi.B} \quad \text{Value\_CPi}$$

$$\frac{}{\mathsf{Value}_R\, \lambda^+ x : A/R_1.a} \quad \text{Value\_AbsRel}$$

$$\frac{}{\mathsf{Value}_R\, \lambda^{R_1,+} x.a} \quad \text{Value\_UAbsRel}$$

$$\frac{\mathbf{CoercedValue}\, R\, a}{\mathsf{Value}_R\, \lambda^{R_1,-} x.a} \quad \text{Value\_UAbsIrrel}$$

$$\frac{\mathbf{CoercedValue}\, R\, a}{\mathsf{Value}_R\, \lambda^- x : A/R_1.a} \quad \text{Value\_AbsIrrel}$$

$$\frac{}{\mathsf{Value}_R\, \Lambda c : \phi.a} \quad \text{Value\_CAbs}$$

$$\frac{}{\mathsf{Value}_R\, \Lambda c.a} \quad \text{Value\_UCAbs}$$

$$\frac{\begin{array}{c} F \sim a : A/R_1 \in \Sigma_0 \\ \neg(R_1 \leq R) \end{array}}{\mathsf{Value}_R\, F} \quad \text{Value\_Ax}$$

$\boxed{\mathbf{ValueType}\, R\, A}$ Types with head forms (erased language)

$$\frac{}{\mathbf{ValueType}\, R\, \star} \quad \text{value\_type\_Star}$$

$$\overline{\textbf{ValueType}\, R\, \Pi^\rho x : A/R_1 \rightarrow B} \quad \text{VALUE\_TYPE\_PI}$$

$$\overline{\textbf{ValueType}\, R\, \forall c : \phi.B} \quad \text{VALUE\_TYPE\_CPI}$$

$$\frac{F \sim a : A/R_1 \,\in\, \Sigma_0 \quad \neg(R_1 \leq R)}{\textbf{ValueType}\, R\, F} \quad \text{VALUE\_TYPE\_AX}$$

$\boxed{\textbf{consistent}\, a\, b\, R}$ (erased) types do not differ in their heads

$$\overline{\textbf{consistent}\, \star\, \star\, R} \quad \text{CONSISTENT\_A\_STAR}$$

$$\overline{\textbf{consistent}\, (\Pi^\rho x_1 : A_1/R \rightarrow B_1)\, (\Pi^\rho x_2 : A_2/R \rightarrow B_2)\, R'} \quad \text{CONSISTENT\_A\_PI}$$

$$\overline{\textbf{consistent}\, (\forall c_1 : \phi_1.A_1)\, (\forall c_2 : \phi_2.A_2)\, R} \quad \text{CONSISTENT\_A\_CPI}$$

$$\overline{\textbf{consistent}\, F\, F\, R'} \quad \text{CONSISTENT\_A\_FAM}$$

$$\frac{\neg \textbf{ValueType}\, R\, b}{\textbf{consistent}\, a\, b\, R} \quad \text{CONSISTENT\_A\_STEP\_R}$$

$$\frac{\neg \textbf{ValueType}\, R\, a}{\textbf{consistent}\, a\, b\, R} \quad \text{CONSISTENT\_A\_STEP\_L}$$

$\boxed{\Omega \vDash erased\_tm\, a\, R}$

$$\frac{rctx\_uniq\Omega}{\Omega \vDash erased\_tm \,\square\, R} \quad \text{ERASED\_A\_BULLET}$$

$$\frac{rctx\_uniq\Omega}{\Omega \vDash erased\_tm \,\star\, R} \quad \text{ERASED\_A\_STAR}$$

$$\frac{rctx\_uniq\Omega \quad x : R \,\in\, \Omega \quad R \leq R_1}{\Omega \vDash erased\_tm\, x\, R_1} \quad \text{ERASED\_A\_VAR}$$

$$\frac{\Omega, x : R_1 \vDash erased\_tm\, a\, R}{\Omega \vDash erased\_tm\, (\lambda^{R_1,\rho} x.a)\, R} \quad \text{ERASED\_A\_ABS}$$

$$\frac{\Omega \vDash erased\_tm\, a\, R \quad \Omega \vDash erased\_tm\, b\, R_1}{\Omega \vDash erased\_tm\, (a\ b^{R_1,\rho})\, R} \quad \text{ERASED\_A\_APP}$$

$$\frac{\Omega \vDash erased\_tm\, A\, R_1 \quad \Omega, x : R_1 \vDash erased\_tm\, B\, R}{\Omega \vDash erased\_tm\, (\Pi^\rho x : A/R_1 \rightarrow B)\, R} \quad \text{ERASED\_A\_PI}$$

$$\frac{\Omega \vDash erased\_tm\, a\, R_1 \quad \Omega \vDash erased\_tm\, b\, R_1 \quad \Omega \vDash erased\_tm\, A\, R_1 \quad \Omega \vDash erased\_tm\, B\, R}{\Omega \vDash erased\_tm\, (\forall c : a \sim_{A/R_1} b.B)\, R} \quad \text{ERASED\_A\_CPI}$$

$$\frac{\Omega \vDash erased\_tm\, b\, R}{\Omega \vDash erased\_tm\, (\Lambda c.b)\, R} \quad \text{ERASED\_A\_CABS}$$

$$\frac{\Omega \vDash erased\_tm\ a\ R}{\Omega \vDash erased\_tm\ (a[\bullet])\ R} \quad \text{ERASED\_A\_CAPP}$$

$$\frac{\begin{array}{l} rctx\_uniq\Omega \\ F \sim a : A/R\ \in\ \Sigma_0 \\ R \leq R_1 \end{array}}{\Omega \vDash erased\_tm\ F\ R_1} \quad \text{ERASED\_A\_FAM}$$

$$\frac{rctx\_uniq\Omega}{\Omega \vDash erased\_tm\ T\ R} \quad \text{ERASED\_A\_CONST}$$

$$\frac{\Omega \vDash erased\_tm\ a\ R}{\Omega \vDash erased\_tm\ (a \triangleright_{R_1} \bullet)\ R} \quad \text{ERASED\_A\_CONV}$$

$\boxed{(\rho = +) \vee (x \notin \mathsf{fv}\ A)}$ irrelevant argument check

$$\frac{}{(+ = +) \vee (x \notin \mathsf{fv}\ A)} \quad \text{RHO\_REL}$$

$$\frac{x\ \notin\ \mathsf{fv} A}{(- = +) \vee (x \notin \mathsf{fv}\ A)} \quad \text{RHO\_IRRREL}$$

$\boxed{\Omega \vDash a \Rightarrow_R b}$ parallel reduction (implicit language)

$$\frac{\Omega \vDash erased\_tm\ a\ R}{\Omega \vDash a \Rightarrow_R a} \quad \text{PAR\_REFL}$$

$$\frac{\begin{array}{l} \Omega \vDash a \Rightarrow_R (\lambda^{R_1,\rho}x.a') \\ \Omega \vDash b \Rightarrow_{R_1} b' \end{array}}{\Omega \vDash a\ b^{R_1,\rho} \Rightarrow_R a'\{b'/x\}} \quad \text{PAR\_BETA}$$

$$\frac{\begin{array}{l} \Omega \vDash a \Rightarrow_R a' \\ \Omega \vDash b \Rightarrow_{R_1} b' \end{array}}{\Omega \vDash a\ b^{R_1,\rho} \Rightarrow_R a'\ b'^{R_1,\rho}} \quad \text{PAR\_APP}$$

$$\frac{\Omega \vDash a \Rightarrow_R (\Lambda c.a')}{\Omega \vDash a[\bullet] \Rightarrow_R a'\{\bullet/c\}} \quad \text{PAR\_CBETA}$$

$$\frac{\Omega \vDash a \Rightarrow_R a'}{\Omega \vDash a[\bullet] \Rightarrow_R a'[\bullet]} \quad \text{PAR\_CAPP}$$

$$\frac{\Omega, x : R_1 \vDash a \Rightarrow_R a'}{\Omega \vDash \lambda^{R_1,\rho}x.a \Rightarrow_R \lambda^{R_1,\rho}x.a'} \quad \text{PAR\_ABS}$$

$$\frac{\begin{array}{l} \Omega \vDash A \Rightarrow_{R_1} A' \\ \Omega, x : R_1 \vDash B \Rightarrow_R B' \end{array}}{\Omega \vDash \Pi^\rho x : A/R_1 \to B \Rightarrow_R \Pi^\rho x : A'/R_1 \to B'} \quad \text{PAR\_PI}$$

$$\frac{\Omega \vDash a \Rightarrow_R a'}{\Omega \vDash \Lambda c.a \Rightarrow_R \Lambda c.a'} \quad \text{PAR\_CABS}$$

$$\frac{\begin{array}{l} \Omega \vDash A \Rightarrow_{R_1} A' \\ \Omega \vDash a \Rightarrow_{R_1} a' \\ \Omega \vDash b \Rightarrow_{R_1} b' \\ \Omega \vDash B \Rightarrow_R B' \end{array}}{\Omega \vDash \forall c : a \sim_{A/R_1} b.B \Rightarrow_R \forall c : a' \sim_{A'/R_1} b'.B'} \quad \text{PAR\_CPI}$$

$$\frac{\begin{array}{l} F \sim a : A/R_1 \in \Sigma_0 \\ R_1 \leq R \\ rctx\_uniq\Omega \end{array}}{\Omega \vDash F \Rightarrow_R a} \quad \text{PAR\_AXIOM}$$

$$\frac{\Omega \vDash a_1 \Rightarrow_{R_1} a_2}{\Omega \vDash a_1 \triangleright_R \bullet \Rightarrow_{R_1} a_2 \triangleright_R \bullet} \quad \text{PAR\_CONG}$$

$$\frac{\Omega \vDash a_1 \Rightarrow_{R_1} (a_2 \triangleright_R \bullet)}{\Omega \vDash (a_1 \triangleright_R \bullet) \Rightarrow_{R_1} (a_2 \triangleright_R \bullet)} \quad \text{PAR\_COMBINE}$$

$$\frac{\begin{array}{l} \Omega \vDash a_1 \Rightarrow_{R_1} (a_2 \triangleright_R \bullet) \\ \Omega \vDash b_1 \Rightarrow_{R_2} b_2 \end{array}}{\Omega \vDash a_1 {b_1}^{R_2,+} \Rightarrow_{R_1} (a_2 (b_2 \triangleright_R \bullet)^{R_2,+}) \triangleright_R \bullet} \quad \text{PAR\_PUSH}$$

$$\frac{\begin{array}{l} \Omega \vDash a_1 \Rightarrow_{R_1} (a_2 \triangleright_R \bullet) \\ \Omega \vDash b_1 \Rightarrow_{R_2} (b_2 \triangleright_R \bullet) \end{array}}{\Omega \vDash a_1 {b_1}^{R_2,+} \Rightarrow_{R_1} (a_2 (b_2 \triangleright_R \bullet)^{R_2,+}) \triangleright_R \bullet} \quad \text{PAR\_PUSHCOMBINE}$$

$$\frac{\Omega \vDash a_1 \Rightarrow_{R_1} (a_2 \triangleright_R \bullet)}{\Omega \vDash a_1[\bullet] \Rightarrow_{R_1} (a_2[\bullet]) \triangleright_R \bullet} \quad \text{PAR\_CPUSH}$$

$\boxed{\Omega \vdash a \Rightarrow_R^* b}$ multistep parallel reduction

$$\frac{}{\Omega \vdash a \Rightarrow_R^* a} \quad \text{MP\_REFL}$$

$$\frac{\begin{array}{l} \Omega \vDash a \Rightarrow_R b \\ \Omega \vdash b \Rightarrow_R^* a' \end{array}}{\Omega \vdash a \Rightarrow_R^* a'} \quad \text{MP\_STEP}$$

$\boxed{\Omega \vdash a \Leftrightarrow_R b}$ parallel reduction to a common term

$$\frac{\begin{array}{l} \Omega \vdash a_1 \Rightarrow_R^* b \\ \Omega \vdash a_2 \Rightarrow_R^* b \end{array}}{\Omega \vdash a_1 \Leftrightarrow_R a_2} \quad \text{JOIN}$$

$\boxed{\vDash a > b/R}$ primitive reductions on erased terms

$$\frac{\mathsf{Value}_{R_1} (\lambda^{R,\rho}x.v)}{\vDash (\lambda^{R,\rho}x.v) \ b^{R,\rho} > v\{b/x\}/R_1} \quad \text{BETA\_APPABS}$$

$$\frac{}{\vDash (\Lambda c.a')[\bullet] > a'\{\bullet/c\}/R} \quad \text{BETA\_CAPPCABS}$$

$$\frac{F \sim a : A/R \in \Sigma_0}{\vDash F > a/R} \quad \text{BETA\_AXIOM}$$

$\boxed{\vDash a \rightsquigarrow b/R}$ single-step head reduction for implicit language

$$\frac{\vDash a \rightsquigarrow a'/R_1}{\vDash \lambda^{R,-}x.a \rightsquigarrow \lambda^{R,-}x.a'/R_1} \quad \text{E\_ABSTERM}$$

$$\frac{\vDash a \rightsquigarrow a'/R_1}{\vDash a \ b^{R,\rho} \rightsquigarrow a' \ b^{R,\rho}/R_1} \quad \text{E\_APPLEFT}$$

$$\frac{\vDash a \rightsquigarrow a'/R}{\vDash a[\bullet] \rightsquigarrow a'[\bullet]/R} \quad \text{E\_CAPPLEFT}$$

$$\frac{\mathsf{Value}_{R_1}\ (\lambda^{R,\rho}x.v)}{\vDash (\lambda^{R,\rho}x.v)\ a^{R,\rho} \rightsquigarrow v\{a/x\}/R_1} \quad \text{E\_AppAbs}$$

$$\frac{}{\vDash (\Lambda c.b)[\bullet] \rightsquigarrow b\{\bullet/c\}/R} \quad \text{E\_CAppCAbs}$$

$$\frac{\begin{array}{c} F \sim a : A/R \in \Sigma_0 \\ R \le R_1 \end{array}}{\vDash F \rightsquigarrow a/R_1} \quad \text{E\_Axiom}$$

$$\frac{\begin{array}{c} \vDash a \rightsquigarrow a'/R_1 \\ \neg(R \le R_1) \end{array}}{\vDash a \rhd_R \bullet \rightsquigarrow a' \rhd_R \bullet/R_1} \quad \text{E\_Cong}$$

$$\frac{}{\vDash (a \rhd_R \bullet) \rhd_R \bullet \rightsquigarrow a \rhd_R \bullet/R_1} \quad \text{E\_Combine}$$

$$\frac{}{\vDash (v_1 \rhd_R \bullet)v_2^{R_1,+} \rightsquigarrow (v_1(v_2 \rhd_R \bullet)^{R_1,+}) \rhd_R \bullet/R_2} \quad \text{E\_Push}$$

$$\frac{}{\vDash (v_1 \rhd_R \bullet)[\bullet] \rightsquigarrow (v_1[\bullet]) \rhd_R \bullet/R_1} \quad \text{E\_CPush}$$

$\boxed{\vDash a \rightsquigarrow^* b/R}$ multistep reduction

$$\frac{}{\vDash a \rightsquigarrow^* a/R} \quad \text{Equal}$$

$$\frac{\begin{array}{c} \vDash a \rightsquigarrow b/R \\ \vDash b \rightsquigarrow^* a'/R \end{array}}{\vDash a \rightsquigarrow^* a'/R} \quad \text{Step}$$

$\boxed{\Gamma \vDash \phi\ \mathsf{ok}}$ Prop wellformedness

$$\frac{\begin{array}{c} \Gamma \vDash a : A/R \\ \Gamma \vDash b : A/R \\ \Gamma \vDash A : \star/R \end{array}}{\Gamma \vDash a \sim_{A/R} b\ \mathsf{ok}} \quad \text{E\_Wff}$$

$\boxed{\Gamma \vDash a : A/R}$ typing

$$\frac{\begin{array}{c} R_1 \le R_2 \\ \Gamma \vDash a : A/R_1 \end{array}}{\Gamma \vDash a : A/R_2} \quad \text{E\_SubRole}$$

$$\frac{\vDash \Gamma}{\Gamma \vDash \star : \star/R} \quad \text{E\_Star}$$

$$\frac{\begin{array}{c} \vDash \Gamma \\ x : A/R \in \Gamma \end{array}}{\Gamma \vDash x : A/R} \quad \text{E\_Var}$$

$$\frac{\begin{array}{c} \Gamma, x : A/R \vDash B : \star/R' \\ \Gamma \vDash A : \star/R \\ R \le R' \end{array}}{\Gamma \vDash \Pi^\rho x : A/R \to B : \star/R'} \quad \text{E\_Pi}$$

$$\frac{\begin{array}{c} \Gamma, x : A/R \vDash a : B/R' \\ \Gamma \vDash A : \star/R \\ (\rho = +) \vee (x \notin \mathsf{fv}\ a) \\ R \le R' \end{array}}{\Gamma \vDash \lambda^{R,\rho}x.a : (\Pi^\rho x : A/R \to B)/R'} \quad \text{E\_Abs}$$

$$\frac{\begin{array}{c}\Gamma \vDash b : \Pi^+ x{:}A/R \to B/R' \\ \Gamma \vDash a : A/R\end{array}}{\Gamma \vDash b \ a^{R,+} : B\{a/x\}/R'} \quad \text{E\_APP}$$

$$\frac{\begin{array}{c}\Gamma \vDash b : \Pi^- x{:}A/R \to B/R' \\ \Gamma \vDash a : A/R\end{array}}{\Gamma \vDash b \ \square^{R,-} : B\{a/x\}/R'} \quad \text{E\_IAPP}$$

$$\frac{\begin{array}{c}\Gamma \vDash a : A/R \\ \Gamma; \widetilde{\Gamma} \vDash A \equiv B : \star/R \\ \Gamma \vDash B : \star/R\end{array}}{\Gamma \vDash a : B/R} \quad \text{E\_CONV}$$

$$\frac{\begin{array}{c}\Gamma, c : \phi \vDash B : \star/R \\ \Gamma \vDash \phi \ \mathsf{ok}\end{array}}{\Gamma \vDash \forall c{:}\phi.B : \star/R} \quad \text{E\_CPI}$$

$$\frac{\begin{array}{c}\Gamma, c : \phi \vDash a : B/R \\ \Gamma \vDash \phi \ \mathsf{ok}\end{array}}{\Gamma \vDash \Lambda c.a : \forall c{:}\phi.B/R} \quad \text{E\_CABS}$$

$$\frac{\begin{array}{c}\Gamma \vDash a_1 : \forall c{:}(a \sim_{A/R} b).B_1/R' \\ \Gamma; \widetilde{\Gamma} \vDash a \equiv b : A/R\end{array}}{\Gamma \vDash a_1[\bullet] : B_1\{\bullet/c\}/R'} \quad \text{E\_CAPP}$$

$$\frac{\begin{array}{c}\vDash \Gamma \\ F \sim a : A/R \ \in \ \Sigma_0 \\ \varnothing \vDash A : \star/R\end{array}}{\Gamma \vDash F : A/R} \quad \text{E\_FAM}$$

$$\frac{\begin{array}{c}\Gamma \vDash a : A_1/R_1 \\ \Gamma; \widetilde{\Gamma} \vDash A_1 \equiv A_2 : \star/R_2 \\ \neg(R_2 \le R_1) \\ \Gamma \vDash A_2 : \star/R_1\end{array}}{\Gamma \vDash a \rhd_{R_2} \bullet : A_2/R_1} \quad \text{E\_TYCAST}$$

$\boxed{\Gamma; \Delta \vDash \phi_1 \equiv \phi_2}$ prop equality

$$\frac{\begin{array}{c}\Gamma; \Delta \vDash A_1 \equiv A_2 : A/R \\ \Gamma; \Delta \vDash B_1 \equiv B_2 : A/R\end{array}}{\Gamma; \Delta \vDash A_1 \sim_{A/R} B_1 \equiv A_2 \sim_{A/R} B_2} \quad \text{E\_PROPCONG}$$

$$\frac{\begin{array}{c}\Gamma; \Delta \vDash A \equiv B : \star/R \\ \Gamma \vDash A_1 \sim_{A/R} A_2 \ \mathsf{ok} \\ \Gamma \vDash A_1 \sim_{B/R} A_2 \ \mathsf{ok}\end{array}}{\Gamma; \Delta \vDash A_1 \sim_{A/R} A_2 \equiv A_1 \sim_{B/R} A_2} \quad \text{E\_ISOCONV}$$

$$\frac{\Gamma; \Delta \vDash \forall c{:}\phi_1.B_1 \equiv \forall c{:}\phi_2.B_2 : \star/R}{\Gamma; \Delta \vDash \phi_1 \equiv \phi_2} \quad \text{E\_CPIFST}$$

$\boxed{\Gamma; \Delta \vDash a \equiv b : A/R}$ definitional equality

$$\frac{\begin{array}{c}\vDash \Gamma \\ c : (a \sim_{A/R} b) \ \in \ \Gamma \\ c \in \Delta\end{array}}{\Gamma; \Delta \vDash a \equiv b : A/R} \quad \text{E\_ASSN}$$

$$\frac{\Gamma \vDash a : A/R}{\Gamma; \Delta \vDash a \equiv a : A/R} \quad \text{E\_Refl}$$

$$\frac{\Gamma; \Delta \vDash b \equiv a : A/R}{\Gamma; \Delta \vDash a \equiv b : A/R} \quad \text{E\_Sym}$$

$$\frac{\begin{array}{c} \Gamma; \Delta \vDash a \equiv a_1 : A/R \\ \Gamma; \Delta \vDash a_1 \equiv b : A/R \end{array}}{\Gamma; \Delta \vDash a \equiv b : A/R} \quad \text{E\_Trans}$$

$$\frac{\begin{array}{c} \Gamma; \Delta \vDash a \equiv b : A/R_1 \\ R_1 \leq R_2 \end{array}}{\Gamma; \Delta \vDash a \equiv b : A/R_2} \quad \text{E\_Sub}$$

$$\frac{\begin{array}{c} \Gamma \vDash a_1 : B/R \\ \Gamma \vDash a_2 : B/R \\ \vDash a_1 > a_2/R \end{array}}{\Gamma; \Delta \vDash a_1 \equiv a_2 : B/R} \quad \text{E\_Beta}$$

$$\frac{\begin{array}{c} \Gamma; \Delta \vDash A_1 \equiv A_2 : \star/R \\ \Gamma, x : A_1/R; \Delta \vDash B_1 \equiv B_2 : \star/R' \\ \Gamma \vDash A_1 : \star/R \\ \Gamma \vDash \Pi^\rho x : A_1/R \to B_1 : \star/R' \\ \Gamma \vDash \Pi^\rho x : A_2/R \to B_2 : \star/R' \\ R \leq R' \end{array}}{\Gamma; \Delta \vDash (\Pi^\rho x : A_1/R \to B_1) \equiv (\Pi^\rho x : A_2/R \to B_2) : \star/R'} \quad \text{E\_PiCong}$$

$$\frac{\begin{array}{c} \Gamma, x : A_1/R; \Delta \vDash b_1 \equiv b_2 : B/R' \\ \Gamma \vDash A_1 : \star/R \\ R \leq R' \\ (\rho = +) \vee (x \notin \mathsf{fv}\ b_1) \\ (\rho = +) \vee (x \notin \mathsf{fv}\ b_2) \end{array}}{\Gamma; \Delta \vDash (\lambda^{R,\rho} x.b_1) \equiv (\lambda^{R,\rho} x.b_2) : (\Pi^\rho x : A_1/R \to B)/R'} \quad \text{E\_AbsCong}$$

$$\frac{\begin{array}{c} \Gamma; \Delta \vDash a_1 \equiv b_1 : (\Pi^+ x : A/R \to B)/R' \\ \Gamma; \Delta \vDash a_2 \equiv b_2 : A/R \end{array}}{\Gamma; \Delta \vDash a_1\ a_2^{R,+} \equiv b_1\ b_2^{R,+} : (B\{a_2/x\})/R'} \quad \text{E\_AppCong}$$

$$\frac{\begin{array}{c} \Gamma; \Delta \vDash a_1 \equiv b_1 : (\Pi^- x : A/R \to B)/R' \\ \Gamma \vDash a : A/R \end{array}}{\Gamma; \Delta \vDash a_1\ \square^{R,-} \equiv b_1\ \square^{R,-} : (B\{a/x\})/R'} \quad \text{E\_IAppCong}$$

$$\frac{\Gamma; \Delta \vDash \Pi^\rho x : A_1/R \to B_1 \equiv \Pi^\rho x : A_2/R \to B_2 : \star/R'}{\Gamma; \Delta \vDash A_1 \equiv A_2 : \star/R} \quad \text{E\_PiFst}$$

$$\frac{\begin{array}{c} \Gamma; \Delta \vDash \Pi^\rho x : A_1/R \to B_1 \equiv \Pi^\rho x : A_2/R \to B_2 : \star/R' \\ \Gamma; \Delta \vDash a_1 \equiv a_2 : A_1/R \end{array}}{\Gamma; \Delta \vDash B_1\{a_1/x\} \equiv B_2\{a_2/x\} : \star/R'} \quad \text{E\_PiSnd}$$

$$\frac{\begin{array}{c} \Gamma; \Delta \vDash a_1 \sim_{A_1/R} b_1 \equiv a_2 \sim_{A_2/R} b_2 \\ \Gamma, c : a_1 \sim_{A_1/R} b_1; \Delta \vDash A \equiv B : \star/R' \\ \Gamma \vDash a_1 \sim_{A_1/R} b_1\ \mathsf{ok} \\ \Gamma \vDash \forall c : a_1 \sim_{A_1/R} b_1.A : \star/R' \\ \Gamma \vDash \forall c : a_2 \sim_{A_2/R} b_2.B : \star/R' \end{array}}{\Gamma; \Delta \vDash \forall c : a_1 \sim_{A_1/R} b_1.A \equiv \forall c : a_2 \sim_{A_2/R} b_2.B : \star/R'} \quad \text{E\_CPiCong}$$

$$\frac{\begin{array}{c}\Gamma, c:\phi_1; \Delta \vDash a \equiv b : B/R \\ \Gamma \vDash \phi_1 \ \mathsf{ok}\end{array}}{\Gamma; \Delta \vDash (\Lambda c.a) \equiv (\Lambda c.b) : \forall c{:}\phi_1.B/R} \quad \text{E\_CABSCONG}$$

$$\frac{\begin{array}{c}\Gamma; \Delta \vDash a_1 \equiv b_1 : (\forall c{:}(a \sim_{A/R} b).B)/R' \\ \Gamma; \widetilde{\Gamma} \vDash a \equiv b : A/R\end{array}}{\Gamma; \Delta \vDash a_1[\bullet] \equiv b_1[\bullet] : (B\{\bullet/c\})/R'} \quad \text{E\_CAPPCONG}$$

$$\frac{\begin{array}{c}\Gamma; \Delta \vDash \forall c{:}(a_1 \sim_{A/R} a_2).B_1 \equiv \forall c{:}(a_1' \sim_{A'/R'} a_2').B_2 : \star/R_0 \\ \Gamma; \widetilde{\Gamma} \vDash a_1 \equiv a_2 : A/R \\ \Gamma; \widetilde{\Gamma} \vDash a_1' \equiv a_2' : A'/R'\end{array}}{\Gamma; \Delta \vDash B_1\{\bullet/c\} \equiv B_2\{\bullet/c\} : \star/R_0} \quad \text{E\_CPISND}$$

$$\frac{\begin{array}{c}\Gamma; \Delta \vDash a \equiv b : A/R \\ \Gamma; \Delta \vDash a \sim_{A/R} b \equiv a' \sim_{A'/R'} b'\end{array}}{\Gamma; \Delta \vDash a' \equiv b' : A'/R'} \quad \text{E\_CAST}$$

$$\frac{\begin{array}{c}\Gamma; \Delta \vDash a \equiv b : A/R_1 \\ \Gamma; \widetilde{\Gamma} \vDash A \equiv B : \star/R_2 \\ R_1 \leq R_2\end{array}}{\Gamma; \Delta \vDash a \equiv b : B/R_2} \quad \text{E\_EQCONV}$$

$$\frac{\Gamma; \Delta \vDash a \sim_{A/R} b \equiv a' \sim_{A'/R} b'}{\Gamma; \Delta \vDash A \equiv A' : \star/R} \quad \text{E\_ISOSND}$$

$$\frac{\begin{array}{c}\Gamma; \Delta \vDash a_1 \equiv a_2 : A/R_1 \\ \Gamma; \Delta \vDash A \equiv B : \star/R_2 \\ \neg(R_2 \leq R_1) \\ \Gamma \vDash B : \star/R_1\end{array}}{\Gamma; \Delta \vDash a_1 \triangleright_{R_2} \bullet \equiv a_2 \triangleright_{R_2} \bullet : B/R_1} \quad \text{E\_CASTCONG}$$

$\boxed{\vDash \Gamma}$ context wellformedness

$$\frac{}{\vDash \varnothing} \quad \text{E\_EMPTY}$$

$$\frac{\begin{array}{c}\vDash \Gamma \\ \Gamma \vDash A : \star/R \\ x \notin \mathsf{dom}\,\Gamma\end{array}}{\vDash \Gamma, x : A/R} \quad \text{E\_CONSTM}$$

$$\frac{\begin{array}{c}\vDash \Gamma \\ \Gamma \vDash \phi \ \mathsf{ok} \\ c \notin \mathsf{dom}\,\Gamma\end{array}}{\vDash \Gamma, c : \phi} \quad \text{E\_CONSCO}$$

$\boxed{\vDash \Sigma}$ signature wellformedness

$$\frac{}{\vDash \varnothing} \quad \text{SIG\_EMPTY}$$

$$\frac{\begin{array}{c}\vDash \Sigma \\ \varnothing \vDash A : \star/R \\ \varnothing \vDash a : A/R' \\ F \notin \mathsf{dom}\,\Sigma \\ R' \leq R\end{array}}{\vDash \Sigma \cup \{F \sim a : A/R'\}} \quad \text{SIG\_CONSAX}$$

$\boxed{\Gamma \vdash \phi \ \mathsf{ok}}$ prop wellformedness

$$\frac{\begin{array}{c} \Gamma \vdash a : A/R \\ \Gamma \vdash b : B/R \\ |A|R = |B|R \end{array}}{\Gamma \vdash a \sim_{A/R} b \ \mathsf{ok}} \quad \textsc{An\_Wff}$$

$\boxed{\Gamma \vdash a : A/R}$ typing

$$\frac{\vdash \Gamma}{\Gamma \vdash \star : \star/R} \quad \textsc{An\_Star}$$

$$\frac{\begin{array}{c} \vdash \Gamma \\ x : A/R \in \Gamma \end{array}}{\Gamma \vdash x : A/R} \quad \textsc{An\_Var}$$

$$\frac{\begin{array}{c} \Gamma, x : A/R \vdash B : \star/R' \\ \Gamma \vdash A : \star/R \end{array}}{\Gamma \vdash \Pi^\rho x{:}A/R \rightarrow B : \star/R'} \quad \textsc{An\_Pi}$$

$$\frac{\begin{array}{c} \Gamma \vdash A : \star/R \\ \Gamma, x : A/R \vdash a : B/R' \\ (\rho = +) \vee (x \notin \mathsf{fv}\ |a|R') \\ R \leq R' \end{array}}{\Gamma \vdash \lambda^\rho x{:}A/R.a : (\Pi^\rho x{:}A/R \rightarrow B)/R'} \quad \textsc{An\_Abs}$$

$$\frac{\begin{array}{c} \Gamma \vdash b : (\Pi^\rho x{:}A/R \rightarrow B)/R' \\ \Gamma \vdash a : A/R \end{array}}{\Gamma \vdash b\ a^{R,\rho} : (B\{a/x\})/R'} \quad \textsc{An\_App}$$

$$\frac{\begin{array}{c} \Gamma \vdash a : A/R \\ \Gamma; \widetilde{\Gamma} \vdash \gamma : A \sim_R B \\ \Gamma \vdash B : \star/R \end{array}}{\Gamma \vdash a \triangleright_R \gamma : B/R} \quad \textsc{An\_Conv}$$

$$\frac{\begin{array}{c} \Gamma \vdash \phi \ \mathsf{ok} \\ \Gamma, c : \phi \vdash B : \star/R \end{array}}{\Gamma \vdash \forall c{:}\phi.B : \star/R} \quad \textsc{An\_CPi}$$

$$\frac{\begin{array}{c} \Gamma \vdash \phi \ \mathsf{ok} \\ \Gamma, c : \phi \vdash a : B/R \end{array}}{\Gamma \vdash \Lambda c{:}\phi.a : (\forall c{:}\phi.B)/R} \quad \textsc{An\_CAbs}$$

$$\frac{\begin{array}{c} \Gamma \vdash a_1 : (\forall c{:}a \sim_{A_1/R} b.B)/R' \\ \Gamma; \widetilde{\Gamma} \vdash \gamma : a \sim_R b \end{array}}{\Gamma \vdash a_1[\gamma] : B\{\gamma/c\}/R'} \quad \textsc{An\_CApp}$$

$$\frac{\begin{array}{c} \vdash \Gamma \\ F \sim a : A/R \in \Sigma_1 \\ \varnothing \vdash A : \star/R \end{array}}{\Gamma \vdash F : A/R} \quad \textsc{An\_Fam}$$

$$\frac{\begin{array}{c} R_1 \leq R_2 \\ \Gamma \vdash a : A/R_1 \end{array}}{\Gamma \vdash a : A/R_2} \quad \textsc{An\_SubRole}$$

$\boxed{\Gamma; \Delta \vdash \gamma : \phi_1 \sim \phi_2}$ coercion between props

$$\frac{\begin{array}{c}\Gamma; \Delta \vdash \gamma_1 : A_1 \sim_R A_2 \\ \Gamma; \Delta \vdash \gamma_2 : B_1 \sim_R B_2 \\ \Gamma \vdash A_1 \sim_{A/R} B_1 \ \mathsf{ok} \\ \Gamma \vdash A_2 \sim_{A/R} B_2 \ \mathsf{ok}\end{array}}{\Gamma; \Delta \vdash (\gamma_1 \sim_A \gamma_2) : (A_1 \sim_{A/R} B_1) \sim (A_2 \sim_{A/R} B_2)} \quad \textsc{An\_PropCong}$$

$$\frac{\Gamma; \Delta \vdash \gamma : \forall c{:}\phi_1.A_2 \sim_R \forall c{:}\phi_2.B_2}{\Gamma; \Delta \vdash \mathbf{cpiFst}\, \gamma : \phi_1 \sim \phi_2} \quad \textsc{An\_CPiFst}$$

$$\frac{\Gamma; \Delta \vdash \gamma : \phi_1 \sim \phi_2}{\Gamma; \Delta \vdash \mathbf{sym}\, \gamma : \phi_2 \sim \phi_1} \quad \textsc{An\_IsoSym}$$

$$\frac{\begin{array}{c}\Gamma; \Delta \vdash \gamma : A \sim_R B \\ \Gamma \vdash a_1 \sim_{A/R} a_2 \ \mathsf{ok} \\ \Gamma \vdash a'_1 \sim_{B/R} a'_2 \ \mathsf{ok} \\ |a_1|R = |a'_1|R \\ |a_2|R = |a'_2|R\end{array}}{\Gamma; \Delta \vdash \mathbf{conv}\, (a_1 \sim_{A/R} a_2) \sim_\gamma (a'_1 \sim_{B/R} a'_2) : (a_1 \sim_{A/R} a_2) \sim (a'_1 \sim_{B/R} a'_2)} \quad \textsc{An\_IsoConv}$$

$\boxed{\Gamma; \Delta \vdash \gamma : A \sim_R B}$ coercion between types

$$\frac{\begin{array}{c}\vdash \Gamma \\ c : a \sim_{A/R} b \in \Gamma \\ c \in \Delta\end{array}}{\Gamma; \Delta \vdash c : a \sim_R b} \quad \textsc{An\_Assn}$$

$$\frac{\Gamma \vdash a : A/R}{\Gamma; \Delta \vdash \mathbf{refl}\, a : a \sim_R a} \quad \textsc{An\_Refl}$$

$$\frac{\begin{array}{c}\Gamma \vdash a : A/R \\ \Gamma \vdash b : B/R \\ |a|R = |b|R \\ \Gamma; \widetilde{\Gamma} \vdash \gamma : A \sim_R B\end{array}}{\Gamma; \Delta \vdash (a \mid=\mid_\gamma b) : a \sim_R b} \quad \textsc{An\_EraseEq}$$

$$\frac{\begin{array}{c}\Gamma \vdash b : B/R \\ \Gamma \vdash a : A/R \\ \Gamma; \widetilde{\Gamma} \vdash \gamma_1 : B \sim_R A \\ \Gamma; \Delta \vdash \gamma : b \sim_R a\end{array}}{\Gamma; \Delta \vdash \mathbf{sym}\, \gamma : a \sim_R b} \quad \textsc{An\_Sym}$$

$$\frac{\begin{array}{c}\Gamma; \Delta \vdash \gamma_1 : a \sim_R a_1 \\ \Gamma; \Delta \vdash \gamma_2 : a_1 \sim_R b \\ \Gamma \vdash a : A/R \\ \Gamma \vdash a_1 : A_1/R \\ \Gamma; \widetilde{\Gamma} \vdash \gamma_3 : A \sim_R A_1\end{array}}{\Gamma; \Delta \vdash (\gamma_1; \gamma_2) : a \sim_R b} \quad \textsc{An\_Trans}$$

$$\frac{\begin{array}{c}\Gamma \vdash a_1 : B_0/R \\ \Gamma \vdash a_2 : B_1/R \\ |B_0|R = |B_1|R \\ \vDash |a_1|R > |a_2|R/R\end{array}}{\Gamma; \Delta \vdash \mathbf{red}\, a_1\, a_2 : a_1 \sim_R a_2} \quad \textsc{An\_Beta}$$

$$\dfrac{\begin{array}{c} \Gamma; \Delta \vdash \gamma_1 : A_1 \sim_{R'} A_2 \\ \Gamma, x : A_1/R; \Delta \vdash \gamma_2 : B_1 \sim_{R'} B_2 \\ B_3 = B_2\{x \triangleright_{R'} \mathbf{sym}\, \gamma_1/x\} \\ \Gamma \vdash \Pi^\rho x : A_1/R \to B_1 : \star/R' \\ \Gamma \vdash \Pi^\rho x : A_1/R \to B_2 : \star/R' \\ \Gamma \vdash \Pi^\rho x : A_2/R \to B_3 : \star/R' \\ R \le R' \end{array}}{\Gamma; \Delta \vdash \Pi^{R,\rho} x : \gamma_1.\gamma_2 : (\Pi^\rho x : A_1/R \to B_1) \sim_{R'} (\Pi^\rho x : A_2/R \to B_3)} \quad \textsc{An\_PiCong}$$

$$\dfrac{\begin{array}{c} \Gamma; \Delta \vdash \gamma_1 : A_1 \sim_R A_2 \\ \Gamma, x : A_1/R; \Delta \vdash \gamma_2 : b_1 \sim_{R'} b_2 \\ b_3 = b_2\{x \triangleright_{R'} \mathbf{sym}\, \gamma_1/x\} \\ \Gamma \vdash A_1 : \star/R \\ \Gamma \vdash A_2 : \star/R \\ (\rho = +) \vee (x \notin \mathsf{fv}\, |b_1|R') \\ (\rho = +) \vee (x \notin \mathsf{fv}\, |b_3|R') \\ \Gamma \vdash (\lambda^\rho x : A_1/R.b_2) : B/R' \\ R \le R' \end{array}}{\Gamma; \Delta \vdash (\lambda^{R,\rho} x : \gamma_1.\gamma_2) : (\lambda^\rho x : A_1/R.b_1) \sim_{R'} (\lambda^\rho x : A_2/R.b_3)} \quad \textsc{An\_AbsCong}$$

$$\dfrac{\begin{array}{c} \Gamma; \Delta \vdash \gamma_1 : a_1 \sim_{R'} b_1 \\ \Gamma; \Delta \vdash \gamma_2 : a_2 \sim_R b_2 \\ \Gamma \vdash a_1\, a_2{}^{R,\rho} : A/R' \\ \Gamma \vdash b_1\, b_2{}^{R,\rho} : B/R' \\ \Gamma; \widetilde{\Gamma} \vdash \gamma_3 : A \sim_{R'} B \end{array}}{\Gamma; \Delta \vdash \gamma_1\, \gamma_2{}^{R,\rho} : a_1\, a_2{}^{R,\rho} \sim_{R'} b_1\, b_2{}^{R,\rho}} \quad \textsc{An\_AppCong}$$

$$\dfrac{\Gamma; \Delta \vdash \gamma : \Pi^\rho x : A_1/R \to B_1 \sim_{R'} \Pi^\rho x : A_2/R \to B_2}{\Gamma; \Delta \vdash \mathbf{piFst}\, \gamma : A_1 \sim_R A_2} \quad \textsc{An\_PiFst}$$

$$\dfrac{\begin{array}{c} \Gamma; \Delta \vdash \gamma_1 : \Pi^\rho x : A_1/R \to B_1 \sim_{R'} \Pi^\rho x : A_2/R \to B_2 \\ \Gamma; \Delta \vdash \gamma_2 : a_1 \sim_R a_2 \\ \Gamma \vdash a_1 : A_1/R \\ \Gamma \vdash a_2 : A_2/R \end{array}}{\Gamma; \Delta \vdash \gamma_1 @ \gamma_2 : B_1\{a_1/x\} \sim_{R'} B_2\{a_2/x\}} \quad \textsc{An\_PiSnd}$$

$$\dfrac{\begin{array}{c} \Gamma; \Delta \vdash \gamma_1 : a_1 \sim_{A_1/R} b_1 \sim a_2 \sim_{A_2/R} b_2 \\ \Gamma, c : a_1 \sim_{A_1/R} b_1; \Delta \vdash \gamma_3 : B_1 \sim_{R'} B_2 \\ B_3 = B_2\{c \triangleright_{R'} \mathbf{sym}\, \gamma_1/c\} \\ \Gamma \vdash \forall c : a_1 \sim_{A_1/R} b_1.B_1 : \star/R' \\ \Gamma \vdash \forall c : a_2 \sim_{A_2/R} b_2.B_3 : \star/R' \\ \Gamma \vdash \forall c : a_1 \sim_{A_1/R} b_1.B_2 : \star/R' \end{array}}{\Gamma; \Delta \vdash (\forall c : \gamma_1.\gamma_3) : (\forall c : a_1 \sim_{A_1/R} b_1.B_1) \sim_R (\forall c : a_2 \sim_{A_2/R} b_2.B_3)} \quad \textsc{An\_CPiCong}$$

$$\dfrac{\begin{array}{c} \Gamma; \Delta \vdash \gamma_1 : b_0 \sim_{A_1/R} b_1 \sim b_2 \sim_{A_2/R} b_3 \\ \Gamma, c : b_0 \sim_{A_1/R} b_1; \Delta \vdash \gamma_3 : a_1 \sim_{R'} a_2 \\ a_3 = a_2\{c \triangleright_{R'} \mathbf{sym}\, \gamma_1/c\} \\ \Gamma \vdash (\Lambda c : b_0 \sim_{A_1/R} b_1.a_1) : \forall c : b_0 \sim_{A_1/R} b_1.B_1/R' \\ \Gamma \vdash (\Lambda c : b_0 \sim_{A_1/R} b_1.a_2) : B/R' \\ \Gamma \vdash (\Lambda c : b_2 \sim_{A_2/R} b_3.a_3) : \forall c : b_2 \sim_{A_2/R} b_3.B_2/R' \\ \Gamma; \widetilde{\Gamma} \vdash \gamma_4 : \forall c : b_0 \sim_{A_1/R} b_1.B_1 \sim_{R'} \forall c : \phi_2.B_2 \end{array}}{\Gamma; \Delta \vdash (\lambda c : \gamma_1.\gamma_3 @ \gamma_4) : (\Lambda c : b_0 \sim_{A_1/R} b_1.a_1) \sim_{R'} (\Lambda c : b_2 \sim_{A_2/R} b_3.a_3)} \quad \textsc{An\_CAbsCong}$$

$$\dfrac{\begin{array}{l}\Gamma;\Delta \vdash \gamma_1 : a_1 \sim_R b_1 \\ \Gamma;\widetilde{\Gamma} \vdash \gamma_2 : a_2 \sim_{R'} b_2 \\ \Gamma;\widetilde{\Gamma} \vdash \gamma_3 : a_3 \sim_{R'} b_3 \\ \Gamma \vdash a_1[\gamma_2] : A/R \\ \Gamma \vdash b_1[\gamma_3] : B/R \\ \Gamma;\widetilde{\Gamma} \vdash \gamma_4 : A \sim_R B\end{array}}{\Gamma;\Delta \vdash \gamma_1(\gamma_2,\gamma_3) : a_1[\gamma_2] \sim_R b_1[\gamma_3]} \quad \textsc{An\_CAppCong}$$

$$\dfrac{\begin{array}{l}\Gamma;\Delta \vdash \gamma_1 : (\forall c_1 : a \sim_{A/R} a'.B_1) \sim_{R_0} (\forall c_2 : b \sim_{B/R'} b'.B_2) \\ \Gamma;\widetilde{\Gamma} \vdash \gamma_2 : a \sim_R a' \\ \Gamma;\widetilde{\Gamma} \vdash \gamma_3 : b \sim_{R'} b'\end{array}}{\Gamma;\Delta \vdash \gamma_1@(\gamma_2 \sim \gamma_3) : B_1\{\gamma_2/c_1\} \sim_{R_0} B_2\{\gamma_3/c_2\}} \quad \textsc{An\_CPiSnd}$$

$$\dfrac{\begin{array}{l}\Gamma;\Delta \vdash \gamma_1 : a \sim_{R_1} a' \\ \Gamma;\Delta \vdash \gamma_2 : a \sim_{A/R_1} a' \sim b \sim_{B/R_2} b'\end{array}}{\Gamma;\Delta \vdash \gamma_1 \triangleright_{R_2} \gamma_2 : b \sim_{R_2} b'} \quad \textsc{An\_Cast}$$

$$\dfrac{\Gamma;\Delta \vdash \gamma : (a \sim_{A/R} a') \sim (b \sim_{B/R} b')}{\Gamma;\Delta \vdash \textbf{isoSnd}\,\gamma : A \sim_R B} \quad \textsc{An\_IsoSnd}$$

$$\dfrac{\begin{array}{l}\Gamma;\Delta \vdash \gamma : a \sim_{R_1} b \\ R_1 \leq R_2\end{array}}{\Gamma;\Delta \vdash \textbf{sub}\,\gamma : a \sim_{R_2} b} \quad \textsc{An\_Sub}$$

$\boxed{\vdash \Gamma}$ context wellformedness

$$\dfrac{}{\vdash \varnothing} \quad \textsc{An\_Empty}$$

$$\dfrac{\begin{array}{l}\vdash \Gamma \\ \Gamma \vdash A : \star/R \\ x \notin \mathsf{dom}\,\Gamma\end{array}}{\vdash \Gamma, x : A/R} \quad \textsc{An\_ConsTm}$$

$$\dfrac{\begin{array}{l}\vdash \Gamma \\ \Gamma \vdash \phi \ \mathsf{ok} \\ c \notin \mathsf{dom}\,\Gamma\end{array}}{\vdash \Gamma, c : \phi} \quad \textsc{An\_ConsCo}$$

$\boxed{\vdash \Sigma}$ signature wellformedness

$$\dfrac{}{\vdash \varnothing} \quad \textsc{An\_Sig\_Empty}$$

$$\dfrac{\begin{array}{l}\vdash \Sigma \\ \varnothing \vdash A : \star/R \\ \varnothing \vdash a : A/R \\ F \notin \mathsf{dom}\,\Sigma\end{array}}{\vdash \Sigma \cup \{F \sim a : A/R\}} \quad \textsc{An\_Sig\_ConsAx}$$

$\boxed{\Gamma \vdash a \rightsquigarrow b/R}$ single-step, weak head reduction to values for annotated language

$$\dfrac{\Gamma \vdash a \rightsquigarrow a'/R_1}{\Gamma \vdash a\ b^{R,\rho} \rightsquigarrow a'\ b^{R,\rho}/R_1} \quad \textsc{An\_AppLeft}$$

$$\dfrac{\mathsf{Value}_R\ (\lambda^\rho x : A/R.w)}{\Gamma \vdash (\lambda^\rho x : A/R.w)\ a^{R,\rho} \rightsquigarrow w\{a/x\}/R} \quad \textsc{An\_AppAbs}$$

$$\frac{\Gamma \vdash a \rightsquigarrow a'/R}{\Gamma \vdash a[\gamma] \rightsquigarrow a'[\gamma]/R} \quad \textsc{An\_CAppLeft}$$

$$\frac{}{\Gamma \vdash (\Lambda c\!:\!\phi.b)[\gamma] \rightsquigarrow b\{\gamma/c\}/R} \quad \textsc{An\_CAppCAbs}$$

$$\frac{\begin{array}{c}\Gamma \vdash A : \star/R \\ \Gamma, x : A/R \vdash b \rightsquigarrow b'/R_1\end{array}}{\Gamma \vdash (\lambda^- x\!:\!A/R.b) \rightsquigarrow (\lambda^- x\!:\!A/R.b')/R_1} \quad \textsc{An\_AbsTerm}$$

$$\frac{F \sim a : A/R \in \Sigma_1}{\Gamma \vdash F \rightsquigarrow a/R} \quad \textsc{An\_Axiom}$$

$$\frac{\Gamma \vdash a \rightsquigarrow a'/R}{\Gamma \vdash a \triangleright_{R_1} \gamma \rightsquigarrow a' \triangleright_{R_1} \gamma/R} \quad \textsc{An\_ConvTerm}$$

$$\frac{\mathsf{Value}_R \ v}{\Gamma \vdash (v \triangleright_{R_2} \gamma_1) \triangleright_{R_2} \gamma_2 \rightsquigarrow v \triangleright_{R_2} (\gamma_1; \gamma_2)/R} \quad \textsc{An\_Combine}$$

$$\frac{\begin{array}{l}\mathsf{Value}_R \ v \\ \Gamma; \widetilde{\Gamma} \vdash \gamma : \Pi^\rho x_1\!:\!A_1/R \to B_1 \sim_{R'} \Pi^\rho x_2\!:\!A_2/R \to B_2 \\ b' = b \triangleright_{R'} \mathbf{sym}\,(\mathbf{piFst}\,\gamma) \\ \gamma' = \gamma@(b' \mathrel{|=|}_{(\mathbf{piFst}\,\gamma)} b)\end{array}}{\Gamma \vdash (v \triangleright_{R'} \gamma)\ b^{R,\rho} \rightsquigarrow ((v\ b'^{R,\rho}) \triangleright_{R'} \gamma')/R} \quad \textsc{An\_Push}$$

$$\frac{\begin{array}{l}\mathsf{Value}_R \ v \\ \Gamma; \widetilde{\Gamma} \vdash \gamma : \forall c_1\!:\!a_1 \sim_{B_1/R} b_1.A_1 \sim_{R'} \forall c_2\!:\!a_2 \sim_{B_2/R} b_2.A_2 \\ \gamma_1' = \gamma_1 \triangleright_{R'} \mathbf{sym}\,(\mathbf{cpiFst}\,\gamma) \\ \gamma' = \gamma@(\gamma_1' \sim \gamma_1)\end{array}}{\Gamma \vdash (v \triangleright_{R'} \gamma)[\gamma_1] \rightsquigarrow ((v[\gamma_1']) \triangleright_{R'} \gamma')/R} \quad \textsc{An\_CPush}$$

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Definition rules:         163 good      0 bad
Definition rule clauses:  483 good      0 bad
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