$tmvar$, x, y, f, m, n	variables
$covar$, c	coercion variables
$datacon$, K	
$const$, T, F	
$index$, i	indices

relflag, ρ	$::=$			relevance flag
	$\vert$	$+$		
	$\vert$	$-$		
appflag, ν	$::=$			applicative flag
	$\vert$	R		
	$\vert$	ρ		
role, R	$::=$			Role
	$\vert$	**Nom**		
	$\vert$	**Rep**		
	$\vert$	$R_1 \cap R_2$	S	
	$\vert$	**param** R_1 R_2	S	
	$\vert$	$\mathit{app\_role}\nu$	S	
	$\vert$	(R)	S	
constraint, ϕ	$::=$			props
	$\vert$	$a \sim_{A/R} b$		
	$\vert$	(ϕ)	S	
	$\vert$	$\phi\{b/x\}$	S	
	$\vert$	$\vert\phi\vert$	S	
	$\vert$	$a \sim_R b$	S	
tm, a, b, v, w, A, B	$::=$			types and kinds
	$\vert$	$\star$		
	$\vert$	x		
	$\vert$	$\lambda^\rho x\!:\!A.b$	bind x in b	
	$\vert$	$\lambda^\rho x.b$	bind x in b	
	$\vert$	$a\ b^\nu$		
	$\vert$	$\Pi^\rho x\!:\!A \to B$	bind x in B	
	$\vert$	$\Lambda c\!:\!\phi.b$	bind c in b	
	$\vert$	$\Lambda c.b$	bind c in b	
	$\vert$	$a[\gamma]$		
	$\vert$	$\forall c\!:\!\phi.B$	bind c in B	
	$\vert$	$a \triangleright_R \gamma$		
	$\vert$	F		
	$\vert$	$\square$		
	$\vert$	$\mathsf{case}_R\ a_1\ \mathsf{of}\ a_2 \to b_1 \Vert_{\_} \to b_2$		
	$\vert$	K		
	$\vert$	**match** a **with** *brs*		
	$\vert$	**sub** R a		
	$\vert$	$a\{b/x\}$	S	
	$\vert$	$a\{\gamma/c\}$	S	
	$\vert$	a	S	
	$\vert$	a	S	
	$\vert$	(a)	S	
	$\vert$	a	S	parsing precedence is hard

	$\mid$	$\lvert a \rvert_R$	S	
	$\mid$	**Int**	S	
	$\mid$	**Bool**	S	
	$\mid$	**Nat**	S	
	$\mid$	**Vec**	S	
	$\mid$	0	S	
	$\mid$	S	S	
	$\mid$	**True**	S	
	$\mid$	**Fix**	S	
	$\mid$	**Age**	S	
	$\mid$	$a \to b$	S	
	$\mid$	$\phi \Rightarrow A$	S	
	$\mid$	$a\ b$	S	
	$\mid$	$\lambda x.a$	S	
	$\mid$	$\lambda x\!:\!A.a$	S	
	$\mid$	$\forall\, x : A \to B$	S	
	$\mid$	**if** ϕ **then** a **else** b	S	
brs	$::=$			case branches
	$\mid$	**none**		
	$\mid$	$K \Rightarrow a; brs$		
	$\mid$	$brs\{a/x\}$	S	
	$\mid$	$brs\{\gamma/c\}$	S	
	$\mid$	(brs)	S	
$co,\ \gamma$	$::=$			explicit coercions
	$\mid$	$\bullet$		
	$\mid$	c		
	$\mid$	**red** $a\ b$		
	$\mid$	**refl** a		
	$\mid$	$(a\ \lvert{=}\rvert_{\gamma}\ b)$		
	$\mid$	**sym** γ		
	$\mid$	$\gamma_1; \gamma_2$		
	$\mid$	**sub** γ		
	$\mid$	$\Pi^{R,\rho} x\!:\!\gamma_1.\gamma_2$	bind x in γ_2	
	$\mid$	$\lambda^{R,\rho} x\!:\!\gamma_1.\gamma_2$	bind x in γ_2	
	$\mid$	$\gamma_1\ \gamma_2^{R,\rho}$		
	$\mid$	**piFst** γ		
	$\mid$	**cpiFst** γ		
	$\mid$	**isoSnd** γ		
	$\mid$	$\gamma_1 @ \gamma_2$		
	$\mid$	$\forall c\!:\!\gamma_1.\gamma_3$	bind c in γ_3	
	$\mid$	$\lambda c\!:\!\gamma_1.\gamma_3 @ \gamma_4$	bind c in γ_3	
	$\mid$	$\gamma(\gamma_1, \gamma_2)$		
	$\mid$	$\gamma @ (\gamma_1 \sim \gamma_2)$		
	$\mid$	$\gamma_1 \triangleright_R \gamma_2$		

	$\|$	$\gamma_1 \sim_A \gamma_2$		
	$\|$	$\mathbf{conv}\ \phi_1 \sim_\gamma \phi_2$		
	$\|$	$\mathbf{eta}\, a$		
	$\|$	$\mathbf{left}\, \gamma\, \gamma'$		
	$\|$	$\mathbf{right}\, \gamma\, \gamma'$		
	$\|$	(γ)	S	
	$\|$	γ	S	
	$\|$	$\gamma\{a/x\}$	S	
$\mathit{role\_context},\ \Omega$	$::=$			$\text{role}_c\mathit{ontexts}$
	$\|$	$\varnothing$		
	$\|$	$x : R$		
	$\|$	$\Omega, x : R$		
	$\|$	(Ω)	M	
	$\|$	Ω	M	
$\mathit{roles},\ Rs$	$::=$			
	$\|$	$\mathbf{nilR}$		
	$\|$	R, Rs		
$\mathit{sig\_sort}$	$::=$			signature classifier
	$\|$	$: A@Rs$		
	$\|$	$\sim a : A/R@Rs$		
sort	$::=$			binding classifier
	$\|$	$\mathbf{Tm}\, A$		
	$\|$	$\mathbf{Co}\, \phi$		
$\mathit{context},\ \Gamma$	$::=$			contexts
	$\|$	$\varnothing$		
	$\|$	$\Gamma, x : A$		
	$\|$	$\Gamma, c : \phi$		
	$\|$	$\Gamma\{b/x\}$	M	
	$\|$	$\Gamma\{\gamma/c\}$	M	
	$\|$	Γ, Γ'	M	
	$\|$	$\|\Gamma\|$	M	
	$\|$	(Γ)	M	
	$\|$	Γ	M	
$\mathit{sig},\ \Sigma$	$::=$			signatures
	$\|$	$\varnothing$		
	$\|$	$\Sigma \cup \{F\, \mathit{sig\_sort}\}$		
	$\|$	Σ_0	M	
	$\|$	Σ_1	M	
	$\|$	$\|\Sigma\|$	M	
$\mathit{available\_props},\ \Delta$	$::=$			

	$\mid$	$\varnothing$
	$\mid$	Δ, c
	$\mid$	$\widetilde{\Gamma}$ M
	$\mid$	(Δ) M
terminals	$::=$	
	$\mid$	$\leftrightarrow$
	$\mid$	$\Leftrightarrow$
	$\mid$	$\longrightarrow$
	$\mid$	min
	$\mid$	$\equiv$
	$\mid$	$\forall$
	$\mid$	$\in$
	$\mid$	$\notin$
	$\mid$	$\Leftarrow$
	$\mid$	$\Rightarrow$
	$\mid$	$\Rightarrow^*$
	$\mid$	$\rightarrow$
	$\mid$	Λ
	$\mid$	
	$\mid$	$\square$
	$\mid$	$\vdash$
	$\mid$	$\dashv$
	$\mid$	$\models$
	$\mid$	$\models$
	$\mid$	$\neq$
	$\mid$	$\triangleright$
	$\mid$	ok
	$\mid$	$\_$
	$\mid$	$\rightsquigarrow$
	$\mid$	$\rightsquigarrow^*$
	$\mid$	$\rightsquigarrow$
	$\mid$	$\varnothing$
	$\mid$	$\circ$
	$\mid$	fv
	$\mid$	dom
	$\mid$	$\sim$
	$\mid$	$\asymp$
	$\mid$	$\mid$
	$\mid$	$\bullet$
	$\mid$	$\mathbf{fst}$
	$\mid$	$\mathbf{snd}$
	$\mid$	$\mid \Rightarrow \mid$
	$\mid$	$\vdash_=$
	$\mid$	$\mathbf{refl_2}$
	$\mid$	$++$

formula, ψ	$::=$		
	$\vert$	*judgement*	
	$\vert$	$x : A \in \Gamma$	
	$\vert$	$x : R \in \Omega$	
	$\vert$	$c : \phi \in \Gamma$	
	$\vert$	$F\ sig\_sort \in \Sigma$	
	$\vert$	$x \in \Delta$	
	$\vert$	$c \in \Delta$	
	$\vert$	$c\ \mathbf{not\ relevant} \in \gamma$	
	$\vert$	$x \notin \mathsf{fv}\, a$	
	$\vert$	$x \notin \mathsf{dom}\, \Gamma$	
	$\vert$	$uniq(\Omega)$	
	$\vert$	$c \notin \mathsf{dom}\, \Gamma$	
	$\vert$	$T \notin \mathsf{dom}\, \Sigma$	
	$\vert$	$F \notin \mathsf{dom}\, \Sigma$	
	$\vert$	$R_1 = R_2$	
	$\vert$	$a = b$	
	$\vert$	$\phi_1 = \phi_2$	
	$\vert$	$\Gamma_1 = \Gamma_2$	
	$\vert$	$\gamma_1 = \gamma_2$	
	$\vert$	$\neg\psi$	
	$\vert$	$\psi_1 \wedge \psi_2$	
	$\vert$	$\psi_1 \vee \psi_2$	
	$\vert$	$\psi_1 \Rightarrow \psi_2$	
	$\vert$	(ψ)	
	$\vert$	ψ	
	$\vert$	$c : (a : A \sim b : B) \in \Gamma$	
	$\vert$		suppress lc hypothesis generated by Ott
	$\vert$	$\{y/x\}B = B_1$	
	$\vert$	$\{c_1/c_2\}B = B_1$	
JSubRole	$::=$		
	$\vert$	$R_1 \leq R_2$	Subroling judgement
JPath	$::=$		
	$\vert$	$\mathsf{Path}_R\ a = F@Rs$	Type headed by constant (partial function)
JPat	$::=$		
	$\vert$	$\Gamma \vDash a : Apat/R$	Pattern judgment
JMatchSubst	$::=$		
	$\vert$	$\mathsf{match}_R\ a_1\ \mathsf{with}\ a_2 \to b_1 = b_2$	match and substitute
JValue	$::=$		
	$\vert$	$\mathsf{Value}_R\ A$	values
JValueType	$::=$		

	$\mid$	$\mathsf{ValueType}_R\ A$	Types with head forms (erased language)
Jconsistent	::=		
	$\mid$	$\mathsf{consistent}_R\ ab$	(erased) types do not differ in their heads
Jroleing	::=		
	$\mid$	$\Omega \vDash a : R$	
JTypeRoleList	::=		
	$\mid$	$\mathsf{Roles}(A) = Rs$	type role list
JChk	::=		
	$\mid$	$(\rho = +) \vee (x \notin \mathsf{fv}\ A)$	irrelevant argument check
Jpar	::=		
	$\mid$	$\Omega \vDash a \Rightarrow_R b$	parallel reduction (implicit language)
	$\mid$	$\Omega \vdash a \Rightarrow_R^* b$	multistep parallel reduction
	$\mid$	$\Omega \vdash a \Leftrightarrow_R b$	parallel reduction to a common term
Jbeta	::=		
	$\mid$	$\vDash a > b/R$	primitive reductions on erased terms
	$\mid$	$\vDash a \leadsto b/R$	single-step head reduction for implicit language
	$\mid$	$\vDash a \leadsto^* b/R$	multistep reduction
Jett	::=		
	$\mid$	$\Gamma \vDash \phi\ \mathsf{ok}$	Prop wellformedness
	$\mid$	$\Gamma \vDash a : A$	typing
	$\mid$	$\Gamma; \Delta \vDash \phi_1 \equiv \phi_2$	prop equality
	$\mid$	$\Gamma; \Delta \vDash a \equiv b : A/R$	definitional equality
	$\mid$	$\vDash \Gamma$	context wellformedness
Jsig	::=		
	$\mid$	$\vDash \Sigma$	signature wellformedness
judgement	::=		
	$\mid$	*JSubRole*	
	$\mid$	*JPath*	
	$\mid$	*JPat*	
	$\mid$	*JMatchSubst*	
	$\mid$	*JValue*	
	$\mid$	*JValueType*	
	$\mid$	*Jconsistent*	
	$\mid$	*Jroleing*	
	$\mid$	*JTypeRoleList*	
	$\mid$	*JChk*	
	$\mid$	*Jpar*	
	$\mid$	*Jbeta*	

| | Jett
| | Jsig

$\mathit{user\_syntax}$ ::=
| | tmvar
| | covar
| | $\mathit{datacon}$
| | const
| | index
| | $\mathit{relflag}$
| | $\mathit{appflag}$
| | role
| | $\mathit{constraint}$
| | tm
| | brs
| | co
| | $\mathit{role\_context}$
| | roles
| | $\mathit{sig\_sort}$
| | sort
| | $\mathit{context}$
| | sig
| | $\mathit{available\_props}$
| | $\mathit{terminals}$
| | $\mathit{formula}$

$\boxed{R_1 \leq R_2}$ Subroling judgement

$$\frac{}{\mathbf{Nom} \leq R} \quad \textsc{NomBot}$$

$$\frac{}{R \leq \mathbf{Rep}} \quad \textsc{RepTop}$$

$$\frac{}{R \leq R} \quad \textsc{Refl}$$

$$\frac{R_1 \leq R_2 \qquad R_2 \leq R_3}{R_1 \leq R_3} \quad \textsc{Trans}$$

$\boxed{\mathsf{Path}_R\ a = F@Rs}$ Type headed by constant (partial function)

$$\frac{F\ : A@Rs\ \in \Sigma_0}{\mathsf{Path}_R\ F = F@Rs} \quad \textsc{Path\_AbsConst}$$

$$\frac{F\ \sim a : A/R_1@Rs\ \in \Sigma_0 \qquad \neg(R_1 \leq R)}{\mathsf{Path}_R\ F = F@Rs} \quad \textsc{Path\_Const}$$

$$\frac{\mathsf{Path}_R\ a = F@R_1, Rs \qquad \mathit{app\_role}\nu = R_1}{\mathsf{Path}_R\ (a\ b'^{\nu}) = F@Rs} \quad \textsc{Path\_App}$$

$$\frac{\mathsf{Path}_R\ a = F@Rs}{\mathsf{Path}_R\ (a[\bullet]) = F@Rs} \quad \textsc{Path\_CApp}$$

$\boxed{\Gamma \vDash a : A pat/R}$ Pattern judgment

$$\frac{F : A@Rs \in \Sigma_0}{\varnothing \vDash F : A pat/R} \quad \textsc{Pat\_AbsConst}$$

$$\frac{\begin{array}{l} F \sim a : A/R_1@Rs \in \Sigma_0 \\ \neg(R_1 \leq R) \end{array}}{\varnothing \vDash F : A pat/R} \quad \textsc{Pat\_Const}$$

$$\frac{\begin{array}{l} \Gamma \vDash a : \Pi^\rho y\!:\!A_1 \to B_1 pat/R \\ x \notin \mathsf{dom}\,\Gamma \\ \{y/x\}B = B_1 \end{array}}{\Gamma, x : A_1 \vDash (a\ x^\rho) : B pat/R} \quad \textsc{Pat\_App}$$

$$\frac{\begin{array}{l} \Gamma \vDash a : \forall c_1\!:\!\phi.B_1 pat/R \\ c \notin \mathsf{dom}\,\Gamma \\ \{c_1/c\}B = B_1 \end{array}}{\Gamma, c : \phi \vDash (a[\bullet]) : B pat/R} \quad \textsc{Pat\_CApp}$$

$\boxed{\mathsf{match}_R\ a_1\ \mathsf{with}\ a_2 \to b_1 = b_2}$ match and substitute

$$\frac{F : A@Rs \in \Sigma_0}{\mathsf{match}_R\ F\ \mathsf{with}\ F \to b = b} \quad \textsc{MatchSubst\_AbsConst}$$

$$\frac{\begin{array}{l} F \sim a : A/R_1@Rs \in \Sigma_0 \\ \neg(R_1 \leq R) \end{array}}{\mathsf{match}_R\ F\ \mathsf{with}\ F \to b = b} \quad \textsc{MatchSubst\_Const}$$

$$\frac{\mathsf{match}_R\ a_1\ \mathsf{with}\ a_2 \to b_1 = b_2}{\mathsf{match}_R\ (a_1\ a^{R'})\ \mathsf{with}\ (a_2\ x^+) \to b_1 = (b_2\{a/x\})} \quad \textsc{MatchSubst\_AppRelR}$$

$$\frac{\mathsf{match}_R\ a_1\ \mathsf{with}\ a_2 \to b_1 = b_2}{\mathsf{match}_R\ (a_1\ a^+)\ \mathsf{with}\ (a_2\ x^+) \to b_1 = (b_2\{a/x\})} \quad \textsc{MatchSubst\_AppRel}$$

$$\frac{\mathsf{match}_R\ a_1\ \mathsf{with}\ a_2 \to b_1 = b_2}{\mathsf{match}_R\ (a_1\ \Box^-)\ \mathsf{with}\ (a_2\ \Box^-) \to b_1 = b_2} \quad \textsc{MatchSubst\_AppIrrel}$$

$$\frac{\mathsf{match}_R\ a_1\ \mathsf{with}\ a_2 \to b_1 = b_2}{\mathsf{match}_R\ (a_1[\bullet])\ \mathsf{with}\ (a_2[\bullet]) \to b_1 = b_2} \quad \textsc{MatchSubst\_CApp}$$

$\boxed{\mathsf{Value}_R\ A}$ values

$$\frac{}{\mathsf{Value}_R\ \star} \quad \textsc{Value\_Star}$$

$$\frac{}{\mathsf{Value}_R\ \Pi^\rho x\!:\!A \to B} \quad \textsc{Value\_Pi}$$

$$\frac{}{\mathsf{Value}_R\ \forall c\!:\!\phi.B} \quad \textsc{Value\_CPi}$$

$$\frac{}{\mathsf{Value}_R\ \lambda^+ x\!:\!A.a} \quad \textsc{Value\_AbsRel}$$

$$\frac{}{\mathsf{Value}_R\ \lambda^+ x.a} \quad \textsc{Value\_UAbsRel}$$

$$\frac{\mathsf{Value}_R\ a}{\mathsf{Value}_R\ \lambda^- x.a} \quad \textsc{Value\_UAbsIrrel}$$

$$\frac{}{\mathsf{Value}_R\ \Lambda c\!:\!\phi.a} \quad \textsc{Value\_CAbs}$$

$$\frac{}{\mathsf{Value}_R\ \Lambda c.a}\quad \text{VALUE\_UCABS}$$

$$\frac{\mathsf{Path}_R\ a = F@Rs}{\mathsf{Value}_R\ a}\quad \text{VALUE\_PATH}$$

$\boxed{\mathsf{ValueType}_R\ A}$ Types with head forms (erased language)

$$\frac{}{\mathsf{ValueType}_R\ \star}\quad \text{VALUE\_TYPE\_STAR}$$

$$\frac{}{\mathsf{ValueType}_R\ \Pi^\rho x\!:\!A \to B}\quad \text{VALUE\_TYPE\_PI}$$

$$\frac{}{\mathsf{ValueType}_R\ \forall c\!:\!\phi.B}\quad \text{VALUE\_TYPE\_CPI}$$

$$\frac{\mathsf{Path}_R\ a = F@Rs}{\mathsf{ValueType}_R\ a}\quad \text{VALUE\_TYPE\_PATH}$$

$\boxed{\mathsf{consistent}_R\ ab}$ (erased) types do not differ in their heads

$$\frac{}{\mathsf{consistent}_R\ \star\star}\quad \text{CONSISTENT\_A\_STAR}$$

$$\frac{}{\mathsf{consistent}_{R'}\ (\Pi^\rho x_1\!:\!A_1 \to B_1)(\Pi^\rho x_2\!:\!A_2 \to B_2)}\quad \text{CONSISTENT\_A\_PI}$$

$$\frac{}{\mathsf{consistent}_R\ (\forall c_1\!:\!\phi_1.A_1)(\forall c_2\!:\!\phi_2.A_2)}\quad \text{CONSISTENT\_A\_CPI}$$

$$\frac{\begin{array}{l}\mathsf{Path}_R\ a_1 = F@Rs\\ \mathsf{Path}_R\ a_2 = F@Rs\end{array}}{\mathsf{consistent}_R\ a_1\, a_2}\quad \text{CONSISTENT\_A\_PATH}$$

$$\frac{\neg\mathsf{ValueType}_R\ b}{\mathsf{consistent}_R\ ab}\quad \text{CONSISTENT\_A\_STEP\_R}$$

$$\frac{\neg\mathsf{ValueType}_R\ a}{\mathsf{consistent}_R\ ab}\quad \text{CONSISTENT\_A\_STEP\_L}$$

$\boxed{\Omega \vDash a : R}$

$$\frac{\mathit{uniq}(\Omega)}{\Omega \vDash \square : R}\quad \text{ROLE\_A\_BULLET}$$

$$\frac{\mathit{uniq}(\Omega)}{\Omega \vDash \star : R}\quad \text{ROLE\_A\_STAR}$$

$$\frac{\begin{array}{l}\mathit{uniq}(\Omega)\\ x : R \in \Omega\\ R \leq R_1\end{array}}{\Omega \vDash x : R_1}\quad \text{ROLE\_A\_VAR}$$

$$\frac{\Omega, x : \mathbf{Nom} \vDash a : R}{\Omega \vDash (\lambda^\rho x.a) : R}\quad \text{ROLE\_A\_ABS}$$

$$\frac{\begin{array}{l}\Omega \vDash a : R\\ \Omega \vDash b : \mathit{app\_role}\nu\end{array}}{\Omega \vDash (a\ b^\nu) : R}\quad \text{ROLE\_A\_APP}$$

$$\frac{\begin{array}{l}\Omega \vDash A : R\\ \Omega, x : \mathbf{Nom} \vDash B : R\end{array}}{\Omega \vDash (\Pi^\rho x\!:\!A \to B) : R}\quad \text{ROLE\_A\_PI}$$

$$\frac{\Omega \vDash a : R_1 \quad \Omega \vDash b : R_1 \quad \Omega \vDash A : R_0 \quad \Omega \vDash B : R}{\Omega \vDash (\forall c\!:\! a \sim_{A/R_1} b.B) : R} \quad \text{ROLE\_A\_CPI}$$

$$\frac{\Omega \vDash b : R}{\Omega \vDash (\Lambda c.b) : R} \quad \text{ROLE\_A\_CABS}$$

$$\frac{\Omega \vDash a : R}{\Omega \vDash (a[\bullet]) : R} \quad \text{ROLE\_A\_CAPP}$$

$$\frac{\mathit{uniq}(\Omega) \quad F\ : A@Rs\ \in\ \Sigma_0}{\Omega \vDash F : R} \quad \text{ROLE\_A\_CONST}$$

$$\frac{\mathit{uniq}(\Omega) \quad F\ \sim a : A/R@Rs\ \in\ \Sigma_0}{\Omega \vDash F : R_1} \quad \text{ROLE\_A\_FAM}$$

$$\frac{\Omega \vDash a_1 : R \quad \Omega \vDash b_1 : R_1 \quad \Omega \vDash b_2 : R_1}{\Omega \vDash \mathsf{case}_R\ a_1\ \mathsf{of}\ a_2 \to b_1 \|_{\_} \to b_2 : R_1} \quad \text{ROLE\_A\_PATTERN}$$

$\boxed{\mathsf{Roles}(A) = Rs}$ type role list

$$\frac{}{\mathsf{Roles}(\star) = \mathbf{nilR}} \quad \text{TYPEROLELIST\_STAR}$$

$$\frac{\mathsf{Roles}(B) = Rs \quad x : \mathbf{Nom} \vDash B : R_1}{\mathsf{Roles}((\Pi^\rho x\!:\! A \to B)) = R_1, Rs} \quad \text{TYPEROLELIST\_PI}$$

$$\frac{\mathsf{Roles}(B) = Rs}{\mathsf{Roles}((\forall c\!:\!\phi.B)) = Rs} \quad \text{TYPEROLELIST\_CPI}$$

$\boxed{(\rho = +) \vee (x \notin \mathsf{fv}\ A)}$ irrelevant argument check

$$\frac{}{(+ = +) \vee (x \notin \mathsf{fv}\ A)} \quad \text{RHO\_REL}$$

$$\frac{x\ \notin\ \mathsf{fv} A}{(- = +) \vee (x \notin \mathsf{fv}\ A)} \quad \text{RHO\_IRRREL}$$

$\boxed{\Omega \vDash a \Rightarrow_R b}$ parallel reduction (implicit language)

$$\frac{\Omega \vDash a : R}{\Omega \vDash a \Rightarrow_R a} \quad \text{PAR\_REFL}$$

$$\frac{\Omega \vDash a \Rightarrow_R (\lambda^\rho x.a') \quad \Omega \vDash b \Rightarrow_{\mathit{app\_role}\,\nu} b'}{\Omega \vDash a\ b^\nu \Rightarrow_R a'\{b'/x\}} \quad \text{PAR\_BETA}$$

$$\frac{\Omega \vDash a \Rightarrow_R a' \quad \Omega \vDash b \Rightarrow_{\mathit{app\_role}\,\nu} b'}{\Omega \vDash a\ b^\nu \Rightarrow_R a'\ b'^\nu} \quad \text{PAR\_APP}$$

$$\frac{\Omega \vDash a \Rightarrow_R (\Lambda c.a')}{\Omega \vDash a[\bullet] \Rightarrow_R a'\{\bullet/c\}} \quad \textsc{Par\_CBeta}$$

$$\frac{\Omega \vDash a \Rightarrow_R a'}{\Omega \vDash a[\bullet] \Rightarrow_R a'[\bullet]} \quad \textsc{Par\_CApp}$$

$$\frac{\Omega, x : \mathbf{Nom} \vDash a \Rightarrow_R a'}{\Omega \vDash \lambda^\rho x.a \Rightarrow_R \lambda^\rho x.a'} \quad \textsc{Par\_Abs}$$

$$\frac{\begin{array}{l}\Omega \vDash A \Rightarrow_R A' \\ \Omega, x : \mathbf{Nom} \vDash B \Rightarrow_R B'\end{array}}{\Omega \vDash \Pi^\rho x{:}A \to B \Rightarrow_R \Pi^\rho x{:}A' \to B'} \quad \textsc{Par\_Pi}$$

$$\frac{\Omega \vDash a \Rightarrow_R a'}{\Omega \vDash \Lambda c.a \Rightarrow_R \Lambda c.a'} \quad \textsc{Par\_CAbs}$$

$$\frac{\begin{array}{l}\Omega \vDash A \Rightarrow_{R_0} A' \\ \Omega \vDash a \Rightarrow_{R_1} a' \\ \Omega \vDash b \Rightarrow_{R_1} b' \\ \Omega \vDash B \Rightarrow_R B'\end{array}}{\Omega \vDash \forall c{:}a \sim_{A/R_1} b.B \Rightarrow_R \forall c{:}a' \sim_{A'/R_1} b'.B'} \quad \textsc{Par\_CPi}$$

$$\frac{\begin{array}{l}F \sim a : A/R_1 @ Rs \in \Sigma_0 \\ R_1 \leq R \\ \mathit{uniq}(\Omega)\end{array}}{\Omega \vDash F \Rightarrow_R a} \quad \textsc{Par\_Axiom}$$

$$\frac{\begin{array}{l}\Omega \vDash a_1 \Rightarrow_R a_1' \\ \Omega \vDash b_1 \Rightarrow_{R_0} b_1' \\ \Omega \vDash b_2 \Rightarrow_{R_0} b_2'\end{array}}{\Omega \vDash (\mathsf{case}_R\ a_1\ \mathsf{of}\ a_2 \to b_1 \| \_ \to b_2) \Rightarrow_{R_0} (\mathsf{case}_R\ a_1'\ \mathsf{of}\ a_2 \to b_1' \| \_ \to b_2')} \quad \textsc{Par\_Pattern}$$

$$\frac{\begin{array}{l}\Omega \vDash a_1 \Rightarrow_R a_1' \\ \Omega \vDash b_1 \Rightarrow_{R_0} b_1' \\ \Omega \vDash b_2 \Rightarrow_{R_0} b_2' \\ \mathsf{match}_R\ a_1'\ \mathsf{with}\ a_2 \to b_1' = b\end{array}}{\Omega \vDash (\mathsf{case}_R\ a_1\ \mathsf{of}\ a_2 \to b_1 \| \_ \to b_2) \Rightarrow_{R_0} b} \quad \textsc{Par\_PatternTrue}$$

$$\frac{\begin{array}{l}\Omega \vDash a_1 \Rightarrow_R a_1' \\ \Omega \vDash b_1 \Rightarrow_{R_0} b_1' \\ \Omega \vDash b_2 \Rightarrow_{R_0} b_2' \\ \mathsf{Value}_R\ a_1' \\ \neg(\mathsf{match}_R\ a_1'\ \mathsf{with}\ a_2 \to b_1' = b)\end{array}}{\Omega \vDash (\mathsf{case}_R\ a_1\ \mathsf{of}\ a_2 \to b_1 \| \_ \to b_2) \Rightarrow_{R_0} b_2'} \quad \textsc{Par\_PatternFalse}$$

$\boxed{\Omega \vdash a \Rightarrow_R^* b}$ multistep parallel reduction

$$\frac{}{\Omega \vdash a \Rightarrow_R^* a} \quad \textsc{MP\_Refl}$$

$$\frac{\begin{array}{l}\Omega \vDash a \Rightarrow_R b \\ \Omega \vdash b \Rightarrow_R^* a'\end{array}}{\Omega \vdash a \Rightarrow_R^* a'} \quad \textsc{MP\_Step}$$

$\boxed{\Omega \vdash a \Leftrightarrow_R b}$ parallel reduction to a common term

$$\frac{\Omega \vdash a_1 \Rightarrow^*_R b \quad \Omega \vdash a_2 \Rightarrow^*_R b}{\Omega \vdash a_1 \Leftrightarrow_R a_2} \quad \text{JOIN}$$

$\boxed{\vDash a > b/R}$ primitive reductions on erased terms

$$\frac{\mathsf{Value}_{R_1}\ (\lambda^\rho x.v)}{\vDash (\lambda^\rho x.v)\ b^\nu > v\{b/x\}/R_1} \quad \textsc{Beta\_AppAbs}$$

$$\frac{}{\vDash (\Lambda c.a')[\bullet] > a'\{\bullet/c\}/R} \quad \textsc{Beta\_CAppCAbs}$$

$$\frac{F \sim a : A/R@Rs \in \Sigma_0 \quad R \leq R_1}{\vDash F > a/R_1} \quad \textsc{Beta\_Axiom}$$

$$\frac{\mathsf{match}_R\ a_1\ \mathsf{with}\ a_2 \to b_1 = b}{\vDash \mathsf{case}_R\ a_1\ \mathsf{of}\ a_2 \to b_1 \|_\_ \to b_2 > b/R_0} \quad \textsc{Beta\_PatternTrue}$$

$$\frac{\mathsf{Value}_R\ a_1 \quad \neg(\mathsf{match}_R\ a_1\ \mathsf{with}\ a_2 \to b_1 = b)}{\vDash \mathsf{case}_R\ a_1\ \mathsf{of}\ a_2 \to b_1 \|_\_ \to b_2 > b_2/R_0} \quad \textsc{Beta\_PatternFalse}$$

$\boxed{\vDash a \leadsto b/R}$ single-step head reduction for implicit language

$$\frac{\vDash a \leadsto a'/R_1}{\vDash \lambda^- x.a \leadsto \lambda^- x.a'/R_1} \quad \textsc{E\_AbsTerm}$$

$$\frac{\vDash a \leadsto a'/R_1}{\vDash a\ b^\nu \leadsto a'\ b^\nu/R_1} \quad \textsc{E\_AppLeft}$$

$$\frac{\vDash a \leadsto a'/R}{\vDash a[\bullet] \leadsto a'[\bullet]/R} \quad \textsc{E\_CAppLeft}$$

$$\frac{\vDash a \leadsto a_1'/R}{\vDash \mathsf{case}_R\ a_1\ \mathsf{of}\ a_2 \to b_1 \|_\_ \to b_2 \leadsto \mathsf{case}_R\ a_1'\ \mathsf{of}\ a_2 \to b_1 \|_\_ \to b_2/R_0} \quad \textsc{E\_Pattern}$$

$$\frac{\vDash a > b/R}{\vDash a \leadsto b/R} \quad \textsc{E\_Prim}$$

$\boxed{\vDash a \leadsto^* b/R}$ multistep reduction

$$\frac{}{\vDash a \leadsto^* a/R} \quad \textsc{Equal}$$

$$\frac{\vDash a \leadsto b/R \quad \vDash b \leadsto^* a'/R}{\vDash a \leadsto^* a'/R} \quad \textsc{Step}$$

$\boxed{\Gamma \vDash \phi\ \mathsf{ok}}$ Prop wellformedness

$$\frac{\Gamma \vDash a : A \quad \Gamma \vDash b : A \quad \Gamma \vDash A : \star}{\Gamma \vDash a \sim_{A/R} b\ \mathsf{ok}} \quad \textsc{E\_Wff}$$

$\boxed{\Gamma \vDash a : A}$ typing

$$\frac{\vDash \Gamma}{\Gamma \vDash \star : \star} \quad \text{E\_Star}$$

$$\frac{\begin{array}{l}\vDash \Gamma \\ x : A \in \Gamma\end{array}}{\Gamma \vDash x : A} \quad \text{E\_Var}$$

$$\frac{\begin{array}{l}\Gamma, x : A \vDash B : \star \\ \Gamma \vDash A : \star\end{array}}{\Gamma \vDash \Pi^\rho x\!:\!A \to B : \star} \quad \text{E\_Pi}$$

$$\frac{\begin{array}{l}\Gamma, x : A \vDash a : B \\ \Gamma \vDash A : \star \\ (\rho = +) \vee (x \notin \mathsf{fv}\ a)\end{array}}{\Gamma \vDash \lambda^\rho x.a : (\Pi^\rho x\!:\!A \to B)} \quad \text{E\_Abs}$$

$$\frac{\begin{array}{l}\Gamma \vDash b : \Pi^+ x\!:\!A \to B \\ \Gamma \vDash a : A\end{array}}{\Gamma \vDash b\ \ a^+ : B\{a/x\}} \quad \text{E\_App}$$

$$\frac{\begin{array}{l}\Gamma \vDash b : \Pi^+ x\!:\!A \to B \\ \Gamma \vDash a : A \\ \mathsf{Path}_{R'}\ \ a = F@R, Rs\end{array}}{\Gamma \vDash b\ \ a^R : B\{a/x\}} \quad \text{E\_TApp}$$

$$\frac{\begin{array}{l}\Gamma \vDash b : \Pi^- x\!:\!A \to B \\ \Gamma \vDash a : A\end{array}}{\Gamma \vDash b\ \square^- : B\{a/x\}} \quad \text{E\_IApp}$$

$$\frac{\begin{array}{l}\Gamma \vDash a : A \\ \Gamma; \widetilde{\Gamma} \vDash A \equiv B : \star/\mathbf{Rep} \\ \Gamma \vDash B : \star\end{array}}{\Gamma \vDash a : B} \quad \text{E\_Conv}$$

$$\frac{\begin{array}{l}\Gamma, c : \phi \vDash B : \star \\ \Gamma \vDash \phi\ \ \mathsf{ok}\end{array}}{\Gamma \vDash \forall c\!:\!\phi.B : \star} \quad \text{E\_CPi}$$

$$\frac{\begin{array}{l}\Gamma, c : \phi \vDash a : B \\ \Gamma \vDash \phi\ \ \mathsf{ok}\end{array}}{\Gamma \vDash \Lambda c.a : \forall c\!:\!\phi.B} \quad \text{E\_CAbs}$$

$$\frac{\begin{array}{l}\Gamma \vDash a_1 : \forall c\!:\!(a \sim_{A/R} b).B_1 \\ \Gamma; \widetilde{\Gamma} \vDash a \equiv b : A/R\end{array}}{\Gamma \vDash a_1[\bullet] : B_1\{\bullet/c\}} \quad \text{E\_CApp}$$

$$\frac{\begin{array}{l}\vDash \Gamma \\ F\ : A@Rs \in \Sigma_0 \\ \varnothing \vDash A : \star\end{array}}{\Gamma \vDash F : A} \quad \text{E\_Const}$$

$$\frac{\begin{array}{l}\vDash \Gamma \\ F\ \sim a : A/R_1@Rs \in \Sigma_0 \\ \varnothing \vDash A : \star\end{array}}{\Gamma \vDash F : A} \quad \text{E\_Fam}$$

$$\frac{\begin{array}{l}\Gamma \vDash a_1 : A \\ \Gamma' \vDash a_2 : Apat/R \\ \Gamma, (\Gamma', c : \phi_1) \vDash b_1 : B \\ \Gamma \vDash b_2 : B \\ \phi_1 = (a_1 \sim_{A/R} a_2)\end{array}}{\Gamma \vDash \mathsf{case}_R\ a_1\ \mathsf{of}\ a_2 \to b_1 \|_{\_} \to b_2 : B} \quad \text{E\_CASE}$$

$\boxed{\Gamma; \Delta \vDash \phi_1 \equiv \phi_2}$ prop equality

$$\frac{\begin{array}{c}\Gamma; \Delta \vDash A_1 \equiv A_2 : A/R \\ \Gamma; \Delta \vDash B_1 \equiv B_2 : A/R\end{array}}{\Gamma; \Delta \vDash A_1 \sim_{A/R} B_1 \equiv A_2 \sim_{A/R} B_2} \quad \text{E\_PropCong}$$

$$\frac{\begin{array}{c}\Gamma; \Delta \vDash A \equiv B : \star/R_0 \\ \Gamma \vDash A_1 \sim_{A/R} A_2\ \ \mathsf{ok} \\ \Gamma \vDash A_1 \sim_{B/R} A_2\ \ \mathsf{ok}\end{array}}{\Gamma; \Delta \vDash A_1 \sim_{A/R} A_2 \equiv A_1 \sim_{B/R} A_2} \quad \text{E\_IsoConv}$$

$$\frac{\Gamma; \Delta \vDash \forall c\!:\!(a_1 \sim_{A/R_1} a_2).B_1 \equiv \forall c\!:\!(b_1 \sim_{B/R_2} b_2).B_2 : \star/R'}{\Gamma; \Delta \vDash a_1 \sim_{A/R_1} a_2 \equiv b_1 \sim_{B/R_2} b_2} \quad \text{E\_CPiFst}$$

$\boxed{\Gamma; \Delta \vDash a \equiv b : A/R}$ definitional equality

$$\frac{\begin{array}{l}\vDash \Gamma \\ c : (a \sim_{A/R} b) \in \Gamma \\ c \in \Delta\end{array}}{\Gamma; \Delta \vDash a \equiv b : A/R} \quad \text{E\_Assn}$$

$$\frac{\Gamma \vDash a : A}{\Gamma; \Delta \vDash a \equiv a : A/\mathbf{Nom}} \quad \text{E\_Refl}$$

$$\frac{\Gamma; \Delta \vDash b \equiv a : A/R}{\Gamma; \Delta \vDash a \equiv b : A/R} \quad \text{E\_Sym}$$

$$\frac{\begin{array}{c}\Gamma; \Delta \vDash a \equiv a_1 : A/R \\ \Gamma; \Delta \vDash a_1 \equiv b : A/R\end{array}}{\Gamma; \Delta \vDash a \equiv b : A/R} \quad \text{E\_Trans}$$

$$\frac{\begin{array}{l}\Gamma; \Delta \vDash a \equiv b : A/R_1 \\ R_1 \leq R_2\end{array}}{\Gamma; \Delta \vDash a \equiv b : A/R_2} \quad \text{E\_Sub}$$

$$\frac{\begin{array}{l}\Gamma \vDash a_1 : B \\ \Gamma \vDash a_2 : B \\ \vDash a_1 > a_2/R\end{array}}{\Gamma; \Delta \vDash a_1 \equiv a_2 : B/R} \quad \text{E\_Beta}$$

$$\frac{\begin{array}{l}\Gamma; \Delta \vDash A_1 \equiv A_2 : \star/R' \\ \Gamma, x : A_1; \Delta \vDash B_1 \equiv B_2 : \star/R' \\ \Gamma \vDash A_1 : \star \\ \Gamma \vDash \Pi^\rho x\!:\!A_1 \to B_1 : \star \\ \Gamma \vDash \Pi^\rho x\!:\!A_2 \to B_2 : \star\end{array}}{\Gamma; \Delta \vDash (\Pi^\rho x\!:\!A_1 \to B_1) \equiv (\Pi^\rho x\!:\!A_2 \to B_2) : \star/R'} \quad \text{E\_PiCong}$$

$$\frac{\begin{gathered}\Gamma, x : A_1; \Delta \vDash b_1 \equiv b_2 : B/R' \\ \Gamma \vDash A_1 : \star \\ (\rho = +) \vee (x \notin \mathsf{fv}\ b_1) \\ (\rho = +) \vee (x \notin \mathsf{fv}\ b_2)\end{gathered}}{\Gamma; \Delta \vDash (\lambda^\rho x.b_1) \equiv (\lambda^\rho x.b_2) : (\Pi^\rho x\!:\!A_1 \to B)/R'} \quad \text{E\_AbsCong}$$

$$\frac{\begin{gathered}\Gamma; \Delta \vDash a_1 \equiv b_1 : (\Pi^+ x\!:\!A \to B)/R' \\ \Gamma; \Delta \vDash a_2 \equiv b_2 : A/\mathbf{Nom}\end{gathered}}{\Gamma; \Delta \vDash a_1\ {a_2}^+ \equiv b_1\ {b_2}^+ : (B\{a_2/x\})/R'} \quad \text{E\_AppCong}$$

$$\frac{\begin{gathered}\Gamma; \Delta \vDash a_1 \equiv b_1 : (\Pi^+ x\!:\!A \to B)/R' \\ \mathsf{Path}_{R'}\ a_1 = F@R, Rs \\ \Gamma; \Delta \vDash a_2 \equiv b_2 : A/\mathbf{param}\, R\, R'\end{gathered}}{\Gamma; \Delta \vDash a_1\ {a_2}^R \equiv b_1\ {b_2}^R : (B\{a_2/x\})/R'} \quad \text{E\_TAppCong}$$

$$\frac{\begin{gathered}\Gamma; \Delta \vDash a_1 \equiv b_1 : (\Pi^- x\!:\!A \to B)/R' \\ \Gamma \vDash a : A\end{gathered}}{\Gamma; \Delta \vDash a_1\ \Box^- \equiv b_1\ \Box^- : (B\{a/x\})/R'} \quad \text{E\_IAppCong}$$

$$\frac{\Gamma; \Delta \vDash \Pi^\rho x\!:\!A_1 \to B_1 \equiv \Pi^\rho x\!:\!A_2 \to B_2 : \star/R'}{\Gamma; \Delta \vDash A_1 \equiv A_2 : \star/R'} \quad \text{E\_PiFst}$$

$$\frac{\begin{gathered}\Gamma; \Delta \vDash \Pi^\rho x\!:\!A_1 \to B_1 \equiv \Pi^\rho x\!:\!A_2 \to B_2 : \star/R' \\ \Gamma; \Delta \vDash a_1 \equiv a_2 : A_1/R'\end{gathered}}{\Gamma; \Delta \vDash B_1\{a_1/x\} \equiv B_2\{a_2/x\} : \star/R'} \quad \text{E\_PiSnd}$$

$$\frac{\begin{gathered}\Gamma; \Delta \vDash a_1 \sim_{A_1/R} b_1 \equiv a_2 \sim_{A_2/R} b_2 \\ \Gamma, c : a_1 \sim_{A_1/R} b_1; \Delta \vDash A \equiv B : \star/R' \\ \Gamma \vDash a_1 \sim_{A_1/R} b_1\ \mathsf{ok} \\ \Gamma \vDash \forall c\!:\!a_1 \sim_{A_1/R} b_1.A : \star \\ \Gamma \vDash \forall c\!:\!a_2 \sim_{A_2/R} b_2.B : \star\end{gathered}}{\Gamma; \Delta \vDash \forall c\!:\!a_1 \sim_{A_1/R} b_1.A \equiv \forall c\!:\!a_2 \sim_{A_2/R} b_2.B : \star/R'} \quad \text{E\_CPiCong}$$

$$\frac{\begin{gathered}\Gamma, c : \phi_1; \Delta \vDash a \equiv b : B/R \\ \Gamma \vDash \phi_1\ \mathsf{ok}\end{gathered}}{\Gamma; \Delta \vDash (\Lambda c.a) \equiv (\Lambda c.b) : \forall c\!:\!\phi_1.B/R} \quad \text{E\_CAbsCong}$$

$$\frac{\begin{gathered}\Gamma; \Delta \vDash a_1 \equiv b_1 : (\forall c\!:\!(a \sim_{A/R} b).B)/R' \\ \Gamma; \widetilde{\Gamma} \vDash a \equiv b : A/\mathbf{param}\, R\, R'\end{gathered}}{\Gamma; \Delta \vDash a_1[\bullet] \equiv b_1[\bullet] : (B\{\bullet/c\})/R'} \quad \text{E\_CAppCong}$$

$$\frac{\begin{gathered}\Gamma; \Delta \vDash \forall c\!:\!(a_1 \sim_{A/R} a_2).B_1 \equiv \forall c\!:\!(a_1' \sim_{A'/R'} a_2').B_2 : \star/R_0 \\ \Gamma; \widetilde{\Gamma} \vDash a_1 \equiv a_2 : A/\mathbf{param}\, R\, R_0 \\ \Gamma; \widetilde{\Gamma} \vDash a_1' \equiv a_2' : A'/\mathbf{param}\, R'\, R_0\end{gathered}}{\Gamma; \Delta \vDash B_1\{\bullet/c\} \equiv B_2\{\bullet/c\} : \star/R_0} \quad \text{E\_CPiSnd}$$

$$\frac{\begin{gathered}\Gamma; \Delta \vDash a \equiv b : A/R \\ \Gamma; \Delta \vDash a \sim_{A/R} b \equiv a' \sim_{A'/R'} b'\end{gathered}}{\Gamma; \Delta \vDash a' \equiv b' : A'/R'} \quad \text{E\_Cast}$$

$$\frac{\begin{gathered}\Gamma; \Delta \vDash a \equiv b : A/R \\ \Gamma; \widetilde{\Gamma} \vDash A \equiv B : \star/\mathbf{Rep} \\ \Gamma \vDash B : \star\end{gathered}}{\Gamma; \Delta \vDash a \equiv b : B/R} \quad \text{E\_EqConv}$$

$$\frac{\Gamma;\Delta \vDash a \sim_{A/R_1} b \equiv a' \sim_{A'/R_1} b'}{\Gamma;\Delta \vDash A \equiv A' : \star/\mathbf{Rep}} \quad \text{E\_IsoSnd}$$

$$\frac{\begin{array}{l}\Gamma;\Delta \vDash a_1 \equiv a_1' : A/R \\ \Gamma;\Delta \vDash b_1 \equiv b_1' : B/R_0 \\ \Gamma;\Delta \vDash b_2 \equiv b_2' : B/R_0\end{array}}{\Gamma;\Delta \vDash \mathsf{case}_R\ a_1\ \mathsf{of}\ a_2 \to b_1 \| \_ \to b_2 \equiv \mathsf{case}_R\ a_1'\ \mathsf{of}\ a_2 \to b_1' \| \_ \to b_2' : B/R_0} \quad \text{E\_PatCong}$$

$$\frac{\begin{array}{l}\mathsf{Path}_{R'}\ a = F@R, Rs \\ \mathsf{Path}_{R'}\ a' = F@R, Rs \\ \Gamma \vDash a : \Pi^+ x\!:\!A \to B \\ \Gamma \vDash b : A \\ \Gamma \vDash a' : \Pi^+ x\!:\!A \to B \\ \Gamma \vDash b' : A \\ \Gamma;\Delta \vDash a\ \ b^{R_1} \equiv a'\ \ b'^{R_1} : B\{b/x\}/R' \\ \Gamma;\widetilde{\Gamma} \vDash B\{b/x\} \equiv B\{b'/x\} : \star/R'\end{array}}{\Gamma;\Delta \vDash a \equiv a' : \Pi^+ x\!:\!A \to B/R'} \quad \text{E\_LeftRel}$$

$$\frac{\begin{array}{l}\mathsf{Path}_{R'}\ a = F@R, Rs \\ \mathsf{Path}_{R'}\ a' = F@R, Rs \\ \Gamma \vDash a : \Pi^- x\!:\!A \to B \\ \Gamma \vDash b : A \\ \Gamma \vDash a' : \Pi^- x\!:\!A \to B \\ \Gamma \vDash b' : A \\ \Gamma;\Delta \vDash a\ \square^- \equiv a'\ \square^- : B\{b/x\}/R' \\ \Gamma;\widetilde{\Gamma} \vDash B\{b/x\} \equiv B\{b'/x\} : \star/R_0\end{array}}{\Gamma;\Delta \vDash a \equiv a' : \Pi^- x\!:\!A \to B/R'} \quad \text{E\_LeftIrrel}$$

$$\frac{\begin{array}{l}\mathsf{Path}_{R'}\ a = F@R, Rs \\ \mathsf{Path}_{R'}\ a' = F@R, Rs \\ \Gamma \vDash a : \Pi^+ x\!:\!A \to B \\ \Gamma \vDash b : A \\ \Gamma \vDash a' : \Pi^+ x\!:\!A \to B \\ \Gamma \vDash b' : A \\ \Gamma;\Delta \vDash a\ \ b^{+} \equiv a'\ \ b'^{+} : B\{b/x\}/R' \\ \Gamma;\widetilde{\Gamma} \vDash B\{b/x\} \equiv B\{b'/x\} : \star/R_0\end{array}}{\Gamma;\Delta \vDash b \equiv b' : A/\mathbf{param}\, R_1\ R'} \quad \text{E\_Right}$$

$$\frac{\begin{array}{l}\mathsf{Path}_{R'}\ a = F@R, Rs \\ \mathsf{Path}_{R'}\ a' = F@R, Rs \\ \Gamma \vDash a : \forall c\!:\!(a_1 \sim_{A/R_1} a_2).B \\ \Gamma \vDash a' : \forall c\!:\!(a_1 \sim_{A/R_1} a_2).B \\ \Gamma;\widetilde{\Gamma} \vDash a_1 \equiv a_2 : A/R' \\ \Gamma;\Delta \vDash a[\bullet] \equiv a'[\bullet] : B\{\bullet/c\}/R'\end{array}}{\Gamma;\Delta \vDash a \equiv a' : \forall c\!:\!(a_1 \sim_{A/R_1} a_2).B/R'} \quad \text{E\_CLeft}$$

$\boxed{\vDash \Gamma}$ context wellformedness

$$\frac{}{\vDash \varnothing} \quad \text{E\_Empty}$$

$$\frac{\begin{array}{l}\vDash \Gamma \\ \Gamma \vDash A : \star \\ x \notin \mathsf{dom}\,\Gamma\end{array}}{\vDash \Gamma, x : A} \quad \text{E\_ConsTm}$$

$$\frac{\models \Gamma \quad \Gamma \models \phi \ \mathsf{ok} \quad c \notin \mathsf{dom}\,\Gamma}{\models \Gamma, c : \phi} \quad \textsc{E\_ConsCo}$$

$\boxed{\models \Sigma}$ signature wellformedness

$$\frac{}{\models \varnothing} \quad \textsc{Sig\_Empty}$$

$$\frac{\models \Sigma \quad \varnothing \models A : \star \quad F \notin \mathsf{dom}\,\Sigma \quad \mathsf{Roles}(A) = Rs}{\models \Sigma \cup \{F : A@Rs\}} \quad \textsc{Sig\_ConsConst}$$

$$\frac{\models \Sigma \quad \varnothing \models a : A \quad F \notin \mathsf{dom}\,\Sigma \quad \mathsf{Roles}(A) = Rs}{\models \Sigma \cup \{F \sim a : A/R@Rs\}} \quad \textsc{Sig\_ConsAx}$$

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Definition rules:        131 good    0 bad
Definition rule clauses: 377 good    0 bad
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