

$tmvar, x, y, f, m, n$	variables
$covar, c$	coercion variables
$datacon, K$	
$const, T$	
$tyfam, F$	
$index, i$	indices



		<b>Fix</b>	S	
		$a \rightarrow b$	S	
		$\phi \Rightarrow A$	S	
		$ab^{R,+}$	S	
		$\lambda^R x. a$	S	
		$\lambda x : A. a$	S	
		$\forall x : A/R \rightarrow B$	S	
$brs$	::=			case branches
		<b>none</b>		
		$K \Rightarrow a; brs$		
		$brs\{a/x\}$	S	
		$brs\{\gamma/c\}$	S	
		$(brs)$	S	
$co, \gamma$	::=			explicit coercions
		•		
		$c$		
		<b>red</b> $a\ b$		
		<b>refl</b> $a$		
		$(a \models_{\gamma} b)$		
		<b>sym</b> $\gamma$		
		$\gamma_1; \gamma_2$		
		<b>sub</b> $\gamma$		
		$\Pi^{R,\rho} x : \gamma_1. \gamma_2$	bind $x$ in $\gamma_2$	
		$\lambda^{R,\rho} x : \gamma_1. \gamma_2$	bind $x$ in $\gamma_2$	
		$\gamma_1 \gamma_2^{R,\rho}$		
		<b>piFst</b> $\gamma$		
		<b>cpiFst</b> $\gamma$		
		<b>isoSnd</b> $\gamma$		
		$\gamma_1 @ \gamma_2$		
		$\forall c : \gamma_1. \gamma_3$	bind $c$ in $\gamma_3$	
		$\lambda c : \gamma_1. \gamma_3 @ \gamma_4$	bind $c$ in $\gamma_3$	
		$\gamma(\gamma_1, \gamma_2)$		
		$\gamma @ (\gamma_1 \sim \gamma_2)$		
		$\gamma_1 \triangleright_R \gamma_2$		
		$\gamma_1 \sim_A \gamma_2$		
		<b>conv</b> $\phi_1 \sim_{\gamma} \phi_2$		
		<b>eta</b> $a$		
		<b>left</b> $\gamma \gamma'$		
		<b>right</b> $\gamma \gamma'$		
		$(\gamma)$	S	
		$\gamma$	S	
		$\gamma\{a/x\}$	S	
$sig\_sort$	::=			signature classifier
		<b>Cs</b> $A$		

		<b>Ax</b> $a A R$	
$sort$	$::=$		binding classifier
		<b>Tm</b> $A R$	
		<b>Co</b> $\phi$	
$context, \Gamma$	$::=$		contexts
		$\emptyset$	
		$\Gamma, x : A/R$	
		$\Gamma, c : \phi$	
		$\Gamma\{b/x\}$	M
		$\Gamma\{\gamma/c\}$	M
		$\Gamma, \Gamma'$	M
		$ \Gamma $	M
		$(\Gamma)$	M
		$\Gamma$	M
$role\_context, \Omega$	$::=$		$role\_contexts$
		$\emptyset$	
		$\Omega, x : R, \Omega'$	
		$\Omega, c : R, \Omega'$	
		$\Omega, \Omega'$	M
		$(\Omega)$	M
		$\Omega$	M
$available\_props, \Delta$	$::=$		
		$\emptyset$	
		$\Delta, c$	
		$\tilde{\Gamma}$	M
		$(\Delta)$	M
$sig, \Sigma$	$::=$		signatures
		$\emptyset$	
		$\Sigma \cup \{T : A/R\}$	
		$\Sigma \cup \{F \sim a : A/R\}$	
		$\Sigma_0$	M
		$\Sigma_1$	M
		$ \Sigma $	M
$terminals$	$::=$		
		$\leftrightarrow$	
		$\Leftrightarrow$	
		$\longrightarrow$	
		<b>min</b>	
		$\equiv$	
		$\forall$	
		$\in$	

	$\notin$
	$\Leftarrow$
	$\Rightarrow$
	$\Rightarrow^*$
	$\rightarrow$
	$\Lambda$
	$\square$
	$\vdash$
	$\dashv$
	$\models$
	$\vDash$
	$\neq$
	$\triangleright$
	<b>ok</b>
	$-$
	$\rightsquigarrow$
	$\rightsquigarrow^*$
	$\rightsquigarrow$
	$\emptyset$
	$\circ$
	<b>fv</b>
	<b>dom</b>
	$\sim$
	$\succ$
	$ $
	$\bullet$
	<b>fst</b>
	<b>snd</b>
	$ \Rightarrow $
	$\vdash=$
	<b>refl<sub>2</sub></b>
	$++$
<i>formula, <math>\psi</math></i>	$::=$
	<i>judgement</i>
	$x : A/R \in \Gamma$
	$c : \phi \in \Gamma$
	$T : A/R \in \Sigma$
	$F \sim a : A/R \in \Sigma$
	$K : T \Gamma \in \Sigma$
	$x \in \Delta$
	$c \in \Delta$
	$c$ <b>not relevant</b> $\in \gamma$
	$x \notin \text{fva}$
	$x \notin \text{dom } \Gamma$

	$ \begin{array}{ l} c \notin \text{dom } \Gamma \\ T \notin \text{dom } \Sigma \\ F \notin \text{dom } \Sigma \\ a = b \\ \phi_1 = \phi_2 \\ \Gamma_1 = \Gamma_2 \\ \gamma_1 = \gamma_2 \\ \neg \psi \\ \psi_1 \wedge \psi_2 \\ \psi_1 \vee \psi_2 \\ \psi_1 \Rightarrow \psi_2 \\ (\psi) \\ \psi \\ c : (a : A \sim b : B) \in \Gamma \end{array} $	suppress lc hypothesis generated by Ott
<i>JSubRole</i>	$ \begin{array}{ l} R_1 \leq R_2 \end{array} $	Subroling judgement
<i>JValue</i>	$ \begin{array}{ l} \mathbf{CoercedValue } R A \\ \mathbf{Value}_R A \\ \mathbf{ValueType } R A \end{array} $	Values with at most one coercion at the top values Types with head forms (erased language)
<i>Jconsistent</i>	$ \begin{array}{ l} \mathbf{consistent } a b \end{array} $	(erased) types do not differ in their heads
<i>Jerased</i>	$ \begin{array}{ l} \Omega \vdash \text{erased\_tma } R \end{array} $	
<i>JChk</i>	$ \begin{array}{ l} (\rho = +) \vee (x \notin \text{fv } A) \end{array} $	irrelevant argument check
<i>Jpar</i>	$ \begin{array}{ l} \Omega \models a \Rightarrow_R b \\ \Omega \vdash a \Rightarrow_R^* b \\ \Omega \vdash a \Leftrightarrow_R b \end{array} $	parallel reduction (implicit language) multistep parallel reduction parallel reduction to a common term
<i>Jbeta</i>	$ \begin{array}{ l} \models a > b/R \\ \models a \rightsquigarrow b/R \\ \models a \rightsquigarrow^* b/R \end{array} $	primitive reductions on erased terms single-step head reduction for implicit language multistep reduction
<i>Jett</i>	$ \begin{array}{ l} \Gamma \models \phi \text{ ok} \\ \Gamma \models a : A/R \\ \Gamma; \Delta \models \phi_1 \equiv \phi_2 \end{array} $	Prop wellformedness typing prop equality

	$\begin{array}{ l} \Gamma; \Delta \models a \equiv b : A/R \\ \models \Gamma \end{array}$	definitional equality context wellformedness
$Jsig$	$\begin{array}{ l} ::= \\ \models \Sigma \end{array}$	signature wellformedness
$Jann$	$\begin{array}{ l} ::= \\ \Gamma \vdash \phi \text{ ok} \\ \Gamma \vdash a : A/R \\ \Gamma; \Delta \vdash \gamma : \phi_1 \sim \phi_2 \\ \Gamma; \Delta \vdash \gamma : A \sim_R B \\ \vdash \Gamma \\ \vdash \Sigma \end{array}$	prop wellformedness typing coercion between props coercion between types context wellformedness signature wellformedness
$Jred$	$\begin{array}{ l} ::= \\ \Gamma \vdash a \rightsquigarrow b/R \end{array}$	single-step, weak head reduction to values for annotated lang
$judgement$	$\begin{array}{ l} ::= \\ JSubRole \\ JValue \\ Jconsistent \\ Jerased \\ JChk \\ Jpar \\ Jbeta \\ Jett \\ Jsig \\ Jann \\ Jred \end{array}$	
$user\_syntax$	$\begin{array}{ l} ::= \\ tmvar \\ covar \\ datacon \\ const \\ tyfam \\ index \\ role \\ relflag \\ constraint \\ tm \\ brs \\ co \\ sig\_sort \\ sort \\ context \\ role\_context \end{array}$	

| *available\_props*  
 | *sig*  
 | *terminals*  
 | *formula*

$R_1 \leq R_2$  Subroling judgement

$$\frac{}{\mathbf{Nom} \leq \mathbf{Rep}} \text{NOMREP}$$

$$\frac{}{\overline{R \leq R}} \text{REFL}$$

$$\frac{R_1 \leq R_2 \quad R_2 \leq R_3}{R_1 \leq R_3} \text{TRANS}$$

$\mathbf{CoercedValue} \ R \ A$  Values with at most one coercion at the top

$$\frac{\mathbf{Value}_R \ a}{\mathbf{CoercedValue} \ R \ a} \text{CV}$$

$$\frac{\mathbf{Value}_R \ a \quad \neg(R_1 \leq R)}{\mathbf{CoercedValue} \ R \ (a \triangleright_{R_1} \gamma)} \text{CC}$$

$\mathbf{Value}_R \ A$  values

$$\frac{}{\mathbf{Value}_R \ \star} \text{VALUE\_STAR}$$

$$\frac{}{\mathbf{Value}_R \ \Pi^\rho x : A / R_1 \rightarrow B} \text{VALUE\_PI}$$

$$\frac{}{\mathbf{Value}_R \ \forall c : \phi. B} \text{VALUE\_CPI}$$

$$\frac{}{\mathbf{Value}_R \ \lambda^+ x : A / R_1. a} \text{VALUE\_ABSR}$$

$$\frac{}{\mathbf{Value}_R \ \lambda^{R_1, +} x. a} \text{VALUE\_UABSR}$$

$$\frac{\mathbf{CoercedValue} \ R \ a}{\mathbf{Value}_R \ \lambda^{R_1, -} x. a} \text{VALUE\_UABSI}$$

$$\frac{\mathbf{CoercedValue} \ R \ a}{\mathbf{Value}_R \ \lambda^- x : A / R_1. a} \text{VALUE\_ABSI}$$

$$\frac{}{\mathbf{Value}_R \ \Lambda c : \phi. a} \text{VALUE\_CABS}$$

$$\frac{}{\mathbf{Value}_R \ \Lambda c. a} \text{VALUE\_UCABS}$$

$$\frac{F \sim a : A / R_1 \in \Sigma_0 \quad \neg(R_1 \leq R)}{\mathbf{Value}_R \ F} \text{VALUE\_AX}$$

$\mathbf{ValueType} \ R \ A$  Types with head forms (erased language)

$$\frac{}{\mathbf{ValueType} \ R \ \star} \text{VALUE\_TYPE\_STAR}$$



$$\frac{}{\mathbf{ValueType} \, R \, \Pi^{\rho} x : A / R_1 \rightarrow B} \text{VALUE\_TYPE\_PI}$$

$$\frac{}{\mathbf{ValueType} \, R \, \forall c : \phi . B} \text{VALUE\_TYPE\_CPI}$$

$$\frac{\begin{array}{c} F \sim a : A / R_1 \in \Sigma_0 \\ \neg(R_1 \leq R) \end{array}}{\mathbf{ValueType} \, R \, F} \text{VALUE\_TYPE\_AX}$$

**consistent**  $a \, b$  (erased) types do not differ in their heads

$$\frac{}{\mathbf{consistent} \, \star \, \star} \text{CONSISTENT\_A\_STAR}$$

$$\frac{}{\mathbf{consistent} \, (\Pi^{\rho} x_1 : A_1 / R \rightarrow B_1) \, (\Pi^{\rho} x_2 : A_2 / R \rightarrow B_2)} \text{CONSISTENT\_A\_PI}$$

$$\frac{}{\mathbf{consistent} \, (\forall c_1 : \phi_1 . A_1) \, (\forall c_2 : \phi_2 . A_2)} \text{CONSISTENT\_A\_CPI}$$

$$\frac{\neg \mathbf{ValueType} \, R \, b}{\mathbf{consistent} \, a \, b} \text{CONSISTENT\_A\_STEP\_R}$$

$$\frac{\neg \mathbf{ValueType} \, R \, a}{\mathbf{consistent} \, a \, b} \text{CONSISTENT\_A\_STEP\_L}$$

$\Omega \vdash \text{erased\_tma} \, R$

$$\frac{}{\Omega \vdash \text{erased\_tm} \, \square \, R} \text{ERASED\_A\_BULLET}$$

$$\frac{}{\Omega \vdash \text{erased\_tm} \, \star \, R} \text{ERASED\_A\_STAR}$$

$$\frac{}{\Omega, x : R, \Omega' \vdash \text{erased\_tm} \, x \, R} \text{ERASED\_A\_VAR}$$

$$\frac{\Omega, x : R_1, \Omega' \vdash \text{erased\_tma} \, R}{\Omega, \Omega' \vdash \text{erased\_tm}(\lambda^{R_1, \rho} x . a) \, R} \text{ERASED\_A\_ABS}$$

$$\frac{\begin{array}{c} \Omega \vdash \text{erased\_tma} \, R \\ \Omega \vdash \text{erased\_tmb} \, R \end{array}}{\Omega \vdash \text{erased\_tm}(a \, b^{R_1, \rho}) \, R} \text{ERASED\_A\_APP}$$

$$\frac{\begin{array}{c} \Omega \vdash \text{erased\_tm} \, A \, R_2 \\ \Omega, x : R_1, \Omega' \vdash \text{erased\_tm} \, B \, R \\ R_1 \leq R_2 \end{array}}{\Omega, \Omega' \vdash \text{erased\_tm}(\Pi^{\rho} x : A / R_1 \rightarrow B) \, R} \text{ERASED\_A\_PI}$$

$$\frac{\begin{array}{c} \Omega \vdash \text{erased\_tma} \, R_1 \\ \Omega \vdash \text{erased\_tmb} \, R_1 \\ \Omega \vdash \text{erased\_tm} \, A \, R_2 \\ \Omega, c : R_1, \Omega' \vdash \text{erased\_tm} \, B \, R \\ R_1 \leq R_2 \end{array}}{\Omega, \Omega' \vdash \text{erased\_tm}(\forall c : a \sim_{A/R_1} b . B) \, R} \text{ERASED\_A\_CPI}$$

$$\frac{\Omega, c : R_1, \Omega' \vdash \text{erased\_tmb} \, R}{\Omega, \Omega' \vdash \text{erased\_tm}(\Lambda c . b) \, R} \text{ERASED\_A\_CABS}$$

$$\frac{\Omega \vdash \text{erased\_tma} \, R}{\Omega \vdash \text{erased\_tm}(a[\bullet]) \, R} \text{ERASED\_A\_CAPP}$$

$$\frac{}{\Omega \vdash \text{erased\_tm} \, F \, R} \text{ERASED\_A\_FAM}$$

$$\begin{array}{c}
\overline{\Omega \vdash \text{erased\_tm } T \ R} \quad \text{ERASED\_A\_CONST} \\
\Omega \vdash \text{erased\_tma } R \\
\overline{\neg(R_1 \leq R)} \\
\Omega \vdash \text{erased\_tm}(a \triangleright_{R_1} \bullet) R \quad \text{ERASED\_A\_CONV}
\end{array}$$

$$\boxed{(\rho = +) \vee (x \notin \text{fv } A)} \quad \text{irrelevant argument check}$$

$$\begin{array}{c}
\overline{(+ = +) \vee (x \notin \text{fv } A)} \quad \text{RHO\_REL} \\
\frac{x \notin \text{fv } A}{(- = +) \vee (x \notin \text{fv } A)} \quad \text{RHO\_IRRREL}
\end{array}$$

$$\boxed{\Omega \models a \Rightarrow_R b} \quad \text{parallel reduction (implicit language)}$$

$$\begin{array}{c}
\overline{\Omega \models a \Rightarrow_R a} \quad \text{PAR\_REFL} \\
\Omega \models a \Rightarrow_R (\lambda^{R_1, \rho} x. a') \\
\Omega \models b \Rightarrow_{R_1} b' \\
R_1 \leq R \\
\Omega \models a \ b^{R_1, \rho} \Rightarrow_R a' \{b'/x\} \quad \text{PAR\_BETA} \\
\Omega \models a \Rightarrow_R a' \\
\Omega \models b \Rightarrow_{R_1} b' \\
R_1 \leq R \\
\Omega \models a \ b^{R_1, \rho} \Rightarrow_R a' \ b'^{R_1, \rho} \quad \text{PAR\_APP} \\
\Omega \models a \Rightarrow_R (\Lambda c. a') \\
\Omega \models a[\bullet] \Rightarrow_R a' \{\bullet/c\} \quad \text{PAR\_CBETA} \\
\Omega \models a \Rightarrow_R a' \\
\Omega \models a[\bullet] \Rightarrow_R a'[\bullet] \quad \text{PAR\_CAPP} \\
\Omega, x : R_1, \Omega' \models a \Rightarrow_R a' \\
\Omega, \Omega' \models \lambda^{R_1, \rho} x. a \Rightarrow_R \lambda^{R_1, \rho} x. a' \quad \text{PAR\_ABS} \\
\Omega \models A \Rightarrow_{R_2} A' \\
\Omega, x : R_1, \Omega' \models B \Rightarrow_R B' \\
R_1 \leq R_2 \\
\Omega, \Omega' \models \Pi^\rho x : A / R_1 \rightarrow B \Rightarrow_R \Pi^\rho x : A' / R_1 \rightarrow B' \quad \text{PAR\_PI} \\
\Omega, c : R_1, \Omega' \models a \Rightarrow_R a' \\
\Omega, \Omega' \models \Lambda c. a \Rightarrow_R \Lambda c. a' \quad \text{PAR\_CABS} \\
\Omega \models A \Rightarrow_{R_2} A' \\
\Omega \models a \Rightarrow_{R_1} a' \\
\Omega \models b \Rightarrow_{R_1} b' \\
\Omega, c : R_1, \Omega' \models B \Rightarrow_R B' \\
R_1 \leq R_2 \\
\Omega, \Omega' \models \forall c : a \sim_{A/R_1} b. B \Rightarrow_R \forall c : a' \sim_{A'/R_1} b'. B' \quad \text{PAR\_CPI} \\
F \sim a : A / R_1 \in \Sigma_0 \\
R_1 \leq R \\
\Omega \models F \Rightarrow_R a \quad \text{PAR\_AXIOM}
\end{array}$$

$$\begin{array}{c}
\frac{\Omega \models a_1 \Rightarrow_{R_1} a_2}{\Omega \models a_1 \triangleright_R \bullet \Rightarrow_{R_1} a_2 \triangleright_R \bullet} \text{PAR\_CONG} \\
\frac{\Omega \models a_1 \Rightarrow_{R_1} (a_2 \triangleright_R \bullet)}{\Omega \models (a_1 \triangleright_R \bullet) \Rightarrow_{R_1} (a_2 \triangleright_R \bullet)} \text{PAR\_COMBINE} \\
\frac{\Omega \models a_1 \Rightarrow_{R_1} (a_2 \triangleright_R \bullet) \quad \Omega \models b_1 \Rightarrow_{R_2} b_2 \quad R_2 \leq R_1}{\Omega \models a_1 b_1^{R_2,+} \Rightarrow_{R_1} (a_2 (b_2 \triangleright_R \bullet)^{R_2,+}) \triangleright_R \bullet} \text{PAR\_PUSH} \\
\frac{\Omega \models a_1 \Rightarrow_{R_1} (a_2 \triangleright_R \bullet)}{\Omega \models a_1[\bullet] \Rightarrow_{R_1} (a_2[\bullet]) \triangleright_R \bullet} \text{PAR\_CPUSH}
\end{array}$$

$\boxed{\Omega \vdash a \Rightarrow_R^* b}$  multistep parallel reduction

$$\begin{array}{c}
\overline{\Omega \vdash a \Rightarrow_R^* a} \text{MP\_REFL} \\
\frac{\Omega \models a \Rightarrow_R b \quad \Omega \vdash b \Rightarrow_R^* a'}{\Omega \vdash a \Rightarrow_R^* a'} \text{MP\_STEP}
\end{array}$$

$\boxed{\Omega \vdash a \Leftrightarrow_R b}$  parallel reduction to a common term

$$\frac{\Omega \vdash a_1 \Rightarrow_R^* b \quad \Omega \vdash a_2 \Rightarrow_R^* b}{\Omega \vdash a_1 \Leftrightarrow_R a_2} \text{JOIN}$$

$\boxed{\models a > b/R}$  primitive reductions on erased terms

$$\begin{array}{c}
\frac{\text{Value}_{R_1}(\lambda^{R,\rho} x.v)}{\models (\lambda^{R,\rho} x.v) \ b^{R,\rho} > v\{b/x\}/R_1} \text{BETA\_APPABS} \\
\frac{}{\models (\Lambda c.a')[\bullet] > a'\{\bullet/c\}/R} \text{BETA\_CAPPCABS} \\
\frac{F \sim a : A/R \in \Sigma_0}{\models F > a/R} \text{BETA\_AXIOM}
\end{array}$$

$\boxed{\models a \rightsquigarrow b/R}$  single-step head reduction for implicit language

$$\begin{array}{c}
\frac{\models a \rightsquigarrow a'/R_1}{\models \lambda^{R,-} x.a \rightsquigarrow \lambda^{R,-} x.a'/R_1} \text{E\_ABSTERM} \\
\frac{\models a \rightsquigarrow a'/R_1}{\models a \ b^{R,\rho} \rightsquigarrow a' \ b^{R,\rho}/R_1} \text{E\_APPLEFT} \\
\frac{\models a \rightsquigarrow a'/R}{\models a[\bullet] \rightsquigarrow a'[\bullet]/R} \text{E\_CAPPLEFT} \\
\frac{\text{Value}_{R_1}(\lambda^{R,\rho} x.v)}{\models (\lambda^{R,\rho} x.v) \ a^{R,\rho} \rightsquigarrow v\{a/x\}/R_1} \text{E\_APPABS} \\
\frac{}{\models (\Lambda c.b)[\bullet] \rightsquigarrow b\{\bullet/c\}/R} \text{E\_CAPPCABS} \\
\frac{F \sim a : A/R \in \Sigma_0 \quad R \leq R_1}{\models F \rightsquigarrow a/R_1} \text{E\_AXIOM}
\end{array}$$

$$\begin{array}{c}
\frac{\frac{\vdash a \rightsquigarrow a'/R_1 \quad \neg(R \leq R_1)}{\vdash a \triangleright_R \bullet \rightsquigarrow a' \triangleright_R \bullet / R_1} \text{E\_CONG}}{\vdash (a \triangleright_R \bullet) \triangleright_R \bullet \rightsquigarrow a \triangleright_R \bullet / R_1} \text{E\_COMBINE} \\
\frac{\vdash (v_1 \triangleright_R \bullet) v_2^{R_1, +} \rightsquigarrow (v_1(v_2 \triangleright_R \bullet)^{R_1, +}) \triangleright_R \bullet / R_2}{} \text{E\_PUSH} \\
\frac{\vdash (v_1 \triangleright_R \bullet)[\bullet] \rightsquigarrow (v_1[\bullet]) \triangleright_R \bullet / R_1}{} \text{E\_CPUSH} \\
\boxed{\vdash a \rightsquigarrow^* b/R} \quad \text{multistep reduction}
\end{array}$$

$$\begin{array}{c}
\frac{}{\vdash a \rightsquigarrow^* a/R} \text{EQUAL} \\
\frac{\vdash a \rightsquigarrow b/R \quad \vdash b \rightsquigarrow^* a'/R}{\vdash a \rightsquigarrow^* a'/R} \text{STEP}
\end{array}$$

$$\boxed{\Gamma \models \phi \text{ ok}} \quad \text{Prop wellformedness}$$

$$\frac{\Gamma \models a : A/R \quad \Gamma \models b : A/R \quad \Gamma \models A : \star/R}{\Gamma \models a \sim_{A/R} b \text{ ok}} \text{E\_WFF}$$

$$\boxed{\Gamma \models a : A/R} \quad \text{typing}$$

$$\begin{array}{c}
\frac{R_1 \leq R_2 \quad \Gamma \models a : A/R_1}{\Gamma \models a : A/R_2} \text{E\_SUBROLE} \\
\frac{\vdash \Gamma}{\Gamma \models \star : \star/R} \text{E\_STAR} \\
\frac{\vdash \Gamma \quad x : A/R \in \Gamma}{\Gamma \models x : A/R} \text{E\_VAR} \\
\frac{\Gamma, x : A/R \models B : \star/R' \quad \Gamma \models A : \star/R \quad R \leq R'}{\Gamma \models \Pi^\rho x : A/R \rightarrow B : \star/R'} \text{E\_PI} \\
\frac{\Gamma, x : A/R \models a : B/R' \quad \Gamma \models A : \star/R \quad (\rho = +) \vee (x \notin \text{fv } a) \quad R \leq R'}{\Gamma \models \lambda^{R, \rho} x. a : (\Pi^\rho x : A/R \rightarrow B)/R'} \text{E\_ABS} \\
\frac{\Gamma \models b : \Pi^+ x : A/R \rightarrow B/R' \quad \Gamma \models a : A/R}{\Gamma \models b \ a^{R, +} : B\{a/x\}/R'} \text{E\_APP} \\
\frac{\Gamma \models b : \Pi^- x : A/R \rightarrow B/R' \quad \Gamma \models a : A/R}{\Gamma \models b \ \Box^{R, -} : B\{a/x\}/R'} \text{E\_IAPP}
\end{array}$$

$$\frac{\begin{array}{l} \Gamma \models a : A/R \\ \Gamma; \tilde{\Gamma} \models A \equiv B : \star/R \\ \Gamma \models B : \star/R \end{array}}{\Gamma \models a : B/R} \quad \text{E\_CONV}$$

$$\frac{\begin{array}{l} \Gamma, c : \phi \models B : \star/R \\ \Gamma \models \phi \text{ ok} \end{array}}{\Gamma \models \forall c : \phi. B : \star/R} \quad \text{E\_CPI}$$

$$\frac{\begin{array}{l} \Gamma, c : \phi \models a : B/R \\ \Gamma \models \phi \text{ ok} \end{array}}{\Gamma \models \Lambda c. a : \forall c : \phi. B/R} \quad \text{E\_CAbs}$$

$$\frac{\begin{array}{l} \Gamma \models a_1 : \forall c : (a \sim_{A/R} b). B_1/R' \\ \Gamma; \tilde{\Gamma} \models a \equiv b : A/R \end{array}}{\Gamma \models a_1[\bullet] : B_1\{\bullet/c\}/R'} \quad \text{E\_CAPP}$$

$$\frac{\begin{array}{l} \models \Gamma \\ F \sim a : A/R \in \Sigma_0 \\ \emptyset \models A : \star/R \end{array}}{\Gamma \models F : A/R} \quad \text{E\_FAM}$$

$$\frac{\begin{array}{l} \Gamma \models a : A_1/R_1 \\ \Gamma; \tilde{\Gamma} \models A_1 \equiv A_2 : \star/R_2 \\ \neg(R_2 \leq R_1) \\ \Gamma \models A_2 : \star/R_2 \end{array}}{\Gamma \models a \triangleright_{R_2} \bullet : A_2/R_1} \quad \text{E\_TYCAST}$$

$$\boxed{\Gamma; \Delta \models \phi_1 \equiv \phi_2} \quad \text{prop equality}$$

$$\frac{\begin{array}{l} \Gamma; \Delta \models A_1 \equiv A_2 : A/R \\ \Gamma; \Delta \models B_1 \equiv B_2 : A/R \end{array}}{\Gamma; \Delta \models A_1 \sim_{A/R} B_1 \equiv A_2 \sim_{A/R} B_2} \quad \text{E\_PROPcong}$$

$$\frac{\begin{array}{l} \Gamma; \Delta \models A \equiv B : \star/R \\ \Gamma \models A_1 \sim_{A/R} A_2 \text{ ok} \\ \Gamma \models A_1 \sim_{B/R} A_2 \text{ ok} \end{array}}{\Gamma; \Delta \models A_1 \sim_{A/R} A_2 \equiv A_1 \sim_{B/R} A_2} \quad \text{E\_ISOCONV}$$

$$\frac{\Gamma; \Delta \models \forall c : \phi_1. B_1 \equiv \forall c : \phi_2. B_2 : \star/R}{\Gamma; \Delta \models \phi_1 \equiv \phi_2} \quad \text{E\_CPIfst}$$

$$\boxed{\Gamma; \Delta \models a \equiv b : A/R} \quad \text{definitional equality}$$

$$\frac{\begin{array}{l} \models \Gamma \\ c : (a \sim_{A/R} b) \in \Gamma \\ c \in \Delta \end{array}}{\Gamma; \Delta \models a \equiv b : A/R} \quad \text{E\_ASSN}$$

$$\frac{\Gamma \models a : A/R}{\Gamma; \Delta \models a \equiv a : A/R} \quad \text{E\_REFL}$$

$$\frac{\Gamma; \Delta \models b \equiv a : A/R}{\Gamma; \Delta \models a \equiv b : A/R} \quad \text{E\_SYM}$$

$$\begin{array}{c}
\frac{\Gamma; \Delta \models a \equiv a_1 : A/R \quad \Gamma; \Delta \models a_1 \equiv b : A/R}{\Gamma; \Delta \models a \equiv b : A/R} \quad \text{E\_TRANS} \\
\\
\frac{\Gamma; \Delta \models a \equiv b : A/R_1 \quad R_1 \leq R_2}{\Gamma; \Delta \models a \equiv b : A/R_2} \quad \text{E\_SUB} \\
\\
\frac{\Gamma \models a_1 : B/R \quad \Gamma \models a_2 : B/R \quad \models a_1 > a_2/R}{\Gamma; \Delta \models a_1 \equiv a_2 : B/R} \quad \text{E\_BETA} \\
\\
\frac{\Gamma; \Delta \models A_1 \equiv A_2 : \star/R \quad \Gamma, x : A_1/R; \Delta \models B_1 \equiv B_2 : \star/R' \quad \Gamma \models A_1 : \star/R \quad \Gamma \models \Pi^\rho x : A_1/R \rightarrow B_1 : \star/R' \quad \Gamma \models \Pi^\rho x : A_2/R \rightarrow B_2 : \star/R' \quad R \leq R'}{\Gamma; \Delta \models (\Pi^\rho x : A_1/R \rightarrow B_1) \equiv (\Pi^\rho x : A_2/R \rightarrow B_2) : \star/R'} \quad \text{E\_PICONG} \\
\\
\frac{\Gamma, x : A_1/R; \Delta \models b_1 \equiv b_2 : B/R' \quad \Gamma \models A_1 : \star/R \quad R \leq R' \quad (\rho = +) \vee (x \notin \text{fv } b_1) \quad (\rho = +) \vee (x \notin \text{fv } b_2)}{\Gamma; \Delta \models (\lambda^{R, \rho} x. b_1) \equiv (\lambda^{R, \rho} x. b_2) : (\Pi^\rho x : A_1/R \rightarrow B)/R'} \quad \text{E\_ABSCONG} \\
\\
\frac{\Gamma; \Delta \models a_1 \equiv b_1 : (\Pi^+ x : A/R \rightarrow B)/R' \quad \Gamma; \Delta \models a_2 \equiv b_2 : A/R}{\Gamma; \Delta \models a_1 \ a_2^{R, +} \equiv b_1 \ b_2^{R, +} : (B\{a_2/x\})/R'} \quad \text{E\_APPCONG} \\
\\
\frac{\Gamma; \Delta \models a_1 \equiv b_1 : (\Pi^- x : A/R \rightarrow B)/R' \quad \Gamma \models a : A/R}{\Gamma; \Delta \models a_1 \ \Box^{R, -} \equiv b_1 \ \Box^{R, -} : (B\{a/x\})/R'} \quad \text{E\_IAPPCONG} \\
\\
\frac{\Gamma; \Delta \models \Pi^\rho x : A_1/R \rightarrow B_1 \equiv \Pi^\rho x : A_2/R \rightarrow B_2 : \star/R'}{\Gamma; \Delta \models A_1 \equiv A_2 : \star/R} \quad \text{E\_PIFST} \\
\\
\frac{\Gamma; \Delta \models \Pi^\rho x : A_1/R \rightarrow B_1 \equiv \Pi^\rho x : A_2/R \rightarrow B_2 : \star/R' \quad \Gamma; \Delta \models a_1 \equiv a_2 : A_1/R}{\Gamma; \Delta \models B_1\{a_1/x\} \equiv B_2\{a_2/x\} : \star/R'} \quad \text{E\_PISND} \\
\\
\frac{\Gamma; \Delta \models a_1 \sim_{A_1/R} b_1 \equiv a_2 \sim_{A_2/R} b_2 \quad \Gamma, c : a_1 \sim_{A_1/R} b_1; \Delta \models A \equiv B : \star/R' \quad \Gamma \models a_1 \sim_{A_1/R} b_1 \text{ ok} \quad \Gamma \models \forall c : a_1 \sim_{A_1/R} b_1. A : \star/R' \quad \Gamma \models \forall c : a_2 \sim_{A_2/R} b_2. B : \star/R'}{\Gamma; \Delta \models \forall c : a_1 \sim_{A_1/R} b_1. A \equiv \forall c : a_2 \sim_{A_2/R} b_2. B : \star/R'} \quad \text{E\_CPICONG} \\
\\
\frac{\Gamma, c : \phi_1; \Delta \models a \equiv b : B/R \quad \Gamma \models \phi_1 \text{ ok}}{\Gamma; \Delta \models (\Lambda c. a) \equiv (\Lambda c. b) : \forall c : \phi_1. B/R} \quad \text{E\_CABSCONG} \\
\\
\frac{\Gamma; \Delta \models a_1 \equiv b_1 : (\forall c : (a \sim_{A/R} b). B)/R' \quad \Gamma; \tilde{\Gamma} \models a \equiv b : A/R}{\Gamma; \Delta \models a_1[\bullet] \equiv b_1[\bullet] : (B\{\bullet/c\})/R'} \quad \text{E\_CAPPCONG}
\end{array}$$

$$\begin{array}{c}
\frac{\Gamma; \Delta \models \forall c : (a_1 \sim_{A/R} a_2). B_1 \equiv \forall c : (a'_1 \sim_{A'/R'} a'_2). B_2 : \star / R_0 \quad \Gamma; \tilde{\Gamma} \models a_1 \equiv a_2 : A/R \quad \Gamma; \tilde{\Gamma} \models a'_1 \equiv a'_2 : A'/R'}{\Gamma; \Delta \models B_1 \{\bullet/c\} \equiv B_2 \{\bullet/c\} : \star / R_0} \text{E\_CPIsND} \\
\\
\frac{\Gamma; \Delta \models a \equiv b : A/R \quad \Gamma; \Delta \models a \sim_{A/R} b \equiv a' \sim_{A'/R'} b'}{\Gamma; \Delta \models a' \equiv b' : A'/R'} \text{E\_CAST} \\
\\
\frac{\Gamma; \Delta \models a \equiv b : A/R_1 \quad \Gamma; \tilde{\Gamma} \models A \equiv B : \star / R_2 \quad R_1 \leq R_2}{\Gamma; \Delta \models a \equiv b : B/R_2} \text{E\_EqCONV} \\
\\
\frac{\Gamma; \Delta \models a \sim_{A/R} b \equiv a' \sim_{A'/R} b'}{\Gamma; \Delta \models A \equiv A' : \star / R} \text{E\_IsoSND} \\
\\
\frac{\Gamma; \Delta \models a_1 \equiv a_2 : A/R_1 \quad \Gamma; \Delta \models A \equiv B : \star / R_2 \quad \neg(R_2 \leq R_1)}{\Gamma; \Delta \models a_1 \triangleright_{R_2} \bullet \equiv a_2 \triangleright_{R_2} \bullet : B/R_1} \text{E\_CASTCONG}
\end{array}$$

$\boxed{\models \Gamma}$  context wellformedness

$$\begin{array}{c}
\frac{}{\models \emptyset} \text{E\_EMPTY} \\
\\
\frac{\models \Gamma \quad \Gamma \models A : \star / R \quad x \notin \text{dom } \Gamma}{\models \Gamma, x : A/R} \text{E\_CONSTM} \\
\\
\frac{\models \Gamma \quad \Gamma \models \phi \text{ ok} \quad c \notin \text{dom } \Gamma}{\models \Gamma, c : \phi} \text{E\_CONSCo}
\end{array}$$

$\boxed{\models \Sigma}$  signature wellformedness

$$\begin{array}{c}
\frac{}{\models \emptyset} \text{SIG\_EMPTY} \\
\\
\frac{\models \Sigma \quad \emptyset \models A : \star / R \quad \emptyset \models a : A/R' \quad F \notin \text{dom } \Sigma \quad R' \leq R}{\models \Sigma \cup \{F \sim a : A/R'\}} \text{SIG\_CONSAx}
\end{array}$$

$\boxed{\Gamma \vdash \phi \text{ ok}}$  prop wellformedness

$$\frac{\Gamma \vdash a : A/R \quad \Gamma \vdash b : B/R \quad |A|R = |B|R}{\Gamma \vdash a \sim_{A/R} b \text{ ok}} \text{AN\_WFF}$$

$\boxed{\Gamma \vdash a : A/R}$  typing

$$\begin{array}{c}
\frac{\vdash \Gamma}{\Gamma \vdash \star : \star / R} \quad \text{AN\_STAR} \\
\\
\frac{\vdash \Gamma \quad x : A/R \in \Gamma}{\Gamma \vdash x : A/R} \quad \text{AN\_VAR} \\
\\
\frac{\Gamma, x : A/R \vdash B : \star / R' \quad \Gamma \vdash A : \star / R}{\Gamma \vdash \Pi^{\rho} x : A/R \rightarrow B : \star / R'} \quad \text{AN\_PI} \\
\\
\frac{\Gamma \vdash A : \star / R \quad \Gamma, x : A/R \vdash a : B/R' \quad (\rho = +) \vee (x \notin \text{fv } |a| R') \quad R \leq R'}{\Gamma \vdash \lambda^{\rho} x : A/R. a : (\Pi^{\rho} x : A/R \rightarrow B)/R'} \quad \text{AN\_ABS} \\
\\
\frac{\Gamma \vdash b : (\Pi^{\rho} x : A/R \rightarrow B)/R' \quad \Gamma \vdash a : A/R}{\Gamma \vdash b \ a^{R, \rho} : (B\{a/x\})/R'} \quad \text{AN\_APP} \\
\\
\frac{\Gamma \vdash a : A/R \quad \Gamma; \tilde{\Gamma} \vdash \gamma : A \sim_R B \quad \Gamma \vdash B : \star / R}{\Gamma \vdash a \triangleright_R \gamma : B/R} \quad \text{AN\_CONV} \\
\\
\frac{\Gamma \vdash \phi \text{ ok} \quad \Gamma, c : \phi \vdash B : \star / R}{\Gamma \vdash \forall c : \phi. B : \star / R} \quad \text{AN\_CPI} \\
\\
\frac{\Gamma \vdash \phi \text{ ok} \quad \Gamma, c : \phi \vdash a : B/R}{\Gamma \vdash \Lambda c : \phi. a : (\forall c : \phi. B)/R} \quad \text{AN\_CABS} \\
\\
\frac{\Gamma \vdash a_1 : (\forall c : a \sim_{A_1/R} b. B)/R' \quad \Gamma; \tilde{\Gamma} \vdash \gamma : a \sim_R b}{\Gamma \vdash a_1[\gamma] : B\{\gamma/c\}/R'} \quad \text{AN\_CAPP} \\
\\
\frac{\vdash \Gamma \quad F \sim a : A/R \in \Sigma_1 \quad \emptyset \vdash A : \star / R}{\Gamma \vdash F : A/R} \quad \text{AN\_FAM} \\
\\
\frac{R_1 \leq R_2 \quad \Gamma \vdash a : A/R_1}{\Gamma \vdash a : A/R_2} \quad \text{AN\_SUBROLE}
\end{array}$$

$$\boxed{\Gamma; \Delta \vdash \gamma : \phi_1 \sim \phi_2}$$

coercion between props

$$\frac{\Gamma; \Delta \vdash \gamma_1 : A_1 \sim_R A_2 \quad \Gamma; \Delta \vdash \gamma_2 : B_1 \sim_R B_2 \quad \Gamma \vdash A_1 \sim_{A/R} B_1 \text{ ok} \quad \Gamma \vdash A_2 \sim_{A/R} B_2 \text{ ok}}{\Gamma; \Delta \vdash (\gamma_1 \sim_A \gamma_2) : (A_1 \sim_{A/R} B_1) \sim (A_2 \sim_{A/R} B_2)} \quad \text{AN\_PROP CONG}$$



$$\begin{array}{c}
\frac{\Gamma; \Delta \vdash \gamma : \forall c : \phi_1. A_2 \sim_R \forall c : \phi_2. B_2}{\Gamma; \Delta \vdash \mathbf{cpiFst} \gamma : \phi_1 \sim \phi_2} \text{AN\_CPiFST} \\
\\
\frac{\Gamma; \Delta \vdash \gamma : \phi_1 \sim \phi_2}{\Gamma; \Delta \vdash \mathbf{sym} \gamma : \phi_2 \sim \phi_1} \text{AN\_ISOsym} \\
\\
\frac{\begin{array}{l} \Gamma; \Delta \vdash \gamma : A \sim_R B \\ \Gamma \vdash a_1 \sim_{A/R} a_2 \text{ ok} \\ \Gamma \vdash a'_1 \sim_{B/R} a'_2 \text{ ok} \\ |a_1|R = |a'_1|R \\ |a_2|R = |a'_2|R \end{array}}{\Gamma; \Delta \vdash \mathbf{conv} (a_1 \sim_{A/R} a_2) \sim_\gamma (a'_1 \sim_{B/R} a'_2) : (a_1 \sim_{A/R} a_2) \sim (a'_1 \sim_{B/R} a'_2)} \text{AN\_ISOConv} \\
\\
\boxed{\Gamma; \Delta \vdash \gamma : A \sim_R B} \quad \text{coercion between types} \\
\\
\frac{\begin{array}{l} \vdash \Gamma \\ c : a \sim_{A/R} b \in \Gamma \\ c \in \Delta \end{array}}{\Gamma; \Delta \vdash c : a \sim_R b} \text{AN\_ASSN} \\
\\
\frac{\Gamma \vdash a : A/R}{\Gamma; \Delta \vdash \mathbf{refl} a : a \sim_R a} \text{AN\_REFL} \\
\\
\frac{\begin{array}{l} \Gamma \vdash a : A/R \\ \Gamma \vdash b : B/R \\ |a|R = |b|R \\ \Gamma; \tilde{\Gamma} \vdash \gamma : A \sim_R B \end{array}}{\Gamma; \Delta \vdash (a \models_\gamma b) : a \sim_R b} \text{AN\_ERASEEQ} \\
\\
\frac{\begin{array}{l} \Gamma \vdash b : B/R \\ \Gamma \vdash a : A/R \\ \Gamma; \tilde{\Gamma} \vdash \gamma_1 : B \sim_R A \\ \Gamma; \Delta \vdash \gamma : b \sim_R a \end{array}}{\Gamma; \Delta \vdash \mathbf{sym} \gamma : a \sim_R b} \text{AN\_SYM} \\
\\
\frac{\begin{array}{l} \Gamma; \Delta \vdash \gamma_1 : a \sim_R a_1 \\ \Gamma; \Delta \vdash \gamma_2 : a_1 \sim_R b \\ \Gamma \vdash a : A/R \\ \Gamma \vdash a_1 : A_1/R \\ \Gamma; \tilde{\Gamma} \vdash \gamma_3 : A \sim_R A_1 \end{array}}{\Gamma; \Delta \vdash (\gamma_1; \gamma_2) : a \sim_R b} \text{AN\_TRANS} \\
\\
\frac{\begin{array}{l} \Gamma \vdash a_1 : B_0/R \\ \Gamma \vdash a_2 : B_1/R \\ |B_0|R = |B_1|R \\ \models |a_1|R > |a_2|R/R \end{array}}{\Gamma; \Delta \vdash \mathbf{red} a_1 a_2 : a_1 \sim_R a_2} \text{AN\_BETA} \\
\\
\frac{\begin{array}{l} \Gamma; \Delta \vdash \gamma_1 : A_1 \sim_{R'} A_2 \\ \Gamma, x : A_1/R; \Delta \vdash \gamma_2 : B_1 \sim_{R'} B_2 \\ B_3 = B_2 \{x \triangleright_{R'} \mathbf{sym} \gamma_1 / x\} \\ \Gamma \vdash \Pi^\rho x : A_1/R \rightarrow B_1 : \star / R' \\ \Gamma \vdash \Pi^\rho x : A_1/R \rightarrow B_2 : \star / R' \\ \Gamma \vdash \Pi^\rho x : A_2/R \rightarrow B_3 : \star / R' \\ R \leq R' \end{array}}{\Gamma; \Delta \vdash \Pi^{R, \rho} x : \gamma_1. \gamma_2 : (\Pi^\rho x : A_1/R \rightarrow B_1) \sim_{R'} (\Pi^\rho x : A_2/R \rightarrow B_3)} \text{AN\_PiCONG}
\end{array}$$

$$\begin{array}{c}
\Gamma; \Delta \vdash \gamma_1 : A_1 \sim_R A_2 \\
\Gamma, x : A_1/R; \Delta \vdash \gamma_2 : b_1 \sim_{R'} b_2 \\
b_3 = b_2\{x \triangleright_{R'} \mathbf{sym} \gamma_1/x\} \\
\Gamma \vdash A_1 : \star/R \\
\Gamma \vdash A_2 : \star/R \\
(\rho = +) \vee (x \notin \mathbf{fv} |b_1| R') \\
(\rho = +) \vee (x \notin \mathbf{fv} |b_3| R') \\
\Gamma \vdash (\lambda^\rho x : A_1/R. b_2) : B/R' \\
R \leq R' \\
\hline
\Gamma; \Delta \vdash (\lambda^{R,\rho} x : \gamma_1. \gamma_2) : (\lambda^\rho x : A_1/R. b_1) \sim_{R'} (\lambda^\rho x : A_2/R. b_3) \quad \text{AN\_ABSCONG}
\end{array}$$

$$\begin{array}{c}
\Gamma; \Delta \vdash \gamma_1 : a_1 \sim_{R'} b_1 \\
\Gamma; \Delta \vdash \gamma_2 : a_2 \sim_R b_2 \\
\Gamma \vdash a_1 \ a_2^{R,\rho} : A/R' \\
\Gamma \vdash b_1 \ b_2^{R,\rho} : B/R' \\
\Gamma; \tilde{\Gamma} \vdash \gamma_3 : A \sim_{R'} B \\
\hline
\Gamma; \Delta \vdash \gamma_1 \ \gamma_2^{R,\rho} : a_1 \ a_2^{R,\rho} \sim_{R'} b_1 \ b_2^{R,\rho} \quad \text{AN\_APPCONG}
\end{array}$$

$$\begin{array}{c}
\Gamma; \Delta \vdash \gamma : \Pi^\rho x : A_1/R \rightarrow B_1 \sim_{R'} \Pi^\rho x : A_2/R \rightarrow B_2 \\
\hline
\Gamma; \Delta \vdash \mathbf{piFst} \gamma : A_1 \sim_R A_2 \quad \text{AN\_PIFST}
\end{array}$$

$$\begin{array}{c}
\Gamma; \Delta \vdash \gamma_1 : \Pi^\rho x : A_1/R \rightarrow B_1 \sim_{R'} \Pi^\rho x : A_2/R \rightarrow B_2 \\
\Gamma; \Delta \vdash \gamma_2 : a_1 \sim_R a_2 \\
\Gamma \vdash a_1 : A_1/R \\
\Gamma \vdash a_2 : A_2/R \\
\hline
\Gamma; \Delta \vdash \gamma_1 @ \gamma_2 : B_1\{a_1/x\} \sim_{R'} B_2\{a_2/x\} \quad \text{AN\_PISND}
\end{array}$$

$$\begin{array}{c}
\Gamma; \Delta \vdash \gamma_1 : a_1 \sim_{A_1/R} b_1 \sim a_2 \sim_{A_2/R} b_2 \\
\Gamma, c : a_1 \sim_{A_1/R} b_1; \Delta \vdash \gamma_3 : B_1 \sim_{R'} B_2 \\
B_3 = B_2\{c \triangleright_{R'} \mathbf{sym} \gamma_1/c\} \\
\Gamma \vdash \forall c : a_1 \sim_{A_1/R} b_1. B_1 : \star/R' \\
\Gamma \vdash \forall c : a_2 \sim_{A_2/R} b_2. B_3 : \star/R' \\
\Gamma \vdash \forall c : a_1 \sim_{A_1/R} b_1. B_2 : \star/R' \\
\hline
\Gamma; \Delta \vdash (\forall c : \gamma_1. \gamma_3) : (\forall c : a_1 \sim_{A_1/R} b_1. B_1) \sim_R (\forall c : a_2 \sim_{A_2/R} b_2. B_3) \quad \text{AN\_CPICONG}
\end{array}$$

$$\begin{array}{c}
\Gamma; \Delta \vdash \gamma_1 : b_0 \sim_{A_1/R} b_1 \sim b_2 \sim_{A_2/R} b_3 \\
\Gamma, c : b_0 \sim_{A_1/R} b_1; \Delta \vdash \gamma_3 : a_1 \sim_{R'} a_2 \\
a_3 = a_2\{c \triangleright_{R'} \mathbf{sym} \gamma_1/c\} \\
\Gamma \vdash (\Lambda c : b_0 \sim_{A_1/R} b_1. a_1) : \forall c : b_0 \sim_{A_1/R} b_1. B_1/R' \\
\Gamma \vdash (\Lambda c : b_0 \sim_{A_1/R} b_1. a_2) : B/R' \\
\Gamma \vdash (\Lambda c : b_2 \sim_{A_2/R} b_3. a_3) : \forall c : b_2 \sim_{A_2/R} b_3. B_2/R' \\
\Gamma; \tilde{\Gamma} \vdash \gamma_4 : \forall c : b_0 \sim_{A_1/R} b_1. B_1 \sim_{R'} \forall c : \phi_2. B_2 \\
\hline
\Gamma; \Delta \vdash (\lambda c : \gamma_1. \gamma_3 @ \gamma_4) : (\Lambda c : b_0 \sim_{A_1/R} b_1. a_1) \sim_{R'} (\Lambda c : b_2 \sim_{A_2/R} b_3. a_3) \quad \text{AN\_CABSCONG}
\end{array}$$

$$\begin{array}{c}
\Gamma; \Delta \vdash \gamma_1 : a_1 \sim_R b_1 \\
\Gamma; \tilde{\Gamma} \vdash \gamma_2 : a_2 \sim_{R'} b_2 \\
\Gamma; \tilde{\Gamma} \vdash \gamma_3 : a_3 \sim_{R'} b_3 \\
\Gamma \vdash a_1[\gamma_2] : A/R \\
\Gamma \vdash b_1[\gamma_3] : B/R \\
\Gamma; \tilde{\Gamma} \vdash \gamma_4 : A \sim_R B \\
\hline
\Gamma; \Delta \vdash \gamma_1(\gamma_2, \gamma_3) : a_1[\gamma_2] \sim_R b_1[\gamma_3] \quad \text{AN\_CAPPCONG}
\end{array}$$

$$\begin{array}{c}
\frac{\Gamma; \Delta \vdash \gamma_1 : (\forall c_1 : a \sim_{A/R} a'. B_1) \sim_{R_0} (\forall c_2 : b \sim_{B/R'} b'. B_2) \quad \Gamma; \tilde{\Gamma} \vdash \gamma_2 : a \sim_R a' \quad \Gamma; \tilde{\Gamma} \vdash \gamma_3 : b \sim_{R'} b'}{\Gamma; \Delta \vdash \gamma_1 @ (\gamma_2 \sim \gamma_3) : B_1\{\gamma_2/c_1\} \sim_{R_0} B_2\{\gamma_3/c_2\}} \quad \text{AN\_CPIsND} \\
\\
\frac{\Gamma; \Delta \vdash \gamma_1 : a \sim_{R_1} a' \quad \Gamma; \Delta \vdash \gamma_2 : a \sim_{A/R_1} a' \sim b \sim_{B/R_2} b'}{\Gamma; \Delta \vdash \gamma_1 \triangleright_{R_2} \gamma_2 : b \sim_{R_2} b'} \quad \text{AN\_CAST} \\
\\
\frac{\Gamma; \Delta \vdash \gamma : (a \sim_{A/R} a') \sim (b \sim_{B/R} b')}{\Gamma; \Delta \vdash \mathbf{isoSnd} \gamma : A \sim_R B} \quad \text{AN\_ISOsND} \\
\\
\frac{\Gamma; \Delta \vdash \gamma : a \sim_{R_1} b \quad R_1 \leq R_2}{\Gamma; \Delta \vdash \mathbf{sub} \gamma : a \sim_{R_2} b} \quad \text{AN\_SUB}
\end{array}$$

$\boxed{\vdash \Gamma}$  context wellformedness

$$\begin{array}{c}
\frac{}{\vdash \emptyset} \quad \text{AN\_EMPTY} \\
\\
\frac{\vdash \Gamma \quad \Gamma \vdash A : \star/R \quad x \notin \text{dom } \Gamma}{\vdash \Gamma, x : A/R} \quad \text{AN\_CONSTM} \\
\\
\frac{\vdash \Gamma \quad \Gamma \vdash \phi \text{ ok} \quad c \notin \text{dom } \Gamma}{\vdash \Gamma, c : \phi} \quad \text{AN\_CONSCo}
\end{array}$$

$\boxed{\vdash \Sigma}$  signature wellformedness

$$\begin{array}{c}
\frac{}{\vdash \emptyset} \quad \text{AN\_SIG\_EMPTY} \\
\\
\frac{\vdash \Sigma \quad \emptyset \vdash A : \star/R \quad \emptyset \vdash a : A/R \quad F \notin \text{dom } \Sigma}{\vdash \Sigma \cup \{F \sim a : A/R\}} \quad \text{AN\_SIG\_CONSAx}
\end{array}$$

$\boxed{\Gamma \vdash a \rightsquigarrow b/R}$  single-step, weak head reduction to values for annotated language

$$\begin{array}{c}
\frac{\Gamma \vdash a \rightsquigarrow a'/R_1}{\Gamma \vdash a \ b^{R,\rho} \rightsquigarrow a' \ b^{R,\rho}/R_1} \quad \text{AN\_APPLEFT} \\
\\
\frac{\text{Value}_R (\lambda^\rho x : A/R.w)}{\Gamma \vdash (\lambda^\rho x : A/R.w) \ a^{R,\rho} \rightsquigarrow w\{a/x\}/R} \quad \text{AN\_APPABS} \\
\\
\frac{\Gamma \vdash a \rightsquigarrow a'/R}{\Gamma \vdash a[\gamma] \rightsquigarrow a'[\gamma]/R} \quad \text{AN\_CAPPLEFT} \\
\\
\frac{}{\Gamma \vdash (\lambda c : \phi.b)[\gamma] \rightsquigarrow b\{\gamma/c\}/R} \quad \text{AN\_CAPPCABS} \\
\\
\frac{\Gamma \vdash A : \star/R \quad \Gamma, x : A/R \vdash b \rightsquigarrow b'/R_1}{\Gamma \vdash (\lambda^- x : A/R.b) \rightsquigarrow (\lambda^- x : A/R.b')/R_1} \quad \text{AN\_ABSTERM}
\end{array}$$

$$\begin{array}{c}
\frac{F \sim a : A/R \in \Sigma_1}{\Gamma \vdash F \rightsquigarrow a/R} \quad \text{AN\_AXIOM} \\
\\
\frac{\Gamma \vdash a \rightsquigarrow a'/R}{\Gamma \vdash a \triangleright_{R_1} \gamma \rightsquigarrow a' \triangleright_{R_1} \gamma/R} \quad \text{AN\_CONVTERM} \\
\\
\frac{\text{Value}_R v}{\Gamma \vdash (v \triangleright_{R_2} \gamma_1) \triangleright_{R_2} \gamma_2 \rightsquigarrow v \triangleright_{R_2} (\gamma_1; \gamma_2)/R} \quad \text{AN\_COMBINE} \\
\\
\begin{array}{l}
\text{Value}_R v \\
\Gamma; \tilde{\Gamma} \vdash \gamma : \Pi^\rho x_1 : A_1/R \rightarrow B_1 \sim_{R'} \Pi^\rho x_2 : A_2/R \rightarrow B_2 \\
b' = b \triangleright_{R'} \mathbf{sym}(\mathbf{piFst} \gamma) \\
\gamma' = \gamma @ (b' \mid_{(\mathbf{piFst} \gamma)} b)
\end{array} \\
\hline
\Gamma \vdash (v \triangleright_{R'} \gamma) \ b^{R,\rho} \rightsquigarrow ((v \ b'^{R,\rho}) \triangleright_{R'} \gamma')/R \quad \text{AN\_PUSH} \\
\\
\begin{array}{l}
\text{Value}_R v \\
\Gamma; \tilde{\Gamma} \vdash \gamma : \forall c_1 : a_1 \sim_{B_1/R} b_1. A_1 \sim_{R'} \forall c_2 : a_2 \sim_{B_2/R} b_2. A_2 \\
\gamma'_1 = \gamma_1 \triangleright_{R'} \mathbf{sym}(\mathbf{cpiFst} \gamma) \\
\gamma' = \gamma @ (\gamma'_1 \sim \gamma_1)
\end{array} \\
\hline
\Gamma \vdash (v \triangleright_{R'} \gamma)[\gamma_1] \rightsquigarrow ((v[\gamma'_1]) \triangleright_{R'} \gamma')/R \quad \text{AN\_CPUSH}
\end{array}$$

Definition rules: 161 good 0 bad  
 Definition rule clauses: 475 good 0 bad