

$tnvar, x, y, f, m, n$	variables
$covar, c$	coercion variables
$datacon, K$	
$const, T, F$	
$index, i$	indices

$role, R$	$::=$ $ $ Nom $ $ Rep $ $ $R_1 \cap R_2$ S $ $ param $R_1 R_2$ S $ $ (R) S	Role
$relflag, \rho$	$::=$ $ $ $+$ $ $ $-$	relevance flag
$constraint, \phi$	$::=$ $ $ $a \sim_{A/R} b$ $ $ (ϕ) S $ $ $\phi\{b/x\}$ S $ $ $ \phi $ S $ $ $a \sim_R b$ S	props
tm, a, b, v, w, A, B	$::=$ $ $ \star $ $ x $ $ $\lambda^\rho x : A/R. b$ bind x in b $ $ $\lambda^{R,\rho} x. b$ bind x in b $ $ $a \ b^{R,\rho}$ $ $ F $ $ $\Pi^\rho x : A/R \rightarrow B$ bind x in B $ $ $a \triangleright_R \gamma$ $ $ $\forall c : \phi. B$ bind c in B $ $ $\Lambda c : \phi. b$ bind c in b $ $ $\Lambda c. b$ bind c in b $ $ $a[\gamma]$ $ $ \square $ $ $case_R \ a \ of \ a' \rightarrow b_1 \parallel_- \rightarrow b_2$ $ $ K $ $ match a with brs $ $ sub $R \ a$ $ $ $a\{b/x\}$ S $ $ a S $ $ $a\{\gamma/c\}$ S $ $ a S $ $ (a) S $ $ a S $ $ $ a _R$ S $ $ Int S $ $ Bool S $ $ Nat S $ $ Vec S	<p>types and kinds</p> <p>parsing precedence is hard</p>

		0	S	
		S	S	
		True	S	
		Fix	S	
		Age	S	
		$a \rightarrow b$	S	
		$a/R \rightarrow b$	S	
		$\phi \Rightarrow A$	S	
		$a \ b$	S	
		$\lambda x. a$	S	
		$\lambda x : A. a$	S	
		$\forall x : A/R \rightarrow B$	S	
		if ϕ then a else b	S	
brs	$::=$			case branches
		none		
		$K \Rightarrow a; brs$		
		$brs\{a/x\}$	S	
		$brs\{\gamma/c\}$	S	
		(brs)	S	
co, γ	$::=$			explicit coercions
		•		
		c		
		red $a \ b$		
		refl a		
		$(a \models_{\gamma} b)$		
		sym γ		
		$\gamma_1; \gamma_2$		
		sub γ		
		$\Pi^{R,\rho} x : \gamma_1. \gamma_2$	bind x in γ_2	
		$\lambda^{R,\rho} x : \gamma_1. \gamma_2$	bind x in γ_2	
		$\gamma_1 \ \gamma_2^{R,\rho}$		
		piFst γ		
		cpiFst γ		
		isoSnd γ		
		$\gamma_1 @ \gamma_2$		
		$\forall c : \gamma_1. \gamma_3$	bind c in γ_3	
		$\lambda c : \gamma_1. \gamma_3 @ \gamma_4$	bind c in γ_3	
		$\gamma(\gamma_1, \gamma_2)$		
		$\gamma @ (\gamma_1 \sim \gamma_2)$		
		$\gamma_1 \triangleright_R \gamma_2$		
		$\gamma_1 \sim_A \gamma_2$		
		conv $\phi_1 \sim_{\gamma} \phi_2$		
		eta a		
		left $\gamma \ \gamma'$		

		right $\gamma \gamma'$	
		(γ)	S
		γ	S
		$\gamma\{a/x\}$	S
$role_context, \Omega$::=		$role_contexts$
		\emptyset	
		$\Omega, x : R$	
		(Ω)	M
		Ω	M
sig_sort	::=		signature classifier
		$: A$	
		$\sim a : A/R$	
$sort$::=		binding classifier
		Tm $A R$	
		Co ϕ	
$context, \Gamma$::=		contexts
		\emptyset	
		$\Gamma, x : A/R$	
		$\Gamma, c : \phi$	
		$\Gamma\{b/x\}$	M
		$\Gamma\{\gamma/c\}$	M
		Γ, Γ'	M
		$ \Gamma $	M
		(Γ)	M
		Γ	M
sig, Σ	::=		signatures
		\emptyset	
		$\Sigma \cup \{F sig_sort\}$	
		Σ_0	M
		Σ_1	M
		$ \Sigma $	M
$available_props, \Delta$::=		
		\emptyset	
		Δ, c	
		$\tilde{\Gamma}$	M
		(Δ)	M
$terminals$::=		
		\leftrightarrow	
		\Leftrightarrow	
		\longrightarrow	

	min
	\equiv
	\forall
	\in
	\notin
	\Leftarrow
	\Rightarrow
	\Rightarrow^*
	\rightarrow
	Λ
	\square
	\vdash
	\dashv
	\models
	\models
	\neq
	\triangleright
	ok
	$-$
	\rightsquigarrow
	\rightsquigarrow^*
	\rightsquigarrow
	\emptyset
	\circ
	fv
	dom
	\sim
	\succ
	$ $
	\bullet
	fst
	snd
	$ \Rightarrow $
	$\vdash=$
	refl₂
	$++$
<i>formula, ψ</i>	$::=$
	<i>judgement</i>
	$x : A/R \in \Gamma$
	$x : R \in \Omega$
	$c : \phi \in \Gamma$
	$F \text{ sig_sort} \in \Sigma$
	$K : T \Gamma \in \Sigma$
	$x \in \Delta$

	$ \begin{array}{ l} c \in \Delta \\ c \text{ not relevant} \in \gamma \\ x \notin \text{fv } a \\ x \notin \text{dom } \Gamma \\ \text{uniq}(\Omega) \\ c \notin \text{dom } \Gamma \\ T \notin \text{dom } \Sigma \\ F \notin \text{dom } \Sigma \\ a = b \\ \phi_1 = \phi_2 \\ \Gamma_1 = \Gamma_2 \\ \gamma_1 = \gamma_2 \\ \neg \psi \\ \psi_1 \wedge \psi_2 \\ \psi_1 \vee \psi_2 \\ \psi_1 \Rightarrow \psi_2 \\ (\psi) \\ \psi \\ c : (a : A \sim b : B) \in \Gamma \end{array} $	suppress lc hypothesis generated by Ott
$JSubRole$	$ \begin{array}{ l} ::= \\ R_1 \leq R_2 \end{array} $	Subroling judgement
$JPath$	$ \begin{array}{ l} ::= \\ \text{Path}_R a = F \end{array} $	Type headed by constant (partial function)
$JValue$	$ \begin{array}{ l} ::= \\ \text{Value}_R A \end{array} $	values
$JValueType$	$ \begin{array}{ l} ::= \\ \text{ValueType}_R A \end{array} $	Types with head forms (erased language)
$Jconsistent$	$ \begin{array}{ l} ::= \\ \text{consistent}_R ab \end{array} $	(erased) types do not differ in their heads
$Jroleing$	$ \begin{array}{ l} ::= \\ \Omega \models a : R \end{array} $	
$JChk$	$ \begin{array}{ l} ::= \\ (\rho = +) \vee (x \notin \text{fv } A) \end{array} $	irrelevant argument check
$Jpar$	$ \begin{array}{ l} ::= \\ \Omega \models a \Rightarrow_R b \\ \Omega \vdash a \Rightarrow_R^* b \\ \Omega \vdash a \Leftrightarrow_R b \end{array} $	parallel reduction (implicit language) multistep parallel reduction parallel reduction to a common term

<i>Jbeta</i>	$::=$ $\mid \models a > b/R$ $\mid \models a \rightsquigarrow b/R$ $\mid \models a \rightsquigarrow^* b/R$	primitive reductions on erased terms single-step head reduction for implicit language multistep reduction
<i>Jett</i>	$::=$ $\mid \Gamma \models \phi \text{ ok}$ $\mid \Gamma \models a : A/R$ $\mid \Gamma; \Delta \models \phi_1 \equiv \phi_2$ $\mid \Gamma; \Delta \models a \equiv b : A/R$ $\mid \models \Gamma$	Prop wellformedness typing prop equality definitional equality context wellformedness
<i>Jsig</i>	$::=$ $\mid \models \Sigma$	signature wellformedness
<i>Jann</i>	$::=$ $\mid \Gamma \vdash \phi \text{ ok}$ $\mid \Gamma \vdash a : A/R$ $\mid \Gamma; \Delta \vdash \gamma : \phi_1 \sim \phi_2$ $\mid \Gamma; \Delta \vdash \gamma : A \sim_R B$ $\mid \vdash \Gamma$ $\mid \vdash \Sigma$	prop wellformedness typing coercion between props coercion between types context wellformedness signature wellformedness
<i>Jred</i>	$::=$ $\mid \Gamma \vdash a \rightsquigarrow b/R$	single-step, weak head reduction to values for annotated lang
<i>judgement</i>	$::=$ $\mid JSubRole$ $\mid JPath$ $\mid JValue$ $\mid JValueType$ $\mid Jconsistent$ $\mid Jroleing$ $\mid JChk$ $\mid Jpar$ $\mid Jbeta$ $\mid Jett$ $\mid Jsig$ $\mid Jann$ $\mid Jred$	
<i>user_syntax</i>	$::=$ $\mid tmvar$ $\mid covar$ $\mid datacon$ $\mid const$ $\mid index$	

\mid *role*
 \mid *relflag*
 \mid *constraint*
 \mid *tm*
 \mid *brs*
 \mid *co*
 \mid *role_context*
 \mid *sig_sort*
 \mid *sort*
 \mid *context*
 \mid *sig*
 \mid *available_props*
 \mid *terminals*
 \mid *formula*

$\boxed{R_1 \leq R_2}$ Subroling judgement

$$\begin{array}{c}
\overline{\mathbf{Nom} \leq R} \quad \text{NOMBOT} \\
\overline{R \leq \mathbf{Rep}} \quad \text{REPTOP} \\
\overline{R \leq R} \quad \text{REFL} \\
\frac{R_1 \leq R_2 \quad R_2 \leq R_3}{R_1 \leq R_3} \quad \text{TRANS}
\end{array}$$

$\boxed{\text{Path}_R a = F}$ Type headed by constant (partial function)

$$\begin{array}{c}
\frac{F : A \in \Sigma_0}{\text{Path}_R F = F} \quad \text{PATH_ABSCONST} \\
\frac{F \sim a : A/R_1 \in \Sigma_0 \quad \neg(R_1 \leq R)}{\text{Path}_R F = F} \quad \text{PATH_CONST} \\
\frac{\text{Path}_R a = F}{\text{Path}_R (a \ b'^{R_1, \rho}) = F} \quad \text{PATH_APP} \\
\frac{\text{Path}_R a = F}{\text{Path}_R (a[\bullet]) = F} \quad \text{PATH_CAPP}
\end{array}$$

$\boxed{\text{Value}_R A}$ values

$$\begin{array}{c}
\overline{\text{Value}_R \star} \quad \text{VALUE_STAR} \\
\overline{\text{Value}_R \Pi^\rho x : A/R_1 \rightarrow B} \quad \text{VALUE_PI} \\
\overline{\text{Value}_R \forall c : \phi. B} \quad \text{VALUE_CPI} \\
\overline{\text{Value}_R \lambda^+ x : A/R_1. a} \quad \text{VALUE_ABsREL} \\
\overline{\text{Value}_R \lambda^{R_1, +} x. a} \quad \text{VALUE_UABsREL}
\end{array}$$

$$\begin{array}{c}
\frac{\text{Value}_R a}{\text{Value}_R \lambda^{R_1, -} x. a} \quad \text{VALUE_UABSIRREL} \\
\frac{}{\text{Value}_R \Lambda c : \phi. a} \quad \text{VALUE_CABS} \\
\frac{}{\text{Value}_R \Lambda c. a} \quad \text{VALUE_UCABS} \\
\frac{\text{Path}_R a = F}{\text{Value}_R a} \quad \text{VALUE_PATH}
\end{array}$$

$\boxed{\text{ValueType}_R A}$ Types with head forms (erased language)

$$\begin{array}{c}
\frac{}{\text{ValueType}_R \star} \quad \text{VALUE_TYPE_STAR} \\
\frac{}{\text{ValueType}_R \Pi^\rho x : A / R_1 \rightarrow B} \quad \text{VALUE_TYPE_PI} \\
\frac{}{\text{ValueType}_R \forall c : \phi. B} \quad \text{VALUE_TYPE_CPI} \\
\frac{\text{Path}_R a = F}{\text{ValueType}_R a} \quad \text{VALUE_TYPE_PATH}
\end{array}$$

$\boxed{\text{consistent}_R ab}$ (erased) types do not differ in their heads

$$\begin{array}{c}
\frac{}{\text{consistent}_R \star \star} \quad \text{CONSISTENT_A_STAR} \\
\frac{}{\text{consistent}_{R'} (\Pi^\rho x_1 : A_1 / R \rightarrow B_1) (\Pi^\rho x_2 : A_2 / R \rightarrow B_2)} \quad \text{CONSISTENT_A_PI} \\
\frac{}{\text{consistent}_R (\forall c_1 : \phi_1. A_1) (\forall c_2 : \phi_2. A_2)} \quad \text{CONSISTENT_A_CPI} \\
\frac{\text{Path}_R a_1 = F \quad \text{Path}_R a_2 = F}{\text{consistent}_R a_1 a_2} \quad \text{CONSISTENT_A_PATH} \\
\frac{\neg \text{ValueType}_R b}{\text{consistent}_R ab} \quad \text{CONSISTENT_A_STEP_R} \\
\frac{\neg \text{ValueType}_R a}{\text{consistent}_R ab} \quad \text{CONSISTENT_A_STEP_L}
\end{array}$$

$\boxed{\Omega \models a : R}$

$$\begin{array}{c}
\frac{\text{uniq}(\Omega)}{\Omega \models \square : R} \quad \text{ROLE_A_BULLET} \\
\frac{\text{uniq}(\Omega)}{\Omega \models \star : R} \quad \text{ROLE_A_STAR} \\
\frac{\text{uniq}(\Omega) \quad x : R \in \Omega \quad R \leq R_1}{\Omega \models x : R_1} \quad \text{ROLE_A_VAR} \\
\frac{\Omega, x : R_1 \models a : R}{\Omega \models (\lambda^{R_1, \rho} x. a) : R} \quad \text{ROLE_A_ABS}
\end{array}$$

$$\frac{\Omega \models a : R \quad \Omega \models b : (\mathbf{param} R_1 R)}{\Omega \models (a \ b^{R_1, \rho}) : R} \quad \text{ROLE_A_APP}$$

$$\frac{\Omega \models A : R \quad \Omega, x : R_1 \models B : R}{\Omega \models (\Pi^{\rho} x : A / R_1 \rightarrow B) : R} \quad \text{ROLE_A_PI}$$

$$\frac{\Omega \models a : R_1 \quad \Omega \models b : R_1 \quad \Omega \models A : R_0 \quad \Omega \models B : R}{\Omega \models (\forall c : a \sim_{A/R_1} b.B) : R} \quad \text{ROLE_A_CPI}$$

$$\frac{\Omega \models b : R}{\Omega \models (\Lambda c.b) : R} \quad \text{ROLE_A_CABS}$$

$$\frac{\Omega \models a : R}{\Omega \models (a[\bullet]) : R} \quad \text{ROLE_A_CAPP}$$

$$\frac{\text{uniq}(\Omega) \quad F : A \in \Sigma_0}{\Omega \models F : R} \quad \text{ROLE_A_CONST}$$

$$\frac{\text{uniq}(\Omega) \quad F \sim a : A / R \in \Sigma_0}{\Omega \models F : R_1} \quad \text{ROLE_A_FAM}$$

$$\frac{F \text{ sig_sort} \in \Sigma_0 \quad \Omega \models a : R \quad \Omega \models b_1 : R_1 \quad \Omega \models b_2 : R_1}{\Omega \models (\text{case}_R a \text{ of } F \rightarrow b_1 \parallel - \rightarrow b_2) : R_1} \quad \text{ROLE_A_PATTERN}$$

$$\boxed{(\rho = +) \vee (x \notin \text{fv } A)} \quad \text{irrelevant argument check}$$

$$\overline{(+ = +) \vee (x \notin \text{fv } A)} \quad \text{RHO_REL}$$

$$\frac{x \notin \text{fv } A}{(- = +) \vee (x \notin \text{fv } A)} \quad \text{RHO_IRRREL}$$

$$\boxed{\Omega \models a \Rightarrow_R b} \quad \text{parallel reduction (implicit language)}$$

$$\frac{\Omega \models a : R}{\Omega \models a \Rightarrow_R a} \quad \text{PAR_REFL}$$

$$\frac{\Omega \models a \Rightarrow_R (\lambda^{R_1, \rho} x. a') \quad \Omega \models b \Rightarrow_{\mathbf{param} R_1 R} b'}{\Omega \models a \ b^{R_1, \rho} \Rightarrow_R a' \{b'/x\}} \quad \text{PAR_BETA}$$

$$\frac{\Omega \models a \Rightarrow_R a' \quad \Omega \models b \Rightarrow_{\mathbf{param} R_1 R} b'}{\Omega \models a \ b^{R_1, \rho} \Rightarrow_R a' \ b'^{R_1, \rho}} \quad \text{PAR_APP}$$

$$\frac{\Omega \models a \Rightarrow_R (\Lambda c. a')}{\Omega \models a[\bullet] \Rightarrow_R a' \{\bullet/c\}} \quad \text{PAR_CBETA}$$

$$\begin{array}{c}
\frac{\Omega \models a \Rightarrow_R a'}{\Omega \models a[\bullet] \Rightarrow_R a'[\bullet]} \text{PAR_CAPP} \\
\\
\frac{\Omega, x : R_1 \models a \Rightarrow_R a'}{\Omega \models \lambda^{R_1, \rho} x. a \Rightarrow_R \lambda^{R_1, \rho} x. a'} \text{PAR_ABS} \\
\\
\frac{\Omega \models A \Rightarrow_R A' \quad \Omega, x : R_1 \models B \Rightarrow_R B'}{\Omega \models \Pi x : A / R_1 \rightarrow B \Rightarrow_R \Pi x : A' / R_1 \rightarrow B'} \text{PAR_PI} \\
\\
\frac{\Omega \models a \Rightarrow_R a'}{\Omega \models \Lambda c. a \Rightarrow_R \Lambda c. a'} \text{PAR_CABS} \\
\\
\frac{\Omega \models A \Rightarrow_{R_0} A' \quad \Omega \models a \Rightarrow_{R_1} a' \quad \Omega \models b \Rightarrow_{R_1} b' \quad \Omega \models B \Rightarrow_R B'}{\Omega \models \forall c : a \sim_{A/R_1} b. B \Rightarrow_R \forall c : a' \sim_{A'/R_1} b'. B'} \text{PAR_CPI} \\
\\
\frac{F \sim a : A / R_1 \in \Sigma_0 \quad R_1 \leq R \quad \text{uniq}(\Omega)}{\Omega \models F \Rightarrow_R a} \text{PAR_AXIOM} \\
\\
\frac{F \text{ sig_sort} \in \Sigma_0 \quad \Omega \models a \Rightarrow_R a' \quad \Omega \models b_1 \Rightarrow_{R_0} b'_1 \quad \Omega \models b_2 \Rightarrow_{R_0} b'_2}{\Omega \models \text{case}_R a \text{ of } F \rightarrow b_1 \parallel \rightarrow b_2 \Rightarrow_{R_0} \text{case}_R a' \text{ of } F \rightarrow b'_1 \parallel \rightarrow b'_2} \text{PAR_PATTERN} \\
\\
\frac{\Omega \models a \Rightarrow_R a' \quad \Omega \models b_1 \Rightarrow_{R_0} b'_1 \quad \Omega \models b_2 \Rightarrow_{R_0} b'_2 \quad \text{Path}_R a' = F}{\Omega \models \text{case}_R a \text{ of } F \rightarrow b_1 \parallel \rightarrow b_2 \Rightarrow_{R_0} b'_1} \text{PAR_PATTERNTRUE} \\
\\
\frac{F \text{ sig_sort} \in \Sigma_0 \quad \Omega \models a \Rightarrow_R a' \quad \Omega \models b_1 \Rightarrow_{R_0} b'_1 \quad \Omega \models b_2 \Rightarrow_{R_0} b'_2 \quad \text{Value}_R a' \quad \neg(\text{Path}_R a' = F)}{\Omega \models \text{case}_R a \text{ of } F \rightarrow b_1 \parallel \rightarrow b_2 \Rightarrow_{R_0} b'_2} \text{PAR_PATTERNFALSE}
\end{array}$$

$$\boxed{\Omega \vdash a \Rightarrow_R^* b}$$

multistep parallel reduction

$$\frac{}{\Omega \vdash a \Rightarrow_R^* a} \text{MP_REFL}$$

$$\frac{\Omega \models a \Rightarrow_R b \quad \Omega \vdash b \Rightarrow_R^* a'}{\Omega \vdash a \Rightarrow_R^* a'} \text{MP_STEP}$$

$$\boxed{\Omega \vdash a \Leftrightarrow_R b}$$

parallel reduction to a common term

$$\frac{\Omega \vdash a_1 \Rightarrow_R^* b \quad \Omega \vdash a_2 \Rightarrow_R^* b}{\Omega \vdash a_1 \Leftrightarrow_R a_2} \text{JOIN}$$

$\boxed{\models a > b/R}$ primitive reductions on erased terms

$$\begin{array}{c}
\frac{\text{Value}_{R_1} (\lambda^{R,\rho} x.v)}{\models (\lambda^{R,\rho} x.v) \ b^{R,\rho} > v\{b/x\}/R_1} \quad \text{BETA_APPABS} \\
\frac{}{\models (\Lambda c.a')[\bullet] > a'\{\bullet/c\}/R} \quad \text{BETA_CAPPCABS} \\
\frac{F \sim a : A/R \in \Sigma_0 \quad R \leq R_1}{\models F > a/R_1} \quad \text{BETA_AXIOM} \\
\frac{\text{Path}_R a = F}{\models \text{case}_R a \text{ of } F \rightarrow b_1 \parallel - \rightarrow b_2 > b_1/R_0} \quad \text{BETA_PATTERNTRUE} \\
\frac{F \text{ sig_sort} \in \Sigma_0 \quad \text{Value}_R a \quad \neg(\text{Path}_R a = F)}{\models \text{case}_R a \text{ of } F \rightarrow b_1 \parallel - \rightarrow b_2 > b_2/R_0} \quad \text{BETA_PATTERNFALSE}
\end{array}$$

$\boxed{\models a \rightsquigarrow b/R}$ single-step head reduction for implicit language

$$\begin{array}{c}
\frac{\models a \rightsquigarrow a'/R_1}{\models \lambda^{R,-} x.a \rightsquigarrow \lambda^{R,-} x.a'/R_1} \quad \text{E_ABSTERM} \\
\frac{\models a \rightsquigarrow a'/R_1}{\models a \ b^{R,\rho} \rightsquigarrow a' \ b^{R,\rho}/R_1} \quad \text{E_APPLEFT} \\
\frac{\models a \rightsquigarrow a'/R}{\models a[\bullet] \rightsquigarrow a'[\bullet]/R} \quad \text{E_CAPPLEFT} \\
\frac{\models a \rightsquigarrow a'/R}{\models \text{case}_R a \text{ of } F \rightarrow b_1 \parallel - \rightarrow b_2 \rightsquigarrow \text{case}_R a' \text{ of } F \rightarrow b_1 \parallel - \rightarrow b_2/R_0} \quad \text{E_PATTERN} \\
\frac{\models a > b/R}{\models a \rightsquigarrow b/R} \quad \text{E_PRIM}
\end{array}$$

$\boxed{\models a \rightsquigarrow^* b/R}$ multistep reduction

$$\begin{array}{c}
\frac{}{\models a \rightsquigarrow^* a/R} \quad \text{EQUAL} \\
\frac{\models a \rightsquigarrow b/R \quad \models b \rightsquigarrow^* a'/R}{\models a \rightsquigarrow^* a'/R} \quad \text{STEP}
\end{array}$$

$\boxed{\Gamma \models \phi \text{ ok}}$ Prop wellformedness

$$\frac{\Gamma \models a : A/R \quad \Gamma \models b : A/R \quad \Gamma \models A : \star/R_0}{\Gamma \models a \sim_{A/R} b \text{ ok}} \quad \text{E_WFF}$$

$\boxed{\Gamma \models a : A/R}$ typing

$$\frac{R_1 \leq R_2 \quad \Gamma \models a : A/R_1}{\Gamma \models a : A/R_2} \quad \text{E_SUBROLE}$$

$$\begin{array}{c}
\frac{\vdash \Gamma}{\Gamma \vdash \star : \star / R} \quad \text{E_STAR} \\
\\
\frac{\vdash \Gamma}{\Gamma \vdash x : A / R} \quad \text{E_VAR} \\
\\
\frac{\Gamma, x : A / R \vdash B : \star / R' \quad \Gamma \vdash A : \star / R'}{\Gamma \vdash \Pi^\rho x : A / R \rightarrow B : \star / R'} \quad \text{E_PI} \\
\\
\frac{\Gamma, x : A / R \vdash a : B / R' \quad \Gamma \vdash A : \star / R_0 \quad (\rho = +) \vee (x \notin \text{fv } a)}{\Gamma \vdash \lambda^{R, \rho} x. a : (\Pi^\rho x : A / R \rightarrow B) / R'} \quad \text{E_ABS} \\
\\
\frac{\Gamma \vdash b : \Pi^+ x : A / R \rightarrow B / R' \quad \Gamma \vdash a : A / \mathbf{param} R R'}{\Gamma \vdash b \ a^{R, +} : B \{a/x\} / R'} \quad \text{E_APP} \\
\\
\frac{\Gamma \vdash b : \Pi^- x : A / R \rightarrow B / R' \quad \Gamma \vdash a : A / \mathbf{param} R R'}{\Gamma \vdash b \ \Box^{R, -} : B \{a/x\} / R'} \quad \text{E_IAAPP} \\
\\
\frac{\Gamma \vdash a : A / R \quad \Gamma; \widetilde{\Gamma} \vdash A \equiv B : \star / \mathbf{Rep} \quad \Gamma \vdash B : \star / R_0}{\Gamma \vdash a : B / R} \quad \text{E_CONV} \\
\\
\frac{\Gamma, c : \phi \vdash B : \star / R \quad \Gamma \vdash \phi \ \mathbf{ok}}{\Gamma \vdash \forall c : \phi. B : \star / R} \quad \text{E_CPI} \\
\\
\frac{\Gamma, c : \phi \vdash a : B / R \quad \Gamma \vdash \phi \ \mathbf{ok}}{\Gamma \vdash \Lambda c. a : \forall c : \phi. B / R} \quad \text{E_CABS} \\
\\
\frac{\Gamma \vdash a_1 : \forall c : (a \sim_{A/R} b). B_1 / R' \quad \Gamma; \widetilde{\Gamma} \vdash a \equiv b : A / R}{\Gamma \vdash a_1[\bullet] : B_1 \{\bullet/c\} / R'} \quad \text{E_CAPP} \\
\\
\frac{\vdash \Gamma \quad F : A \in \Sigma_0 \quad \emptyset \vdash A : \star / R_0}{\Gamma \vdash F : A / R_1} \quad \text{E_CONST} \\
\\
\frac{\vdash \Gamma \quad F \sim a : A / R \in \Sigma_0 \quad \emptyset \vdash A : \star / R_0}{\Gamma \vdash F : A / R_1} \quad \text{E_FAM} \\
\\
\frac{F \text{ sig_sort} \in \Sigma_0 \quad \Gamma \vdash a : A / R \quad \Gamma \vdash b_1 : B / R_0 \quad \Gamma \vdash b_2 : B / R_0}{\Gamma \vdash \text{case}_R a \text{ of } F \rightarrow b_1 \parallel - \rightarrow b_2 : B / R_0} \quad \text{E_PAT}
\end{array}$$

$\boxed{\Gamma; \Delta \models \phi_1 \equiv \phi_2}$ prop equality

$$\frac{\begin{array}{c} \Gamma; \Delta \models A_1 \equiv A_2 : A/R \\ \Gamma; \Delta \models B_1 \equiv B_2 : A/R \end{array}}{\Gamma; \Delta \models A_1 \sim_{A/R} B_1 \equiv A_2 \sim_{A/R} B_2} \text{E_PROP_CONG}$$

$$\frac{\begin{array}{c} \Gamma; \Delta \models A \equiv B : \star/R_0 \\ \Gamma \models A_1 \sim_{A/R} A_2 \text{ ok} \\ \Gamma \models A_1 \sim_{B/R} A_2 \text{ ok} \end{array}}{\Gamma; \Delta \models A_1 \sim_{A/R} A_2 \equiv A_1 \sim_{B/R} A_2} \text{E_ISO_CONV}$$

$$\frac{\Gamma; \Delta \models \forall c : (a_1 \sim_{A/R_1} a_2). B_1 \equiv \forall c : (b_1 \sim_{B/R_2} b_2). B_2 : \star/R'}{\Gamma; \Delta \models a_1 \sim_{A/R_1} a_2 \equiv b_1 \sim_{B/R_2} b_2} \text{E_CPI_FST}$$

$\boxed{\Gamma; \Delta \models a \equiv b : A/R}$ definitional equality

$$\frac{\begin{array}{c} \models \Gamma \\ c : (a \sim_{A/R} b) \in \Gamma \\ c \in \Delta \end{array}}{\Gamma; \Delta \models a \equiv b : A/R} \text{E_ASSN}$$

$$\frac{\Gamma \models a : A/R}{\Gamma; \Delta \models a \equiv a : A/R} \text{E_REFL}$$

$$\frac{\Gamma; \Delta \models b \equiv a : A/R}{\Gamma; \Delta \models a \equiv b : A/R} \text{E_SYM}$$

$$\frac{\begin{array}{c} \Gamma; \Delta \models a \equiv a_1 : A/R \\ \Gamma; \Delta \models a_1 \equiv b : A/R \end{array}}{\Gamma; \Delta \models a \equiv b : A/R} \text{E_TRANS}$$

$$\frac{\begin{array}{c} \Gamma; \Delta \models a \equiv b : A/R_1 \\ R_1 \leq R_2 \end{array}}{\Gamma; \Delta \models a \equiv b : A/R_2} \text{E_SUB}$$

$$\frac{\begin{array}{c} \Gamma \models a_1 : B/R \\ \Gamma \models a_2 : B/R \\ \models a_1 > a_2/R \end{array}}{\Gamma; \Delta \models a_1 \equiv a_2 : B/R} \text{E_BETA}$$

$$\frac{\begin{array}{c} \Gamma; \Delta \models A_1 \equiv A_2 : \star/R' \\ \Gamma, x : A_1/R; \Delta \models B_1 \equiv B_2 : \star/R' \\ \Gamma \models A_1 : \star/R' \\ \Gamma \models \Pi^\rho x : A_1/R \rightarrow B_1 : \star/R' \\ \Gamma \models \Pi^\rho x : A_2/R \rightarrow B_2 : \star/R' \end{array}}{\Gamma; \Delta \models (\Pi^\rho x : A_1/R \rightarrow B_1) \equiv (\Pi^\rho x : A_2/R \rightarrow B_2) : \star/R'} \text{E_PI_CONG}$$

$$\frac{\begin{array}{c} \Gamma, x : A_1/R; \Delta \models b_1 \equiv b_2 : B/R' \\ \Gamma \models A_1 : \star/R_0 \\ (\rho = +) \vee (x \notin \text{fv } b_1) \\ (\rho = +) \vee (x \notin \text{fv } b_2) \end{array}}{\Gamma; \Delta \models (\lambda^{R, \rho} x. b_1) \equiv (\lambda^{R, \rho} x. b_2) : (\Pi^\rho x : A_1/R \rightarrow B)/R'} \text{E_ABS_CONG}$$

$$\frac{\begin{array}{c} \Gamma; \Delta \models a_1 \equiv b_1 : (\Pi^+ x : A/R \rightarrow B)/R' \\ \Gamma; \Delta \models a_2 \equiv b_2 : A/\mathbf{param} R R' \end{array}}{\Gamma; \Delta \models a_1 a_2^{R, +} \equiv b_1 b_2^{R, +} : (B\{a_2/x\})/R'} \text{E_APP_CONG}$$

$$\begin{array}{c}
\frac{\Gamma; \Delta \models a_1 \equiv b_1 : (\Pi^- x : A/R \rightarrow B)/R' \quad \Gamma \models a : A/\mathbf{param} R R'}{\Gamma; \Delta \models a_1 \sqcap^{R,-} \equiv b_1 \sqcap^{R,-} : (B\{a/x\})/R'} \quad \text{E_IAPP_CONG} \\
\\
\frac{\Gamma; \Delta \models \Pi^\rho x : A_1/R \rightarrow B_1 \equiv \Pi^\rho x : A_2/R \rightarrow B_2 : \star/R'}{\Gamma; \Delta \models A_1 \equiv A_2 : \star/R'} \quad \text{E_PIFST} \\
\\
\frac{\Gamma; \Delta \models \Pi^\rho x : A_1/R \rightarrow B_1 \equiv \Pi^\rho x : A_2/R \rightarrow B_2 : \star/R' \quad \Gamma; \Delta \models a_1 \equiv a_2 : A_1/\mathbf{param} R R'}{\Gamma; \Delta \models B_1\{a_1/x\} \equiv B_2\{a_2/x\} : \star/R'} \quad \text{E_PISND} \\
\\
\frac{\begin{array}{l} \Gamma; \Delta \models a_1 \sim_{A_1/R} b_1 \equiv a_2 \sim_{A_2/R} b_2 \\ \Gamma, c : a_1 \sim_{A_1/R} b_1; \Delta \models A \equiv B : \star/R' \\ \Gamma \models a_1 \sim_{A_1/R} b_1 \text{ ok} \\ \Gamma \models \forall c : a_1 \sim_{A_1/R} b_1.A : \star/R' \\ \Gamma \models \forall c : a_2 \sim_{A_2/R} b_2.B : \star/R' \end{array}}{\Gamma; \Delta \models \forall c : a_1 \sim_{A_1/R} b_1.A \equiv \forall c : a_2 \sim_{A_2/R} b_2.B : \star/R'} \quad \text{E_CPI_CONG} \\
\\
\frac{\begin{array}{l} \Gamma, c : \phi_1; \Delta \models a \equiv b : B/R \\ \Gamma \models \phi_1 \text{ ok} \end{array}}{\Gamma; \Delta \models (\Lambda c.a) \equiv (\Lambda c.b) : \forall c : \phi_1.B/R} \quad \text{E_CABS_CONG} \\
\\
\frac{\begin{array}{l} \Gamma; \Delta \models a_1 \equiv b_1 : (\forall c : (a \sim_{A/R} b).B)/R' \\ \Gamma; \tilde{\Gamma} \models a \equiv b : A/R \end{array}}{\Gamma; \Delta \models a_1[\bullet] \equiv b_1[\bullet] : (B\{\bullet/c\})/R'} \quad \text{E_CAPP_CONG} \\
\\
\frac{\begin{array}{l} \Gamma; \Delta \models \forall c : (a_1 \sim_{A/R} a_2).B_1 \equiv \forall c : (a'_1 \sim_{A'/R'} a'_2).B_2 : \star/R_0 \\ \Gamma; \tilde{\Gamma} \models a_1 \equiv a_2 : A/R \\ \Gamma; \tilde{\Gamma} \models a'_1 \equiv a'_2 : A'/R' \end{array}}{\Gamma; \Delta \models B_1\{\bullet/c\} \equiv B_2\{\bullet/c\} : \star/R_0} \quad \text{E_CPI_SND} \\
\\
\frac{\begin{array}{l} \Gamma; \Delta \models a \equiv b : A/R \\ \Gamma; \Delta \models a \sim_{A/R} b \equiv a' \sim_{A'/R'} b' \end{array}}{\Gamma; \Delta \models a' \equiv b' : A'/R'} \quad \text{E_CAST} \\
\\
\frac{\begin{array}{l} \Gamma; \Delta \models a \equiv b : A/R \\ \Gamma; \tilde{\Gamma} \models A \equiv B : \star/\mathbf{Rep} \\ \Gamma \models B : \star/R_0 \end{array}}{\Gamma; \Delta \models a \equiv b : B/R} \quad \text{E_EQ_CONV} \\
\\
\frac{\Gamma; \Delta \models a \sim_{A/R_1} b \equiv a' \sim_{A'/R_1} b'}{\Gamma; \Delta \models A \equiv A' : \star/\mathbf{Rep}} \quad \text{E_ISO_SND} \\
\\
\frac{\begin{array}{l} F \text{ sig_sort} \in \Sigma_0 \\ \Gamma; \Delta \models a \equiv a' : A/R \\ \Gamma; \Delta \models b_1 \equiv b'_1 : B/R_0 \\ \Gamma; \Delta \models b_2 \equiv b'_2 : B/R_0 \end{array}}{\Gamma; \Delta \models \text{case}_R a \text{ of } F \rightarrow b_1 \parallel - \rightarrow b_2 \equiv \text{case}_R a' \text{ of } F \rightarrow b'_1 \parallel - \rightarrow b'_2 : B/R_0} \quad \text{E_PAT_CONG}
\end{array}$$

$$\begin{array}{c}
\text{Path}_{R'} a = F \\
\text{Path}_{R'} a' = F \\
\Gamma \models a : \Pi^+ x : A/R_1 \rightarrow B/R' \\
\Gamma \models b : A/\mathbf{param} R_1 R' \\
\Gamma \models a' : \Pi^+ x : A/R_1 \rightarrow B/R' \\
\Gamma \models b' : A/\mathbf{param} R_1 R' \\
\Gamma; \Delta \models a \ b^{R_1, +} \equiv a' \ b'^{R_1, +} : B\{b/x\}/R' \\
\Gamma; \tilde{\Gamma} \models B\{b/x\} \equiv B\{b'/x\} : \star/R_0 \\
\hline
\Gamma; \Delta \models a \equiv a' : \Pi^+ x : A/R_1 \rightarrow B/R' \quad \text{E_LEFTREL}
\end{array}$$

$$\begin{array}{c}
\text{Path}_{R'} a = F \\
\text{Path}_{R'} a' = F \\
\Gamma \models a : \Pi^- x : A/R_1 \rightarrow B/R' \\
\Gamma \models b : A/\mathbf{param} R_1 R' \\
\Gamma \models a' : \Pi^- x : A/R_1 \rightarrow B/R' \\
\Gamma \models b' : A/\mathbf{param} R_1 R' \\
\Gamma; \Delta \models a \ \square^{R_1, -} \equiv a' \ \square'^{R_1, -} : B\{b/x\}/R' \\
\Gamma; \tilde{\Gamma} \models B\{b/x\} \equiv B\{b'/x\} : \star/R_0 \\
\hline
\Gamma; \Delta \models a \equiv a' : \Pi^- x : A/R_1 \rightarrow B/R' \quad \text{E_LEFTIRREL}
\end{array}$$

$$\begin{array}{c}
\text{Path}_{R'} a = F \\
\text{Path}_{R'} a' = F \\
\Gamma \models a : \Pi^+ x : A/R_1 \rightarrow B/R' \\
\Gamma \models b : A/\mathbf{param} R_1 R' \\
\Gamma \models a' : \Pi^+ x : A/R_1 \rightarrow B/R' \\
\Gamma \models b' : A/\mathbf{param} R_1 R' \\
\Gamma; \Delta \models a \ b^{R_1, +} \equiv a' \ b'^{R_1, +} : B\{b/x\}/R' \\
\Gamma; \tilde{\Gamma} \models B\{b/x\} \equiv B\{b'/x\} : \star/R_0 \\
\hline
\Gamma; \Delta \models b \equiv b' : A/\mathbf{param} R_1 R' \quad \text{E_RIGHT}
\end{array}$$

$$\begin{array}{c}
\text{Path}_{R'} a = F \\
\text{Path}_{R'} a' = F \\
\Gamma \models a : \forall c : (a_1 \sim_{A/R_1} a_2). B/R' \\
\Gamma \models a' : \forall c : (a_1 \sim_{A/R_1} a_2). B/R' \\
\Gamma; \tilde{\Gamma} \models a_1 \equiv a_2 : A/R_1 \\
\Gamma; \Delta \models a[\bullet] \equiv a'[\bullet] : B\{\bullet/c\}/R' \\
\hline
\Gamma; \Delta \models a \equiv a' : \forall c : (a_1 \sim_{A/R_1} a_2). B/R' \quad \text{E_CLEFT}
\end{array}$$

$\boxed{\models \Gamma}$ context wellformedness

$$\frac{}{\models \emptyset} \quad \text{E_EMPTY}$$

$$\frac{\begin{array}{c} \models \Gamma \\ \Gamma \models A : \star/R' \\ x \notin \text{dom } \Gamma \end{array}}{\models \Gamma, x : A/R} \quad \text{E_CONSTM}$$

$$\frac{\begin{array}{c} \models \Gamma \\ \Gamma \models \phi \text{ ok} \\ c \notin \text{dom } \Gamma \end{array}}{\models \Gamma, c : \phi} \quad \text{E_CONSCo}$$

$\boxed{\models \Sigma}$ signature wellformedness

$$\frac{}{\models \emptyset} \text{SIG_EMPTY}$$

$$\frac{\begin{array}{l} \models \Sigma \\ \emptyset \models A : \star/R' \\ F \notin \text{dom } \Sigma \end{array}}{\models \Sigma \cup \{F : A\}} \text{SIG_CONSCONST}$$

$$\frac{\begin{array}{l} \models \Sigma \\ \emptyset \models a : A/R' \\ F \notin \text{dom } \Sigma \end{array}}{\models \Sigma \cup \{F \sim a : A/R'\}} \text{SIG_CONSAx}$$

$$\boxed{\Gamma \vdash \phi \text{ ok}} \quad \text{prop wellformedness}$$

$$\frac{\begin{array}{l} \Gamma \vdash a : A/R \\ \Gamma \vdash b : B/R \\ |A|_R = |B|_R \end{array}}{\Gamma \vdash a \sim_{A/R} b \text{ ok}} \text{AN_WFF}$$

$$\boxed{\Gamma \vdash a : A/R} \quad \text{typing}$$

$$\frac{\vdash \Gamma}{\Gamma \vdash \star : \star/R} \text{AN_STAR}$$

$$\frac{\begin{array}{l} \vdash \Gamma \\ x : A/R \in \Gamma \end{array}}{\Gamma \vdash x : A/R} \text{AN_VAR}$$

$$\frac{\begin{array}{l} \Gamma, x : A/R \vdash B : \star/R' \\ \Gamma \vdash A : \star/R \end{array}}{\Gamma \vdash \Pi^{\rho} x : A/R \rightarrow B : \star/R'} \text{AN_PI}$$

$$\frac{\begin{array}{l} \Gamma \vdash A : \star/R \\ \Gamma, x : A/R \vdash a : B/R' \\ (\rho = +) \vee (x \notin \text{fv } |a|_{R'}) \\ R \leq R' \end{array}}{\Gamma \vdash \lambda^{\rho} x : A/R. a : (\Pi^{\rho} x : A/R \rightarrow B)/R'} \text{AN_ABS}$$

$$\frac{\begin{array}{l} \Gamma \vdash b : (\Pi^{\rho} x : A/R \rightarrow B)/R' \\ \Gamma \vdash a : A/R \end{array}}{\Gamma \vdash b \ a^{R,\rho} : (B\{a/x\})/R'} \text{AN_APP}$$

$$\frac{\begin{array}{l} \Gamma \vdash a : A/R \\ \Gamma; \tilde{\Gamma} \vdash \gamma : A \sim_R B \\ \Gamma \vdash B : \star/R \end{array}}{\Gamma \vdash a \triangleright_R \gamma : B/R} \text{AN_CONV}$$

$$\frac{\begin{array}{l} \Gamma \vdash \phi \text{ ok} \\ \Gamma, c : \phi \vdash B : \star/R \end{array}}{\Gamma \vdash \forall c : \phi. B : \star/R} \text{AN_CPI}$$

$$\frac{\begin{array}{l} \Gamma \vdash \phi \text{ ok} \\ \Gamma, c : \phi \vdash a : B/R \end{array}}{\Gamma \vdash \Lambda c : \phi. a : (\forall c : \phi. B)/R} \text{AN_CABS}$$

$$\frac{\Gamma \vdash a_1 : (\forall c : a \sim_{A_1/R} b.B)/R' \quad \Gamma; \tilde{\Gamma} \vdash \gamma : a \sim_R b}{\Gamma \vdash a_1[\gamma] : B\{\gamma/c\}/R'} \quad \text{AN_CAPP}$$

$$\frac{\vdash \Gamma \quad F \sim a : A/R \in \Sigma_1 \quad \emptyset \vdash A : \star/R_0}{\Gamma \vdash F : A/R_1} \quad \text{AN_FAM}$$

$$\frac{R_1 \leq R_2 \quad \Gamma \vdash a : A/R_1}{\Gamma \vdash \mathbf{sub} R_1 a : A/R_2} \quad \text{AN_SUBROLE}$$

$$\boxed{\Gamma; \Delta \vdash \gamma : \phi_1 \sim \phi_2} \quad \text{coercion between props}$$

$$\frac{\Gamma; \Delta \vdash \gamma_1 : A_1 \sim_R A_2 \quad \Gamma; \Delta \vdash \gamma_2 : B_1 \sim_R B_2 \quad \Gamma \vdash A_1 \sim_{A/R} B_1 \text{ ok} \quad \Gamma \vdash A_2 \sim_{A/R} B_2 \text{ ok}}{\Gamma; \Delta \vdash (\gamma_1 \sim_A \gamma_2) : (A_1 \sim_{A/R} B_1) \sim (A_2 \sim_{A/R} B_2)} \quad \text{AN_PROPCONG}$$

$$\frac{\Gamma; \Delta \vdash \gamma : \forall c : \phi_1.A_2 \sim_R \forall c : \phi_2.B_2}{\Gamma; \Delta \vdash \mathbf{cpiFst} \gamma : \phi_1 \sim \phi_2} \quad \text{AN_CPIFST}$$

$$\frac{\Gamma; \Delta \vdash \gamma : \phi_1 \sim \phi_2}{\Gamma; \Delta \vdash \mathbf{sym} \gamma : \phi_2 \sim \phi_1} \quad \text{AN_ISOSYM}$$

$$\frac{\Gamma; \Delta \vdash \gamma : A \sim_R B \quad \Gamma \vdash a_1 \sim_{A/R} a_2 \text{ ok} \quad \Gamma \vdash a'_1 \sim_{B/R} a'_2 \text{ ok} \quad |a_1|_R = |a'_1|_R \quad |a_2|_R = |a'_2|_R}{\Gamma; \Delta \vdash \mathbf{conv} (a_1 \sim_{A/R} a_2) \sim_\gamma (a'_1 \sim_{B/R} a'_2) : (a_1 \sim_{A/R} a_2) \sim (a'_1 \sim_{B/R} a'_2)} \quad \text{AN_ISOCONV}$$

$$\boxed{\Gamma; \Delta \vdash \gamma : A \sim_R B} \quad \text{coercion between types}$$

$$\frac{\vdash \Gamma \quad c : a \sim_{A/R} b \in \Gamma \quad c \in \Delta}{\Gamma; \Delta \vdash c : a \sim_R b} \quad \text{AN_ASSN}$$

$$\frac{\Gamma \vdash a : A/R}{\Gamma; \Delta \vdash \mathbf{refl} a : a \sim_R a} \quad \text{AN_REFL}$$

$$\frac{\Gamma \vdash a : A/R \quad \Gamma \vdash b : B/R \quad |a|_R = |b|_R \quad \Gamma; \tilde{\Gamma} \vdash \gamma : A \sim_R B}{\Gamma; \Delta \vdash (a \models_\gamma b) : a \sim_R b} \quad \text{AN_ERASEEQ}$$

$$\frac{\Gamma \vdash b : B/R \quad \Gamma \vdash a : A/R \quad \Gamma; \tilde{\Gamma} \vdash \gamma_1 : B \sim_R A \quad \Gamma; \Delta \vdash \gamma : b \sim_R a}{\Gamma; \Delta \vdash \mathbf{sym} \gamma : a \sim_R b} \quad \text{AN_SYM}$$

$$\begin{array}{c}
\Gamma; \Delta \vdash \gamma_1 : a \sim_R a_1 \\
\Gamma; \Delta \vdash \gamma_2 : a_1 \sim_R b \\
\Gamma \vdash a : A/R \\
\Gamma \vdash a_1 : A_1/R \\
\Gamma; \tilde{\Gamma} \vdash \gamma_3 : A \sim_R A_1 \\
\hline
\Gamma; \Delta \vdash (\gamma_1; \gamma_2) : a \sim_R b \quad \text{AN_TRANS}
\end{array}$$

$$\begin{array}{c}
\Gamma \vdash a_1 : B_0/R \\
\Gamma \vdash a_2 : B_1/R \\
|B_0|_R = |B_1|_R \\
\models |a_1|_R > |a_2|_R/R \\
\hline
\Gamma; \Delta \vdash \mathbf{red} \ a_1 \ a_2 : a_1 \sim_R a_2 \quad \text{AN_BETA}
\end{array}$$

$$\begin{array}{c}
\Gamma; \Delta \vdash \gamma_1 : A_1 \sim_{R'} A_2 \\
\Gamma, x : A_1/R; \Delta \vdash \gamma_2 : B_1 \sim_{R'} B_2 \\
B_3 = B_2\{x \triangleright_{R'} \mathbf{sym} \ \gamma_1/x\} \\
\Gamma \vdash \Pi^\rho x : A_1/R \rightarrow B_1 : \star/R' \\
\Gamma \vdash \Pi^\rho x : A_1/R \rightarrow B_2 : \star/R' \\
\Gamma \vdash \Pi^\rho x : A_2/R \rightarrow B_3 : \star/R' \\
R \leq R' \\
\hline
\Gamma; \Delta \vdash \Pi^{R,\rho} x : \gamma_1. \gamma_2 : (\Pi^\rho x : A_1/R \rightarrow B_1) \sim_{R'} (\Pi^\rho x : A_2/R \rightarrow B_3) \quad \text{AN_PICONG}
\end{array}$$

$$\begin{array}{c}
\Gamma; \Delta \vdash \gamma_1 : A_1 \sim_R A_2 \\
\Gamma, x : A_1/R; \Delta \vdash \gamma_2 : b_1 \sim_{R'} b_2 \\
b_3 = b_2\{x \triangleright_{R'} \mathbf{sym} \ \gamma_1/x\} \\
\Gamma \vdash A_1 : \star/R \\
\Gamma \vdash A_2 : \star/R \\
(\rho = +) \vee (x \notin \mathbf{fv} \ |b_1|_{R'}) \\
(\rho = +) \vee (x \notin \mathbf{fv} \ |b_3|_{R'}) \\
\Gamma \vdash (\lambda^\rho x : A_1/R. b_2) : B/R' \\
R \leq R' \\
\hline
\Gamma; \Delta \vdash (\lambda^{R,\rho} x : \gamma_1. \gamma_2) : (\lambda^\rho x : A_1/R. b_1) \sim_{R'} (\lambda^\rho x : A_2/R. b_3) \quad \text{AN_ABSCONG}
\end{array}$$

$$\begin{array}{c}
\Gamma; \Delta \vdash \gamma_1 : a_1 \sim_{R'} b_1 \\
\Gamma; \Delta \vdash \gamma_2 : a_2 \sim_R b_2 \\
\Gamma \vdash a_1 \ a_2^{R,\rho} : A/R' \\
\Gamma \vdash b_1 \ b_2^{R,\rho} : B/R' \\
\Gamma; \tilde{\Gamma} \vdash \gamma_3 : A \sim_{R'} B \\
\hline
\Gamma; \Delta \vdash \gamma_1 \ \gamma_2^{R,\rho} : a_1 \ a_2^{R,\rho} \sim_{R'} b_1 \ b_2^{R,\rho} \quad \text{AN_APPCONG}
\end{array}$$

$$\begin{array}{c}
\Gamma; \Delta \vdash \gamma : \Pi^\rho x : A_1/R \rightarrow B_1 \sim_{R'} \Pi^\rho x : A_2/R \rightarrow B_2 \\
\hline
\Gamma; \Delta \vdash \mathbf{piFst} \ \gamma : A_1 \sim_R A_2 \quad \text{AN_PIFST}
\end{array}$$

$$\begin{array}{c}
\Gamma; \Delta \vdash \gamma_1 : \Pi^\rho x : A_1/R \rightarrow B_1 \sim_{R'} \Pi^\rho x : A_2/R \rightarrow B_2 \\
\Gamma; \Delta \vdash \gamma_2 : a_1 \sim_R a_2 \\
\Gamma \vdash a_1 : A_1/R \\
\Gamma \vdash a_2 : A_2/R \\
\hline
\Gamma; \Delta \vdash \gamma_1 @ \gamma_2 : B_1\{a_1/x\} \sim_{R'} B_2\{a_2/x\} \quad \text{AN_PISND}
\end{array}$$

$$\begin{array}{c}
\Gamma; \Delta \vdash \gamma_1 : a_1 \sim_{A_1/R} b_1 \sim_{A_2/R} b_2 \\
\Gamma, c : a_1 \sim_{A_1/R} b_1; \Delta \vdash \gamma_3 : B_1 \sim_{R'} B_2 \\
B_3 = B_2\{c \triangleright_{R'} \mathbf{sym} \gamma_1 / c\} \\
\Gamma \vdash \forall c : a_1 \sim_{A_1/R} b_1.B_1 : \star / R' \\
\Gamma \vdash \forall c : a_2 \sim_{A_2/R} b_2.B_3 : \star / R' \\
\Gamma \vdash \forall c : a_1 \sim_{A_1/R} b_1.B_2 : \star / R' \\
\hline
\Gamma; \Delta \vdash (\forall c : \gamma_1.\gamma_3) : (\forall c : a_1 \sim_{A_1/R} b_1.B_1) \sim_R (\forall c : a_2 \sim_{A_2/R} b_2.B_3) \quad \text{AN_CPICONG}
\end{array}$$

$$\begin{array}{c}
\Gamma; \Delta \vdash \gamma_1 : b_0 \sim_{A_1/R} b_1 \sim_{A_2/R} b_3 \\
\Gamma, c : b_0 \sim_{A_1/R} b_1; \Delta \vdash \gamma_3 : a_1 \sim_{R'} a_2 \\
a_3 = a_2\{c \triangleright_{R'} \mathbf{sym} \gamma_1 / c\} \\
\Gamma \vdash (\Lambda c : b_0 \sim_{A_1/R} b_1.a_1) : \forall c : b_0 \sim_{A_1/R} b_1.B_1 / R' \\
\Gamma \vdash (\Lambda c : b_0 \sim_{A_1/R} b_1.a_2) : B / R' \\
\Gamma \vdash (\Lambda c : b_2 \sim_{A_2/R} b_3.a_3) : \forall c : b_2 \sim_{A_2/R} b_3.B_2 / R' \\
\Gamma; \tilde{\Gamma} \vdash \gamma_4 : \forall c : b_0 \sim_{A_1/R} b_1.B_1 \sim_{R'} \forall c : \phi_2.B_2 \\
\hline
\Gamma; \Delta \vdash (\lambda c : \gamma_1.\gamma_3 @ \gamma_4) : (\Lambda c : b_0 \sim_{A_1/R} b_1.a_1) \sim_{R'} (\Lambda c : b_2 \sim_{A_2/R} b_3.a_3) \quad \text{AN_CABSCONG}
\end{array}$$

$$\begin{array}{c}
\Gamma; \Delta \vdash \gamma_1 : a_1 \sim_R b_1 \\
\Gamma; \tilde{\Gamma} \vdash \gamma_2 : a_2 \sim_{R'} b_2 \\
\Gamma; \tilde{\Gamma} \vdash \gamma_3 : a_3 \sim_{R'} b_3 \\
\Gamma \vdash a_1[\gamma_2] : A / R \\
\Gamma \vdash b_1[\gamma_3] : B / R \\
\Gamma; \tilde{\Gamma} \vdash \gamma_4 : A \sim_R B \\
\hline
\Gamma; \Delta \vdash \gamma_1(\gamma_2, \gamma_3) : a_1[\gamma_2] \sim_R b_1[\gamma_3] \quad \text{AN_CAPPCONG}
\end{array}$$

$$\begin{array}{c}
\Gamma; \Delta \vdash \gamma_1 : (\forall c_1 : a \sim_{A/R} a'.B_1) \sim_{R_0} (\forall c_2 : b \sim_{B/R'} b'.B_2) \\
\Gamma; \tilde{\Gamma} \vdash \gamma_2 : a \sim_R a' \\
\Gamma; \tilde{\Gamma} \vdash \gamma_3 : b \sim_{R'} b' \\
\hline
\Gamma; \Delta \vdash \gamma_1 @ (\gamma_2 \sim \gamma_3) : B_1\{\gamma_2 / c_1\} \sim_{R_0} B_2\{\gamma_3 / c_2\} \quad \text{AN_CPIsND}
\end{array}$$

$$\begin{array}{c}
\Gamma; \Delta \vdash \gamma_1 : a \sim_{R_1} a' \\
\Gamma; \Delta \vdash \gamma_2 : a \sim_{A/R_1} a' \sim b \sim_{B/R_1} b' \\
\hline
\Gamma; \Delta \vdash \gamma_1 \triangleright_{R_1} \gamma_2 : b \sim_{R_1} b' \quad \text{AN_CAST}
\end{array}$$

$$\begin{array}{c}
\Gamma; \Delta \vdash \gamma : (a \sim_{A/R} a') \sim (b \sim_{B/R} b') \\
\hline
\Gamma; \Delta \vdash \mathbf{isoSnd} \gamma : A \sim_R B \quad \text{AN_ISOsND}
\end{array}$$

$$\begin{array}{c}
\Gamma; \Delta \vdash \gamma : a \sim_{R_1} b \\
R_1 \leq R_2 \\
\hline
\Gamma; \Delta \vdash \mathbf{sub} \gamma : a \sim_{R_2} b \quad \text{AN_SUB}
\end{array}$$

$\boxed{\vdash \Gamma}$ context wellformedness

$$\begin{array}{c}
\overline{\vdash \emptyset} \quad \text{AN_EMPTY} \\
\\
\vdash \Gamma \\
\Gamma \vdash A : \star / R \\
x \notin \text{dom } \Gamma \\
\hline
\vdash \Gamma, x : A / R \quad \text{AN_CONSTM} \\
\\
\vdash \Gamma \\
\Gamma \vdash \phi \text{ ok} \\
c \notin \text{dom } \Gamma \\
\hline
\vdash \Gamma, c : \phi \quad \text{AN_CONSCo}
\end{array}$$

$\boxed{\vdash \Sigma}$ signature wellformedness

$$\frac{}{\vdash \emptyset} \text{AN_SIG_EMPTY}$$

$$\frac{\begin{array}{c} \vdash \Sigma \\ \emptyset \vdash A : \star / R \\ \emptyset \vdash a : A / R \\ F \notin \text{dom } \Sigma \end{array}}{\vdash \Sigma \cup \{F \sim a : A / R\}} \text{AN_SIG_CONSAx}$$

$\boxed{\Gamma \vdash a \rightsquigarrow b / R}$ single-step, weak head reduction to values for annotated language

$$\frac{\Gamma \vdash a \rightsquigarrow a' / R_1}{\Gamma \vdash a \ b^{R,\rho} \rightsquigarrow a' \ b^{R,\rho} / R_1} \text{AN_APPLEFT}$$

$$\frac{\text{Value}_R (\lambda^\rho x : A / R.w)}{\Gamma \vdash (\lambda^\rho x : A / R.w) \ a^{R,\rho} \rightsquigarrow w\{a/x\} / R} \text{AN_APPABS}$$

$$\frac{\Gamma \vdash a \rightsquigarrow a' / R}{\Gamma \vdash a[\gamma] \rightsquigarrow a'[\gamma] / R} \text{AN_CAPPLEFT}$$

$$\frac{}{\Gamma \vdash (\Lambda c : \phi.b)[\gamma] \rightsquigarrow b\{\gamma/c\} / R} \text{AN_CAPPcABS}$$

$$\frac{\begin{array}{c} \Gamma \vdash A : \star / R \\ \Gamma, x : A / R \vdash b \rightsquigarrow b' / R_1 \end{array}}{\Gamma \vdash (\lambda^- x : A / R.b) \rightsquigarrow (\lambda^- x : A / R.b') / R_1} \text{AN_ABSTERM}$$

$$\frac{F \sim a : A / R \in \Sigma_1}{\Gamma \vdash F \rightsquigarrow a / R} \text{AN_AXIOM}$$

$$\frac{\Gamma \vdash a \rightsquigarrow a' / R}{\Gamma \vdash a \triangleright_{R_1} \gamma \rightsquigarrow a' \triangleright_{R_1} \gamma / R} \text{AN_CONVTERM}$$

$$\frac{\text{Value}_R v}{\Gamma \vdash (v \triangleright_{R_2} \gamma_1) \triangleright_{R_2} \gamma_2 \rightsquigarrow v \triangleright_{R_2} (\gamma_1; \gamma_2) / R} \text{AN_COMBINE}$$

$$\frac{\begin{array}{c} \text{Value}_R v \\ \Gamma; \tilde{\Gamma} \vdash \gamma : \Pi^\rho x_1 : A_1 / R \rightarrow B_1 \sim_{R'} \Pi^\rho x_2 : A_2 / R \rightarrow B_2 \\ b' = b \triangleright_{R'} \mathbf{sym}(\mathbf{piFst} \gamma) \\ \gamma' = \gamma @ (b' \mid_{(\mathbf{piFst} \gamma)} b) \end{array}}{\Gamma \vdash (v \triangleright_{R'} \gamma) \ b^{R,\rho} \rightsquigarrow ((v \ b'^{R,\rho}) \triangleright_{R'} \gamma') / R} \text{AN_PUSH}$$

$$\frac{\begin{array}{c} \text{Value}_R v \\ \Gamma; \tilde{\Gamma} \vdash \gamma : \forall c_1 : a_1 \sim_{B_1/R} b_1.A_1 \sim_{R'} \forall c_2 : a_2 \sim_{B_2/R} b_2.A_2 \\ \gamma'_1 = \gamma_1 \triangleright_{R'} \mathbf{sym}(\mathbf{cpiFst} \gamma) \\ \gamma' = \gamma @ (\gamma'_1 \sim \gamma_1) \end{array}}{\Gamma \vdash (v \triangleright_{R'} \gamma)[\gamma_1] \rightsquigarrow ((v[\gamma'_1]) \triangleright_{R'} \gamma') / R} \text{AN_CPUSH}$$

Definition rules: 166 good 0 bad

Definition rule clauses: 529 good 0 bad