

| | |
|------------------------|--------------------|
| $tmvar, x, y, f, m, n$ | variables |
| $covar, c$ | coercion variables |
| $datacon, K$ | |
| $const, T$ | |
| $tyfam, F$ | |
| $index, i$ | indices |

| | | | | |
|--------------|-----|---|------------------------|----------------------|
| | | Fix | S | |
| | | $a \rightarrow b$ | S | |
| | | $\phi \Rightarrow A$ | S | |
| | | $ab^{R,+}$ | S | |
| | | $\lambda^R x. a$ | S | |
| | | $\lambda x : A. a$ | S | |
| | | $\forall x : A/R \rightarrow B$ | S | |
| brs | ::= | | | case branches |
| | | none | | |
| | | $K \Rightarrow a; brs$ | | |
| | | $brs\{a/x\}$ | S | |
| | | $brs\{\gamma/c\}$ | S | |
| | | (brs) | S | |
| co, γ | ::= | | | explicit coercions |
| | | • | | |
| | | c | | |
| | | red $a\ b$ | | |
| | | refl a | | |
| | | $(a \models_{\gamma} b)$ | | |
| | | sym γ | | |
| | | $\gamma_1; \gamma_2$ | | |
| | | sub γ | | |
| | | $\Pi^{R,\rho} x : \gamma_1. \gamma_2$ | bind x in γ_2 | |
| | | $\lambda^{R,\rho} x : \gamma_1. \gamma_2$ | bind x in γ_2 | |
| | | $\gamma_1 \gamma_2^{R,\rho}$ | | |
| | | piFst γ | | |
| | | cpiFst γ | | |
| | | isoSnd γ | | |
| | | $\gamma_1 @ \gamma_2$ | | |
| | | $\forall c : \gamma_1. \gamma_3$ | bind c in γ_3 | |
| | | $\lambda c : \gamma_1. \gamma_3 @ \gamma_4$ | bind c in γ_3 | |
| | | $\gamma(\gamma_1, \gamma_2)$ | | |
| | | $\gamma @ (\gamma_1 \sim \gamma_2)$ | | |
| | | $\gamma_1 \triangleright_R \gamma_2$ | | |
| | | $\gamma_1 \sim_A \gamma_2$ | | |
| | | conv $\phi_1 \sim_{\gamma} \phi_2$ | | |
| | | eta a | | |
| | | left $\gamma \gamma'$ | | |
| | | right $\gamma \gamma'$ | | |
| | | (γ) | S | |
| | | γ | S | |
| | | $\gamma\{a/x\}$ | S | |
| sig_sort | ::= | | | signature classifier |
| | | Cs A | | |

| | | | |
|------------------------------|-------|----------------------------------|--------------------|
| | | $\mathbf{Ax} \ a \ A \ R$ | |
| $sort$ | $::=$ | | binding classifier |
| | | $\mathbf{Tm} \ A \ R$ | |
| | | $\mathbf{Co} \ \phi$ | |
| $context, \ \Gamma$ | $::=$ | | contexts |
| | | \emptyset | |
| | | $\Gamma, x : A/R$ | |
| | | $\Gamma, c : \phi$ | |
| | | $\Gamma\{b/x\}$ | M |
| | | $\Gamma\{\gamma/c\}$ | M |
| | | Γ, Γ' | M |
| | | $ \Gamma $ | M |
| | | (Γ) | M |
| | | Γ | M |
| $sig, \ \Sigma$ | $::=$ | | signatures |
| | | \emptyset | |
| | | $\Sigma \cup \{T : A/R\}$ | |
| | | $\Sigma \cup \{F \sim a : A/R\}$ | |
| | | Σ_0 | M |
| | | Σ_1 | M |
| | | $ \Sigma $ | M |
| $available_props, \ \Delta$ | $::=$ | | |
| | | \emptyset | |
| | | Δ, c | |
| | | $\tilde{\Gamma}$ | M |
| | | (Δ) | M |
| $role_context, \ \Omega$ | $::=$ | | $role_contexts$ |
| | | \emptyset | |
| | | $\Omega, x : R$ | |
| | | (Ω) | M |
| | | Ω | M |
| $terminals$ | $::=$ | | |
| | | \leftrightarrow | |
| | | \Leftrightarrow | |
| | | \longrightarrow | |
| | | \mathbf{min} | |
| | | \equiv | |
| | | \forall | |
| | | \in | |
| | | \notin | |
| | | \Leftarrow | |

| | |
|-----------------------------------|-------------------------------------|
| | \Rightarrow |
| | \Rightarrow^* |
| | \rightarrow |
| | Λ |
| | \square |
| | \vdash |
| | \vdash |
| | \models |
| | \models |
| | \neq |
| | \triangleright |
| | ok |
| | - |
| | \rightsquigarrow |
| | \rightsquigarrow^* |
| | \rightsquigarrow |
| | \emptyset |
| | \circ |
| | fv |
| | dom |
| | \sim |
| | \succ |
| | |
| | • |
| | fst |
| | snd |
| | $ \Rightarrow $ |
| | $\vdash_{=}$ |
| | refl₂ |
| | ++ |
| <i>formula, ψ</i> | $::=$ |
| | <i>judgement</i> |
| | $x : A/R \in \Gamma$ |
| | $x : R \in \Omega$ |
| | $c : \phi \in \Gamma$ |
| | $T : A/R \in \Sigma$ |
| | $F \sim a : A/R \in \Sigma$ |
| | $K : T\Gamma \in \Sigma$ |
| | $x \in \Delta$ |
| | $c \in \Delta$ |
| | $c \text{ not relevant} \in \gamma$ |
| | $x \notin \text{fva}$ |
| | $x \notin \text{dom } \Gamma$ |
| | $\text{rctx_uniq}\Omega$ |

| | | |
|---------------|--|---|
| | $ \begin{array}{ l} c \notin \text{dom } \Gamma \\ T \notin \text{dom } \Sigma \\ F \notin \text{dom } \Sigma \\ a = b \\ \phi_1 = \phi_2 \\ \Gamma_1 = \Gamma_2 \\ \gamma_1 = \gamma_2 \\ \neg \psi \\ \psi_1 \wedge \psi_2 \\ \psi_1 \vee \psi_2 \\ \psi_1 \Rightarrow \psi_2 \\ (\psi) \\ \psi \\ c : (a : A \sim b : B) \in \Gamma \end{array} $ | suppress lc hypothesis generated by Ott |
| $JSubRole$ | $ \begin{array}{ l} R_1 \leq R_2 \end{array} $ | Subroling judgement |
| $JPath$ | $ \begin{array}{ l} \text{Path}_R Fa \end{array} $ | Type headed by constant |
| $JValue$ | $ \begin{array}{ l} \mathbf{CoercedValue } R A \\ \mathbf{Value}_R A \\ \mathbf{ValueType } R A \end{array} $ | Values with at most one coercion at the top values Types with head forms (erased language) |
| $Jconsistent$ | $ \begin{array}{ l} \mathbf{consistent } a b R \end{array} $ | (erased) types do not differ in their heads |
| $Jerased$ | $ \begin{array}{ l} \Omega \models \text{erased_tm } a R \end{array} $ | |
| $Jchk$ | $ \begin{array}{ l} (\rho = +) \vee (x \notin \text{fv } A) \end{array} $ | irrelevant argument check |
| $Jpar$ | $ \begin{array}{ l} \Omega \models a \Rightarrow_R b \\ \Omega \vdash a \Rightarrow_R^* b \\ \Omega \vdash a \Leftrightarrow_R b \end{array} $ | parallel reduction (implicit language) multistep parallel reduction parallel reduction to a common term |
| $Jbeta$ | $ \begin{array}{ l} \models a > b/R \\ \models a \rightsquigarrow b/R \\ \models a \rightsquigarrow^* b/R \end{array} $ | primitive reductions on erased terms single-step head reduction for implicit language multistep reduction |
| $Jett$ | $ \begin{array}{ l} \end{array} $ | |

| | | | |
|----------------|-------|---|---|
| | | $\Gamma \models \phi \text{ ok}$ | Prop wellformedness |
| | | $\Gamma \models a : A/R$ | typing |
| | | $\Gamma; \Delta \models \phi_1 \equiv \phi_2$ | prop equality |
| | | $\Gamma; \Delta \models a \equiv b : A/R$ | definitional equality |
| | | $\models \Gamma$ | context wellformedness |
| $Jsig$ | $::=$ | | |
| | | $\models \Sigma$ | signature wellformedness |
| $Jann$ | $::=$ | | |
| | | $\Gamma \vdash \phi \text{ ok}$ | prop wellformedness |
| | | $\Gamma \vdash a : A/R$ | typing |
| | | $\Gamma; \Delta \vdash \gamma : \phi_1 \sim \phi_2$ | coercion between props |
| | | $\Gamma; \Delta \vdash \gamma : A \sim_R B$ | coercion between types |
| | | $\vdash \Gamma$ | context wellformedness |
| | | $\vdash \Sigma$ | signature wellformedness |
| $Jred$ | $::=$ | | |
| | | $\Gamma \vdash a \rightsquigarrow b/R$ | single-step, weak head reduction to values for annotated lang |
| $judgement$ | $::=$ | | |
| | | $JSubRole$ | |
| | | $JPath$ | |
| | | $JValue$ | |
| | | $Jconsistent$ | |
| | | $Jerased$ | |
| | | $JChk$ | |
| | | $Jpar$ | |
| | | $Jbeta$ | |
| | | $Jett$ | |
| | | $Jsig$ | |
| | | $Jann$ | |
| | | $Jred$ | |
| $user_syntax$ | $::=$ | | |
| | | $tmvar$ | |
| | | $covar$ | |
| | | $datacon$ | |
| | | $const$ | |
| | | $tyfam$ | |
| | | $index$ | |
| | | $role$ | |
| | | $relflag$ | |
| | | $constraint$ | |
| | | tm | |
| | | brs | |
| | | co | |

\mid *sig_sort*
 \mid *sort*
 \mid *context*
 \mid *sig*
 \mid *available_props*
 \mid *role_context*
 \mid *terminals*
 \mid *formula*

$\boxed{R_1 \leq R_2}$ Subroling judgement

$$\begin{array}{c}
\overline{\mathbf{Nom} \leq \mathbf{Rep}} \quad \text{NOMREP} \\
\overline{R \leq R} \quad \text{REFL} \\
\frac{R_1 \leq R_2 \quad R_2 \leq R_3}{R_1 \leq R_3} \quad \text{TRANS}
\end{array}$$

$\boxed{\text{Path}_R Fa}$ Type headed by constant

$$\begin{array}{c}
\frac{F \sim a : A / R_1 \in \Sigma_0 \quad \neg(R_1 \leq R)}{\text{Path}_R FF} \quad \text{PATH_CONST} \\
\frac{\text{Path}_R Fa}{\text{Path}_R F(a \triangleright_{R_1}^{\rho})} \quad \text{PATH_APP} \\
\frac{\text{Path}_R Fa}{\text{Path}_R F(a[\bullet])} \quad \text{PATH_CAPP} \\
\frac{\text{Path}_R Fa}{\text{Path}_R F(a \triangleright_{R_1} \bullet)} \quad \text{PATH_CONV}
\end{array}$$

$\boxed{\mathbf{CoercedValue} R A}$ Values with at most one coercion at the top

$$\begin{array}{c}
\frac{\text{Value}_R a}{\mathbf{CoercedValue} R a} \quad \text{CV} \\
\frac{\text{Value}_R a}{\mathbf{CoercedValue} R (a \triangleright_{R_1} \bullet)} \quad \text{CC} \\
\frac{\mathbf{CoercedValue} R (a \triangleright_{R_1} \bullet) \quad \neg(R_1 \leq R_2)}{\mathbf{CoercedValue} R ((a \triangleright_{R_1} \bullet) \triangleright_{R_2} \bullet)} \quad \text{CCV}
\end{array}$$

$\boxed{\text{Value}_R A}$ values

$$\begin{array}{c}
\overline{\text{Value}_R \star} \quad \text{VALUE_STAR} \\
\overline{\text{Value}_R \Pi^\rho x : A / R_1 \rightarrow B} \quad \text{VALUE_PI} \\
\overline{\text{Value}_R \forall c : \phi. B} \quad \text{VALUE_CPI} \\
\overline{\text{Value}_R \lambda^+ x : A / R_1. a} \quad \text{VALUE_ABSR}
\end{array}$$

$$\begin{array}{c}
\frac{}{\text{Value}_R \lambda^{R_1, +} x. a} \quad \text{VALUE_UABSREL} \\
\frac{\text{CoercedValue } R \ a}{\text{Value}_R \lambda^{R_1, -} x. a} \quad \text{VALUE_UABSIRREL} \\
\frac{}{\text{Value}_R \Lambda c : \phi. a} \quad \text{VALUE_CABS} \\
\frac{}{\text{Value}_R \Lambda c. a} \quad \text{VALUE_UCABS} \\
\frac{F \sim a : A/R_1 \in \Sigma_0 \quad \neg(R_1 \leq R)}{\text{Value}_R F} \quad \text{VALUE_AX} \\
\frac{\text{Path}_R Fa \quad \text{Value}_R a}{\text{Value}_R (a \ b'^{R_1, \rho})} \quad \text{VALUE_APP} \\
\frac{\text{Path}_R Fa \quad \text{Value}_R a}{\text{Value}_R (a[\bullet])} \quad \text{VALUE_CAPP}
\end{array}$$

ValueType $R \ A$ Types with head forms (erased language)

$$\begin{array}{c}
\frac{}{\text{ValueType } R \star} \quad \text{VALUE_TYPE_STAR} \\
\frac{}{\text{ValueType } R \Pi^\rho x : A/R_1 \rightarrow B} \quad \text{VALUE_TYPE_PI} \\
\frac{}{\text{ValueType } R \forall c : \phi. B} \quad \text{VALUE_TYPE_CPI} \\
\frac{\text{Path}_R FA \quad \text{Value}_R A}{\text{ValueType } R \ A} \quad \text{VALUE_TYPE_PATH}
\end{array}$$

consistent $a \ b \ R$ (erased) types do not differ in their heads

$$\begin{array}{c}
\frac{}{\text{consistent } \star \star R} \quad \text{CONSISTENT_A_STAR} \\
\frac{}{\text{consistent } (\Pi^\rho x_1 : A_1/R \rightarrow B_1) (\Pi^\rho x_2 : A_2/R \rightarrow B_2) R'} \quad \text{CONSISTENT_A_PI} \\
\frac{}{\text{consistent } (\forall c_1 : \phi_1. A_1) (\forall c_2 : \phi_2. A_2) R} \quad \text{CONSISTENT_A_CPI} \\
\frac{\text{Path}_R F a_1 \quad \text{Path}_R F a_2}{\text{consistent } a_1 \ a_2 \ R} \quad \text{CONSISTENT_A_PATH} \\
\frac{\neg \text{ValueType } R \ b}{\text{consistent } a \ b \ R} \quad \text{CONSISTENT_A_STEP_R} \\
\frac{\neg \text{ValueType } R \ a}{\text{consistent } a \ b \ R} \quad \text{CONSISTENT_A_STEP_L}
\end{array}$$

$\Omega \models \text{erased_tm } a \ R$

$$\frac{rctx_uniq \Omega}{\Omega \models \text{erased_tm } \square \ R} \quad \text{ERASED_A_BULLET}$$

$$\begin{array}{c}
\frac{rctx_uniq\Omega}{\Omega \models erased_tm \star R} \quad \text{ERASED_A_STAR} \\
\\
\frac{rctx_uniq\Omega \quad x : R \in \Omega \quad R \leq R_1}{\Omega \models erased_tm x R_1} \quad \text{ERASED_A_VAR} \\
\\
\frac{\Omega, x : R_1 \models erased_tm a R}{\Omega \models erased_tm (\lambda^{R_1, \rho} x. a) R} \quad \text{ERASED_A_ABS} \\
\\
\frac{\Omega \models erased_tm a R \quad \Omega \models erased_tm b R_1}{\Omega \models erased_tm (a \ b^{R_1, \rho}) R} \quad \text{ERASED_A_APP} \\
\\
\frac{\Omega \models erased_tm A R_1 \quad \Omega, x : R_1 \models erased_tm B R}{\Omega \models erased_tm (\Pi^\rho x : A/R_1 \rightarrow B) R} \quad \text{ERASED_A_PI} \\
\\
\frac{\Omega \models erased_tm a R_1 \quad \Omega \models erased_tm b R_1 \quad \Omega \models erased_tm A R_1 \quad \Omega \models erased_tm B R}{\Omega \models erased_tm (\forall c : a \sim_{A/R_1} b. B) R} \quad \text{ERASED_A_CPI} \\
\\
\frac{\Omega \models erased_tm b R}{\Omega \models erased_tm (\Lambda c. b) R} \quad \text{ERASED_A_CABS} \\
\\
\frac{\Omega \models erased_tm a R}{\Omega \models erased_tm (a[\bullet]) R} \quad \text{ERASED_A_CAPP} \\
\\
\frac{rctx_uniq\Omega \quad F \sim a : A/R \in \Sigma_0}{\Omega \models erased_tm F R_1} \quad \text{ERASED_A_FAM} \\
\\
\frac{rctx_uniq\Omega}{\Omega \models erased_tm T R} \quad \text{ERASED_A_CONST} \\
\\
\frac{\Omega \models erased_tm a R}{\Omega \models erased_tm (a \triangleright_{R_1} \bullet) R} \quad \text{ERASED_A_CONV}
\end{array}$$

$\boxed{(\rho = +) \vee (x \notin \text{fv } A)}$ irrelevant argument check

$$\frac{}{(+ = +) \vee (x \notin \text{fv } A)} \quad \text{RHO_REL}$$

$$\frac{x \notin \text{fv } A}{(- = +) \vee (x \notin \text{fv } A)} \quad \text{RHO_IRRREL}$$

$\boxed{\Omega \models a \Rightarrow_R b}$ parallel reduction (implicit language)

$$\frac{\Omega \models erased_tm a R}{\Omega \models a \Rightarrow_R a} \quad \text{PAR_REFL}$$

$$\frac{\Omega \models a \Rightarrow_R (\lambda^{R_1, \rho} x. a') \quad \Omega \models b \Rightarrow_{R_1} b'}{\Omega \models a \ b^{R_1, \rho} \Rightarrow_R a' \{b'/x\}} \quad \text{PAR_BETA}$$

$$\begin{array}{c}
\frac{\Omega \models a \Rightarrow_R a' \quad \Omega \models b \Rightarrow_{R_1} b'}{\Omega \models a \ b^{R_1, \rho} \Rightarrow_R a' \ b'^{R_1, \rho}} \text{PAR_APP} \\
\\
\frac{\Omega \models a \Rightarrow_R (\Lambda c. a')}{\Omega \models a[\bullet] \Rightarrow_R a' \{ \bullet / c \}} \text{PAR_CBETA} \\
\\
\frac{\Omega \models a \Rightarrow_R a'}{\Omega \models a[\bullet] \Rightarrow_R a'[\bullet]} \text{PAR_CAPP} \\
\\
\frac{\Omega, x : R_1 \models a \Rightarrow_R a'}{\Omega \models \lambda^{R_1, \rho} x. a \Rightarrow_R \lambda^{R_1, \rho} x. a'} \text{PAR_ABS} \\
\\
\frac{\Omega \models A \Rightarrow_{R_1} A' \quad \Omega, x : R_1 \models B \Rightarrow_R B'}{\Omega \models \Pi^{\rho} x : A / R_1 \rightarrow B \Rightarrow_R \Pi^{\rho} x : A' / R_1 \rightarrow B'} \text{PAR_PI} \\
\\
\frac{\Omega \models a \Rightarrow_R a'}{\Omega \models \Lambda c. a \Rightarrow_R \Lambda c. a'} \text{PAR_CABS} \\
\\
\frac{\Omega \models A \Rightarrow_{R_1} A' \quad \Omega \models a \Rightarrow_{R_1} a' \quad \Omega \models b \Rightarrow_{R_1} b' \quad \Omega \models B \Rightarrow_R B'}{\Omega \models \forall c : a \sim_{A/R_1} b. B \Rightarrow_R \forall c : a' \sim_{A'/R_1} b'. B'} \text{PAR_CPI} \\
\\
\frac{F \sim a : A / R_1 \in \Sigma_0 \quad R_1 \leq R \quad rctx_uniq \Omega}{\Omega \models F \Rightarrow_R a} \text{PAR_AXIOM} \\
\\
\frac{\Omega \models a_1 \Rightarrow_{R_1} a_2}{\Omega \models a_1 \triangleright_R \bullet \Rightarrow_{R_1} a_2 \triangleright_R \bullet} \text{PAR_CONG} \\
\\
\frac{\Omega \models a_1 \Rightarrow_{R_1} (a_2 \triangleright_R \bullet)}{\Omega \models (a_1 \triangleright_R \bullet) \Rightarrow_{R_1} (a_2 \triangleright_R \bullet)} \text{PAR_COMBINE} \\
\\
\frac{\Omega \models a_1 \Rightarrow_{R_1} (a_2 \triangleright_R \bullet) \quad \Omega \models b_1 \Rightarrow_{R_2} b_2}{\Omega \models a_1 b_1^{R_2, +} \Rightarrow_{R_1} (a_2 (b_2 \triangleright_R \bullet)^{R_2, +}) \triangleright_R \bullet} \text{PAR_PUSH} \\
\\
\frac{\Omega \models a_1 \Rightarrow_{R_1} (a_2 \triangleright_R \bullet) \quad \Omega \models b_1 \Rightarrow_{R_2} (b_2 \triangleright_R \bullet)}{\Omega \models a_1 b_1^{R_2, +} \Rightarrow_{R_1} (a_2 (b_2 \triangleright_R \bullet)^{R_2, +}) \triangleright_R \bullet} \text{PAR_PUSHCOMBINE} \\
\\
\frac{\Omega \models a_1 \Rightarrow_{R_1} (a_2 \triangleright_R \bullet)}{\Omega \models a_1[\bullet] \Rightarrow_{R_1} (a_2[\bullet]) \triangleright_R \bullet} \text{PAR_CPUSH}
\end{array}$$

$$\boxed{\Omega \vdash a \Rightarrow_R^* b}$$

multistep parallel reduction

$$\frac{}{\Omega \vdash a \Rightarrow_R^* a} \text{MP_REFL}$$

$$\frac{\Omega \models a \Rightarrow_R b \quad \Omega \vdash b \Rightarrow_R^* a'}{\Omega \vdash a \Rightarrow_R^* a'} \text{MP_STEP}$$

$$\boxed{\Omega \vdash a \Leftrightarrow_R b}$$

parallel reduction to a common term

$$\frac{\frac{\Omega \vdash a_1 \Rightarrow_R^* b}{\Omega \vdash a_2 \Rightarrow_R^* b}}{\Omega \vdash a_1 \Leftrightarrow_R a_2} \text{ JOIN}$$

$\boxed{\models a > b/R}$ primitive reductions on erased terms

$$\frac{\text{Value}_{R_1}(\lambda^{R,\rho}x.v)}{\models (\lambda^{R,\rho}x.v) \ b^{R,\rho} > v\{b/x\}/R_1} \text{ BETA_APPABS}$$

$$\frac{}{\models (\Lambda c.a')[\bullet] > a'\{\bullet/c\}/R} \text{ BETA_CAPPCABS}$$

$$\frac{F \sim a : A/R \in \Sigma_0}{\models F > a/R} \text{ BETA_AXIOM}$$

$\boxed{\models a \rightsquigarrow b/R}$ single-step head reduction for implicit language

$$\frac{\models a \rightsquigarrow a'/R_1}{\models \lambda^{R,-}x.a \rightsquigarrow \lambda^{R,-}x.a'/R_1} \text{ E_ABSTERM}$$

$$\frac{\models a \rightsquigarrow a'/R_1}{\models a \ b^{R,\rho} \rightsquigarrow a' \ b^{R,\rho}/R_1} \text{ E_APPLEFT}$$

$$\frac{\models a \rightsquigarrow a'/R}{\models a[\bullet] \rightsquigarrow a'[\bullet]/R} \text{ E_CAPPLEFT}$$

$$\frac{\text{Value}_{R_1}(\lambda^{R,\rho}x.v)}{\models (\lambda^{R,\rho}x.v) \ a^{R,\rho} \rightsquigarrow v\{a/x\}/R_1} \text{ E_APPABS}$$

$$\frac{}{\models (\Lambda c.b)[\bullet] \rightsquigarrow b\{\bullet/c\}/R} \text{ E_CAPPCABS}$$

$$\frac{F \sim a : A/R \in \Sigma_0 \quad R \leq R_1}{\models F \rightsquigarrow a/R_1} \text{ E_AXIOM}$$

$$\frac{\models a \rightsquigarrow a'/R_1}{\models a \triangleright_R \bullet \rightsquigarrow a' \triangleright_R \bullet /R_1} \text{ E_CONG}$$

$$\frac{\text{CoercedValue } R(v \triangleright_{R_1} \bullet) \quad R_1 \leq R_2}{\models (v \triangleright_{R_1} \bullet) \triangleright_{R_2} \bullet \rightsquigarrow v \triangleright_{R_2} \bullet /R} \text{ E_COMBINE}$$

$$\frac{\text{CoercedValue } R_2(v_1 \triangleright_R \bullet)}{\models (v_1 \triangleright_R \bullet) \ b^{R_1,\rho} \rightsquigarrow (v_1 (b \triangleright_R \bullet)^{R_1,\rho}) \triangleright_R \bullet /R_2} \text{ E_PUSH}$$

$$\frac{\text{CoercedValue } R_1(v_1 \triangleright_R \bullet)}{\models (v_1 \triangleright_R \bullet)[\bullet] \rightsquigarrow (v_1[\bullet]) \triangleright_R \bullet /R_1} \text{ E_CPUSH}$$

$\boxed{\models a \rightsquigarrow^* b/R}$ multistep reduction

$$\frac{}{\models a \rightsquigarrow^* a/R} \text{ EQUAL}$$

$$\frac{\models a \rightsquigarrow b/R \quad \models b \rightsquigarrow^* a'/R}{\models a \rightsquigarrow^* a'/R} \text{ STEP}$$

$\boxed{\Gamma \models \phi \text{ ok}}$ Prop wellformedness

$$\frac{\begin{array}{c} \Gamma \models a : A/R \\ \Gamma \models b : A/R \\ \Gamma \models A : \star/R \end{array}}{\Gamma \models a \sim_{A/R} b \text{ ok}} \quad \text{E_WFF}$$

$\boxed{\Gamma \models a : A/R}$ typing

$$\frac{\begin{array}{c} R_1 \leq R_2 \\ \Gamma \models a : A/R_1 \end{array}}{\Gamma \models a : A/R_2} \quad \text{E_SUBROLE}$$

$$\frac{\vdash \Gamma}{\Gamma \models \star : \star/R} \quad \text{E_STAR}$$

$$\frac{\begin{array}{c} \vdash \Gamma \\ x : A/R \in \Gamma \end{array}}{\Gamma \models x : A/R} \quad \text{E_VAR}$$

$$\frac{\begin{array}{c} \Gamma, x : A/R \models B : \star/R' \\ \Gamma \models A : \star/R \end{array}}{\Gamma \models \Pi^\rho x : A/R \rightarrow B : \star/R'} \quad \text{E_PI}$$

$$\frac{\begin{array}{c} \Gamma, x : A/R \models a : B/R' \\ \Gamma \models A : \star/R \\ (\rho = +) \vee (x \notin \text{fv } a) \end{array}}{\Gamma \models \lambda^{R,\rho} x. a : (\Pi^\rho x : A/R \rightarrow B)/R'} \quad \text{E_ABS}$$

$$\frac{\begin{array}{c} \Gamma \models b : \Pi^+ x : A/R \rightarrow B/R' \\ \Gamma \models a : A/R \end{array}}{\Gamma \models b \ a^{R,+} : B\{a/x\}/R'} \quad \text{E_APP}$$

$$\frac{\begin{array}{c} \Gamma \models b : \Pi^- x : A/R \rightarrow B/R' \\ \Gamma \models a : A/R \end{array}}{\Gamma \models b \ \Box^{R,-} : B\{a/x\}/R'} \quad \text{E_IAPP}$$

$$\frac{\begin{array}{c} \Gamma \models a : A/R \\ \Gamma; \tilde{\Gamma} \models A \equiv B : \star/R \\ \Gamma \models B : \star/R \end{array}}{\Gamma \models a : B/R} \quad \text{E_CONV}$$

$$\frac{\begin{array}{c} \Gamma, c : \phi \models B : \star/R \\ \Gamma \models \phi \text{ ok} \end{array}}{\Gamma \models \forall c : \phi. B : \star/R} \quad \text{E_CPI}$$

$$\frac{\begin{array}{c} \Gamma, c : \phi \models a : B/R \\ \Gamma \models \phi \text{ ok} \end{array}}{\Gamma \models \Lambda c. a : \forall c : \phi. B/R} \quad \text{E_CABS}$$

$$\frac{\begin{array}{c} \Gamma \models a_1 : \forall c : (a \sim_{A/R} b). B_1/R' \\ \Gamma; \tilde{\Gamma} \models a \equiv b : A/R \end{array}}{\Gamma \models a_1[\bullet] : B_1\{\bullet/c\}/R'} \quad \text{E_CAPP}$$

$$\frac{\begin{array}{c} \vdash \Gamma \\ F \sim a : A/R \in \Sigma_0 \\ \emptyset \models A : \star/R_1 \end{array}}{\Gamma \models F : A/R_1} \quad \text{E_FAM}$$

$$\frac{\begin{array}{l} \Gamma \vdash a : A_1/R_1 \\ \Gamma; \tilde{\Gamma} \vdash A_1 \equiv A_2 : \star/R_2 \\ \Gamma \vdash A_2 : \star/R_1 \end{array}}{\Gamma \vdash a \triangleright_{R_2} \bullet : A_2/R_1} \quad \text{E_TYCAST}$$

$$\boxed{\Gamma; \Delta \vdash \phi_1 \equiv \phi_2} \quad \text{prop equality}$$

$$\frac{\begin{array}{l} \Gamma; \Delta \vdash A_1 \equiv A_2 : A/R \\ \Gamma; \Delta \vdash B_1 \equiv B_2 : A/R \end{array}}{\Gamma; \Delta \vdash A_1 \sim_{A/R} B_1 \equiv A_2 \sim_{A/R} B_2} \quad \text{E_PROPCONG}$$

$$\frac{\begin{array}{l} \Gamma; \Delta \vdash A \equiv B : \star/R \\ \Gamma \vdash A_1 \sim_{A/R} A_2 \text{ ok} \\ \Gamma \vdash A_1 \sim_{B/R} A_2 \text{ ok} \end{array}}{\Gamma; \Delta \vdash A_1 \sim_{A/R} A_2 \equiv A_1 \sim_{B/R} A_2} \quad \text{E_ISOCONV}$$

$$\frac{\Gamma; \Delta \vdash \forall c : (a_1 \sim_{A/R} a_2). B_1 \equiv \forall c : (b_1 \sim_{B/R} b_2). B_2 : \star/R'}{\Gamma; \Delta \vdash a_1 \sim_{A/R} a_2 \equiv b_1 \sim_{B/R} b_2} \quad \text{E_CPIFST}$$

$$\boxed{\Gamma; \Delta \vdash a \equiv b : A/R} \quad \text{definitional equality}$$

$$\frac{\begin{array}{l} \vdash \Gamma \\ c : (a \sim_{A/R} b) \in \Gamma \\ c \in \Delta \end{array}}{\Gamma; \Delta \vdash a \equiv b : A/R} \quad \text{E_ASSN}$$

$$\frac{\Gamma \vdash a : A/R}{\Gamma; \Delta \vdash a \equiv a : A/R} \quad \text{E_REFL}$$

$$\frac{\Gamma; \Delta \vdash b \equiv a : A/R}{\Gamma; \Delta \vdash a \equiv b : A/R} \quad \text{E_SYM}$$

$$\frac{\begin{array}{l} \Gamma; \Delta \vdash a \equiv a_1 : A/R \\ \Gamma; \Delta \vdash a_1 \equiv b : A/R \end{array}}{\Gamma; \Delta \vdash a \equiv b : A/R} \quad \text{E_TRANS}$$

$$\frac{\begin{array}{l} \Gamma; \Delta \vdash a \equiv b : A/R_1 \\ R_1 \leq R_2 \end{array}}{\Gamma; \Delta \vdash a \equiv b : A/R_2} \quad \text{E_SUB}$$

$$\frac{\begin{array}{l} \Gamma \vdash a_1 : B/R \\ \Gamma \vdash a_2 : B/R \\ \vdash a_1 > a_2/R \end{array}}{\Gamma; \Delta \vdash a_1 \equiv a_2 : B/R} \quad \text{E_BETA}$$

$$\frac{\begin{array}{l} \Gamma; \Delta \vdash A_1 \equiv A_2 : \star/R \\ \Gamma, x : A_1/R; \Delta \vdash B_1 \equiv B_2 : \star/R' \\ \Gamma \vdash A_1 : \star/R \\ \Gamma \vdash \Pi^\rho x : A_1/R \rightarrow B_1 : \star/R' \\ \Gamma \vdash \Pi^\rho x : A_2/R \rightarrow B_2 : \star/R' \end{array}}{\Gamma; \Delta \vdash (\Pi^\rho x : A_1/R \rightarrow B_1) \equiv (\Pi^\rho x : A_2/R \rightarrow B_2) : \star/R'} \quad \text{E_PICONG}$$

$$\frac{\begin{array}{l} \Gamma, x : A_1/R; \Delta \vdash b_1 \equiv b_2 : B/R' \\ \Gamma \vdash A_1 : \star/R \\ (\rho = +) \vee (x \notin \text{fv } b_1) \\ (\rho = +) \vee (x \notin \text{fv } b_2) \end{array}}{\Gamma; \Delta \vdash (\lambda^{R, \rho} x. b_1) \equiv (\lambda^{R, \rho} x. b_2) : (\Pi^\rho x : A_1/R \rightarrow B)/R'} \quad \text{E_ABSCONG}$$

$$\begin{array}{c}
\frac{\Gamma; \Delta \models a_1 \equiv b_1 : (\Pi^+ x : A/R \rightarrow B)/R' \quad \Gamma; \Delta \models a_2 \equiv b_2 : A/R}{\Gamma; \Delta \models a_1 \ a_2^{R,+} \equiv b_1 \ b_2^{R,+} : (B\{a_2/x\})/R'} \quad \text{E_APP_CONG} \\
\\
\frac{\Gamma; \Delta \models a_1 \equiv b_1 : (\Pi^- x : A/R \rightarrow B)/R' \quad \Gamma \models a : A/R}{\Gamma; \Delta \models a_1 \ \square^{R,-} \equiv b_1 \ \square^{R,-} : (B\{a/x\})/R'} \quad \text{E_IA_APP_CONG} \\
\\
\frac{\Gamma; \Delta \models \Pi^\rho x : A_1/R \rightarrow B_1 \equiv \Pi^\rho x : A_2/R \rightarrow B_2 : \star/R'}{\Gamma; \Delta \models A_1 \equiv A_2 : \star/R} \quad \text{E_PI_FST} \\
\\
\frac{\Gamma; \Delta \models \Pi^\rho x : A_1/R \rightarrow B_1 \equiv \Pi^\rho x : A_2/R \rightarrow B_2 : \star/R' \quad \Gamma; \Delta \models a_1 \equiv a_2 : A_1/R}{\Gamma; \Delta \models B_1\{a_1/x\} \equiv B_2\{a_2/x\} : \star/R'} \quad \text{E_PI_SND} \\
\\
\frac{\Gamma; \Delta \models a_1 \sim_{A_1/R} b_1 \equiv a_2 \sim_{A_2/R} b_2 \quad \Gamma, c : a_1 \sim_{A_1/R} b_1; \Delta \models A \equiv B : \star/R' \quad \Gamma \models a_1 \sim_{A_1/R} b_1 \text{ ok} \quad \Gamma \models \forall c : a_1 \sim_{A_1/R} b_1.A : \star/R' \quad \Gamma \models \forall c : a_2 \sim_{A_2/R} b_2.B : \star/R'}{\Gamma; \Delta \models \forall c : a_1 \sim_{A_1/R} b_1.A \equiv \forall c : a_2 \sim_{A_2/R} b_2.B : \star/R'} \quad \text{E_CPI_CONG} \\
\\
\frac{\Gamma, c : \phi_1; \Delta \models a \equiv b : B/R \quad \Gamma \models \phi_1 \text{ ok}}{\Gamma; \Delta \models (\Lambda c.a) \equiv (\Lambda c.b) : \forall c : \phi_1.B/R} \quad \text{E_CABS_CONG} \\
\\
\frac{\Gamma; \Delta \models a_1 \equiv b_1 : (\forall c : (a \sim_{A/R} b).B)/R' \quad \Gamma; \tilde{\Gamma} \models a \equiv b : A/R}{\Gamma; \Delta \models a_1[\bullet] \equiv b_1[\bullet] : (B\{\bullet/c\})/R'} \quad \text{E_CAPP_CONG} \\
\\
\frac{\Gamma; \Delta \models \forall c : (a_1 \sim_{A/R} a_2).B_1 \equiv \forall c : (a'_1 \sim_{A'/R'} a'_2).B_2 : \star/R_0 \quad \Gamma; \tilde{\Gamma} \models a_1 \equiv a_2 : A/R \quad \Gamma; \tilde{\Gamma} \models a'_1 \equiv a'_2 : A'/R'}{\Gamma; \Delta \models B_1\{\bullet/c\} \equiv B_2\{\bullet/c\} : \star/R_0} \quad \text{E_CPI_SND} \\
\\
\frac{\Gamma; \Delta \models a \equiv b : A/R \quad \Gamma; \Delta \models a \sim_{A/R} b \equiv a' \sim_{A'/R} b'}{\Gamma; \Delta \models a' \equiv b' : A'/R} \quad \text{E_CAST} \\
\\
\frac{\Gamma; \Delta \models a \equiv b : A/R_1 \quad \Gamma; \tilde{\Gamma} \models A \equiv B : \star/R_2 \quad R_1 \leq R_2}{\Gamma; \Delta \models a \equiv b : B/R_2} \quad \text{E_EQ_CONV} \\
\\
\frac{\Gamma; \Delta \models a \sim_{A/R} b \equiv a' \sim_{A'/R} b'}{\Gamma; \Delta \models A \equiv A' : \star/R} \quad \text{E_ISO_SND} \\
\\
\frac{\Gamma; \Delta \models a_1 \equiv a_2 : A/R_1 \quad \Gamma; \tilde{\Gamma} \models A \equiv B : \star/R_2 \quad \Gamma \models B : \star/R_1}{\Gamma; \Delta \models a_1 \triangleright_{R_2} \bullet \equiv a_2 \triangleright_{R_2} \bullet : B/R_1} \quad \text{E_CAST_CONG}
\end{array}$$

$\boxed{\models \Gamma}$ context wellformedness

$$\frac{}{\models \emptyset} \quad \text{E_EMPTY}$$

$$\frac{\begin{array}{l} \models \Gamma \\ \Gamma \models A : \star/R \\ x \notin \text{dom } \Gamma \end{array}}{\models \Gamma, x : A/R} \quad \text{E_CONSTM}$$

$$\frac{\begin{array}{l} \models \Gamma \\ \Gamma \models \phi \text{ ok} \\ c \notin \text{dom } \Gamma \end{array}}{\models \Gamma, c : \phi} \quad \text{E_CONSCo}$$

$\boxed{\models \Sigma}$ signature wellformedness

$$\frac{}{\models \emptyset} \quad \text{SIG_EMPTY}$$

$$\frac{\begin{array}{l} \models \Sigma \\ \emptyset \models A : \star/R \\ \emptyset \models a : A/R' \\ F \notin \text{dom } \Sigma \\ R' \leq R \end{array}}{\models \Sigma \cup \{F \sim a : A/R'\}} \quad \text{SIG_CONSAx}$$

$\boxed{\Gamma \vdash \phi \text{ ok}}$ prop wellformedness

$$\frac{\begin{array}{l} \Gamma \vdash a : A/R \\ \Gamma \vdash b : B/R \\ |A|R = |B|R \end{array}}{\Gamma \vdash a \sim_{A/R} b \text{ ok}} \quad \text{AN_WFF}$$

$\boxed{\Gamma \vdash a : A/R}$ typing

$$\frac{\vdash \Gamma}{\Gamma \vdash \star : \star/R} \quad \text{AN_STAR}$$

$$\frac{\begin{array}{l} \vdash \Gamma \\ x : A/R \in \Gamma \end{array}}{\Gamma \vdash x : A/R} \quad \text{AN_VAR}$$

$$\frac{\begin{array}{l} \Gamma, x : A/R \vdash B : \star/R' \\ \Gamma \vdash A : \star/R \end{array}}{\Gamma \vdash \Pi^{\rho} x : A/R \rightarrow B : \star/R'} \quad \text{AN_PI}$$

$$\frac{\begin{array}{l} \Gamma \vdash A : \star/R \\ \Gamma, x : A/R \vdash a : B/R' \\ (\rho = +) \vee (x \notin \text{fv } |a|R') \\ R \leq R' \end{array}}{\Gamma \vdash \lambda^{\rho} x : A/R. a : (\Pi^{\rho} x : A/R \rightarrow B)/R'} \quad \text{AN_ABS}$$

$$\frac{\begin{array}{l} \Gamma \vdash b : (\Pi^{\rho} x : A/R \rightarrow B)/R' \\ \Gamma \vdash a : A/R \end{array}}{\Gamma \vdash b \ a^{R,\rho} : (B\{a/x\})/R'} \quad \text{AN_APP}$$

$$\frac{\begin{array}{l} \Gamma \vdash a : A/R \\ \Gamma; \tilde{\Gamma} \vdash \gamma : A \sim_R B \\ \Gamma \vdash B : \star/R \end{array}}{\Gamma \vdash a \triangleright_R \gamma : B/R} \quad \text{AN_CONV}$$

$$\begin{array}{c}
\frac{\Gamma \vdash \phi \text{ ok} \quad \Gamma, c : \phi \vdash B : \star / R}{\Gamma \vdash \forall c : \phi. B : \star / R} \text{ AN_CPI} \\
\\
\frac{\Gamma \vdash \phi \text{ ok} \quad \Gamma, c : \phi \vdash a : B / R}{\Gamma \vdash \Lambda c : \phi. a : (\forall c : \phi. B) / R} \text{ AN_CABS} \\
\\
\frac{\Gamma \vdash a_1 : (\forall c : a \sim_{A_1/R} b. B) / R' \quad \Gamma; \tilde{\Gamma} \vdash \gamma : a \sim_R b}{\Gamma \vdash a_1[\gamma] : B\{\gamma/c\} / R'} \text{ AN_CAPP} \\
\\
\frac{\vdash \Gamma \quad F \sim a : A / R \in \Sigma_1 \quad \emptyset \vdash A : \star / R}{\Gamma \vdash F : A / R} \text{ AN_FAM} \\
\\
\frac{R_1 \leq R_2 \quad \Gamma \vdash a : A / R_1}{\Gamma \vdash \mathbf{sub} \, R_1 \, a : A / R_2} \text{ AN_SUBROLE} \\
\\
\boxed{\Gamma; \Delta \vdash \gamma : \phi_1 \sim \phi_2} \quad \text{coercion between props} \\
\\
\frac{\Gamma; \Delta \vdash \gamma_1 : A_1 \sim_R A_2 \quad \Gamma; \Delta \vdash \gamma_2 : B_1 \sim_R B_2 \quad \Gamma \vdash A_1 \sim_{A/R} B_1 \text{ ok} \quad \Gamma \vdash A_2 \sim_{A/R} B_2 \text{ ok}}{\Gamma; \Delta \vdash (\gamma_1 \sim_A \gamma_2) : (A_1 \sim_{A/R} B_1) \sim (A_2 \sim_{A/R} B_2)} \text{ AN_PROPCONG} \\
\\
\frac{\Gamma; \Delta \vdash \gamma : \forall c : \phi_1. A_2 \sim_R \forall c : \phi_2. B_2}{\Gamma; \Delta \vdash \mathbf{cpiFst} \, \gamma : \phi_1 \sim \phi_2} \text{ AN_CPIFST} \\
\\
\frac{\Gamma; \Delta \vdash \gamma : \phi_1 \sim \phi_2}{\Gamma; \Delta \vdash \mathbf{sym} \, \gamma : \phi_2 \sim \phi_1} \text{ AN_ISOSYM} \\
\\
\frac{\Gamma; \Delta \vdash \gamma : A \sim_R B \quad \Gamma \vdash a_1 \sim_{A/R} a_2 \text{ ok} \quad \Gamma \vdash a'_1 \sim_{B/R} a'_2 \text{ ok} \quad |a_1|R = |a'_1|R \quad |a_2|R = |a'_2|R}{\Gamma; \Delta \vdash \mathbf{conv} \, (a_1 \sim_{A/R} a_2) \sim_\gamma (a'_1 \sim_{B/R} a'_2) : (a_1 \sim_{A/R} a_2) \sim (a'_1 \sim_{B/R} a'_2)} \text{ AN_ISOCONV} \\
\\
\boxed{\Gamma; \Delta \vdash \gamma : A \sim_R B} \quad \text{coercion between types} \\
\\
\frac{\vdash \Gamma \quad c : a \sim_{A/R} b \in \Gamma \quad c \in \Delta}{\Gamma; \Delta \vdash c : a \sim_R b} \text{ AN_ASSN} \\
\\
\frac{\Gamma \vdash a : A / R}{\Gamma; \Delta \vdash \mathbf{refl} \, a : a \sim_R a} \text{ AN_REFL} \\
\\
\frac{\Gamma \vdash a : A / R \quad \Gamma \vdash b : B / R \quad |a|R = |b|R \quad \Gamma; \tilde{\Gamma} \vdash \gamma : A \sim_R B}{\Gamma; \Delta \vdash (a \models_\gamma b) : a \sim_R b} \text{ AN_ERASEEQ}
\end{array}$$

$$\begin{array}{c}
\frac{\Gamma \vdash b : B/R \quad \Gamma \vdash a : A/R \quad \Gamma; \tilde{\Gamma} \vdash \gamma_1 : B \sim_R A \quad \Gamma; \Delta \vdash \gamma : b \sim_R a}{\Gamma; \Delta \vdash \mathbf{sym} \gamma : a \sim_R b} \text{AN_SYM} \\
\\
\frac{\Gamma; \Delta \vdash \gamma_1 : a \sim_R a_1 \quad \Gamma; \Delta \vdash \gamma_2 : a_1 \sim_R b \quad \Gamma \vdash a : A/R \quad \Gamma \vdash a_1 : A_1/R \quad \Gamma; \tilde{\Gamma} \vdash \gamma_3 : A \sim_R A_1}{\Gamma; \Delta \vdash (\gamma_1; \gamma_2) : a \sim_R b} \text{AN_TRANS} \\
\\
\frac{\Gamma \vdash a_1 : B_0/R \quad \Gamma \vdash a_2 : B_1/R \quad |B_0|R = |B_1|R \quad \models |a_1|R > |a_2|R/R}{\Gamma; \Delta \vdash \mathbf{red} a_1 a_2 : a_1 \sim_R a_2} \text{AN_BETA} \\
\\
\frac{\Gamma; \Delta \vdash \gamma_1 : A_1 \sim_{R'} A_2 \quad \Gamma, x : A_1/R; \Delta \vdash \gamma_2 : B_1 \sim_{R'} B_2 \quad B_3 = B_2\{x \triangleright_{R'} \mathbf{sym} \gamma_1/x\} \quad \Gamma \vdash \Pi^\rho x : A_1/R \rightarrow B_1 : \star/R' \quad \Gamma \vdash \Pi^\rho x : A_1/R \rightarrow B_2 : \star/R' \quad \Gamma \vdash \Pi^\rho x : A_2/R \rightarrow B_3 : \star/R' \quad R \leq R'}{\Gamma; \Delta \vdash \Pi^{R, \rho} x : \gamma_1. \gamma_2 : (\Pi^\rho x : A_1/R \rightarrow B_1) \sim_{R'} (\Pi^\rho x : A_2/R \rightarrow B_3)} \text{AN_PICONG} \\
\\
\frac{\Gamma; \Delta \vdash \gamma_1 : A_1 \sim_R A_2 \quad \Gamma, x : A_1/R; \Delta \vdash \gamma_2 : b_1 \sim_{R'} b_2 \quad b_3 = b_2\{x \triangleright_{R'} \mathbf{sym} \gamma_1/x\} \quad \Gamma \vdash A_1 : \star/R \quad \Gamma \vdash A_2 : \star/R \quad (\rho = +) \vee (x \notin \mathbf{fv} |b_1|R') \quad (\rho = +) \vee (x \notin \mathbf{fv} |b_3|R') \quad \Gamma \vdash (\lambda^\rho x : A_1/R. b_2) : B/R' \quad R \leq R'}{\Gamma; \Delta \vdash (\lambda^{R, \rho} x : \gamma_1. \gamma_2) : (\lambda^\rho x : A_1/R. b_1) \sim_{R'} (\lambda^\rho x : A_2/R. b_3)} \text{AN_ABSCONG} \\
\\
\frac{\Gamma; \Delta \vdash \gamma_1 : a_1 \sim_{R'} b_1 \quad \Gamma; \Delta \vdash \gamma_2 : a_2 \sim_R b_2 \quad \Gamma \vdash a_1 a_2^{R, \rho} : A/R' \quad \Gamma \vdash b_1 b_2^{R, \rho} : B/R' \quad \Gamma; \tilde{\Gamma} \vdash \gamma_3 : A \sim_{R'} B}{\Gamma; \Delta \vdash \gamma_1 \gamma_2^{R, \rho} : a_1 a_2^{R, \rho} \sim_{R'} b_1 b_2^{R, \rho}} \text{AN_APPCONG} \\
\\
\frac{\Gamma; \Delta \vdash \gamma : \Pi^\rho x : A_1/R \rightarrow B_1 \sim_{R'} \Pi^\rho x : A_2/R \rightarrow B_2}{\Gamma; \Delta \vdash \mathbf{piFst} \gamma : A_1 \sim_R A_2} \text{AN_PIFST} \\
\\
\frac{\Gamma; \Delta \vdash \gamma_1 : \Pi^\rho x : A_1/R \rightarrow B_1 \sim_{R'} \Pi^\rho x : A_2/R \rightarrow B_2 \quad \Gamma; \Delta \vdash \gamma_2 : a_1 \sim_R a_2 \quad \Gamma \vdash a_1 : A_1/R \quad \Gamma \vdash a_2 : A_2/R}{\Gamma; \Delta \vdash \gamma_1 @ \gamma_2 : B_1\{a_1/x\} \sim_{R'} B_2\{a_2/x\}} \text{AN_PISND}
\end{array}$$

$$\begin{array}{c}
\Gamma; \Delta \vdash \gamma_1 : a_1 \sim_{A_1/R} b_1 \sim_{A_2/R} b_2 \\
\Gamma, c : a_1 \sim_{A_1/R} b_1; \Delta \vdash \gamma_3 : B_1 \sim_{R'} B_2 \\
B_3 = B_2 \{c \triangleright_{R'} \mathbf{sym} \gamma_1 / c\} \\
\Gamma \vdash \forall c : a_1 \sim_{A_1/R} b_1.B_1 : \star / R' \\
\Gamma \vdash \forall c : a_2 \sim_{A_2/R} b_2.B_3 : \star / R' \\
\Gamma \vdash \forall c : a_1 \sim_{A_1/R} b_1.B_2 : \star / R' \\
\hline
\Gamma; \Delta \vdash (\forall c : \gamma_1.\gamma_3) : (\forall c : a_1 \sim_{A_1/R} b_1.B_1) \sim_R (\forall c : a_2 \sim_{A_2/R} b_2.B_3) \quad \text{AN_CPICONG}
\end{array}$$

$$\begin{array}{c}
\Gamma; \Delta \vdash \gamma_1 : b_0 \sim_{A_1/R} b_1 \sim_{A_2/R} b_3 \\
\Gamma, c : b_0 \sim_{A_1/R} b_1; \Delta \vdash \gamma_3 : a_1 \sim_{R'} a_2 \\
a_3 = a_2 \{c \triangleright_{R'} \mathbf{sym} \gamma_1 / c\} \\
\Gamma \vdash (\Lambda c : b_0 \sim_{A_1/R} b_1.a_1) : \forall c : b_0 \sim_{A_1/R} b_1.B_1 / R' \\
\Gamma \vdash (\Lambda c : b_0 \sim_{A_1/R} b_1.a_2) : B / R' \\
\Gamma \vdash (\Lambda c : b_2 \sim_{A_2/R} b_3.a_3) : \forall c : b_2 \sim_{A_2/R} b_3.B_2 / R' \\
\Gamma; \tilde{\Gamma} \vdash \gamma_4 : \forall c : b_0 \sim_{A_1/R} b_1.B_1 \sim_{R'} \forall c : \phi_2.B_2 \\
\hline
\Gamma; \Delta \vdash (\lambda c : \gamma_1.\gamma_3 @ \gamma_4) : (\Lambda c : b_0 \sim_{A_1/R} b_1.a_1) \sim_{R'} (\Lambda c : b_2 \sim_{A_2/R} b_3.a_3) \quad \text{AN_CABSCONG}
\end{array}$$

$$\begin{array}{c}
\Gamma; \Delta \vdash \gamma_1 : a_1 \sim_R b_1 \\
\Gamma; \tilde{\Gamma} \vdash \gamma_2 : a_2 \sim_{R'} b_2 \\
\Gamma; \tilde{\Gamma} \vdash \gamma_3 : a_3 \sim_{R'} b_3 \\
\Gamma \vdash a_1[\gamma_2] : A / R \\
\Gamma \vdash b_1[\gamma_3] : B / R \\
\Gamma; \tilde{\Gamma} \vdash \gamma_4 : A \sim_R B \\
\hline
\Gamma; \Delta \vdash \gamma_1(\gamma_2, \gamma_3) : a_1[\gamma_2] \sim_R b_1[\gamma_3] \quad \text{AN_CAPPCONG}
\end{array}$$

$$\begin{array}{c}
\Gamma; \Delta \vdash \gamma_1 : (\forall c_1 : a \sim_{A/R} a'.B_1) \sim_{R_0} (\forall c_2 : b \sim_{B/R'} b'.B_2) \\
\Gamma; \tilde{\Gamma} \vdash \gamma_2 : a \sim_R a' \\
\Gamma; \tilde{\Gamma} \vdash \gamma_3 : b \sim_{R'} b' \\
\hline
\Gamma; \Delta \vdash \gamma_1 @ (\gamma_2 \sim \gamma_3) : B_1 \{ \gamma_2 / c_1 \} \sim_{R_0} B_2 \{ \gamma_3 / c_2 \} \quad \text{AN_CPIsND}
\end{array}$$

$$\begin{array}{c}
\Gamma; \Delta \vdash \gamma_1 : a \sim_{R_1} a' \\
\Gamma; \Delta \vdash \gamma_2 : a \sim_{A/R_1} a' \sim b \sim_{B/R_1} b' \\
\hline
\Gamma; \Delta \vdash \gamma_1 \triangleright_{R_1} \gamma_2 : b \sim_{R_1} b' \quad \text{AN_CAST}
\end{array}$$

$$\begin{array}{c}
\Gamma; \Delta \vdash \gamma : (a \sim_{A/R} a') \sim (b \sim_{B/R} b') \\
\hline
\Gamma; \Delta \vdash \mathbf{isoSnd} \gamma : A \sim_R B \quad \text{AN_ISOsND}
\end{array}$$

$$\begin{array}{c}
\Gamma; \Delta \vdash \gamma : a \sim_{R_1} b \\
R_1 \leq R_2 \\
\hline
\Gamma; \Delta \vdash \mathbf{sub} \gamma : a \sim_{R_2} b \quad \text{AN_SUB}
\end{array}$$

$\boxed{\vdash \Gamma}$ context wellformedness

$$\begin{array}{c}
\overline{\vdash \emptyset} \quad \text{AN_EMPTY} \\
\\
\vdash \Gamma \\
\Gamma \vdash A : \star / R \\
x \notin \text{dom } \Gamma \\
\hline
\vdash \Gamma, x : A / R \quad \text{AN_CONSTM} \\
\\
\vdash \Gamma \\
\Gamma \vdash \phi \text{ ok} \\
c \notin \text{dom } \Gamma \\
\hline
\vdash \Gamma, c : \phi \quad \text{AN_CONSCo}
\end{array}$$

$\boxed{\vdash \Sigma}$ signature wellformedness

$$\frac{}{\vdash \emptyset} \text{AN_SIG_EMPTY}$$

$$\frac{\begin{array}{c} \vdash \Sigma \\ \emptyset \vdash A : \star / R \\ \emptyset \vdash a : A / R \\ F \notin \text{dom } \Sigma \end{array}}{\vdash \Sigma \cup \{F \sim a : A / R\}} \text{AN_SIG_CONSAx}$$

$\boxed{\Gamma \vdash a \rightsquigarrow b / R}$ single-step, weak head reduction to values for annotated language

$$\frac{\Gamma \vdash a \rightsquigarrow a' / R_1}{\Gamma \vdash a \ b^{R,\rho} \rightsquigarrow a' \ b^{R,\rho} / R_1} \text{AN_APPLEFT}$$

$$\frac{\text{Value}_R (\lambda^\rho x : A / R.w)}{\Gamma \vdash (\lambda^\rho x : A / R.w) \ a^{R,\rho} \rightsquigarrow w\{a/x\} / R} \text{AN_APPABS}$$

$$\frac{\Gamma \vdash a \rightsquigarrow a' / R}{\Gamma \vdash a[\gamma] \rightsquigarrow a'[\gamma] / R} \text{AN_CAPPLEFT}$$

$$\frac{}{\Gamma \vdash (\Lambda c : \phi.b)[\gamma] \rightsquigarrow b\{\gamma/c\} / R} \text{AN_CAPPcABS}$$

$$\frac{\begin{array}{c} \Gamma \vdash A : \star / R \\ \Gamma, x : A / R \vdash b \rightsquigarrow b' / R_1 \end{array}}{\Gamma \vdash (\lambda^- x : A / R.b) \rightsquigarrow (\lambda^- x : A / R.b') / R_1} \text{AN_ABSTERM}$$

$$\frac{F \sim a : A / R \in \Sigma_1}{\Gamma \vdash F \rightsquigarrow a / R} \text{AN_AXIOM}$$

$$\frac{\Gamma \vdash a \rightsquigarrow a' / R}{\Gamma \vdash a \triangleright_{R_1} \gamma \rightsquigarrow a' \triangleright_{R_1} \gamma / R} \text{AN_CONVTERM}$$

$$\frac{\text{Value}_R v}{\Gamma \vdash (v \triangleright_{R_2} \gamma_1) \triangleright_{R_2} \gamma_2 \rightsquigarrow v \triangleright_{R_2} (\gamma_1; \gamma_2) / R} \text{AN_COMBINE}$$

$$\frac{\begin{array}{c} \text{Value}_R v \\ \Gamma; \tilde{\Gamma} \vdash \gamma : \Pi^\rho x_1 : A_1 / R \rightarrow B_1 \sim_{R'} \Pi^\rho x_2 : A_2 / R \rightarrow B_2 \\ b' = b \triangleright_{R'} \mathbf{sym}(\mathbf{piFst} \gamma) \\ \gamma' = \gamma @ (b' \mid_{(\mathbf{piFst} \gamma)} b) \end{array}}{\Gamma \vdash (v \triangleright_{R'} \gamma) \ b^{R,\rho} \rightsquigarrow ((v \ b'^{R,\rho}) \triangleright_{R'} \gamma') / R} \text{AN_PUSH}$$

$$\frac{\begin{array}{c} \text{Value}_R v \\ \Gamma; \tilde{\Gamma} \vdash \gamma : \forall c_1 : a_1 \sim_{B_1/R} b_1.A_1 \sim_{R'} \forall c_2 : a_2 \sim_{B_2/R} b_2.A_2 \\ \gamma'_1 = \gamma_1 \triangleright_{R'} \mathbf{sym}(\mathbf{cpiFst} \gamma) \\ \gamma' = \gamma @ (\gamma'_1 \sim \gamma_1) \end{array}}{\Gamma \vdash (v \triangleright_{R'} \gamma)[\gamma_1] \rightsquigarrow ((v[\gamma'_1]) \triangleright_{R'} \gamma') / R} \text{AN_CPUSH}$$

Definition rules: 169 good 0 bad

Definition rule clauses: 496 good 0 bad