$tmvar$, x, y, f, m, n	variables
$covar$, c	coercion variables
$datacon$, K	
$const$, T, F	
$index$, i	indices

relflag, ρ	::=			relevance flag
	|	$+$		
	|	$-$		
	|	$\mathit{app\_rho}\nu$	S	
	|	(ρ)	S	
appflag, ν	::=			applicative flag
	|	R		
	|	ρ		
role, R	::=			Role
	|	**Nom**		
	|	**Rep**		
	|	$R_1 \cap R_2$	S	
	|	$\mathbf{param}\, R_1\, R_2$	S	
	|	$\mathit{app\_role}\nu$	S	
	|	(R)	S	
constraint, ϕ	::=			props
	|	$a \sim_{A/R} b$		
	|	(ϕ)	S	
	|	$\phi\{b/x\}$	S	
	|	$\lvert\phi\rvert$	S	
	|	$a \sim_R b$	S	
tm, a, b, p, v, w, A, B, C	::=			types and kinds
	|	$\star$		
	|	x		
	|	$\lambda^\rho x\!:\!A.b$	bind x in b	
	|	$\lambda^\rho x.b$	bind x in b	
	|	$a\ b^\nu$		
	|	$\Pi^\rho x\!:\!A \to B$	bind x in B	
	|	$\Lambda c\!:\!\phi.b$	bind c in b	
	|	$\Lambda c.b$	bind c in b	
	|	$a[\gamma]$		
	|	$\forall c\!:\!\phi.B$	bind c in B	
	|	$a \triangleright_R \gamma$		
	|	F		
	|	$\square$		
	|	$\mathsf{case}_R\ a\ \mathsf{of}\ F \to b_1 \Vert_{\_} \to b_2$		
	|	K		
	|	$\mathbf{match}\, a\, \mathbf{with}\, \mathit{brs}$		
	|	$\mathbf{sub}\, R\, a$		
	|	$a\{b/x\}$	S	
	|	$a\{\gamma/c\}$	S	
	|	a	S	
	|	a	S	

	|	(a)	S	
	|	a	S	parsing precedence is hard
	|	$\|a\|_R$	S	
	|	**Int**	S	
	|	**Bool**	S	
	|	**Nat**	S	
	|	**Vec**	S	
	|	0	S	
	|	S	S	
	|	**True**	S	
	|	**Fix**	S	
	|	**Age**	S	
	|	$a \to b$	S	
	|	$\phi \Rightarrow A$	S	
	|	$a\ b$	S	
	|	$\lambda x.a$	S	
	|	$\lambda x\!:\!A.a$	S	
	|	$\forall\, x : A \to B$	S	
	|	**if** ϕ **then** a **else** b	S	
brs	::=			case branches
	|	**none**		
	|	$K \Rightarrow a; brs$		
	|	$brs\{a/x\}$	S	
	|	$brs\{\gamma/c\}$	S	
	|	(brs)	S	
$co,\ \gamma$	::=			explicit coercions
	|	$\bullet$		
	|	c		
	|	**red** $a\ b$		
	|	**refl** a		
	|	$(a \mid=\mid_{\gamma}\ b)$		
	|	**sym** γ		
	|	$\gamma_1; \gamma_2$		
	|	**sub** γ		
	|	$\Pi^{R,\rho} x\!:\!\gamma_1.\gamma_2$	bind x in γ_2	
	|	$\lambda^{R,\rho} x\!:\!\gamma_1.\gamma_2$	bind x in γ_2	
	|	$\gamma_1\ {\gamma_2}^{R,\rho}$		
	|	**piFst** γ		
	|	**cpiFst** γ		
	|	**isoSnd** γ		
	|	$\gamma_1 @ \gamma_2$		
	|	$\forall c\!:\!\gamma_1.\gamma_3$	bind c in γ_3	
	|	$\lambda c\!:\!\gamma_1.\gamma_3 @ \gamma_4$	bind c in γ_3	
	|	$\gamma(\gamma_1, \gamma_2)$		

	|	$\gamma@(\gamma_1 \sim \gamma_2)$		
	|	$\gamma_1 \triangleright_R \gamma_2$		
	|	$\gamma_1 \sim_A \gamma_2$		
	|	$\mathbf{conv}\ \phi_1 \sim_\gamma \phi_2$		
	|	$\mathbf{eta}\, a$		
	|	$\mathbf{left}\, \gamma\, \gamma'$		
	|	$\mathbf{right}\, \gamma\, \gamma'$		
	|	(γ)	S	
	|	γ	S	
	|	$\gamma\{a/x\}$	S	
$role\_context,\ \Omega$	::=			role$_{c}ontexts$
	|	$\varnothing$		
	|	$x : R$		
	|	$\Omega, x : R$		
	|	Ω, Ω'	M	
	|	Γ_{Nom}		
	|	(Ω)	M	
	|	Ω	M	
$roles,\ Rs$	::=			
	|	$\mathbf{nilR}$		
	|	R, Rs		
	|	$\mathbf{range}\, \Omega$	S	
sig_sort	::=			signature classifier
	|	$A@Rs$		
	|	$p \sim a : A/R@Rs$		
$sort$	::=			binding classifier
	|	$\mathbf{Tm}\, A$		
	|	$\mathbf{Co}\, \phi$		
$context,\ \Gamma$	::=			contexts
	|	$\varnothing$		
	|	$\Gamma, x : A$		
	|	$\Gamma, c : \phi$		
	|	$\Gamma\{b/x\}$	M	
	|	$\Gamma\{\gamma/c\}$	M	
	|	Γ, Γ'	M	
	|	$\|\Gamma\|$	M	
	|	(Γ)	M	
	|	Γ	M	
$sig,\ \Sigma$	::=			signatures
	|	$\varnothing$		
	|	$\Sigma \cup \{F : sig\_sort\}$		

	$\mid$	Σ_0	M
	$\mid$	Σ_1	M
	$\mid$	$\lvert\Sigma\rvert$	M
$available\_props,\ \Delta$	::=		
	$\mid$	$\varnothing$	
	$\mid$	Δ, c	
	$\mid$	$\widetilde{\Gamma}$	M
	$\mid$	(Δ)	M
$terminals$	::=		
	$\mid$	$\leftrightarrow$	
	$\mid$	$\Leftrightarrow$	
	$\mid$	$\longrightarrow$	
	$\mid$	min	
	$\mid$	$\equiv$	
	$\mid$	$\forall$	
	$\mid$	$\in$	
	$\mid$	$\not\in$	
	$\mid$	$\Leftarrow$	
	$\mid$	$\Rightarrow$	
	$\mid$	$\Rightarrow^*$	
	$\mid$	$\rightarrow$	
	$\mid$	Λ	
	$\mid$		
	$\mid$	$\square$	
	$\mid$	$\vdash$	
	$\mid$	$\dashv$	
	$\mid$	$\models$	
	$\mid$	$\models$	
	$\mid$	$\neq$	
	$\mid$	$\triangleright$	
	$\mid$	ok	
	$\mid$	$\_$	
	$\mid$	$\leadsto$	
	$\mid$	$\leadsto^*$	
	$\mid$	$\leadsto$	
	$\mid$	$\varnothing$	
	$\mid$	$\circ$	
	$\mid$	fv	
	$\mid$	dom	
	$\mid$	$\sim$	
	$\mid$	$\asymp$	
	$\mid$	$\mid$	
	$\mid$	$\bullet$	
	$\mid$	**fst**	

	$\mid$	**snd**	
	$\mid$	**as**	
	$\mid$	$\mid \Rightarrow \mid$	
	$\mid$	$\vdash_{=}$	
	$\mid$	$\mathbf{refl_2}$	
	$\mid$	$++$	
formula, ψ	::=		
	$\mid$	*judgement*	
	$\mid$	$x : A \in \Gamma$	
	$\mid$	$x : R \in \Omega$	
	$\mid$	$c : \phi \in \Gamma$	
	$\mid$	$F : sig\_sort \in \Sigma$	
	$\mid$	$x \in \Delta$	
	$\mid$	$c \in \Delta$	
	$\mid$	$c\,\mathbf{not\ relevant} \in \gamma$	
	$\mid$	$x \notin \mathsf{fv}\, a$	
	$\mid$	$x \notin \mathsf{dom}\, \Gamma$	
	$\mid$	$uniq\ \Gamma$	
	$\mid$	$uniq(\Omega)$	
	$\mid$	$c \notin \mathsf{dom}\, \Gamma$	
	$\mid$	$T \notin \mathsf{dom}\, \Sigma$	
	$\mid$	$F \notin \mathsf{dom}\, \Sigma$	
	$\mid$	$R_1 = R_2$	
	$\mid$	$a = b$	
	$\mid$	$\phi_1 = \phi_2$	
	$\mid$	$\Gamma_1 = \Gamma_2$	
	$\mid$	$\gamma_1 = \gamma_2$	
	$\mid$	$\neg\psi$	
	$\mid$	$\psi_1 \wedge \psi_2$	
	$\mid$	$\psi_1 \vee \psi_2$	
	$\mid$	$\psi_1 \Rightarrow \psi_2$	
	$\mid$	(ψ)	
	$\mid$	ψ	
	$\mid$	$c : (a : A \sim b : B) \in \Gamma$	
	$\mid$		suppress lc hypothesis generated by Ott
JSubRole	::=		
	$\mid$	$R_1 \leq R_2$	Subroling judgement
JPath	::=		
	$\mid$	$\mathsf{Path}\ a = F@Rs$	Type headed by constant (partial function)
JRoledPath	::=		
	$\mid$	$\mathsf{Path}_R\ a = F@Rs$	Type headed by constant (role-sensitive partial function)
JPatCtx	::=		

	|	$\Omega; \Gamma \vDash p : A$	Contexts generated by a pattern (variables b
JMatchSubst	::=		
	|	$\mathsf{match}\ a_1\ \mathsf{with}\ p \rightarrow b_1 = b_2$	match and substitute
JApplyArgs	::=		
	|	$\mathsf{apply\ args}\ a\ \mathsf{to}\ b \mapsto b'$	apply arguments of a (headed by a constant
JValue	::=		
	|	$\mathsf{Value}_R\ A$	values
JValueType	::=		
	|	$\mathsf{ValueType}_R\ A$	Types with head forms (erased language)
Jconsistent	::=		
	|	$\mathsf{consistent}_R\ a\ b$	(erased) types do not differ in their heads
Jroleing	::=		
	|	$\Omega \vDash a : R$	Roleing judgment
JChk	::=		
	|	$(\rho = +) \vee (x \notin \mathsf{fv}\ A)$	irrelevant argument check
Jpar	::=		
	|	$\Omega \vDash a \Rightarrow_R b$	parallel reduction (implicit language)
	|	$\Omega \vDash a \Rightarrow_R^* b$	multistep parallel reduction
	|	$\Omega \vDash a \Leftrightarrow_R b$	parallel reduction to a common term
Jbeta	::=		
	|	$\vDash a > b/R$	primitive reductions on erased terms
	|	$\vDash a \rightsquigarrow b/R$	single-step head reduction for implicit langu
	|	$\vDash a \rightsquigarrow^* b/R$	multistep reduction
JBranchTyping	::=		
	|	$\Gamma \vDash \mathsf{case}_R\ a : A\ \mathsf{of}\ b : B \Rightarrow C \mid C'$	Branch Typing (aligning the types of case)
Jett	::=		
	|	$\Gamma \vDash \phi\ \mathsf{ok}$	Prop wellformedness
	|	$\Gamma \vDash a : A$	typing
	|	$\Gamma; \Delta \vDash \phi_1 \equiv \phi_2$	prop equality
	|	$\Gamma; \Delta \vDash a \equiv b : A/R$	definitional equality
	|	$\vDash \Gamma$	context wellformedness
Jsig	::=		
	|	$\vDash \Sigma$	signature wellformedness
judgement	::=		

$$
\begin{array}{lll}
 & | & \mathit{JSubRole} \\
 & | & \mathit{JPath} \\
 & | & \mathit{JRoledPath} \\
 & | & \mathit{JPatCtx} \\
 & | & \mathit{JMatchSubst} \\
 & | & \mathit{JApplyArgs} \\
 & | & \mathit{JValue} \\
 & | & \mathit{JValueType} \\
 & | & \mathit{Jconsistent} \\
 & | & \mathit{Jroleing} \\
 & | & \mathit{JChk} \\
 & | & \mathit{Jpar} \\
 & | & \mathit{Jbeta} \\
 & | & \mathit{JBranchTyping} \\
 & | & \mathit{Jett} \\
 & | & \mathit{Jsig}
\end{array}
$$

$$
\begin{array}{lll}
\mathit{user\_syntax} & ::= & \\
 & | & \mathit{tmvar} \\
 & | & \mathit{covar} \\
 & | & \mathit{datacon} \\
 & | & \mathit{const} \\
 & | & \mathit{index} \\
 & | & \mathit{relflag} \\
 & | & \mathit{appflag} \\
 & | & \mathit{role} \\
 & | & \mathit{constraint} \\
 & | & \mathit{tm} \\
 & | & \mathit{brs} \\
 & | & \mathit{co} \\
 & | & \mathit{role\_context} \\
 & | & \mathit{roles} \\
 & | & \mathit{sig\_sort} \\
 & | & \mathit{sort} \\
 & | & \mathit{context} \\
 & | & \mathit{sig} \\
 & | & \mathit{available\_props} \\
 & | & \mathit{terminals} \\
 & | & \mathit{formula}
\end{array}
$$

$\boxed{R_1 \leq R_2}$ Subroling judgement

$$\frac{}{\mathbf{Nom} \leq R} \quad \textsc{NomBot}$$

$$\frac{}{R \leq \mathbf{Rep}} \quad \textsc{RepTop}$$

$$\frac{}{R \leq R} \quad \textsc{Refl}$$

$$\frac{\begin{array}{c} R_1 \leq R_2 \\ R_2 \leq R_3 \end{array}}{R_1 \leq R_3} \quad \textsc{Trans}$$

$\boxed{\mathsf{Path}\ a = F@Rs}$ Type headed by constant (partial function)

$$\frac{F : A@Rs \in \Sigma_0}{\mathsf{Path}\ F = F@Rs} \quad \textsc{Path\_AbsConst}$$

$$\frac{F : p \sim a : A/R_1@Rs \in \Sigma_0}{\mathsf{Path}\ F = F@Rs} \quad \textsc{Path\_Const}$$

$$\frac{\begin{array}{l}\mathsf{Path}\ a = F@R_1, Rs \\ app\_role \nu = R_1\end{array}}{\mathsf{Path}\ (a\ b'^{\nu}) = F@Rs} \quad \textsc{Path\_App}$$

$$\frac{\mathsf{Path}\ a = F@Rs}{\mathsf{Path}\ (a[\bullet]) = F@Rs} \quad \textsc{Path\_CApp}$$

$\boxed{\mathsf{Path}_R\ a = F@Rs}$ Type headed by constant (role-sensitive partial function)

$$\frac{F : A@Rs \in \Sigma_0}{\mathsf{Path}_R\ F = F@Rs} \quad \textsc{RoledPath\_AbsConst}$$

$$\frac{\begin{array}{l}F : p \sim a : A/R_1@Rs \in \Sigma_0 \\ \neg(R_1 \leq R)\end{array}}{\mathsf{Path}_R\ F = F@Rs} \quad \textsc{RoledPath\_Const}$$

$$\frac{\begin{array}{l}\mathsf{Path}_R\ a = F@R_1, Rs \\ app\_role \nu = R_1\end{array}}{\mathsf{Path}_R\ (a\ b'^{\nu}) = F@Rs} \quad \textsc{RoledPath\_App}$$

$$\frac{\mathsf{Path}_R\ a = F@Rs}{\mathsf{Path}_R\ (a[\bullet]) = F@Rs} \quad \textsc{RoledPath\_CApp}$$

$\boxed{\Omega; \Gamma \vDash p : A}$ Contexts generated by a pattern (variables bound by the pattern)

$$\frac{}{\varnothing; \varnothing \vDash F : A} \quad \textsc{PatCtx\_Const}$$

$$\frac{\Omega; \Gamma \vDash p : \Pi^{+} x : A' \to A}{\Omega, x : R; \Gamma, x : A' \vDash p\ x^{+} : A} \quad \textsc{PatCtx\_PiRel}$$

$$\frac{\Omega; \Gamma \vDash p : \Pi^{-} x : A' \to A}{\Omega; \Gamma, x : A' \vDash p\ \Box^{-} : A} \quad \textsc{PatCtx\_PiIrr}$$

$$\frac{\Omega; \Gamma \vDash p : \forall c : \phi . A}{\Omega; \Gamma, c : \phi \vDash p[\bullet] : A} \quad \textsc{PatCtx\_CPi}$$

$\boxed{\mathsf{match}\ a_1\ \mathsf{with}\ p \to b_1 = b_2}$ match and substitute

$$\frac{}{\mathsf{match}\ F\ \mathsf{with}\ F \to b = b} \quad \textsc{MatchSubst\_Const}$$

$$\frac{\mathsf{match}\ a_1\ \mathsf{with}\ a_2 \to b_1 = b_2}{\mathsf{match}\ (a_1\ a^{R'})\ \mathsf{with}\ (a_2\ x^{+}) \to b_1 = (b_2\{a/x\})} \quad \textsc{MatchSubst\_AppRelR}$$

$$\frac{\mathsf{match}\ a_1\ \mathsf{with}\ a_2 \to b_1 = b_2}{\mathsf{match}\ (a_1\ a^{+})\ \mathsf{with}\ (a_2\ x^{+}) \to b_1 = (b_2\{a/x\})} \quad \textsc{MatchSubst\_AppRel}$$

$$\frac{\mathsf{match}\ a_1\ \mathsf{with}\ a_2 \to b_1 = b_2}{\mathsf{match}\ (a_1\ \Box^{-})\ \mathsf{with}\ (a_2\ \Box^{-}) \to b_1 = b_2} \quad \textsc{MatchSubst\_AppIrrel}$$

$$\frac{\mathsf{match}\ a_1\ \mathsf{with}\ a_2 \to b_1 = b_2}{\mathsf{match}\ (a_1[\bullet])\ \mathsf{with}\ (a_2[\bullet]) \to b_1 = b_2} \quad \textsc{MatchSubst\_CApp}$$

$\boxed{\textsf{apply args } a \textsf{ to } b \mapsto b'}$ apply arguments of a (headed by a constant) to b

$$\frac{}{\textsf{apply args } F \textsf{ to } b \mapsto b} \quad \textsc{ApplyArgs\_Const}$$

$$\frac{\textsf{apply args } a \textsf{ to } b \mapsto b'}{\textsf{apply args } a \ a'^{\nu} \textsf{ to } b \mapsto b' \ a'^{(app\_rho\nu)}} \quad \textsc{ApplyArgs\_App}$$

$$\frac{\textsf{apply args } a \textsf{ to } b \mapsto b'}{\textsf{apply args } a[\gamma] \textsf{ to } b \mapsto b'[\gamma]} \quad \textsc{ApplyArgs\_CApp}$$

$\boxed{\textsf{Value}_R \ A}$ values

$$\frac{}{\textsf{Value}_R \ \star} \quad \textsc{Value\_Star}$$

$$\frac{}{\textsf{Value}_R \ \Pi^{\rho} x\!:\!A \to B} \quad \textsc{Value\_Pi}$$

$$\frac{}{\textsf{Value}_R \ \forall c\!:\!\phi.B} \quad \textsc{Value\_CPi}$$

$$\frac{}{\textsf{Value}_R \ \lambda^{+} x\!:\!A.a} \quad \textsc{Value\_AbsRel}$$

$$\frac{}{\textsf{Value}_R \ \lambda^{+} x.a} \quad \textsc{Value\_UAbsRel}$$

$$\frac{\textsf{Value}_R \ a}{\textsf{Value}_R \ \lambda^{-} x.a} \quad \textsc{Value\_UAbsIrrel}$$

$$\frac{}{\textsf{Value}_R \ \Lambda c\!:\!\phi.a} \quad \textsc{Value\_CAbs}$$

$$\frac{}{\textsf{Value}_R \ \Lambda c.a} \quad \textsc{Value\_UCAbs}$$

$$\frac{\textsf{Path}_R \ a = F@Rs}{\textsf{Value}_R \ a} \quad \textsc{Value\_RolePath}$$

$$\frac{\begin{array}{c}\neg(\textsf{Path}_R \ a = F@Rs) \\ \textsf{Path} \ a = F@R', Rs'\end{array}}{\textsf{Value}_R \ a} \quad \textsc{Value\_Path}$$

$\boxed{\textsf{ValueType}_R \ A}$ Types with head forms (erased language)

$$\frac{}{\textsf{ValueType}_R \ \star} \quad \textsc{value\_type\_Star}$$

$$\frac{}{\textsf{ValueType}_R \ \Pi^{\rho} x\!:\!A \to B} \quad \textsc{value\_type\_Pi}$$

$$\frac{}{\textsf{ValueType}_R \ \forall c\!:\!\phi.B} \quad \textsc{value\_type\_CPi}$$

$$\frac{\textsf{Path}_R \ a = F@Rs}{\textsf{ValueType}_R \ a} \quad \textsc{value\_type\_RoledPath}$$

$$\frac{\begin{array}{c}\neg(\textsf{Path}_R \ a = F@Rs) \\ \textsf{Path} \ a = F@R', Rs'\end{array}}{\textsf{ValueType}_R \ a} \quad \textsc{value\_type\_Path}$$

$\boxed{\textsf{consistent}_R \ a \ b}$ (erased) types do not differ in their heads

$$\frac{}{\textsf{consistent}_R \ \star \ \star} \quad \textsc{consistent\_a\_Star}$$

$$\frac{}{\mathsf{consistent}_{R'}\ (\Pi^\rho x_1\!:\!A_1 \to B_1)\ (\Pi^\rho x_2\!:\!A_2 \to B_2)}\quad \textsc{consistent\_a\_Pi}$$

$$\frac{}{\mathsf{consistent}_{R}\ (\forall c_1\!:\!\phi_1.A_1)\ (\forall c_2\!:\!\phi_2.A_2)}\quad \textsc{consistent\_a\_CPi}$$

$$\frac{\begin{array}{l}\mathsf{Path}_R\ a_1 = F@Rs\\ \mathsf{Path}_R\ a_2 = F@Rs\end{array}}{\mathsf{consistent}_{R}\ a_1\ a_2}\quad \textsc{consistent\_a\_RoledPath}$$

$$\frac{\begin{array}{l}\neg(\mathsf{Path}_R\ a = F@Rs')\\ \mathsf{Path}\ a_1 = F@R', Rs\\ \mathsf{Path}\ a_2 = F@R', Rs\end{array}}{\mathsf{consistent}_{R}\ a_1\ a_2}\quad \textsc{consistent\_a\_Path}$$

$$\frac{\neg\mathsf{ValueType}_R\ b}{\mathsf{consistent}_{R}\ a\ b}\quad \textsc{consistent\_a\_Step\_R}$$

$$\frac{\neg\mathsf{ValueType}_R\ a}{\mathsf{consistent}_{R}\ a\ b}\quad \textsc{consistent\_a\_Step\_L}$$

$\boxed{\Omega \vDash a : R}$ Roleing judgment

$$\frac{\mathit{uniq}(\Omega)}{\Omega \vDash \Box : R}\quad \textsc{role\_a\_Bullet}$$

$$\frac{\mathit{uniq}(\Omega)}{\Omega \vDash \star : R}\quad \textsc{role\_a\_Star}$$

$$\frac{\begin{array}{l}\mathit{uniq}(\Omega)\\ x : R \in \Omega\\ R \leq R_1\end{array}}{\Omega \vDash x : R_1}\quad \textsc{role\_a\_Var}$$

$$\frac{\Omega, x : \mathbf{Nom} \vDash a : R}{\Omega \vDash (\lambda^\rho x.a) : R}\quad \textsc{role\_a\_Abs}$$

$$\frac{\begin{array}{l}\Omega \vDash a : R\\ \Omega \vDash b : \mathbf{Nom}\end{array}}{\Omega \vDash (a\ b^+) : R}\quad \textsc{role\_a\_App}$$

$$\frac{\Omega \vDash a : R}{\Omega \vDash a\ \Box^- : R}\quad \textsc{role\_a\_IApp}$$

$$\frac{\begin{array}{l}\Omega \vDash a : R\\ \mathsf{Path}\ a = F@R_1, Rs\\ \Omega \vDash b : R_1\end{array}}{\Omega \vDash a\ b^{R_1} : R}\quad \textsc{role\_a\_TApp}$$

$$\frac{\begin{array}{l}\Omega \vDash A : R\\ \Omega, x : \mathbf{Nom} \vDash B : R\end{array}}{\Omega \vDash (\Pi^\rho x\!:\!A \to B) : R}\quad \textsc{role\_a\_Pi}$$

$$\frac{\begin{array}{l}\Omega \vDash a : R_1\\ \Omega \vDash b : R_1\\ \Omega \vDash A : R_0\\ \Omega \vDash B : R\end{array}}{\Omega \vDash (\forall c\!:\!a \sim_{A/R_1} b.B) : R}\quad \textsc{role\_a\_CPi}$$

$$\frac{\Omega \vDash b : R}{\Omega \vDash (\Lambda c.b) : R} \quad \text{ROLE\_A\_CABS}$$

$$\frac{\Omega \vDash a : R}{\Omega \vDash (a[\bullet]) : R} \quad \text{ROLE\_A\_CAPP}$$

$$\frac{\begin{array}{l} uniq(\Omega) \\ F : A@Rs \in \Sigma_0 \end{array}}{\Omega \vDash F : R} \quad \text{ROLE\_A\_CONST}$$

$$\frac{\begin{array}{l} uniq(\Omega) \\ F : p \sim a : A/R@Rs \in \Sigma_0 \end{array}}{\Omega \vDash F : R_1} \quad \text{ROLE\_A\_FAM}$$

$$\frac{\begin{array}{l} \Omega \vDash a : R \\ \Omega \vDash b_1 : R_1 \\ \Omega \vDash b_2 : R_1 \end{array}}{\Omega \vDash \mathsf{case}_R\ a\ \mathsf{of}\ F \to b_1 \|_{\_} \to b_2 : R_1} \quad \text{ROLE\_A\_PATTERN}$$

$\boxed{(\rho = +) \vee (x \notin \mathsf{fv}\ A)}$ irrelevant argument check

$$\frac{}{(+ = +) \vee (x \notin \mathsf{fv}\ A)} \quad \text{RHO\_REL}$$

$$\frac{x \notin \mathsf{fv} A}{(- = +) \vee (x \notin \mathsf{fv}\ A)} \quad \text{RHO\_IRRREL}$$

$\boxed{\Omega \vDash a \Rightarrow_R b}$ parallel reduction (implicit language)

$$\frac{\Omega \vDash a : R}{\Omega \vDash a \Rightarrow_R a} \quad \text{PAR\_REFL}$$

$$\frac{\begin{array}{l} \Omega \vDash a \Rightarrow_R (\lambda^\rho x.a') \\ \Omega \vDash b \Rightarrow_{app\_role\nu} b' \end{array}}{\Omega \vDash a\ b^\nu \Rightarrow_R a'\{b'/x\}} \quad \text{PAR\_BETA}$$

$$\frac{\begin{array}{l} \Omega \vDash a \Rightarrow_R a' \\ \Omega \vDash b \Rightarrow_{app\_role\nu} b' \end{array}}{\Omega \vDash a\ b^\nu \Rightarrow_R a'\ b'^\nu} \quad \text{PAR\_APP}$$

$$\frac{\Omega \vDash a \Rightarrow_R (\Lambda c.a')}{\Omega \vDash a[\bullet] \Rightarrow_R a'\{\bullet/c\}} \quad \text{PAR\_CBETA}$$

$$\frac{\Omega \vDash a \Rightarrow_R a'}{\Omega \vDash a[\bullet] \Rightarrow_R a'[\bullet]} \quad \text{PAR\_CAPP}$$

$$\frac{\Omega, x : \mathbf{Nom} \vDash a \Rightarrow_R a'}{\Omega \vDash \lambda^\rho x.a \Rightarrow_R \lambda^\rho x.a'} \quad \text{PAR\_ABS}$$

$$\frac{\begin{array}{l} \Omega \vDash A \Rightarrow_R A' \\ \Omega, x : \mathbf{Nom} \vDash B \Rightarrow_R B' \end{array}}{\Omega \vDash \Pi^\rho x\!:\!A \to B \Rightarrow_R \Pi^\rho x\!:\!A' \to B'} \quad \text{PAR\_PI}$$

$$\frac{\Omega \vDash a \Rightarrow_R a'}{\Omega \vDash \Lambda c.a \Rightarrow_R \Lambda c.a'} \quad \text{PAR\_CABS}$$

$$\frac{\begin{array}{c}\Omega \vDash A \Rightarrow_{R_0} A' \\ \Omega \vDash a \Rightarrow_{R_1} a' \\ \Omega \vDash b \Rightarrow_{R_1} b' \\ \Omega \vDash B \Rightarrow_{R} B'\end{array}}{\Omega \vDash \forall c\!:\! a \sim_{A/R_1} b.B \Rightarrow_{R} \forall c\!:\! a' \sim_{A'/R_1} b'.B'} \quad \textsc{Par\_CPi}$$

$$\frac{\begin{array}{l}F : p \sim b : A/R_1@Rs \in \Sigma_0 \\ \mathsf{match}\ a'\ \mathsf{with}\ p \rightarrow b = b' \\ R_1 \leq R \\ \mathit{uniq}(\Omega)\end{array}}{\Omega \vDash a \Rightarrow_R b'} \quad \textsc{Par\_Axiom}$$

$$\frac{\begin{array}{l}\Omega \vDash a \Rightarrow_R a' \\ \Omega \vDash b_1 \Rightarrow_{R_0} b_1' \\ \Omega \vDash b_2 \Rightarrow_{R_0} b_2'\end{array}}{\Omega \vDash (\mathsf{case}_R\ a\ \mathsf{of}\ F \rightarrow b_1 \| \_ \rightarrow b_2) \Rightarrow_{R_0} (\mathsf{case}_R\ a'\ \mathsf{of}\ F \rightarrow b_1' \| \_ \rightarrow b_2')} \quad \textsc{Par\_Pattern}$$

$$\frac{\begin{array}{l}\Omega \vDash a \Rightarrow_R a' \\ \Omega \vDash b_1 \Rightarrow_{R_0} b_1' \\ \mathsf{Path}_R\ a' = F@Rs \\ \mathsf{apply\ args}\ a'\ \mathsf{to}\ b_1' \mapsto b\end{array}}{\Omega \vDash (\mathsf{case}_R\ a\ \mathsf{of}\ F \rightarrow b_1 \| \_ \rightarrow b_2) \Rightarrow_{R_0} b[\bullet]} \quad \textsc{Par\_PatternTrue}$$

$$\frac{\begin{array}{l}\Omega \vDash a \Rightarrow_R a' \\ \Omega \vDash b_2 \Rightarrow_{R_0} b_2' \\ \mathsf{Value}_R\ a' \\ \neg(\mathsf{Path}_R\ a' = F@Rs)\end{array}}{\Omega \vDash (\mathsf{case}_R\ a\ \mathsf{of}\ F \rightarrow b_1 \| \_ \rightarrow b_2) \Rightarrow_{R_0} b_2'} \quad \textsc{Par\_PatternFalse}$$

$\boxed{\Omega \vDash a \Rightarrow^*_R b}$ multistep parallel reduction

$$\frac{}{\Omega \vDash a \Rightarrow^*_R a} \quad \textsc{MP\_Refl}$$

$$\frac{\begin{array}{c}\Omega \vDash a \Rightarrow_R b \\ \Omega \vDash b \Rightarrow^*_R a'\end{array}}{\Omega \vDash a \Rightarrow^*_R a'} \quad \textsc{MP\_Step}$$

$\boxed{\Omega \vDash a \Leftrightarrow_R b}$ parallel reduction to a common term

$$\frac{\begin{array}{c}\Omega \vDash a_1 \Rightarrow^*_R b \\ \Omega \vDash a_2 \Rightarrow^*_R b\end{array}}{\Omega \vDash a_1 \Leftrightarrow_R a_2} \quad \textsc{join}$$

$\boxed{\vDash a > b/R}$ primitive reductions on erased terms

$$\frac{\mathsf{Value}_{R_1}\ (\lambda^\rho x.v)}{\vDash (\lambda^\rho x.v)\ b^\nu > v\{b/x\}/R_1} \quad \textsc{Beta\_AppAbs}$$

$$\frac{}{\vDash (\Lambda c.a')[\bullet] > a'\{\bullet/c\}/R} \quad \textsc{Beta\_CAppCAbs}$$

$$\frac{\begin{array}{l}F : p \sim b : A/R_1@Rs \in \Sigma_0 \\ \mathsf{match}\ a\ \mathsf{with}\ p \rightarrow b = b' \\ R_1 \leq R\end{array}}{\vDash a > b'/R} \quad \textsc{Beta\_Axiom}$$

$$\frac{\begin{array}{c}\mathsf{Path}_R\ a = F@Rs\\ \mathsf{apply\ args}\ a\ \mathsf{to}\ b_1 \mapsto b_1'\end{array}}{\vDash \mathsf{case}_R\ a\ \mathsf{of}\ F \to b_1 \|\_ \to b_2 > b_1'[\bullet]/R_0}\quad \textsc{Beta\_PatternTrue}$$

$$\frac{\begin{array}{c}\mathsf{Value}_R\ a\\ \neg(\mathsf{Path}_R\ a = F@Rs)\end{array}}{\vDash \mathsf{case}_R\ a\ \mathsf{of}\ F \to b_1 \|\_ \to b_2 > b_2/R_0}\quad \textsc{Beta\_PatternFalse}$$

$\boxed{\vDash a \leadsto b/R}$ single-step head reduction for implicit language

$$\frac{\vDash a \leadsto a'/R_1}{\vDash \lambda^{-}x.a \leadsto \lambda^{-}x.a'/R_1}\quad \textsc{E\_AbsTerm}$$

$$\frac{\vDash a \leadsto a'/R_1}{\vDash a\ b^{\nu} \leadsto a'\ b^{\nu}/R_1}\quad \textsc{E\_AppLeft}$$

$$\frac{\vDash a \leadsto a'/R}{\vDash a[\bullet] \leadsto a'[\bullet]/R}\quad \textsc{E\_CAppLeft}$$

$$\frac{\vDash a \leadsto a'/R}{\vDash \mathsf{case}_R\ a\ \mathsf{of}\ F \to b_1 \|\_ \to b_2 \leadsto \mathsf{case}_R\ a'\ \mathsf{of}\ F \to b_1 \|\_ \to b_2/R_0}\quad \textsc{E\_Pattern}$$

$$\frac{\vDash a > b/R}{\vDash a \leadsto b/R}\quad \textsc{E\_Prim}$$

$\boxed{\vDash a \leadsto^{*} b/R}$ multistep reduction

$$\frac{}{\vDash a \leadsto^{*} a/R}\quad \textsc{Equal}$$

$$\frac{\begin{array}{c}\vDash a \leadsto b/R\\ \vDash b \leadsto^{*} a'/R\end{array}}{\vDash a \leadsto^{*} a'/R}\quad \textsc{Step}$$

$\boxed{\Gamma \vDash \mathsf{case}_R\ a : A\ \mathsf{of}\ b : B \Rightarrow C \mid C'}$ Branch Typing (aligning the types of case)

$$\frac{\mathit{uniq}\ \Gamma}{\Gamma \vDash \mathsf{case}_R\ a : A\ \mathsf{of}\ b : A \Rightarrow \forall c{:}(a \sim_{A/R} b).C \mid C}\quad \textsc{BranchTyping\_Base}$$

$$\frac{\Gamma, x : A \vDash \mathsf{case}_R\ a : A_1\ \mathsf{of}\ b\ x^{+} : B \Rightarrow C \mid C'}{\Gamma \vDash \mathsf{case}_R\ a : A_1\ \mathsf{of}\ b : \Pi^{+}x{:}A \to B \Rightarrow \Pi^{+}x{:}A \to C \mid C'}\quad \textsc{BranchTyping\_PiRel}$$

$$\frac{\Gamma, x : A \vDash \mathsf{case}_R\ a : A_1\ \mathsf{of}\ b\ \square^{-} : B \Rightarrow C \mid C'}{\Gamma \vDash \mathsf{case}_R\ a : A_1\ \mathsf{of}\ b : \Pi^{-}x{:}A \to B \Rightarrow \Pi^{-}x{:}A \to C \mid C'}\quad \textsc{BranchTyping\_PiIrrel}$$

$$\frac{\Gamma, c : \phi \vDash \mathsf{case}_R\ a : A\ \mathsf{of}\ b[\bullet] : B \Rightarrow C \mid C'}{\Gamma \vDash \mathsf{case}_R\ a : A\ \mathsf{of}\ b : \forall c{:}\phi.B \Rightarrow \forall c{:}\phi.C \mid C'}\quad \textsc{BranchTyping\_CPi}$$

$\boxed{\Gamma \vDash \phi\ \mathsf{ok}}$ Prop wellformedness

$$\frac{\begin{array}{c}\Gamma \vDash a : A\\ \Gamma \vDash b : A\\ \Gamma \vDash A : \star\end{array}}{\Gamma \vDash a \sim_{A/R} b\ \ \mathsf{ok}}\quad \textsc{E\_Wff}$$

$\boxed{\Gamma \vDash a : A}$ typing

$$\frac{\vDash \Gamma}{\Gamma \vDash \star : \star}\quad \textsc{E\_Star}$$

$$\frac{\begin{array}{l}\vDash \Gamma \\ x : A \in \Gamma\end{array}}{\Gamma \vDash x : A}\ \textsc{E\_Var}$$

$$\frac{\begin{array}{l}\Gamma, x : A \vDash B : \star \\ \Gamma \vDash A : \star\end{array}}{\Gamma \vDash \Pi^{\rho} x\!:\!A \to B : \star}\ \textsc{E\_Pi}$$

$$\frac{\begin{array}{l}\Gamma, x : A \vDash a : B \\ \Gamma \vDash A : \star \\ (\rho = +) \vee (x \notin \mathsf{fv}\ a)\end{array}}{\Gamma \vDash \lambda^{\rho} x.a : (\Pi^{\rho} x\!:\!A \to B)}\ \textsc{E\_Abs}$$

$$\frac{\begin{array}{l}\Gamma \vDash b : \Pi^{+} x\!:\!A \to B \\ \Gamma \vDash a : A\end{array}}{\Gamma \vDash b\ \ a^{+} : B\{a/x\}}\ \textsc{E\_App}$$

$$\frac{\begin{array}{l}\Gamma \vDash b : \Pi^{+} x\!:\!A \to B \\ \Gamma \vDash a : A\end{array}}{\Gamma \vDash b\ \ a^{R} : B\{a/x\}}\ \textsc{E\_TApp}$$

$$\frac{\begin{array}{l}\Gamma \vDash b : \Pi^{-} x\!:\!A \to B \\ \Gamma \vDash a : A\end{array}}{\Gamma \vDash b\ \Box^{-} : B\{a/x\}}\ \textsc{E\_IApp}$$

$$\frac{\begin{array}{l}\Gamma \vDash a : A \\ \Gamma; \widetilde{\Gamma} \vDash A \equiv B : \star/\mathbf{Rep} \\ \Gamma \vDash B : \star\end{array}}{\Gamma \vDash a : B}\ \textsc{E\_Conv}$$

$$\frac{\begin{array}{l}\Gamma, c : \phi \vDash B : \star \\ \Gamma \vDash \phi\ \ \mathsf{ok}\end{array}}{\Gamma \vDash \forall c\!:\!\phi.B : \star}\ \textsc{E\_CPi}$$

$$\frac{\begin{array}{l}\Gamma, c : \phi \vDash a : B \\ \Gamma \vDash \phi\ \ \mathsf{ok}\end{array}}{\Gamma \vDash \Lambda c.a : \forall c\!:\!\phi.B}\ \textsc{E\_CAbs}$$

$$\frac{\begin{array}{l}\Gamma \vDash a_1 : \forall c\!:\!(a \sim_{A/R} b).B_1 \\ \Gamma; \widetilde{\Gamma} \vDash a \equiv b : A/R\end{array}}{\Gamma \vDash a_1[\bullet] : B_1\{\bullet/c\}}\ \textsc{E\_CApp}$$

$$\frac{\begin{array}{l}\vDash \Gamma \\ F : A@Rs \in \Sigma_0 \\ \varnothing \vDash A : \star\end{array}}{\Gamma \vDash F : A}\ \textsc{E\_Const}$$

$$\frac{\begin{array}{l}\vDash \Gamma \\ F : p \sim a : A/R_1@Rs \in \Sigma_0 \\ \varnothing \vDash A : \star \\ \Omega; \Gamma' \vDash p : A\end{array}}{\Gamma \vDash F : A}\ \textsc{E\_Fam}$$

$$\frac{\begin{array}{l}\Gamma \vDash a : A \\ \Gamma \vDash F : A_1 \\ \Gamma \vDash b_1 : B \\ \Gamma \vDash b_2 : C \\ \Gamma \vDash \mathsf{case}_R\ a : A\ \mathsf{of}\ F : A_1 \Rightarrow B \mid C\end{array}}{\Gamma \vDash \mathsf{case}_R\ a\ \mathsf{of}\ F \to b_1 \|_{\_} \to b_2 : C}\ \textsc{E\_Case}$$

$\boxed{\Gamma; \Delta \vDash \phi_1 \equiv \phi_2}$ prop equality

$$\frac{\begin{array}{c}\Gamma; \Delta \vDash A_1 \equiv A_2 : A/R \\ \Gamma; \Delta \vDash B_1 \equiv B_2 : A/R\end{array}}{\Gamma; \Delta \vDash A_1 \sim_{A/R} B_1 \equiv A_2 \sim_{A/R} B_2} \quad \text{E\_PropCong}$$

$$\frac{\begin{array}{c}\Gamma; \Delta \vDash A \equiv B : \star/R_0 \\ \Gamma \vDash A_1 \sim_{A/R} A_2 \ \mathsf{ok} \\ \Gamma \vDash A_1 \sim_{B/R} A_2 \ \mathsf{ok}\end{array}}{\Gamma; \Delta \vDash A_1 \sim_{A/R} A_2 \equiv A_1 \sim_{B/R} A_2} \quad \text{E\_IsoConv}$$

$$\frac{\Gamma; \Delta \vDash \forall c\!:\!(a_1 \sim_{A/R_1} a_2).B_1 \equiv \forall c\!:\!(b_1 \sim_{B/R_2} b_2).B_2 : \star/R'}{\Gamma; \Delta \vDash a_1 \sim_{A/R_1} a_2 \equiv b_1 \sim_{B/R_2} b_2} \quad \text{E\_CPiFst}$$

$\boxed{\Gamma; \Delta \vDash a \equiv b : A/R}$ definitional equality

$$\frac{\begin{array}{l}\vDash \Gamma \\ c : (a \sim_{A/R} b) \in \Gamma \\ c \in \Delta\end{array}}{\Gamma; \Delta \vDash a \equiv b : A/R} \quad \text{E\_Assn}$$

$$\frac{\Gamma \vDash a : A}{\Gamma; \Delta \vDash a \equiv a : A/\mathbf{Nom}} \quad \text{E\_Refl}$$

$$\frac{\Gamma; \Delta \vDash b \equiv a : A/R}{\Gamma; \Delta \vDash a \equiv b : A/R} \quad \text{E\_Sym}$$

$$\frac{\begin{array}{c}\Gamma; \Delta \vDash a \equiv a_1 : A/R \\ \Gamma; \Delta \vDash a_1 \equiv b : A/R\end{array}}{\Gamma; \Delta \vDash a \equiv b : A/R} \quad \text{E\_Trans}$$

$$\frac{\begin{array}{l}\Gamma; \Delta \vDash a \equiv b : A/R_1 \\ R_1 \leq R_2\end{array}}{\Gamma; \Delta \vDash a \equiv b : A/R_2} \quad \text{E\_Sub}$$

$$\frac{\begin{array}{l}\Gamma \vDash a_1 : B \\ \Gamma \vDash a_2 : B \\ \vDash a_1 > a_2/R\end{array}}{\Gamma; \Delta \vDash a_1 \equiv a_2 : B/R} \quad \text{E\_Beta}$$

$$\frac{\begin{array}{l}\Gamma; \Delta \vDash A_1 \equiv A_2 : \star/R' \\ \Gamma, x : A_1; \Delta \vDash B_1 \equiv B_2 : \star/R' \\ \Gamma \vDash A_1 : \star \\ \Gamma \vDash \Pi^\rho x\!:\!A_1 \to B_1 : \star \\ \Gamma \vDash \Pi^\rho x\!:\!A_2 \to B_2 : \star\end{array}}{\Gamma; \Delta \vDash (\Pi^\rho x\!:\!A_1 \to B_1) \equiv (\Pi^\rho x\!:\!A_2 \to B_2) : \star/R'} \quad \text{E\_PiCong}$$

$$\frac{\begin{array}{l}\Gamma, x : A_1; \Delta \vDash b_1 \equiv b_2 : B/R' \\ \Gamma \vDash A_1 : \star \\ (\rho = +) \vee (x \notin \mathsf{fv}\ b_1) \\ (\rho = +) \vee (x \notin \mathsf{fv}\ b_2)\end{array}}{\Gamma; \Delta \vDash (\lambda^\rho x.b_1) \equiv (\lambda^\rho x.b_2) : (\Pi^\rho x\!:\!A_1 \to B)/R'} \quad \text{E\_AbsCong}$$

$$\frac{\begin{array}{l}\Gamma; \Delta \vDash a_1 \equiv b_1 : (\Pi^+ x\!:\!A \to B)/R' \\ \Gamma; \Delta \vDash a_2 \equiv b_2 : A/\mathbf{Nom}\end{array}}{\Gamma; \Delta \vDash a_1\ {a_2}^+ \equiv b_1\ {b_2}^+ : (B\{a_2/x\})/R'} \quad \text{E\_AppCong}$$

$$\frac{\begin{array}{l}\Gamma;\Delta \vDash a_1 \equiv b_1 : (\Pi^+ x\!:\!A \to B)/R' \\ \mathsf{Path}_{R'}\ a_1 = F@R, Rs \\ \Gamma;\Delta \vDash a_2 \equiv b_2 : A/\mathbf{param}\,R\,R'\end{array}}{\Gamma;\Delta \vDash a_1\ {a_2}^R \equiv b_1\ {b_2}^R : (B\{a_2/x\})/R'} \quad \text{E\_TAppCong}$$

$$\frac{\begin{array}{l}\Gamma;\Delta \vDash a_1 \equiv b_1 : (\Pi^- x\!:\!A \to B)/R' \\ \Gamma \vDash a : A\end{array}}{\Gamma;\Delta \vDash a_1\ \Box^- \equiv b_1\ \Box^- : (B\{a/x\})/R'} \quad \text{E\_IAppCong}$$

$$\frac{\Gamma;\Delta \vDash \Pi^\rho x\!:\!A_1 \to B_1 \equiv \Pi^\rho x\!:\!A_2 \to B_2 : \star/R'}{\Gamma;\Delta \vDash A_1 \equiv A_2 : \star/R'} \quad \text{E\_PiFst}$$

$$\frac{\begin{array}{l}\Gamma;\Delta \vDash \Pi^\rho x\!:\!A_1 \to B_1 \equiv \Pi^\rho x\!:\!A_2 \to B_2 : \star/R' \\ \Gamma;\Delta \vDash a_1 \equiv a_2 : A_1/R'\end{array}}{\Gamma;\Delta \vDash B_1\{a_1/x\} \equiv B_2\{a_2/x\} : \star/R'} \quad \text{E\_PiSnd}$$

$$\frac{\begin{array}{l}\Gamma;\Delta \vDash a_1 \sim_{A_1/R} b_1 \equiv a_2 \sim_{A_2/R} b_2 \\ \Gamma, c : a_1 \sim_{A_1/R} b_1;\Delta \vDash A \equiv B : \star/R' \\ \Gamma \vDash a_1 \sim_{A_1/R} b_1\ \ \mathsf{ok} \\ \Gamma \vDash \forall c\!:\!a_1 \sim_{A_1/R} b_1.A : \star \\ \Gamma \vDash \forall c\!:\!a_2 \sim_{A_2/R} b_2.B : \star\end{array}}{\Gamma;\Delta \vDash \forall c\!:\!a_1 \sim_{A_1/R} b_1.A \equiv \forall c\!:\!a_2 \sim_{A_2/R} b_2.B : \star/R'} \quad \text{E\_CPiCong}$$

$$\frac{\begin{array}{l}\Gamma, c : \phi_1;\Delta \vDash a \equiv b : B/R \\ \Gamma \vDash \phi_1\ \ \mathsf{ok}\end{array}}{\Gamma;\Delta \vDash (\Lambda c.a) \equiv (\Lambda c.b) : \forall c\!:\!\phi_1.B/R} \quad \text{E\_CAbsCong}$$

$$\frac{\begin{array}{l}\Gamma;\Delta \vDash a_1 \equiv b_1 : (\forall c\!:\!(a \sim_{A/R} b).B)/R' \\ \Gamma;\widetilde{\Gamma} \vDash a \equiv b : A/\mathbf{param}\,R\,R'\end{array}}{\Gamma;\Delta \vDash a_1[\bullet] \equiv b_1[\bullet] : (B\{\bullet/c\})/R'} \quad \text{E\_CAppCong}$$

$$\frac{\begin{array}{l}\Gamma;\Delta \vDash \forall c\!:\!(a_1 \sim_{A/R} a_2).B_1 \equiv \forall c\!:\!(a_1' \sim_{A'/R'} a_2').B_2 : \star/R_0 \\ \Gamma;\widetilde{\Gamma} \vDash a_1 \equiv a_2 : A/\mathbf{param}\,R\,R_0 \\ \Gamma;\widetilde{\Gamma} \vDash a_1' \equiv a_2' : A'/\mathbf{param}\,R'\,R_0\end{array}}{\Gamma;\Delta \vDash B_1\{\bullet/c\} \equiv B_2\{\bullet/c\} : \star/R_0} \quad \text{E\_CPiSnd}$$

$$\frac{\begin{array}{l}\Gamma;\Delta \vDash a \equiv b : A/R \\ \Gamma;\Delta \vDash a \sim_{A/R} b \equiv a' \sim_{A'/R'} b'\end{array}}{\Gamma;\Delta \vDash a' \equiv b' : A'/R'} \quad \text{E\_Cast}$$

$$\frac{\begin{array}{l}\Gamma;\Delta \vDash a \equiv b : A/R \\ \Gamma;\widetilde{\Gamma} \vDash A \equiv B : \star/\mathbf{Rep} \\ \Gamma \vDash B : \star\end{array}}{\Gamma;\Delta \vDash a \equiv b : B/R} \quad \text{E\_EqConv}$$

$$\frac{\Gamma;\Delta \vDash a \sim_{A/R_1} b \equiv a' \sim_{A'/R_1} b'}{\Gamma;\Delta \vDash A \equiv A' : \star/\mathbf{Rep}} \quad \text{E\_IsoSnd}$$

$$\frac{\begin{array}{l}\Gamma;\Delta \vDash a \equiv a' : A/R \\ \Gamma;\Delta \vDash b_1 \equiv b_1' : B/R_0 \\ \Gamma;\Delta \vDash b_2 \equiv b_2' : B/R_0\end{array}}{\Gamma;\Delta \vDash \mathsf{case}_R\ a\ \mathsf{of}\ F \to b_1 \|\_ \to b_2 \equiv \mathsf{case}_R\ a'\ \mathsf{of}\ F \to b_1' \|\_ \to b_2' : B/R_0} \quad \text{E\_PatCong}$$

$$\frac{\begin{array}{l}\mathsf{Path}_{R'}\ a = F@R, Rs\\ \mathsf{Path}_{R'}\ a' = F@R, Rs\\ \Gamma \vDash a : \Pi^{+}x\!:\!A \to B\\ \Gamma \vDash b : A\\ \Gamma \vDash a' : \Pi^{+}x\!:\!A \to B\\ \Gamma \vDash b' : A\\ \Gamma ; \Delta \vDash a\ b^{R_1} \equiv a'\ b'^{R_1} : B\{b/x\}/R'\\ \Gamma ; \widetilde{\Gamma} \vDash B\{b/x\} \equiv B\{b'/x\} : \star/R'\end{array}}{\Gamma ; \Delta \vDash a \equiv a' : \Pi^{+}x\!:\!A \to B/R'} \quad \textsc{E\_LeftRel}$$

$$\frac{\begin{array}{l}\mathsf{Path}_{R'}\ a = F@R, Rs\\ \mathsf{Path}_{R'}\ a' = F@R, Rs\\ \Gamma \vDash a : \Pi^{-}x\!:\!A \to B\\ \Gamma \vDash b : A\\ \Gamma \vDash a' : \Pi^{-}x\!:\!A \to B\\ \Gamma \vDash b' : A\\ \Gamma ; \Delta \vDash a\ \Box^{-} \equiv a'\ \Box^{-} : B\{b/x\}/R'\\ \Gamma ; \widetilde{\Gamma} \vDash B\{b/x\} \equiv B\{b'/x\} : \star/R_0\end{array}}{\Gamma ; \Delta \vDash a \equiv a' : \Pi^{-}x\!:\!A \to B/R'} \quad \textsc{E\_LeftIrrel}$$

$$\frac{\begin{array}{l}\mathsf{Path}_{R'}\ a = F@R, Rs\\ \mathsf{Path}_{R'}\ a' = F@R, Rs\\ \Gamma \vDash a : \Pi^{+}x\!:\!A \to B\\ \Gamma \vDash b : A\\ \Gamma \vDash a' : \Pi^{+}x\!:\!A \to B\\ \Gamma \vDash b' : A\\ \Gamma ; \Delta \vDash a\ b^{+} \equiv a'\ b'^{+} : B\{b/x\}/R'\\ \Gamma ; \widetilde{\Gamma} \vDash B\{b/x\} \equiv B\{b'/x\} : \star/R_0\end{array}}{\Gamma ; \Delta \vDash b \equiv b' : A/\mathbf{param}\ R_1\ R'} \quad \textsc{E\_Right}$$

$$\frac{\begin{array}{l}\mathsf{Path}_{R'}\ a = F@R, Rs\\ \mathsf{Path}_{R'}\ a' = F@R, Rs\\ \Gamma \vDash a : \forall c\!:\!(a_1 \sim_{A/R_1} a_2).B\\ \Gamma \vDash a' : \forall c\!:\!(a_1 \sim_{A/R_1} a_2).B\\ \Gamma ; \widetilde{\Gamma} \vDash a_1 \equiv a_2 : A/R'\\ \Gamma ; \Delta \vDash a[\bullet] \equiv a'[\bullet] : B\{\bullet/c\}/R'\end{array}}{\Gamma ; \Delta \vDash a \equiv a' : \forall c\!:\!(a_1 \sim_{A/R_1} a_2).B/R'} \quad \textsc{E\_CLeft}$$

$\boxed{\vDash \Gamma}$ context wellformedness

$$\frac{}{\vDash \varnothing} \quad \textsc{E\_Empty}$$

$$\frac{\begin{array}{l}\vDash \Gamma\\ \Gamma \vDash A : \star\\ x \notin \mathsf{dom}\,\Gamma\end{array}}{\vDash \Gamma, x : A} \quad \textsc{E\_ConsTm}$$

$$\frac{\begin{array}{l}\vDash \Gamma\\ \Gamma \vDash \phi\ \ \mathsf{ok}\\ c \notin \mathsf{dom}\,\Gamma\end{array}}{\vDash \Gamma, c : \phi} \quad \textsc{E\_ConsCo}$$

$\boxed{\vDash \Sigma}$ signature wellformedness

$$\frac{}{\vDash \varnothing} \quad \textsc{Sig\_Empty}$$

$$\frac{\begin{array}{l} \vDash \Sigma \\ \varnothing \vDash A : \star \\ F \notin \mathsf{dom}\, \Sigma \end{array}}{\vDash \Sigma \cup \{F : A @ Rs\}} \quad \textsc{Sig\_ConsConst}$$

$$\frac{\begin{array}{l} \vDash \Sigma \\ F \notin \mathsf{dom}\, \Sigma \\ \Omega; \Gamma \vDash p : A \\ \Gamma \vDash a : A \\ \Omega \vDash a : \mathbf{Rep} \end{array}}{\vDash \Sigma \cup \{F : \ p \sim a : A / R @ \mathbf{range}\, \Omega\}} \quad \textsc{Sig\_ConsAx}$$

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Definition rules:         143 good    0 bad
Definition rule clauses:  401 good    0 bad
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