

$tnvar, x, y, f, m, n$	variables
$covar, c$	coercion variables
$datacon, K$	
$const, T, F$	
$index, i$	indices

$relflag, \rho$	$::=$ $ \quad +$ $ \quad -$ $ \quad app_rho \nu$ S $ \quad (\rho)$ S	relevance flag
$appflag, \nu$	$::=$ $ \quad R$ $ \quad \rho$	applicative flag
$role, R$	$::=$ $ \quad \mathbf{Nom}$ $ \quad \mathbf{Rep}$ $ \quad R_1 \cap R_2$ S $ \quad \mathbf{param} R_1 R_2$ S $ \quad app_role \nu$ S $ \quad (R)$ S	Role
$constraint, \phi$	$::=$ $ \quad a \sim_{A/R} b$ $ \quad (\phi)$ S $ \quad \phi\{b/x\}$ S $ \quad \phi $ S $ \quad a \sim_R b$ S	props
$tm, a, b, p, v, w, A, B, C$	$::=$ $ \quad \star$ $ \quad x$ $ \quad \lambda^\rho x:A. b$ bind x in b $ \quad \lambda^\rho x. b$ bind x in b $ \quad a \ b^\nu$ $ \quad \Pi^\rho x:A \rightarrow B$ bind x in B $ \quad \Lambda c:\phi. b$ bind c in b $ \quad \Lambda c. b$ bind c in b $ \quad a[\gamma]$ $ \quad \forall c:\phi. B$ bind c in B $ \quad a \triangleright_R \gamma$ $ \quad F$ $ \quad \square$ $ \quad \mathbf{case}_R a \text{ of } F \rightarrow b_1 \parallel - \rightarrow b_2$ $ \quad K$ $ \quad \mathbf{match} a \text{ with } brs$ $ \quad \mathbf{sub} R a$ $ \quad a\{b/x\}$ S $ \quad a\{\gamma/c\}$ S $ \quad a\{b/x\}$ S $ \quad a\{\gamma/c\}$ S	types and kinds

		a	S	
		a	S	
		(a)	S	
		a	S	parsing precedence is hard
		$ a _R$	S	
		Int	S	
		Bool	S	
		Nat	S	
		Vec	S	
		0	S	
		S	S	
		True	S	
		Fix	S	
		Age	S	
		$a \rightarrow b$	S	
		$\phi \Rightarrow A$	S	
		$a \ b$	S	
		$\lambda x. a$	S	
		$\lambda x : A. a$	S	
		$\forall x : A \rightarrow B$	S	
		if ϕ then a else b	S	
brs	$::=$			case branches
		none		
		$K \Rightarrow a; brs$		
		$brs\{a/x\}$	S	
		$brs\{\gamma/c\}$	S	
		(brs)	S	
co, γ	$::=$			explicit coercions
		•		
		c		
		red $a \ b$		
		refl a		
		$(a \models_\gamma b)$		
		sym γ		
		$\gamma_1; \gamma_2$		
		sub γ		
		$\Pi^{R,\rho} x : \gamma_1. \gamma_2$	bind x in γ_2	
		$\lambda^{R,\rho} x : \gamma_1. \gamma_2$	bind x in γ_2	
		$\gamma_1 \ \gamma_2^{R,\rho}$		
		piFst γ		
		cpiFst γ		
		isoSnd γ		
		$\gamma_1 @ \gamma_2$		
		$\forall c : \gamma_1. \gamma_3$	bind c in γ_3	

	$\lambda c : \gamma_1. \gamma_3 @ \gamma_4$ $\gamma(\gamma_1, \gamma_2)$ $\gamma @ (\gamma_1 \sim \gamma_2)$ $\gamma_1 \triangleright_R \gamma_2$ $\gamma_1 \sim_A \gamma_2$ $\mathbf{conv} \ \phi_1 \sim_\gamma \phi_2$ $\mathbf{eta} \ a$ $\mathbf{left} \ \gamma \ \gamma'$ $\mathbf{right} \ \gamma \ \gamma'$ (γ) γ $\gamma\{a/x\}$	 S S S
$role_context, \ \Omega$	$::=$ \emptyset $x : R$ $\Omega, x : R$ Ω, Ω' Γ_{Nom} (Ω) Ω	$role_contexts$ M M M
$roles, \ Rs$	$::=$ \mathbf{nilR} R, Rs $\mathbf{range} \ \Omega$	 S
sig_sort	$::=$ $A @ Rs$ $p \sim a : A / R @ Rs$	$signature \ classifier$
$sort$	$::=$ $\mathbf{Tm} \ A$ $\mathbf{Co} \ \phi$	$binding \ classifier$
$context, \ \Gamma$	$::=$ \emptyset $\Gamma, x : A$ $\Gamma, c : \phi$ $\Gamma\{b/x\}$ $\Gamma\{\gamma/c\}$ Γ, Γ' $ \Gamma $ (Γ) Γ	$contexts$ M M M M M M
$sig, \ \Sigma$	$::=$	$signatures$

			\emptyset	
			$\Sigma \cup \{F : sig_sort\}$	
			Σ_0	M
			Σ_1	M
			$ \Sigma $	M
$available_props, \Delta$	$::=$		\emptyset	
			Δ, c	
			$\tilde{\Gamma}$	M
			(Δ)	M
$terminals$	$::=$		\leftrightarrow	
			\Leftrightarrow	
			\longrightarrow	
			min	
			\equiv	
			\forall	
			\in	
			\notin	
			\Leftarrow	
			\Rightarrow	
			\Rightarrow^*	
			\rightarrow	
			Λ	
			\square	
			\vdash	
			\vdash	
			\models	
			\models	
			\neq	
			\triangleright	
			ok	
			$-$	
			\rightsquigarrow	
			\rightsquigarrow^*	
			\rightsquigarrow	
			\emptyset	
			\circ	
			fv	
			dom	
			\sim	
			\langle	
			$ $	

	<div> <div>•</div> <div>fst</div> <div>snd</div> <div>as</div> <div>\Rightarrow</div> <div>$\vdash_{=}$</div> <div>refl₂</div> <div>$++$</div> <div>{</div> <div>}</div> </div>	
<i>formula, ψ</i>	<div> <div>$::=$</div> <div> <div><i>judgement</i></div> <div>$x : A \in \Gamma$</div> <div>$x : R \in \Omega$</div> <div>$c : \phi \in \Gamma$</div> <div>$F : sig_sort \in \Sigma$</div> <div>$x \in \Delta$</div> <div>$c \in \Delta$</div> <div>$c \text{ not relevant} \in \gamma$</div> <div>$x \notin fva$</div> <div>$x \notin \text{dom } \Gamma$</div> <div>$uniq \ \Gamma$</div> <div>$uniq(\Omega)$</div> <div>$c \notin \text{dom } \Gamma$</div> <div>$T \notin \text{dom } \Sigma$</div> <div>$F \notin \text{dom } \Sigma$</div> <div>$R_1 = R_2$</div> <div>$a = b$</div> <div>$\phi_1 = \phi_2$</div> <div>$\Gamma_1 = \Gamma_2$</div> <div>$\gamma_1 = \gamma_2$</div> <div>$\neg \psi$</div> <div>$\psi_1 \wedge \psi_2$</div> <div>$\psi_1 \vee \psi_2$</div> <div>$\psi_1 \Rightarrow \psi_2$</div> <div>$(\psi)$</div> <div>$\psi$</div> <div>$c : (a : A \sim b : B) \in \Gamma$</div> </div> </div> <div>suppress lc hypothesis generated by Ott</div>	
<i>JSubRole</i>	<div> <div>$::=$</div> <div> <div>$R_1 \leq R_2$</div> </div> </div> <div>Subroling judgement</div>	
<i>JPath</i>	<div> <div>$::=$</div> <div> <div>Path $a = F@Rs$</div> </div> </div> <div>Type headed by constant (partial function)</div>	

$JValuePath$	$::=$ $ \quad \text{ValuePath}_R \ a = F$	Type headed by constant (role-sensitive part)
$JPatCtx$	$::=$ $ \quad \Omega; \Gamma \vdash p :_F B \Rightarrow A$	Contexts generated by a pattern (variables bound)
$JMatchSubst$	$::=$ $ \quad \text{match } a_1 \text{ with } p \rightarrow b_1 = b_2$	match and substitute
$JApplyArgs$	$::=$ $ \quad \text{apply args } a \text{ to } b \mapsto b'$	apply arguments of a (headed by a constant)
$JValue$	$::=$ $ \quad \text{Value}_R \ A$	values
$JValueType$	$::=$ $ \quad \text{ValueType}_R \ A$	Types with head forms (erased language)
$Jconsistent$	$::=$ $ \quad \text{consistent}_R \ a \ b$	(erased) types do not differ in their heads
$Jroleing$	$::=$ $ \quad \Omega \vdash a : R$	Roleing judgment
$Jchk$	$::=$ $ \quad (\rho = +) \vee (x \notin \text{fv } A)$	irrelevant argument check
$Jpar$	$::=$ $ \quad \Omega \vdash a \Rightarrow_R b$ $ \quad \Omega \vdash a \Rightarrow_R^* b$ $ \quad \Omega \vdash a \Leftrightarrow_R b$	parallel reduction (implicit language) multistep parallel reduction parallel reduction to a common term
$Jbeta$	$::=$ $ \quad \vdash a > b / R$ $ \quad \vdash a \rightsquigarrow b / R$ $ \quad \vdash a \rightsquigarrow^* b / R$	primitive reductions on erased terms single-step head reduction for implicit language multistep reduction
$JBranchTyping$	$::=$ $ \quad \Gamma \vdash \text{case}_R \ a : A \text{ of } b : B \Rightarrow C \mid C'$	Branch Typing (aligning the types of case)
$Jett$	$::=$ $ \quad \Gamma \vdash \phi \text{ ok}$ $ \quad \Gamma \vdash a : A$ $ \quad \Gamma; \Delta \vdash \phi_1 \equiv \phi_2$ $ \quad \Gamma; \Delta \vdash a \equiv b : A / R$ $ \quad \vdash \Gamma$	Prop wellformedness typing prop equality definitional equality context wellformedness

$Jsig$	$::=$ $ \quad \models \Sigma$	signature wellformedness
$Jann$	$::=$ $ \quad \Gamma \vdash \phi \text{ ok}$ $ \quad \Gamma \vdash a : A/R$ $ \quad \Gamma; \Delta \vdash \gamma : \phi_1 \sim \phi_2$ $ \quad \Gamma; \Delta \vdash \gamma : A \sim_R B$ $ \quad \vdash \Gamma$	prop wellformedness typing coercion between props coercion between types context wellformedness
$Jred$	$::=$ $ \quad \Gamma \vdash a \rightsquigarrow b/R$	single-step, weak head reduction to values for annotated language
$judgement$	$::=$ $ \quad JSubRole$ $ \quad JPath$ $ \quad JValuePath$ $ \quad JPatCtx$ $ \quad JMatchSubst$ $ \quad JApplyArgs$ $ \quad JValue$ $ \quad JValueType$ $ \quad Jconsistent$ $ \quad Jroleing$ $ \quad JChk$ $ \quad Jpar$ $ \quad Jbeta$ $ \quad JBranchTyping$ $ \quad Jett$ $ \quad Jsig$ $ \quad Jann$ $ \quad Jred$	
$user_syntax$	$::=$ $ \quad tmvar$ $ \quad covar$ $ \quad datacon$ $ \quad const$ $ \quad index$ $ \quad relflag$ $ \quad appflag$ $ \quad role$ $ \quad constraint$ $ \quad tm$ $ \quad brs$ $ \quad co$ $ \quad role_context$	

$|$ *roles*
 $|$ *sig_sort*
 $|$ *sort*
 $|$ *context*
 $|$ *sig*
 $|$ *available_props*
 $|$ *terminals*
 $|$ *formula*

$\boxed{R_1 \leq R_2}$ Subroling judgement

$$\begin{array}{c}
\overline{\mathbf{Nom} \leq R} \quad \text{NOMBOT} \\
\overline{R \leq \mathbf{Rep}} \quad \text{REPTOP} \\
\overline{R \leq R} \quad \text{REFL} \\
\frac{R_1 \leq R_2 \quad R_2 \leq R_3}{R_1 \leq R_3} \quad \text{TRANS}
\end{array}$$

$\boxed{\text{Path } a = F@Rs}$ Type headed by constant (partial function)

$$\begin{array}{c}
\frac{F : A@Rs \in \Sigma_0}{\text{Path } F = F@Rs} \quad \text{PATH_ABSCONST} \\
\frac{F : p \sim a : A/R_1@Rs \in \Sigma_0}{\text{Path } F = F@Rs} \quad \text{PATH_CONST} \\
\frac{\text{Path } a = F@R_1, Rs \quad \text{app_role}\nu = R_1}{\text{Path } (a \ b^\nu) = F@Rs} \quad \text{PATH_APP} \\
\frac{\text{Path } a = F@Rs}{\text{Path } (a \ \square^-) = F@Rs} \quad \text{PATH_IAPP} \\
\frac{\text{Path } a = F@Rs}{\text{Path } (a[\bullet]) = F@Rs} \quad \text{PATH_CAPP}
\end{array}$$

$\boxed{\text{ValuePath}_R a = F}$ Type headed by constant (role-sensitive partial function)

$$\begin{array}{c}
\frac{F : A@Rs \in \Sigma_0}{\text{ValuePath}_R F = F} \quad \text{VALUEPATH_ABSCONST} \\
\frac{F : p \sim a : A/R_1@Rs \in \Sigma_0 \quad \neg(R_1 \leq R)}{\text{ValuePath}_R F = F} \quad \text{VALUEPATH_CONST} \\
\frac{\text{ValuePath}_R a = F}{\text{ValuePath}_R (a \ b^\nu) = F} \quad \text{VALUEPATH_APP} \\
\frac{\text{ValuePath}_R a = F}{\text{ValuePath}_R (a[\bullet]) = F} \quad \text{VALUEPATH_CAPP}
\end{array}$$

$\boxed{\Omega; \Gamma \vdash p :_F B \Rightarrow A}$ Contexts generated by a pattern (variables bound by the pattern)

$$\frac{}{\emptyset; \emptyset \models F :_F A \Rightarrow A} \text{PATCTX_CONST}$$

$$\frac{\Omega; \Gamma \models p :_F \Pi^+ x : A' \rightarrow A \Rightarrow B}{\Omega, x : R; \Gamma, x : A' \models p \ x^+ :_F A \Rightarrow B} \text{PATCTX_PIREL}$$

$$\frac{\Omega; \Gamma \models p :_F \Pi^- x : A' \rightarrow A \Rightarrow B}{\Omega; \Gamma, x : A' \models p \ \Box^- :_F A \Rightarrow B} \text{PATCTX_PIIRR}$$

$$\frac{\Omega; \Gamma \models p :_F \forall c : \phi. A \Rightarrow B}{\Omega; \Gamma, c : \phi \models p[\bullet] :_F A \Rightarrow B} \text{PATCTX_CPI}$$

$\boxed{\text{match } a_1 \text{ with } p \rightarrow b_1 = b_2}$ match and substitute

$$\frac{}{\text{match } F \text{ with } F \rightarrow b = b} \text{MATCHSUBST_CONST}$$

$$\frac{\text{match } a_1 \text{ with } a_2 \rightarrow b_1 = b_2}{\text{match } (a_1 \ a^{R'}) \text{ with } (a_2 \ x^+) \rightarrow b_1 = (b_2\{a/x\})} \text{MATCHSUBST_APPRELR}$$

$$\frac{\text{match } a_1 \text{ with } a_2 \rightarrow b_1 = b_2}{\text{match } (a_1 \ a^+) \text{ with } (a_2 \ x^+) \rightarrow b_1 = (b_2\{a/x\})} \text{MATCHSUBST_APPREL}$$

$$\frac{\text{match } a_1 \text{ with } a_2 \rightarrow b_1 = b_2}{\text{match } (a_1 \ \Box^-) \text{ with } (a_2 \ \Box^-) \rightarrow b_1 = b_2} \text{MATCHSUBST_APPIRREL}$$

$$\frac{\text{match } a_1 \text{ with } a_2 \rightarrow b_1 = b_2}{\text{match } (a_1[\bullet]) \text{ with } (a_2[\bullet]) \rightarrow b_1 = b_2} \text{MATCHSUBST_CAPP}$$

$\boxed{\text{apply args } a \text{ to } b \mapsto b'}$ apply arguments of a (headed by a constant) to b

$$\frac{}{\text{apply args } F \text{ to } b \mapsto b} \text{APPLYARGS_CONST}$$

$$\frac{\text{apply args } a \text{ to } b \mapsto b'}{\text{apply args } a \ a'^{\nu} \text{ to } b \mapsto b' \ a'^{(app.rhov)}} \text{APPLYARGS_APP}$$

$$\frac{\text{apply args } a \text{ to } b \mapsto b'}{\text{apply args } a[\gamma] \text{ to } b \mapsto b'[\gamma]} \text{APPLYARGS_CAPP}$$

$\boxed{\text{Value}_R \ A}$ values

$$\frac{}{\text{Value}_R \ \star} \text{VALUE_STAR}$$

$$\frac{}{\text{Value}_R \ \Pi^{\rho} x : A \rightarrow B} \text{VALUE_PI}$$

$$\frac{}{\text{Value}_R \ \forall c : \phi. B} \text{VALUE_CPI}$$

$$\frac{}{\text{Value}_R \ \lambda^+ x : A. a} \text{VALUE_ABSREL}$$

$$\frac{}{\text{Value}_R \ \lambda^+ x. a} \text{VALUE_UABSREL}$$

$$\frac{\text{Value}_R \ a}{\text{Value}_R \ \lambda^- x. a} \text{VALUE_UABSIRREL}$$

$$\frac{}{\text{Value}_R \ \Lambda c : \phi. a} \text{VALUE_CABS}$$

$$\frac{}{\text{Value}_R \ \Lambda c. a} \text{VALUE_UCABS}$$

$$\begin{array}{c}
\frac{\text{ValuePath}_R a = F}{\text{Value}_R a} \quad \text{VALUE_ROLEPATH} \\
\frac{\neg(\text{ValuePath}_R a = F) \quad \text{Path } a = F@R', Rs}{\text{Value}_R a} \quad \text{VALUE_PATH}
\end{array}$$

$\boxed{\text{ValueType}_R A}$

Types with head forms (erased language)

$$\begin{array}{c}
\frac{}{\text{ValueType}_R \star} \quad \text{VALUE_TYPE_STAR} \\
\frac{}{\text{ValueType}_R \Pi^\rho x : A \rightarrow B} \quad \text{VALUE_TYPE_PI} \\
\frac{}{\text{ValueType}_R \forall c : \phi. B} \quad \text{VALUE_TYPE_CPI} \\
\frac{\text{ValuePath}_R a = F}{\text{ValueType}_R a} \quad \text{VALUE_TYPE_VALUEPATH} \\
\frac{\neg(\text{ValuePath}_R a = F) \quad \text{Path } a = F@R', Rs}{\text{ValueType}_R a} \quad \text{VALUE_TYPE_PATH}
\end{array}$$

$\boxed{\text{consistent}_R a b}$

(erased) types do not differ in their heads

$$\begin{array}{c}
\frac{}{\text{consistent}_R \star \star} \quad \text{CONSISTENT_A_STAR} \\
\frac{}{\text{consistent}_{R'} (\Pi^\rho x_1 : A_1 \rightarrow B_1) (\Pi^\rho x_2 : A_2 \rightarrow B_2)} \quad \text{CONSISTENT_A_PI} \\
\frac{}{\text{consistent}_R (\forall c_1 : \phi_1. A_1) (\forall c_2 : \phi_2. A_2)} \quad \text{CONSISTENT_A_CPI} \\
\frac{\text{ValuePath}_R a_1 = F \quad \text{ValuePath}_R a_2 = F}{\text{consistent}_R a_1 a_2} \quad \text{CONSISTENT_A_VALUEPATH} \\
\frac{\neg(\text{ValuePath}_R a = F) \quad \text{Path } a_1 = F@R', Rs \quad \text{Path } a_2 = F@R', Rs}{\text{consistent}_R a_1 a_2} \quad \text{CONSISTENT_A_PATH} \\
\frac{\neg \text{ValueType}_R b}{\text{consistent}_R a b} \quad \text{CONSISTENT_A_STEP_R} \\
\frac{\neg \text{ValueType}_R a}{\text{consistent}_R a b} \quad \text{CONSISTENT_A_STEP_L}
\end{array}$$

$\boxed{\Omega \models a : R}$

Roleing judgment

$$\begin{array}{c}
\frac{\text{uniq}(\Omega)}{\Omega \models \square : R} \quad \text{ROLE_A_BULLET} \\
\frac{\text{uniq}(\Omega)}{\Omega \models \star : R} \quad \text{ROLE_A_STAR} \\
\frac{\text{uniq}(\Omega) \quad x : R \in \Omega \quad R \leq R_1}{\Omega \models x : R_1} \quad \text{ROLE_A_VAR}
\end{array}$$

$$\frac{\Omega, x : \mathbf{Nom} \models a : R}{\Omega \models (\lambda^\rho x. a) : R} \quad \text{ROLE_A_ABS}$$

$$\frac{\begin{array}{l} \Omega \models a : R \\ \Omega \models b : \mathbf{Nom} \end{array}}{\Omega \models (a \ b^\rho) : R} \quad \text{ROLE_A_APP}$$

$$\frac{\begin{array}{l} \Omega \models a : R \\ \text{Path } a = F @ R_1, Rs \\ \Omega \models b : R_1 \end{array}}{\Omega \models a \ b^{R_1} : R} \quad \text{ROLE_A_TAPP}$$

$$\frac{\begin{array}{l} \Omega \models A : R \\ \Omega, x : \mathbf{Nom} \models B : R \end{array}}{\Omega \models (\Pi^\rho x : A \rightarrow B) : R} \quad \text{ROLE_A_PI}$$

$$\frac{\begin{array}{l} \Omega \models a : R_1 \\ \Omega \models b : R_1 \\ \Omega \models A : R_0 \\ \Omega \models B : R \end{array}}{\Omega \models (\forall c : a \sim_{A/R_1} b. B) : R} \quad \text{ROLE_A_CPI}$$

$$\frac{\Omega \models b : R}{\Omega \models (\Lambda c. b) : R} \quad \text{ROLE_A_CABS}$$

$$\frac{\Omega \models a : R}{\Omega \models (a[\bullet]) : R} \quad \text{ROLE_A_CAPP}$$

$$\frac{\begin{array}{l} \text{uniq}(\Omega) \\ F : A @ Rs \in \Sigma_0 \end{array}}{\Omega \models F : R} \quad \text{ROLE_A_CONST}$$

$$\frac{\begin{array}{l} \text{uniq}(\Omega) \\ F : p \sim a : A / R @ Rs \in \Sigma_0 \end{array}}{\Omega \models F : R_1} \quad \text{ROLE_A_FAM}$$

$$\frac{\begin{array}{l} \Omega \models a : R \\ \Omega \models b_1 : R_1 \\ \Omega \models b_2 : R_1 \end{array}}{\Omega \models \text{case}_R a \text{ of } F \rightarrow b_1 \parallel - \rightarrow b_2 : R_1} \quad \text{ROLE_A_PATTERN}$$

$$\boxed{(\rho = +) \vee (x \notin \text{fv } A)} \quad \text{irrelevant argument check}$$

$$\overline{(+ = +) \vee (x \notin \text{fv } A)} \quad \text{RHO_REL}$$

$$\frac{x \notin \text{fv } A}{(- = +) \vee (x \notin \text{fv } A)} \quad \text{RHO_IRRREL}$$

$$\boxed{\Omega \models a \Rightarrow_R b} \quad \text{parallel reduction (implicit language)}$$

$$\frac{\Omega \models a : R}{\Omega \models a \Rightarrow_R a} \quad \text{PAR_REFL}$$

$$\frac{\begin{array}{l} \Omega \models a \Rightarrow_R (\lambda^\rho x. a') \\ \Omega \models b \Rightarrow_{\mathbf{Nom}} b' \end{array}}{\Omega \models a \ b^\rho \Rightarrow_R a' \{b'/x\}} \quad \text{PAR_BETA}$$

$$\begin{array}{c}
\frac{\Omega \models a \Rightarrow_R a' \quad \Omega \models b \Rightarrow_{\mathbf{Nom}} b'}{\Omega \models a \ b^\rho \Rightarrow_R a' \ b'^\rho} \text{PAR_APP} \\
\frac{\Omega \models a \Rightarrow_R (\Lambda c. a')}{\Omega \models a[\bullet] \Rightarrow_R a' \{ \bullet / c \}} \text{PAR_CBETA} \\
\frac{\Omega \models a \Rightarrow_R a'}{\Omega \models a[\bullet] \Rightarrow_R a'[\bullet]} \text{PAR_CAPP} \\
\frac{\Omega, x : \mathbf{Nom} \models a \Rightarrow_R a'}{\Omega \models \lambda^\rho x. a \Rightarrow_R \lambda^\rho x. a'} \text{PAR_ABS} \\
\frac{\Omega \models A \Rightarrow_R A' \quad \Omega, x : \mathbf{Nom} \models B \Rightarrow_R B'}{\Omega \models \Pi^\rho x : A \rightarrow B \Rightarrow_R \Pi^\rho x : A' \rightarrow B'} \text{PAR_PI} \\
\frac{\Omega \models a \Rightarrow_R a'}{\Omega \models \Lambda c. a \Rightarrow_R \Lambda c. a'} \text{PAR_CABS} \\
\frac{\Omega \models A \Rightarrow_{R_0} A' \quad \Omega \models a \Rightarrow_{R_1} a' \quad \Omega \models b \Rightarrow_{R_1} b' \quad \Omega \models B \Rightarrow_R B'}{\Omega \models \forall c : a \sim_{A/R_1} b. B \Rightarrow_R \forall c : a' \sim_{A'/R_1} b'. B'} \text{PAR_CPI} \\
\frac{\begin{array}{l} F : p \sim b : A/R_1 @ R_s \in \Sigma_0 \\ \Omega \models a : R \\ \text{match } a \text{ with } p \rightarrow b = b' \\ R_1 \leq R \\ \text{uniq}(\Omega) \end{array}}{\Omega \models a \Rightarrow_R b'} \text{PAR_AXIOM} \\
\frac{\Omega \models a \Rightarrow_R a' \quad \Omega \models b_1 \Rightarrow_{R_0} b'_1 \quad \Omega \models b_2 \Rightarrow_{R_0} b'_2}{\Omega \models (\text{case}_R a \text{ of } F \rightarrow b_1 \parallel_- \rightarrow b_2) \Rightarrow_{R_0} (\text{case}_R a' \text{ of } F \rightarrow b'_1 \parallel_- \rightarrow b'_2)} \text{PAR_PATTERN} \\
\frac{\Omega \models a \Rightarrow_R a' \quad \Omega \models b_1 \Rightarrow_{R_0} b'_1 \quad \text{ValuePath}_R a' = F \quad \text{apply args } a' \text{ to } b'_1 \mapsto b}{\Omega \models (\text{case}_R a \text{ of } F \rightarrow b_1 \parallel_- \rightarrow b_2) \Rightarrow_{R_0} b[\bullet]} \text{PAR_PATTERNTRUE} \\
\frac{\Omega \models a \Rightarrow_R a' \quad \Omega \models b_2 \Rightarrow_{R_0} b'_2 \quad \text{Value}_R a' \quad \neg(\text{ValuePath}_R a' = F)}{\Omega \models (\text{case}_R a \text{ of } F \rightarrow b_1 \parallel_- \rightarrow b_2) \Rightarrow_{R_0} b'_2} \text{PAR_PATTERNFALSE}
\end{array}$$

$$\boxed{\Omega \models a \Rightarrow_R^* b}$$

multistep parallel reduction

$$\begin{array}{c}
\overline{\Omega \models a \Rightarrow_R^* a} \text{MP_REFL} \\
\frac{\Omega \models a \Rightarrow_R b \quad \Omega \models b \Rightarrow_R^* a'}{\Omega \models a \Rightarrow_R^* a'} \text{MP_STEP}
\end{array}$$

$\boxed{\Omega \models a \Leftrightarrow_R b}$ parallel reduction to a common term

$$\frac{\Omega \models a_1 \Rightarrow_R^* b \quad \Omega \models a_2 \Rightarrow_R^* b}{\Omega \models a_1 \Leftrightarrow_R a_2} \text{ JOIN}$$

$\boxed{\models a > b/R}$ primitive reductions on erased terms

$$\frac{\text{Value}_{R_1} (\lambda^\rho x.v)}{\models (\lambda^\rho x.v) \ b^\rho > v\{b/x\}/R_1} \text{ BETA_APPAbs}$$

$$\frac{}{\models (\Lambda c.a')[\bullet] > a'\{\bullet/c\}/R} \text{ BETA_CAPPCAbs}$$

$$\frac{\begin{array}{l} F : p \sim b : A/R_1 @ Rs \in \Sigma_0 \\ \text{match } a \text{ with } p \rightarrow b = b' \\ R_1 \leq R \end{array}}{\models a > b'/R} \text{ BETA_AXIOM}$$

$$\frac{\begin{array}{l} \text{ValuePath}_R a = F \\ \text{apply args } a \text{ to } b_1 \mapsto b'_1 \end{array}}{\models \text{case}_R a \text{ of } F \rightarrow b_1 \parallel _ \rightarrow b_2 > b'_1[\bullet]/R_0} \text{ BETA_PATTERNTRUE}$$

$$\frac{\begin{array}{l} \text{Value}_R a \\ \neg(\text{ValuePath}_R a = F) \end{array}}{\models \text{case}_R a \text{ of } F \rightarrow b_1 \parallel _ \rightarrow b_2 > b_2/R_0} \text{ BETA_PATTERNFALSE}$$

$\boxed{\models a \rightsquigarrow b/R}$ single-step head reduction for implicit language

$$\frac{\models a \rightsquigarrow a'/R_1}{\models \lambda^- x.a \rightsquigarrow \lambda^- x.a'/R_1} \text{ E_ABSTERM}$$

$$\frac{\models a \rightsquigarrow a'/R_1}{\models a \ b^\rho \rightsquigarrow a' \ b^\rho/R_1} \text{ E_APPLEFT}$$

$$\frac{\models a \rightsquigarrow a'/R}{\models a[\bullet] \rightsquigarrow a'[\bullet]/R} \text{ E_CAPPLEFT}$$

$$\frac{\models a \rightsquigarrow a'/R}{\models \text{case}_R a \text{ of } F \rightarrow b_1 \parallel _ \rightarrow b_2 \rightsquigarrow \text{case}_R a' \text{ of } F \rightarrow b_1 \parallel _ \rightarrow b_2/R_0} \text{ E_PATTERN}$$

$$\frac{\models a > b/R}{\models a \rightsquigarrow b/R} \text{ E_PRIM}$$

$\boxed{\models a \rightsquigarrow^* b/R}$ multistep reduction

$$\frac{}{\models a \rightsquigarrow^* a/R} \text{ EQUAL}$$

$$\frac{\models a \rightsquigarrow b/R \quad \models b \rightsquigarrow^* a'/R}{\models a \rightsquigarrow^* a'/R} \text{ STEP}$$

$\boxed{\Gamma \models \text{case}_R a : A \text{ of } b : B \Rightarrow C \mid C'}$ Branch Typing (aligning the types of case)

$$\frac{\begin{array}{l} \text{uniq } \Gamma \\ \text{lc_tm } C \end{array}}{\Gamma \models \text{case}_R a : A \text{ of } b : A \Rightarrow \forall c:(a \sim_{A/R} b).C \mid C} \text{ BRANCHTYPING_BASE}$$

$$\frac{\Gamma, x : A \models \text{case}_R a : A_1 \text{ of } b \ x^+ : B \Rightarrow C \mid C'}{\Gamma \models \text{case}_R a : A_1 \text{ of } b : \Pi^+ x : A \rightarrow B \Rightarrow \Pi^+ x : A \rightarrow C \mid C'} \quad \text{BRANCH_TYPING_PIREL}$$

$$\frac{\Gamma, x : A \models \text{case}_R a : A_1 \text{ of } b \ \Box^- : B \Rightarrow C \mid C'}{\Gamma \models \text{case}_R a : A_1 \text{ of } b : \Pi^- x : A \rightarrow B \Rightarrow \Pi^- x : A \rightarrow C \mid C'} \quad \text{BRANCH_TYPING_PIRREL}$$

$$\frac{\Gamma, c : \phi \models \text{case}_R a : A \text{ of } b[\bullet] : B \Rightarrow C \mid C'}{\Gamma \models \text{case}_R a : A \text{ of } b : \forall c : \phi. B \Rightarrow \forall c : \phi. C \mid C'} \quad \text{BRANCH_TYPING_CPI}$$

$$\boxed{\Gamma \models \phi \text{ ok}} \quad \text{Prop wellformedness}$$

$$\frac{\begin{array}{c} \Gamma \models a : A \\ \Gamma \models b : A \\ \Gamma \models A : \star \end{array}}{\Gamma \models a \sim_{A/R} b \text{ ok}} \quad \text{E_WFF}$$

$$\boxed{\Gamma \models a : A} \quad \text{typing}$$

$$\frac{\vdash \Gamma}{\Gamma \vdash \star : \star} \quad \text{E_STAR}$$

$$\frac{\begin{array}{c} \vdash \Gamma \\ x : A \in \Gamma \end{array}}{\Gamma \vdash x : A} \quad \text{E_VAR}$$

$$\frac{\begin{array}{c} \Gamma, x : A \vdash B : \star \\ \Gamma \vdash A : \star \end{array}}{\Gamma \vdash \Pi^\rho x : A \rightarrow B : \star} \quad \text{E_PI}$$

$$\frac{\begin{array}{c} \Gamma, x : A \vdash a : B \\ \Gamma \vdash A : \star \\ (\rho = +) \vee (x \notin \text{fv } a) \end{array}}{\Gamma \vdash \lambda^\rho x. a : (\Pi^\rho x : A \rightarrow B)} \quad \text{E_ABS}$$

$$\frac{\begin{array}{c} \Gamma \vdash b : \Pi^+ x : A \rightarrow B \\ \Gamma \vdash a : A \end{array}}{\Gamma \vdash b \ a^+ : B\{a/x\}} \quad \text{E_APP}$$

$$\frac{\begin{array}{c} \Gamma \vdash b : \Pi^+ x : A \rightarrow B \\ \Gamma \vdash a : A \end{array}}{\Gamma \vdash b \ a^R : B\{a/x\}} \quad \text{E_TAPP}$$

$$\frac{\begin{array}{c} \Gamma \vdash b : \Pi^- x : A \rightarrow B \\ \Gamma \vdash a : A \end{array}}{\Gamma \vdash b \ \Box^- : B\{a/x\}} \quad \text{E_IAPP}$$

$$\frac{\begin{array}{c} \Gamma \vdash a : A \\ \Gamma; \tilde{\Gamma} \vdash A \equiv B : \star / \mathbf{Rep} \\ \Gamma \vdash B : \star \end{array}}{\Gamma \vdash a : B} \quad \text{E_CONV}$$

$$\frac{\begin{array}{c} \Gamma, c : \phi \vdash B : \star \\ \Gamma \vdash \phi \text{ ok} \end{array}}{\Gamma \vdash \forall c : \phi. B : \star} \quad \text{E_CPI}$$

$$\frac{\begin{array}{c} \Gamma, c : \phi \vdash a : B \\ \Gamma \vdash \phi \text{ ok} \end{array}}{\Gamma \vdash \Lambda c. a : \forall c : \phi. B} \quad \text{E_CABS}$$

$$\frac{\begin{array}{c} \Gamma \models a_1 : \forall c : (a \sim_{A/R} b). B_1 \\ \Gamma; \tilde{\Gamma} \models a \equiv b : A/R \end{array}}{\Gamma \models a_1[\bullet] : B_1\{\bullet/c\}} \quad \text{E_CAPP}$$

$$\frac{\begin{array}{c} \models \Gamma \\ F : A @ R_s \in \Sigma_0 \\ \emptyset \models A : \star \end{array}}{\Gamma \models F : A} \quad \text{E_CONST}$$

$$\frac{\begin{array}{c} \models \Gamma \\ F : p \sim a : A/R_1 @ R_s \in \Sigma_0 \end{array}}{\Gamma \models F : A} \quad \text{E_FAM}$$

$$\frac{\begin{array}{c} \Gamma \models a : A \\ \Gamma \models F : A_1 \\ \Gamma \models b_1 : B \\ \Gamma \models b_2 : C \\ \Gamma \models \text{case}_R a : A \text{ of } F : A_1 \Rightarrow B \mid C \end{array}}{\Gamma \models \text{case}_R a \text{ of } F \rightarrow b_1 \parallel - \rightarrow b_2 : C} \quad \text{E_CASE}$$

$$\boxed{\Gamma; \Delta \models \phi_1 \equiv \phi_2} \quad \text{prop equality}$$

$$\frac{\begin{array}{c} \Gamma; \Delta \models A_1 \equiv A_2 : A/R \\ \Gamma; \Delta \models B_1 \equiv B_2 : A/R \end{array}}{\Gamma; \Delta \models A_1 \sim_{A/R} B_1 \equiv A_2 \sim_{A/R} B_2} \quad \text{E_PROP_CONG}$$

$$\frac{\begin{array}{c} \Gamma; \Delta \models A \equiv B : \star / R_0 \\ \Gamma \models A_1 \sim_{A/R} A_2 \text{ ok} \\ \Gamma \models A_1 \sim_{B/R} A_2 \text{ ok} \end{array}}{\Gamma; \Delta \models A_1 \sim_{A/R} A_2 \equiv A_1 \sim_{B/R} A_2} \quad \text{E_ISO_CONV}$$

$$\frac{\Gamma; \Delta \models \forall c : (a_1 \sim_{A/R_1} a_2). B_1 \equiv \forall c : (b_1 \sim_{B/R_2} b_2). B_2 : \star / R'}{\Gamma; \Delta \models a_1 \sim_{A/R_1} a_2 \equiv b_1 \sim_{B/R_2} b_2} \quad \text{E_CPI_FST}$$

$$\boxed{\Gamma; \Delta \models a \equiv b : A/R} \quad \text{definitional equality}$$

$$\frac{\begin{array}{c} \models \Gamma \\ c : (a \sim_{A/R} b) \in \Gamma \\ c \in \Delta \end{array}}{\Gamma; \Delta \models a \equiv b : A/R} \quad \text{E_ASSN}$$

$$\frac{\Gamma \models a : A}{\Gamma; \Delta \models a \equiv a : A/R} \quad \text{E_REFL}$$

$$\frac{\Gamma; \Delta \models b \equiv a : A/R}{\Gamma; \Delta \models a \equiv b : A/R} \quad \text{E_SYM}$$

$$\frac{\begin{array}{c} \Gamma; \Delta \models a \equiv a_1 : A/R \\ \Gamma; \Delta \models a_1 \equiv b : A/R \end{array}}{\Gamma; \Delta \models a \equiv b : A/R} \quad \text{E_TRANS}$$

$$\frac{\begin{array}{c} \Gamma; \Delta \models a \equiv b : A/R_1 \\ R_1 \leq R_2 \end{array}}{\Gamma; \Delta \models a \equiv b : A/R_2} \quad \text{E_SUB}$$

$$\begin{array}{c}
\frac{\Gamma \vdash a_1 : B \quad \Gamma \vdash a_2 : B \quad \vdash a_1 > a_2 / R}{\Gamma; \Delta \vdash a_1 \equiv a_2 : B / R} \text{E_BETA} \\
\\
\frac{\Gamma; \Delta \vdash A_1 \equiv A_2 : \star / R' \quad \Gamma, x : A_1; \Delta \vdash B_1 \equiv B_2 : \star / R' \quad \Gamma \vdash A_1 : \star \quad \Gamma \vdash \Pi^\rho x : A_1 \rightarrow B_1 : \star \quad \Gamma \vdash \Pi^\rho x : A_2 \rightarrow B_2 : \star}{\Gamma; \Delta \vdash (\Pi^\rho x : A_1 \rightarrow B_1) \equiv (\Pi^\rho x : A_2 \rightarrow B_2) : \star / R'} \text{E_PiCONG} \\
\\
\frac{\Gamma, x : A_1; \Delta \vdash b_1 \equiv b_2 : B / R' \quad \Gamma \vdash A_1 : \star \quad (\rho = +) \vee (x \notin \text{fv } b_1) \quad (\rho = +) \vee (x \notin \text{fv } b_2)}{\Gamma; \Delta \vdash (\lambda^\rho x. b_1) \equiv (\lambda^\rho x. b_2) : (\Pi^\rho x : A_1 \rightarrow B) / R'} \text{E_ABSCONG} \\
\\
\frac{\Gamma; \Delta \vdash a_1 \equiv b_1 : (\Pi^+ x : A \rightarrow B) / R' \quad \Gamma; \Delta \vdash a_2 \equiv b_2 : A / \mathbf{Nom}}{\Gamma; \Delta \vdash a_1 \ a_2^+ \equiv b_1 \ b_2^+ : (B\{a_2/x\}) / R'} \text{E_APPCONG} \\
\\
\frac{\Gamma; \Delta \vdash a_1 \equiv b_1 : (\Pi^+ x : A \rightarrow B) / R' \quad \Gamma; \Delta \vdash a_2 \equiv b_2 : A / \mathbf{param } R \ R'}{\Gamma; \Delta \vdash a_1 \ a_2^R \equiv b_1 \ b_2^R : (B\{a_2/x\}) / R'} \text{E_TAPPCONG} \\
\\
\frac{\Gamma; \Delta \vdash a_1 \equiv b_1 : (\Pi^- x : A \rightarrow B) / R' \quad \Gamma \vdash a : A}{\Gamma; \Delta \vdash a_1 \ \Box^- \equiv b_1 \ \Box^- : (B\{a/x\}) / R'} \text{E_IAPPCONG} \\
\\
\frac{\Gamma; \Delta \vdash \Pi^\rho x : A_1 \rightarrow B_1 \equiv \Pi^\rho x : A_2 \rightarrow B_2 : \star / R' \quad \Gamma; \Delta \vdash A_1 \equiv A_2 : \star / R'}{\Gamma; \Delta \vdash \Pi^\rho x : A_1 \rightarrow B_1 \equiv \Pi^\rho x : A_2 \rightarrow B_2 : \star / R'} \text{E_PiFST} \\
\\
\frac{\Gamma; \Delta \vdash \Pi^\rho x : A_1 \rightarrow B_1 \equiv \Pi^\rho x : A_2 \rightarrow B_2 : \star / R' \quad \Gamma; \Delta \vdash a_1 \equiv a_2 : A_1 / R'}{\Gamma; \Delta \vdash B_1\{a_1/x\} \equiv B_2\{a_2/x\} : \star / R'} \text{E_PiSND} \\
\\
\frac{\Gamma; \Delta \vdash a_1 \sim_{A_1/R} b_1 \equiv a_2 \sim_{A_2/R} b_2 \quad \Gamma, c : a_1 \sim_{A_1/R} b_1; \Delta \vdash A \equiv B : \star / R' \quad \Gamma \vdash a_1 \sim_{A_1/R} b_1 \text{ ok} \quad \Gamma \vdash \forall c : a_1 \sim_{A_1/R} b_1. A : \star \quad \Gamma \vdash \forall c : a_2 \sim_{A_2/R} b_2. B : \star}{\Gamma; \Delta \vdash \forall c : a_1 \sim_{A_1/R} b_1. A \equiv \forall c : a_2 \sim_{A_2/R} b_2. B : \star / R'} \text{E_CPiCONG} \\
\\
\frac{\Gamma, c : \phi_1; \Delta \vdash a \equiv b : B / R \quad \Gamma \vdash \phi_1 \text{ ok}}{\Gamma; \Delta \vdash (\Lambda c. a) \equiv (\Lambda c. b) : \forall c : \phi_1. B / R} \text{E_CAbsCONG} \\
\\
\frac{\Gamma; \Delta \vdash a_1 \equiv b_1 : (\forall c : (a \sim_{A/R} b). B) / R' \quad \Gamma; \tilde{\Gamma} \vdash a \equiv b : A / \mathbf{param } R \ R'}{\Gamma; \Delta \vdash a_1[\bullet] \equiv b_1[\bullet] : (B\{\bullet/c\}) / R'} \text{E_CAppCONG} \\
\\
\frac{\Gamma; \Delta \vdash \forall c : (a_1 \sim_{A/R} a_2). B_1 \equiv \forall c : (a'_1 \sim_{A'/R'} a'_2). B_2 : \star / R_0 \quad \Gamma; \tilde{\Gamma} \vdash a_1 \equiv a_2 : A / \mathbf{param } R \ R_0 \quad \Gamma; \tilde{\Gamma} \vdash a'_1 \equiv a'_2 : A' / \mathbf{param } R' \ R_0}{\Gamma; \Delta \vdash B_1\{\bullet/c\} \equiv B_2\{\bullet/c\} : \star / R_0} \text{E_CPiSND}
\end{array}$$

$$\begin{array}{c}
\frac{\Gamma; \Delta \vdash a \equiv b : A/R \quad \Gamma; \Delta \vdash a \sim_{A/R} b \equiv a' \sim_{A'/R'} b'}{\Gamma; \Delta \vdash a' \equiv b' : A'/R'} \quad \text{E_CAST} \\
\\
\frac{\Gamma; \Delta \vdash a \equiv b : A/R \quad \Gamma; \tilde{\Gamma} \vdash A \equiv B : \star/\mathbf{Rep} \quad \Gamma \vdash B : \star}{\Gamma; \Delta \vdash a \equiv b : B/R} \quad \text{E_EQCONV} \\
\\
\frac{\Gamma; \Delta \vdash a \sim_{A/R_1} b \equiv a' \sim_{A'/R_1} b'}{\Gamma; \Delta \vdash A \equiv A' : \star/\mathbf{Rep}} \quad \text{E_ISOSND} \\
\\
\frac{\Gamma; \Delta \vdash a \equiv a' : A/R \quad \Gamma; \Delta \vdash b_1 \equiv b'_1 : B/R_0 \quad \Gamma; \Delta \vdash b_2 \equiv b'_2 : B/R_0}{\Gamma; \Delta \vdash \text{case}_R a \text{ of } F \rightarrow b_1 \parallel \rightarrow b_2 \equiv \text{case}_R a' \text{ of } F \rightarrow b'_1 \parallel \rightarrow b'_2 : B/R_0} \quad \text{E_PATCONG} \\
\\
\frac{\begin{array}{l} \text{ValuePath}_{R'} a = F \\ \text{ValuePath}_{R'} a' = F \\ \Gamma \vdash a : \Pi^+ x : A \rightarrow B \\ \Gamma \vdash b : A \\ \Gamma \vdash a' : \Pi^+ x : A \rightarrow B \\ \Gamma \vdash b' : A \\ \Gamma; \Delta \vdash a \ b^{R_1} \equiv a' \ b'^{R_1} : B\{b/x\}/R' \\ \Gamma; \tilde{\Gamma} \vdash B\{b/x\} \equiv B\{b'/x\} : \star/R' \end{array}}{\Gamma; \Delta \vdash a \equiv a' : \Pi^+ x : A \rightarrow B/R'} \quad \text{E_LEFTREL} \\
\\
\frac{\begin{array}{l} \text{ValuePath}_{R'} a = F \\ \text{ValuePath}_{R'} a' = F \\ \Gamma \vdash a : \Pi^- x : A \rightarrow B \\ \Gamma \vdash b : A \\ \Gamma \vdash a' : \Pi^- x : A \rightarrow B \\ \Gamma \vdash b' : A \\ \Gamma; \Delta \vdash a \ \square^- \equiv a' \ \square^- : B\{b/x\}/R' \\ \Gamma; \tilde{\Gamma} \vdash B\{b/x\} \equiv B\{b'/x\} : \star/R_0 \end{array}}{\Gamma; \Delta \vdash a \equiv a' : \Pi^- x : A \rightarrow B/R'} \quad \text{E_LEFTIRREL} \\
\\
\frac{\begin{array}{l} \text{ValuePath}_{R'} a = F \\ \text{ValuePath}_{R'} a' = F \\ \Gamma \vdash a : \Pi^+ x : A \rightarrow B \\ \Gamma \vdash b : A \\ \Gamma \vdash a' : \Pi^+ x : A \rightarrow B \\ \Gamma \vdash b' : A \\ \Gamma; \Delta \vdash a \ b^+ \equiv a' \ b'^+ : B\{b/x\}/R' \\ \Gamma; \tilde{\Gamma} \vdash B\{b/x\} \equiv B\{b'/x\} : \star/R_0 \end{array}}{\Gamma; \Delta \vdash b \equiv b' : A/\mathbf{param} \ R_1 \ R'} \quad \text{E_RIGHT} \\
\\
\frac{\begin{array}{l} \text{ValuePath}_{R'} a = F \\ \text{ValuePath}_{R'} a' = F \\ \Gamma \vdash a : \forall c : (a_1 \sim_{A/R_1} a_2). B \\ \Gamma \vdash a' : \forall c : (a_1 \sim_{A/R_1} a_2). B \\ \Gamma; \tilde{\Gamma} \vdash a_1 \equiv a_2 : A/R' \\ \Gamma; \Delta \vdash a[\bullet] \equiv a'[\bullet] : B\{\bullet/c\}/R' \end{array}}{\Gamma; \Delta \vdash a \equiv a' : \forall c : (a_1 \sim_{A/R_1} a_2). B/R'} \quad \text{E_CLEFT}
\end{array}$$

$\boxed{\models \Gamma}$ context wellformedness

$$\begin{array}{c}
\overline{\models \emptyset} \quad \text{E_EMPTY} \\
\\
\begin{array}{c}
\models \Gamma \\
\Gamma \models A : \star \\
x \notin \text{dom } \Gamma \\
\hline
\models \Gamma, x : A
\end{array} \quad \text{E_CONSTM} \\
\\
\begin{array}{c}
\models \Gamma \\
\Gamma \models \phi \text{ ok} \\
c \notin \text{dom } \Gamma \\
\hline
\models \Gamma, c : \phi
\end{array} \quad \text{E_CONSCo}
\end{array}$$

$\boxed{\models \Sigma}$ signature wellformedness

$$\begin{array}{c}
\overline{\models \emptyset} \quad \text{SIG_EMPTY} \\
\\
\begin{array}{c}
\models \Sigma \\
\emptyset \models A : \star \\
F \notin \text{dom } \Sigma \\
\hline
\models \Sigma \cup \{F : A @ R_s\}
\end{array} \quad \text{SIG_CONSTCONST} \\
\\
\begin{array}{c}
\models \Sigma \\
F \notin \text{dom } \Sigma \\
\emptyset \models A : \star \\
\Omega; \Gamma \models p :_F B \Rightarrow A \\
\Gamma \models a : B \\
\Omega \models a : \mathbf{Rep} \\
\hline
\models \Sigma \cup \{F : p \sim a : A/R @ \mathbf{range } \Omega\}
\end{array} \quad \text{SIG_CONSAX}
\end{array}$$

$\boxed{\Gamma \vdash \phi \text{ ok}}$ prop wellformedness

$\boxed{\Gamma \vdash a : A/R}$ typing

$\boxed{\Gamma; \Delta \vdash \gamma : \phi_1 \sim \phi_2}$ coercion between props

$\boxed{\Gamma; \Delta \vdash \gamma : A \sim_R B}$ coercion between types

$\boxed{\vdash \Gamma}$ context wellformedness

$\boxed{\Gamma \vdash a \rightsquigarrow b/R}$ single-step, weak head reduction to values for annotated language

Definition rules: 143 good 0 bad

Definition rule clauses: 400 good 0 bad