$tmvar$, x, y, f, m, n	variables
$covar$, c	coercion variables
$datacon$, K	
$const$, T, F	
$index$, i	indices

role, R	$::=$			Role
	$\mid$	**Nom**		
	$\mid$	**Rep**		
	$\mid$	$R_1 \cap R_2$	S	
relflag, ρ	$::=$			relevance flag
	$\mid$	$+$		
	$\mid$	$-$		
constraint, ϕ	$::=$			props
	$\mid$	$a \sim_{A/R} b$		
	$\mid$	(ϕ)	S	
	$\mid$	$\phi\{b/x\}$	S	
	$\mid$	$\lvert\phi\rvert$	S	
	$\mid$	$a \sim_R b$	S	
tm, a, b, v, w, A, B	$::=$			types and kinds
	$\mid$	$\star$		
	$\mid$	x		
	$\mid$	$\lambda^{\rho} x\!:\!A/R.b$	bind x in b	
	$\mid$	$\lambda^{R,\rho} x.b$	bind x in b	
	$\mid$	$a\ b^{R,\rho}$		
	$\mid$	F		
	$\mid$	$\Pi^{\rho} x\!:\!A/R \to B$	bind x in B	
	$\mid$	$a \triangleright_R \gamma$		
	$\mid$	$\forall c\!:\!\phi.B$	bind c in B	
	$\mid$	$\Lambda c\!:\!\phi.b$	bind c in b	
	$\mid$	$\Lambda c.b$	bind c in b	
	$\mid$	$a[\gamma]$		
	$\mid$	$\square$		
	$\mid$	**ifPath** $R\ a'\ a\ b$		
	$\mid$	K		
	$\mid$	**match** a **with** brs		
	$\mid$	**sub** $R\ a$		
	$\mid$	$a\{b/x\}$	S	
	$\mid$	a	S	
	$\mid$	$a\{\gamma/c\}$	S	
	$\mid$	a	S	
	$\mid$	(a)	S	
	$\mid$	a	S	parsing precedence is hard
	$\mid$	$\lvert a\rvert_R$	S	
	$\mid$	**Int**	S	
	$\mid$	**Bool**	S	
	$\mid$	**Nat**	S	
	$\mid$	**Vec**	S	
	$\mid$	0	S	
	$\mid$	S	S	

	$\mid$	**True**	S	
	$\mid$	**Fix**	S	
	$\mid$	**Age**	S	
	$\mid$	$a \rightarrow b$	S	
	$\mid$	$\phi \Rightarrow A$	S	
	$\mid$	$a\ \ b$	S	
	$\mid$	$\lambda x.a$	S	
	$\mid$	$\lambda x\!:\!A.a$	S	
	$\mid$	$\forall\, x : A/R \rightarrow B$	S	
	$\mid$	**if** ϕ **then** a **else** b	S	
brs	$::=$			case branches
	$\mid$	**none**		
	$\mid$	$K \Rightarrow a; brs$		
	$\mid$	$brs\{a/x\}$	S	
	$\mid$	$brs\{\gamma/c\}$	S	
	$\mid$	(brs)	S	
$co,\ \gamma$	$::=$			explicit coercions
	$\mid$	$\bullet$		
	$\mid$	c		
	$\mid$	**red** $a\ b$		
	$\mid$	**refl** a		
	$\mid$	$(a \mid=\mid_{\gamma}\ b)$		
	$\mid$	**sym** γ		
	$\mid$	$\gamma_1;\gamma_2$		
	$\mid$	**sub** γ		
	$\mid$	$\Pi^{R,\rho} x\!:\!\gamma_1.\gamma_2$	bind x in γ_2	
	$\mid$	$\lambda^{R,\rho} x\!:\!\gamma_1.\gamma_2$	bind x in γ_2	
	$\mid$	$\gamma_1\ \ \gamma_2^{R,\rho}$		
	$\mid$	**piFst** γ		
	$\mid$	**cpiFst** γ		
	$\mid$	**isoSnd** γ		
	$\mid$	$\gamma_1 @ \gamma_2$		
	$\mid$	$\forall c\!:\!\gamma_1.\gamma_3$	bind c in γ_3	
	$\mid$	$\lambda c\!:\!\gamma_1.\gamma_3 @ \gamma_4$	bind c in γ_3	
	$\mid$	$\gamma(\gamma_1, \gamma_2)$		
	$\mid$	$\gamma@(\gamma_1 \sim \gamma_2)$		
	$\mid$	$\gamma_1 \triangleright_R \gamma_2$		
	$\mid$	$\gamma_1 \sim_A \gamma_2$		
	$\mid$	**conv** $\phi_1 \sim_{\gamma} \phi_2$		
	$\mid$	**eta** a		
	$\mid$	**left** $\gamma\,\gamma'$		
	$\mid$	**right** $\gamma\,\gamma'$		
	$\mid$	(γ)	S	
	$\mid$	γ	S	

	$\vert$	$\gamma\{a/x\}$	S
role_context, Ω	::=		role$_c$*ontexts*
	$\vert$	$\varnothing$	
	$\vert$	$\Omega, x : R$	
	$\vert$	(Ω)	M
	$\vert$	Ω	M
sig_sort	::=		signature classifier
	$\vert$	$: A/R$	
	$\vert$	$\sim a : A/R$	
sort	::=		binding classifier
	$\vert$	$\mathbf{Tm}\, A\, R$	
	$\vert$	$\mathbf{Co}\, \phi$	
context, Γ	::=		contexts
	$\vert$	$\varnothing$	
	$\vert$	$\Gamma, x : A/R$	
	$\vert$	$\Gamma, c : \phi$	
	$\vert$	$\Gamma\{b/x\}$	M
	$\vert$	$\Gamma\{\gamma/c\}$	M
	$\vert$	Γ, Γ'	M
	$\vert$	$\vert\Gamma\vert$	M
	$\vert$	(Γ)	M
	$\vert$	Γ	M
sig, Σ	::=		signatures
	$\vert$	$\varnothing$	
	$\vert$	$\Sigma \cup \{F\, \mathit{sig\_sort}\}$	
	$\vert$	Σ_0	M
	$\vert$	Σ_1	M
	$\vert$	$\vert\Sigma\vert$	M
available_props, Δ	::=		
	$\vert$	$\varnothing$	
	$\vert$	Δ, c	
	$\vert$	$\widetilde{\Gamma}$	M
	$\vert$	(Δ)	M
terminals	::=		
	$\vert$	$\leftrightarrow$	
	$\vert$	$\Leftrightarrow$	
	$\vert$	$\longrightarrow$	
	$\vert$	min	
	$\vert$	$\equiv$	
	$\vert$	$\forall$	

| $\in$
| $\notin$
| $\Leftarrow$
| $\Rightarrow$
| $\Rightarrow^*$
| $\rightarrow$
| Λ
|
| $\Box$
| $\vdash$
| $\dashv$
| $\models$
| $\models$
| $\neq$
| $\triangleright$
| ok
| $\_$
| $\leadsto$
| $\leadsto^*$
| $\leadsto$
| $\varnothing$
| $\circ$
| fv
| dom
| $\sim$
| $\asymp$
| $|$
| $\bullet$
| $\mathbf{fst}$
| $\mathbf{snd}$
| $|\Rightarrow|$
| $\vdash_{=}$
| $\mathbf{refl_2}$
| $++$

$formula,\ \psi$ ::=
| $judgement$
| $x : A/R \ \in \ \Gamma$
| $x : R \ \in \ \Omega$
| $c : \phi \ \in \ \Gamma$
| $F\, sig\_sort \ \in \ \Sigma$
| $K : T\, \Gamma \ \in \ \Sigma$
| $x \ \in \ \Delta$
| $c \ \in \ \Delta$
| $c\, \mathbf{not\ relevant} \ \in \ \gamma$
| $x \ \notin \ \mathsf{fv} a$

	$\mid$	$x \notin \mathsf{dom}\,\Gamma$	
	$\mid$	$uniq(\Omega)$	
	$\mid$	$c \notin \mathsf{dom}\,\Gamma$	
	$\mid$	$T \notin \mathsf{dom}\,\Sigma$	
	$\mid$	$F \notin \mathsf{dom}\,\Sigma$	
	$\mid$	$a = b$	
	$\mid$	$\phi_1 = \phi_2$	
	$\mid$	$\Gamma_1 = \Gamma_2$	
	$\mid$	$\gamma_1 = \gamma_2$	
	$\mid$	$\neg\psi$	
	$\mid$	$\psi_1 \wedge \psi_2$	
	$\mid$	$\psi_1 \vee \psi_2$	
	$\mid$	$\psi_1 \Rightarrow \psi_2$	
	$\mid$	(ψ)	
	$\mid$	ψ	
	$\mid$	$c : (a : A \sim b : B) \in \Gamma$	
	$\mid$		suppress lc hypothesis generated by Ott
JSubRole	::=		
	$\mid$	$R_1 \leq R_2$	Subroling judgement
JPath	::=		
	$\mid$	$\mathsf{Path}_R\ a = F$	Type headed by constant (partial function)
JValue	::=		
	$\mid$	$\mathsf{Value}_R\ A$	values
JValueType	::=		
	$\mid$	$\mathsf{ValueType}_R\ A$	Types with head forms (erased language)
Jconsistent	::=		
	$\mid$	$\mathsf{consistent}_R\ ab$	(erased) types do not differ in their heads
Jerased	::=		
	$\mid$	$\Omega \vDash a : R$	
JChk	::=		
	$\mid$	$(\rho = +) \vee (x \notin \mathsf{fv}\ A)$	irrelevant argument check
Jpar	::=		
	$\mid$	$\Omega \vDash a \Rightarrow_R b$	parallel reduction (implicit language)
	$\mid$	$\Omega \vdash a \Rightarrow^*_R b$	multistep parallel reduction
	$\mid$	$\Omega \vdash a \Leftrightarrow_R b$	parallel reduction to a common term
Jbeta	::=		
	$\mid$	$\vDash a > b/R$	primitive reductions on erased terms
	$\mid$	$\vDash a \rightsquigarrow b/R$	single-step head reduction for implicit language

	$\mid$	$\vDash a \leadsto^* b/R$	multistep reduction
Jett	$::=$		
	$\mid$	$\Gamma \vDash \phi$ ok	Prop wellformedness
	$\mid$	$\Gamma \vDash a : A/R$	typing
	$\mid$	$\Gamma; \Delta \vDash \phi_1 \equiv \phi_2$	prop equality
	$\mid$	$\Gamma; \Delta \vDash a \equiv b : A/R$	definitional equality
	$\mid$	$\vDash \Gamma$	context wellformedness
Jsig	$::=$		
	$\mid$	$\vDash \Sigma$	signature wellformedness
Jann	$::=$		
	$\mid$	$\Gamma \vdash \phi$ ok	prop wellformedness
	$\mid$	$\Gamma \vdash a : A/R$	typing
	$\mid$	$\Gamma; \Delta \vdash \gamma : \phi_1 \sim \phi_2$	coercion between props
	$\mid$	$\Gamma; \Delta \vdash \gamma : A \sim_R B$	coercion between types
	$\mid$	$\vdash \Gamma$	context wellformedness
	$\mid$	$\vdash \Sigma$	signature wellformedness
Jred	$::=$		
	$\mid$	$\Gamma \vdash a \leadsto b/R$	single-step, weak head reduction to values for annotated lang
judgement	$::=$		
	$\mid$	*JSubRole*	
	$\mid$	*JPath*	
	$\mid$	*JValue*	
	$\mid$	*JValueType*	
	$\mid$	*Jconsistent*	
	$\mid$	*Jerased*	
	$\mid$	*JChk*	
	$\mid$	*Jpar*	
	$\mid$	*Jbeta*	
	$\mid$	*Jett*	
	$\mid$	*Jsig*	
	$\mid$	*Jann*	
	$\mid$	*Jred*	
user_syntax	$::=$		
	$\mid$	*tmvar*	
	$\mid$	*covar*	
	$\mid$	*datacon*	
	$\mid$	*const*	
	$\mid$	*index*	
	$\mid$	*role*	
	$\mid$	*relflag*	
	$\mid$	*constraint*	

| *tm*
| *brs*
| *co*
| *role_context*
| *sig_sort*
| *sort*
| *context*
| *sig*
| *available_props*
| *terminals*
| *formula*

$\boxed{R_1 \leq R_2}$ Subroling judgement

$$\frac{}{\mathbf{Nom} \leq R} \quad \textsc{NomBot}$$

$$\frac{}{R \leq \mathbf{Rep}} \quad \textsc{RepTop}$$

$$\frac{}{R \leq R} \quad \textsc{Refl}$$

$$\frac{R_1 \leq R_2 \quad R_2 \leq R_3}{R_1 \leq R_3} \quad \textsc{Trans}$$

$\boxed{\mathsf{Path}_R\ a = F}$ Type headed by constant (partial function)

$$\frac{F \sim a : A/R_1 \in \Sigma_0 \quad \neg(R_1 \leq R)}{\mathsf{Path}_R\ F = F} \quad \textsc{Path\_Const}$$

$$\frac{\mathsf{Path}_R\ a = F}{\mathsf{Path}_R\ (a\ b'^{R_1,\rho}) = F} \quad \textsc{Path\_App}$$

$$\frac{\mathsf{Path}_R\ a = F}{\mathsf{Path}_R\ (a[\bullet]) = F} \quad \textsc{Path\_CApp}$$

$\boxed{\mathsf{Value}_R\ A}$ values

$$\frac{}{\mathsf{Value}_R\ \star} \quad \textsc{Value\_Star}$$

$$\frac{}{\mathsf{Value}_R\ \Pi^\rho x\!:\!A/R_1 \to B} \quad \textsc{Value\_Pi}$$

$$\frac{}{\mathsf{Value}_R\ \forall c\!:\!\phi.B} \quad \textsc{Value\_CPi}$$

$$\frac{}{\mathsf{Value}_R\ \lambda^+ x\!:\!A/R_1.a} \quad \textsc{Value\_AbsRel}$$

$$\frac{}{\mathsf{Value}_R\ \lambda^{R_1,+} x.a} \quad \textsc{Value\_UAbsRel}$$

$$\frac{\mathsf{Value}_R\ a}{\mathsf{Value}_R\ \lambda^{R_1,-} x.a} \quad \textsc{Value\_UAbsIrrel}$$

$$\frac{}{\mathsf{Value}_R\ \Lambda c\!:\!\phi.a} \quad \textsc{Value\_CAbs}$$

$$\frac{}{\mathsf{Value}_R\ \Lambda c.a} \quad \textsc{Value\_UCAbs}$$

$$\frac{\mathsf{Path}_R\ a = F}{\mathsf{Value}_R\ a}\quad\textsc{Value\_Path}$$

$\boxed{\mathsf{ValueType}_R\ A}$ Types with head forms (erased language)

$$\frac{}{\mathsf{ValueType}_R\ \star}\quad\textsc{value\_type\_Star}$$

$$\frac{}{\mathsf{ValueType}_R\ \Pi^\rho x\!:\!A/R_1 \to B}\quad\textsc{value\_type\_Pi}$$

$$\frac{}{\mathsf{ValueType}_R\ \forall c\!:\!\phi.B}\quad\textsc{value\_type\_CPi}$$

$$\frac{\begin{array}{l}\mathsf{Path}_R\ A = F\\ \mathsf{Value}_R\ A\end{array}}{\mathsf{ValueType}_R\ A}\quad\textsc{value\_type\_Path}$$

$\boxed{\mathsf{consistent}_R\ ab}$ (erased) types do not differ in their heads

$$\frac{}{\mathsf{consistent}_R\ \star\star}\quad\textsc{consistent\_a\_Star}$$

$$\frac{}{\mathsf{consistent}_{R'}\ (\Pi^\rho x_1\!:\!A_1/R \to B_1)(\Pi^\rho x_2\!:\!A_2/R \to B_2)}\quad\textsc{consistent\_a\_Pi}$$

$$\frac{}{\mathsf{consistent}_R\ (\forall c_1\!:\!\phi_1.A_1)(\forall c_2\!:\!\phi_2.A_2)}\quad\textsc{consistent\_a\_CPi}$$

$$\frac{\begin{array}{l}\mathsf{Path}_R\ a_1 = F\\ \mathsf{Path}_R\ a_2 = F\end{array}}{\mathsf{consistent}_R\ a_1 a_2}\quad\textsc{consistent\_a\_Path}$$

$$\frac{\neg\mathsf{ValueType}_R\ b}{\mathsf{consistent}_R\ ab}\quad\textsc{consistent\_a\_Step\_R}$$

$$\frac{\neg\mathsf{ValueType}_R\ a}{\mathsf{consistent}_R\ ab}\quad\textsc{consistent\_a\_Step\_L}$$

$\boxed{\Omega \vDash a : R}$

$$\frac{\mathit{uniq}(\Omega)}{\Omega \vDash \Box : R}\quad\textsc{erased\_a\_Bullet}$$

$$\frac{\mathit{uniq}(\Omega)}{\Omega \vDash \star : R}\quad\textsc{erased\_a\_Star}$$

$$\frac{\begin{array}{l}\mathit{uniq}(\Omega)\\ x : R \in \Omega\\ R \leq R_1\end{array}}{\Omega \vDash x : R_1}\quad\textsc{erased\_a\_Var}$$

$$\frac{\Omega, x : R_1 \vDash a : R}{\Omega \vDash (\lambda^{R_1,\rho} x.a) : R}\quad\textsc{erased\_a\_Abs}$$

$$\frac{\begin{array}{l}\Omega \vDash a : R\\ \Omega \vDash b : R_1\end{array}}{\Omega \vDash (a\ b^{R_1,\rho}) : R}\quad\textsc{erased\_a\_App}$$

$$\frac{\begin{array}{l}\Omega \vDash A : R_1\\ \Omega, x : R_1 \vDash B : R\end{array}}{\Omega \vDash (\Pi^\rho x\!:\!A/R_1 \to B) : R}\quad\textsc{erased\_a\_Pi}$$

$$\frac{\begin{array}{l}\Omega \vDash a : R_1\\ \Omega \vDash b : R_1\\ \Omega \vDash A : R_1\\ \Omega \vDash B : R\end{array}}{\Omega \vDash (\forall c\!:\!a \sim_{A/R_1} b.B) : R} \quad \textsc{erased\_a\_CPi}$$

$$\frac{\Omega \vDash b : R}{\Omega \vDash (\Lambda c.b) : R} \quad \textsc{erased\_a\_CAbs}$$

$$\frac{\Omega \vDash a : R}{\Omega \vDash (a[\bullet]) : R} \quad \textsc{erased\_a\_CApp}$$

$$\frac{\begin{array}{l}\mathit{uniq}(\Omega)\\ F \sim a : A/R \in \Sigma_0\end{array}}{\Omega \vDash F : R_1} \quad \textsc{erased\_a\_Fam}$$

$$\frac{\begin{array}{l}\Omega \vDash F : R'\\ \Omega \vDash a : R\\ \Omega \vDash b : R\end{array}}{\Omega \vDash (\mathbf{ifPath}\, R\, F\, a\, b) : R} \quad \textsc{erased\_a\_Pattern}$$

$\boxed{(\rho = +) \vee (x \notin \mathsf{fv}\ A)}$ irrelevant argument check

$$\frac{}{(+ = +) \vee (x \notin \mathsf{fv}\ A)} \quad \textsc{Rho\_Rel}$$

$$\frac{x \notin \mathsf{fv} A}{(- = +) \vee (x \notin \mathsf{fv}\ A)} \quad \textsc{Rho\_IrrRel}$$

$\boxed{\Omega \vDash a \Rightarrow_R b}$ parallel reduction (implicit language)

$$\frac{\Omega \vDash a : R}{\Omega \vDash a \Rightarrow_R a} \quad \textsc{Par\_Refl}$$

$$\frac{\begin{array}{l}\Omega \vDash a \Rightarrow_R (\lambda^{R_1,\rho} x.a')\\ \Omega \vDash b \Rightarrow_{R_1} b'\end{array}}{\Omega \vDash a\ b^{R_1,\rho} \Rightarrow_R a'\{b'/x\}} \quad \textsc{Par\_Beta}$$

$$\frac{\begin{array}{l}\Omega \vDash a \Rightarrow_R a'\\ \Omega \vDash b \Rightarrow_{R_1} b'\end{array}}{\Omega \vDash a\ b^{R_1,\rho} \Rightarrow_R a'\ b'^{R_1,\rho}} \quad \textsc{Par\_App}$$

$$\frac{\Omega \vDash a \Rightarrow_R (\Lambda c.a')}{\Omega \vDash a[\bullet] \Rightarrow_R a'\{\bullet/c\}} \quad \textsc{Par\_CBeta}$$

$$\frac{\Omega \vDash a \Rightarrow_R a'}{\Omega \vDash a[\bullet] \Rightarrow_R a'[\bullet]} \quad \textsc{Par\_CApp}$$

$$\frac{\Omega, x : R_1 \vDash a \Rightarrow_R a'}{\Omega \vDash \lambda^{R_1,\rho} x.a \Rightarrow_R \lambda^{R_1,\rho} x.a'} \quad \textsc{Par\_Abs}$$

$$\frac{\begin{array}{l}\Omega \vDash A \Rightarrow_{R_1} A'\\ \Omega, x : R_1 \vDash B \Rightarrow_R B'\end{array}}{\Omega \vDash \Pi^\rho x\!:\!A/R_1 \to B \Rightarrow_R \Pi^\rho x\!:\!A'/R_1 \to B'} \quad \textsc{Par\_Pi}$$

$$\frac{\Omega \vDash a \Rightarrow_R a'}{\Omega \vDash \Lambda c.a \Rightarrow_R \Lambda c.a'} \quad \textsc{Par\_CAbs}$$

$$\frac{\Omega \vDash A \Rightarrow_{R_1} A' \quad \Omega \vDash a \Rightarrow_{R_1} a' \quad \Omega \vDash b \Rightarrow_{R_1} b' \quad \Omega \vDash B \Rightarrow_R B'}{\Omega \vDash \forall c\!:\!a \sim_{A/R_1} b.B \Rightarrow_R \forall c\!:\!a' \sim_{A'/R_1} b'.B'} \quad \text{PAR\_CPI}$$

$$\frac{F \sim a : A/R_1 \in \Sigma_0 \quad R_1 \leq R \quad \mathit{uniq}(\Omega)}{\Omega \vDash F \Rightarrow_R a} \quad \text{PAR\_AXIOM}$$

$$\frac{\Omega \vDash a \Rightarrow_R a' \quad \Omega \vDash b \Rightarrow_R b'}{\Omega \vDash \mathbf{ifPath}\, R\, F\ a\ b \Rightarrow_R \mathbf{ifPath}\, R\, F\ a'\ b'} \quad \text{PAR\_PATTERN}$$

$$\frac{\Omega \vDash a \Rightarrow_R a' \quad \mathsf{Path}_R\ a' = F}{\Omega \vDash \mathbf{ifPath}\, R\, F\ a\ b \Rightarrow_R a'} \quad \text{PAR\_PATTERNTRUE}$$

$$\frac{\Omega \vDash a \Rightarrow_R a' \quad \Omega \vDash b \Rightarrow_R b' \quad \mathsf{Value}_R\ a' \quad \neg(\mathsf{Path}_R\ a' = F)}{\Omega \vDash \mathbf{ifPath}\, R\, F\ a\ b \Rightarrow_R b'} \quad \text{PAR\_PATTERNFALSE}$$

$\boxed{\Omega \vdash a \Rightarrow^*_R b}$ multistep parallel reduction

$$\frac{}{\Omega \vdash a \Rightarrow^*_R a} \quad \text{MP\_REFL}$$

$$\frac{\Omega \vDash a \Rightarrow_R b \quad \Omega \vdash b \Rightarrow^*_R a'}{\Omega \vdash a \Rightarrow^*_R a'} \quad \text{MP\_STEP}$$

$\boxed{\Omega \vdash a \Leftrightarrow_R b}$ parallel reduction to a common term

$$\frac{\Omega \vdash a_1 \Rightarrow^*_R b \quad \Omega \vdash a_2 \Rightarrow^*_R b}{\Omega \vdash a_1 \Leftrightarrow_R a_2} \quad \text{JOIN}$$

$\boxed{\vDash a > b/R}$ primitive reductions on erased terms

$$\frac{\mathsf{Value}_{R_1}\ (\lambda^{R,\rho} x.v)}{\vDash (\lambda^{R,\rho} x.v)\ b^{R,\rho} > v\{b/x\}/R_1} \quad \text{BETA\_APPABS}$$

$$\frac{}{\vDash (\Lambda c.a')[\bullet] > a'\{\bullet/c\}/R} \quad \text{BETA\_CAPPCABS}$$

$$\frac{F \sim a : A/R \in \Sigma_0 \quad R \leq R_1}{\vDash F > a/R_1} \quad \text{BETA\_AXIOM}$$

$$\frac{\mathsf{Path}_R\ a = F}{\vDash \mathbf{ifPath}\, R\, F\ a\ b > a/R} \quad \text{BETA\_PATTERNTRUE}$$

$$\frac{\mathsf{Value}_R\ a \quad \neg(\mathsf{Path}_R\ a = F)}{\vDash \mathbf{ifPath}\, R\, F\ a\ b > b/R} \quad \text{BETA\_PATTERNFALSE}$$

$\boxed{\vDash a \leadsto b/R}$ single-step head reduction for implicit language

$$\frac{\vDash a \leadsto a'/R_1}{\vDash \lambda^{R,-}x.a \leadsto \lambda^{R,-}x.a'/R_1} \quad \text{E\_AbsTerm}$$

$$\frac{\vDash a \leadsto a'/R_1}{\vDash a\ b^{R,\rho} \leadsto a'\ b^{R,\rho}/R_1} \quad \text{E\_AppLeft}$$

$$\frac{\vDash a \leadsto a'/R}{\vDash a[\bullet] \leadsto a'[\bullet]/R} \quad \text{E\_CAppLeft}$$

$$\frac{\vDash a \leadsto a'/R}{\vDash \textbf{ifPath}\, R\, F\, a\, b \leadsto \textbf{ifPath}\, R\, F\, a'\, b/R} \quad \text{E\_Pattern}$$

$$\frac{\vDash a > b/R}{\vDash a \leadsto b/R} \quad \text{E\_Prim}$$

$\boxed{\vDash a \leadsto^* b/R}$ multistep reduction

$$\frac{}{\vDash a \leadsto^* a/R} \quad \text{Equal}$$

$$\frac{\begin{array}{c}\vDash a \leadsto b/R \\ \vDash b \leadsto^* a'/R\end{array}}{\vDash a \leadsto^* a'/R} \quad \text{Step}$$

$\boxed{\Gamma \vDash \phi\ \mathsf{ok}}$ Prop wellformedness

$$\frac{\begin{array}{c}\Gamma \vDash a : A/R \\ \Gamma \vDash b : A/R \\ \Gamma \vDash A : \star/R\end{array}}{\Gamma \vDash a \sim_{A/R} b\ \ \mathsf{ok}} \quad \text{E\_Wff}$$

$\boxed{\Gamma \vDash a : A/R}$ typing

$$\frac{\begin{array}{l}R_1 \leq R_2 \\ \Gamma \vDash a : A/R_1\end{array}}{\Gamma \vDash a : A/R_2} \quad \text{E\_SubRole}$$

$$\frac{\vDash \Gamma}{\Gamma \vDash \star : \star/R} \quad \text{E\_Star}$$

$$\frac{\begin{array}{l}\vDash \Gamma \\ x : A/R\ \in\ \Gamma\end{array}}{\Gamma \vDash x : A/R} \quad \text{E\_Var}$$

$$\frac{\begin{array}{l}\Gamma, x : A/R \vDash B : \star/R' \\ \Gamma \vDash A : \star/R\end{array}}{\Gamma \vDash \Pi^\rho x\!:\!A/R \to B : \star/R'} \quad \text{E\_Pi}$$

$$\frac{\begin{array}{l}\Gamma, x : A/R \vDash a : B/R' \\ \Gamma \vDash A : \star/R \\ (\rho = +) \vee (x \notin \mathsf{fv}\ a)\end{array}}{\Gamma \vDash \lambda^{R,\rho}x.a : (\Pi^\rho x\!:\!A/R \to B)/R'} \quad \text{E\_Abs}$$

$$\frac{\begin{array}{l}\Gamma \vDash b : \Pi^+ x\!:\!A/R \to B/R' \\ \Gamma \vDash a : A/R\end{array}}{\Gamma \vDash b\ \ a^{R,+} : B\{a/x\}/R'} \quad \text{E\_App}$$

$$\frac{\Gamma \vDash b : \Pi^{-}x\!:\!A/R \to B/R' \quad \Gamma \vDash a : A/R}{\Gamma \vDash b\ \Box^{R,-} : B\{a/x\}/R'}\ \text{E\_IApp}$$

$$\frac{\Gamma \vDash a : A/R \quad \Gamma;\widetilde{\Gamma} \vDash A \equiv B : \star/\mathbf{Rep} \quad \Gamma \vDash B : \star/R}{\Gamma \vDash a : B/R}\ \text{E\_Conv}$$

$$\frac{\Gamma, c : \phi \vDash B : \star/R \quad \Gamma \vDash \phi\ \mathsf{ok}}{\Gamma \vDash \forall c\!:\!\phi.B : \star/R}\ \text{E\_CPi}$$

$$\frac{\Gamma, c : \phi \vDash a : B/R \quad \Gamma \vDash \phi\ \mathsf{ok}}{\Gamma \vDash \Lambda c.a : \forall c\!:\!\phi.B/R}\ \text{E\_CAbs}$$

$$\frac{\Gamma \vDash a_1 : \forall c\!:\!(a \sim_{A/R} b).B_1/R' \quad \Gamma;\widetilde{\Gamma} \vDash a \equiv b : A/R}{\Gamma \vDash a_1[\bullet] : B_1\{\bullet/c\}/R'}\ \text{E\_CApp}$$

$$\frac{\vDash \Gamma \quad F \sim a : A/R \in \Sigma_0 \quad \varnothing \vDash A : \star/R_1}{\Gamma \vDash F : A/R_1}\ \text{E\_Fam}$$

$$\frac{\Gamma \vDash F : \star/R' \quad \Gamma \vDash a : A/R \quad \Gamma \vDash b : A/R}{\Gamma \vDash \mathbf{ifPath}\, R\, F\ a\ b : A/R}\ \text{E\_Pat}$$

$\boxed{\Gamma;\Delta \vDash \phi_1 \equiv \phi_2}$ prop equality

$$\frac{\Gamma;\Delta \vDash A_1 \equiv A_2 : A/R \quad \Gamma;\Delta \vDash B_1 \equiv B_2 : A/R}{\Gamma;\Delta \vDash A_1 \sim_{A/R} B_1 \equiv A_2 \sim_{A/R} B_2}\ \text{E\_PropCong}$$

$$\frac{\Gamma;\Delta \vDash A \equiv B : \star/R \quad \Gamma \vDash A_1 \sim_{A/R} A_2\ \mathsf{ok} \quad \Gamma \vDash A_1 \sim_{B/R} A_2\ \mathsf{ok}}{\Gamma;\Delta \vDash A_1 \sim_{A/R} A_2 \equiv A_1 \sim_{B/R} A_2}\ \text{E\_IsoConv}$$

$$\frac{\Gamma;\Delta \vDash \forall c\!:\!(a_1 \sim_{A/R_1} a_2).B_1 \equiv \forall c\!:\!(b_1 \sim_{B/R_2} b_2).B_2 : \star/R'}{\Gamma;\Delta \vDash a_1 \sim_{A/R_1} a_2 \equiv b_1 \sim_{B/R_2} b_2}\ \text{E\_CPiFst}$$

$\boxed{\Gamma;\Delta \vDash a \equiv b : A/R}$ definitional equality

$$\frac{\vDash \Gamma \quad c : (a \sim_{A/R} b) \in \Gamma \quad c \in \Delta}{\Gamma;\Delta \vDash a \equiv b : A/R}\ \text{E\_Assn}$$

$$\frac{\Gamma \vDash a : A/R}{\Gamma;\Delta \vDash a \equiv a : A/R}\ \text{E\_Refl}$$

$$\frac{\Gamma;\Delta \vDash b \equiv a : A/R}{\Gamma;\Delta \vDash a \equiv b : A/R}\ \text{E\_Sym}$$

$$\frac{\begin{array}{l}\Gamma; \Delta \vDash a \equiv a_1 : A/R \\ \Gamma; \Delta \vDash a_1 \equiv b : A/R\end{array}}{\Gamma; \Delta \vDash a \equiv b : A/R} \quad \text{E\_TRANS}$$

$$\frac{\begin{array}{l}\Gamma; \Delta \vDash a \equiv b : A/R_1 \\ R_1 \leq R_2\end{array}}{\Gamma; \Delta \vDash a \equiv b : A/R_2} \quad \text{E\_SUB}$$

$$\frac{\begin{array}{l}\Gamma \vDash a_1 : B/R \\ \Gamma \vDash a_2 : B/R \\ \vDash a_1 > a_2/R\end{array}}{\Gamma; \Delta \vDash a_1 \equiv a_2 : B/R} \quad \text{E\_BETA}$$

$$\frac{\begin{array}{l}\Gamma; \Delta \vDash A_1 \equiv A_2 : \star/R \\ \Gamma, x : A_1/R; \Delta \vDash B_1 \equiv B_2 : \star/R' \\ \Gamma \vDash A_1 : \star/R \\ \Gamma \vDash \Pi^\rho x\!:\!A_1/R \to B_1 : \star/R' \\ \Gamma \vDash \Pi^\rho x\!:\!A_2/R \to B_2 : \star/R'\end{array}}{\Gamma; \Delta \vDash (\Pi^\rho x\!:\!A_1/R \to B_1) \equiv (\Pi^\rho x\!:\!A_2/R \to B_2) : \star/R'} \quad \text{E\_PICONG}$$

$$\frac{\begin{array}{l}\Gamma, x : A_1/R; \Delta \vDash b_1 \equiv b_2 : B/R' \\ \Gamma \vDash A_1 : \star/R \\ (\rho = +) \vee (x \notin \mathsf{fv}\ b_1) \\ (\rho = +) \vee (x \notin \mathsf{fv}\ b_2)\end{array}}{\Gamma; \Delta \vDash (\lambda^{R,\rho} x.b_1) \equiv (\lambda^{R,\rho} x.b_2) : (\Pi^\rho x\!:\!A_1/R \to B)/R'} \quad \text{E\_ABSCONG}$$

$$\frac{\begin{array}{l}\Gamma; \Delta \vDash a_1 \equiv b_1 : (\Pi^+ x\!:\!A/R \to B)/R' \\ \Gamma; \Delta \vDash a_2 \equiv b_2 : A/R\end{array}}{\Gamma; \Delta \vDash a_1\ {a_2}^{R,+} \equiv b_1\ {b_2}^{R,+} : (B\{a_2/x\})/R'} \quad \text{E\_APPCONG}$$

$$\frac{\begin{array}{l}\Gamma; \Delta \vDash a_1 \equiv b_1 : (\Pi^- x\!:\!A/R \to B)/R' \\ \Gamma \vDash a : A/R\end{array}}{\Gamma; \Delta \vDash a_1\ \square^{R,-} \equiv b_1\ \square^{R,-} : (B\{a/x\})/R'} \quad \text{E\_IAPPCONG}$$

$$\frac{\Gamma; \Delta \vDash \Pi^\rho x\!:\!A_1/R \to B_1 \equiv \Pi^\rho x\!:\!A_2/R \to B_2 : \star/R'}{\Gamma; \Delta \vDash A_1 \equiv A_2 : \star/R} \quad \text{E\_PIFST}$$

$$\frac{\begin{array}{l}\Gamma; \Delta \vDash \Pi^\rho x\!:\!A_1/R \to B_1 \equiv \Pi^\rho x\!:\!A_2/R \to B_2 : \star/R' \\ \Gamma; \Delta \vDash a_1 \equiv a_2 : A_1/R\end{array}}{\Gamma; \Delta \vDash B_1\{a_1/x\} \equiv B_2\{a_2/x\} : \star/R'} \quad \text{E\_PISND}$$

$$\frac{\begin{array}{l}\Gamma; \Delta \vDash a_1 \sim_{A_1/R} b_1 \equiv a_2 \sim_{A_2/R} b_2 \\ \Gamma, c : a_1 \sim_{A_1/R} b_1; \Delta \vDash A \equiv B : \star/R' \\ \Gamma \vDash a_1 \sim_{A_1/R} b_1\ \ \mathsf{ok} \\ \Gamma \vDash \forall c\!:\!a_1 \sim_{A_1/R} b_1.A : \star/R' \\ \Gamma \vDash \forall c\!:\!a_2 \sim_{A_2/R} b_2.B : \star/R'\end{array}}{\Gamma; \Delta \vDash \forall c\!:\!a_1 \sim_{A_1/R} b_1.A \equiv \forall c\!:\!a_2 \sim_{A_2/R} b_2.B : \star/R'} \quad \text{E\_CPICONG}$$

$$\frac{\begin{array}{l}\Gamma, c : \phi_1; \Delta \vDash a \equiv b : B/R \\ \Gamma \vDash \phi_1\ \ \mathsf{ok}\end{array}}{\Gamma; \Delta \vDash (\Lambda c.a) \equiv (\Lambda c.b) : \forall c\!:\!\phi_1.B/R} \quad \text{E\_CABSCONG}$$

$$\frac{\begin{array}{l}\Gamma; \Delta \vDash a_1 \equiv b_1 : (\forall c\!:\!(a \sim_{A/R} b).B)/R' \\ \Gamma; \widetilde{\Gamma} \vDash a \equiv b : A/R\end{array}}{\Gamma; \Delta \vDash a_1[\bullet] \equiv b_1[\bullet] : (B\{\bullet/c\})/R'} \quad \text{E\_CAPPCONG}$$

$$\frac{\begin{array}{l}\Gamma;\Delta \vDash \forall c{:}(a_1 \sim_{A/R} a_2).B_1 \equiv \forall c{:}(a_1' \sim_{A'/R'} a_2').B_2 : \star/R_0\\ \Gamma;\widetilde{\Gamma} \vDash a_1 \equiv a_2 : A/R\\ \Gamma;\widetilde{\Gamma} \vDash a_1' \equiv a_2' : A'/R'\end{array}}{\Gamma;\Delta \vDash B_1\{\bullet/c\} \equiv B_2\{\bullet/c\} : \star/R_0} \quad \textsc{E\_CPiSnd}$$

$$\frac{\begin{array}{l}\Gamma;\Delta \vDash a \equiv b : A/R\\ \Gamma;\Delta \vDash a \sim_{A/R} b \equiv a' \sim_{A'/R'} b'\end{array}}{\Gamma;\Delta \vDash a' \equiv b' : A'/R'} \quad \textsc{E\_Cast}$$

$$\frac{\begin{array}{l}\Gamma;\Delta \vDash a \equiv b : A/R\\ \Gamma;\widetilde{\Gamma} \vDash A \equiv B : \star/\mathbf{Rep}\\ \Gamma \vDash B : \star/R\end{array}}{\Gamma;\Delta \vDash a \equiv b : B/R} \quad \textsc{E\_EqConv}$$

$$\frac{\Gamma;\Delta \vDash a \sim_{A/R_1} b \equiv a' \sim_{A'/R_1} b'}{\Gamma;\Delta \vDash A \equiv A' : \star/R_1} \quad \textsc{E\_IsoSnd}$$

$$\frac{\begin{array}{l}\Gamma \vDash F : \star/R'\\ \Gamma;\Delta \vDash a \equiv a' : A/R\\ \Gamma;\Delta \vDash b \equiv b' : A/R\end{array}}{\Gamma;\Delta \vDash \mathbf{ifPath}\, R\, F\, a\, b \equiv \mathbf{ifPath}\, R\, F\, a'\, b' : A/R} \quad \textsc{E\_PatCong}$$

$\boxed{\vDash \Gamma}$ context wellformedness

$$\frac{}{\vDash \varnothing} \quad \textsc{E\_Empty}$$

$$\frac{\begin{array}{l}\vDash \Gamma\\ \Gamma \vDash A : \star/R\\ x \notin \mathsf{dom}\,\Gamma\end{array}}{\vDash \Gamma, x : A/R} \quad \textsc{E\_ConsTm}$$

$$\frac{\begin{array}{l}\vDash \Gamma\\ \Gamma \vDash \phi\ \mathsf{ok}\\ c \notin \mathsf{dom}\,\Gamma\end{array}}{\vDash \Gamma, c : \phi} \quad \textsc{E\_ConsCo}$$

$\boxed{\vDash \Sigma}$ signature wellformedness

$$\frac{}{\vDash \varnothing} \quad \textsc{Sig\_Empty}$$

$$\frac{\begin{array}{l}\vDash \Sigma\\ \varnothing \vDash a : A/R'\\ F \notin \mathsf{dom}\,\Sigma\end{array}}{\vDash \Sigma \cup \{F \sim a : A/R'\}} \quad \textsc{Sig\_ConsAx}$$

$\boxed{\Gamma \vdash \phi\ \mathsf{ok}}$ prop wellformedness

$$\frac{\begin{array}{l}\Gamma \vdash a : A/R\\ \Gamma \vdash b : B/R\\ |A|_R = |B|_R\end{array}}{\Gamma \vdash a \sim_{A/R} b\ \mathsf{ok}} \quad \textsc{An\_Wff}$$

$\boxed{\Gamma \vdash a : A/R}$ typing

$$\frac{\vdash \Gamma}{\Gamma \vdash \star : \star/R} \quad \textsc{An\_Star}$$

$$\frac{\begin{array}{l}\vdash \Gamma \\ x : A/R \in \Gamma\end{array}}{\Gamma \vdash x : A/R}\ \textsc{An\_Var}$$

$$\frac{\begin{array}{l}\Gamma, x : A/R \vdash B : \star/R' \\ \Gamma \vdash A : \star/R\end{array}}{\Gamma \vdash \Pi^{\rho} x\!:\!A/R \to B : \star/R'}\ \textsc{An\_Pi}$$

$$\frac{\begin{array}{l}\Gamma \vdash A : \star/R \\ \Gamma, x : A/R \vdash a : B/R' \\ (\rho = +) \vee (x \notin \mathsf{fv}\ |a|_{R'}) \\ R \leq R'\end{array}}{\Gamma \vdash \lambda^{\rho} x\!:\!A/R.a : (\Pi^{\rho} x\!:\!A/R \to B)/R'}\ \textsc{An\_Abs}$$

$$\frac{\begin{array}{l}\Gamma \vdash b : (\Pi^{\rho} x\!:\!A/R \to B)/R' \\ \Gamma \vdash a : A/R\end{array}}{\Gamma \vdash b\ \ a^{R,\rho} : (B\{a/x\})/R'}\ \textsc{An\_App}$$

$$\frac{\begin{array}{l}\Gamma \vdash a : A/R \\ \Gamma; \widetilde{\Gamma} \vdash \gamma : A \sim_R B \\ \Gamma \vdash B : \star/R\end{array}}{\Gamma \vdash a \triangleright_R \gamma : B/R}\ \textsc{An\_Conv}$$

$$\frac{\begin{array}{l}\Gamma \vdash \phi\ \ \mathsf{ok} \\ \Gamma, c : \phi \vdash B : \star/R\end{array}}{\Gamma \vdash \forall c\!:\!\phi.B : \star/R}\ \textsc{An\_CPi}$$

$$\frac{\begin{array}{l}\Gamma \vdash \phi\ \ \mathsf{ok} \\ \Gamma, c : \phi \vdash a : B/R\end{array}}{\Gamma \vdash \Lambda c\!:\!\phi.a : (\forall c\!:\!\phi.B)/R}\ \textsc{An\_CAbs}$$

$$\frac{\begin{array}{l}\Gamma \vdash a_1 : (\forall c\!:\!a \sim_{A_1/R} b.B)/R' \\ \Gamma; \widetilde{\Gamma} \vdash \gamma : a \sim_R b\end{array}}{\Gamma \vdash a_1[\gamma] : B\{\gamma/c\}/R'}\ \textsc{An\_CApp}$$

$$\frac{\begin{array}{l}\vdash \Gamma \\ F\ \sim a : A/R \in \Sigma_1 \\ \varnothing \vdash A : \star/R_1\end{array}}{\Gamma \vdash F : A/R_1}\ \textsc{An\_Fam}$$

$$\frac{\begin{array}{l}R_1 \leq R_2 \\ \Gamma \vdash a : A/R_1\end{array}}{\Gamma \vdash \mathbf{sub}\, R_1\ a : A/R_2}\ \textsc{An\_SubRole}$$

$\boxed{\Gamma; \Delta \vdash \gamma : \phi_1 \sim \phi_2}$ coercion between props

$$\frac{\begin{array}{l}\Gamma; \Delta \vdash \gamma_1 : A_1 \sim_R A_2 \\ \Gamma; \Delta \vdash \gamma_2 : B_1 \sim_R B_2 \\ \Gamma \vdash A_1 \sim_{A/R} B_1\ \ \mathsf{ok} \\ \Gamma \vdash A_2 \sim_{A/R} B_2\ \ \mathsf{ok}\end{array}}{\Gamma; \Delta \vdash (\gamma_1 \sim_A \gamma_2) : (A_1 \sim_{A/R} B_1) \sim (A_2 \sim_{A/R} B_2)}\ \textsc{An\_PropCong}$$

$$\frac{\Gamma; \Delta \vdash \gamma : \forall c\!:\!\phi_1.A_2 \sim_R \forall c\!:\!\phi_2.B_2}{\Gamma; \Delta \vdash \mathbf{cpiFst}\, \gamma : \phi_1 \sim \phi_2}\ \textsc{An\_CPiFst}$$

$$\frac{\Gamma; \Delta \vdash \gamma : \phi_1 \sim \phi_2}{\Gamma; \Delta \vdash \mathbf{sym}\, \gamma : \phi_2 \sim \phi_1}\ \textsc{An\_IsoSym}$$

$$\frac{\begin{array}{l}\Gamma; \Delta \vdash \gamma : A \sim_R B \\ \Gamma \vdash a_1 \sim_{A/R} a_2 \;\; \mathsf{ok} \\ \Gamma \vdash a_1' \sim_{B/R} a_2' \;\; \mathsf{ok} \\ |a_1|_R = |a_1'|_R \\ |a_2|_R = |a_2'|_R\end{array}}{\Gamma; \Delta \vdash \mathbf{conv}\ (a_1 \sim_{A/R} a_2) \sim_\gamma (a_1' \sim_{B/R} a_2') : (a_1 \sim_{A/R} a_2) \sim (a_1' \sim_{B/R} a_2')} \quad \textsc{An\_IsoConv}$$

$\boxed{\Gamma; \Delta \vdash \gamma : A \sim_R B}$ coercion between types

$$\frac{\begin{array}{l}\vdash \Gamma \\ c : a \sim_{A/R} b \ \in\ \Gamma \\ c \ \in\ \Delta\end{array}}{\Gamma; \Delta \vdash c : a \sim_R b} \quad \textsc{An\_Assn}$$

$$\frac{\Gamma \vdash a : A/R}{\Gamma; \Delta \vdash \mathbf{refl}\, a : a \sim_R a} \quad \textsc{An\_Refl}$$

$$\frac{\begin{array}{l}\Gamma \vdash a : A/R \\ \Gamma \vdash b : B/R \\ |a|_R = |b|_R \\ \Gamma; \widetilde{\Gamma} \vdash \gamma : A \sim_R B\end{array}}{\Gamma; \Delta \vdash (a \mathrel{|{=}|_\gamma} b) : a \sim_R b} \quad \textsc{An\_EraseEq}$$

$$\frac{\begin{array}{l}\Gamma \vdash b : B/R \\ \Gamma \vdash a : A/R \\ \Gamma; \widetilde{\Gamma} \vdash \gamma_1 : B \sim_R A \\ \Gamma; \Delta \vdash \gamma : b \sim_R a\end{array}}{\Gamma; \Delta \vdash \mathbf{sym}\, \gamma : a \sim_R b} \quad \textsc{An\_Sym}$$

$$\frac{\begin{array}{l}\Gamma; \Delta \vdash \gamma_1 : a \sim_R a_1 \\ \Gamma; \Delta \vdash \gamma_2 : a_1 \sim_R b \\ \Gamma \vdash a : A/R \\ \Gamma \vdash a_1 : A_1/R \\ \Gamma; \widetilde{\Gamma} \vdash \gamma_3 : A \sim_R A_1\end{array}}{\Gamma; \Delta \vdash (\gamma_1; \gamma_2) : a \sim_R b} \quad \textsc{An\_Trans}$$

$$\frac{\begin{array}{l}\Gamma \vdash a_1 : B_0/R \\ \Gamma \vdash a_2 : B_1/R \\ |B_0|_R = |B_1|_R \\ \vDash |a_1|_R > |a_2|_R/R\end{array}}{\Gamma; \Delta \vdash \mathbf{red}\, a_1\ a_2 : a_1 \sim_R a_2} \quad \textsc{An\_Beta}$$

$$\frac{\begin{array}{l}\Gamma; \Delta \vdash \gamma_1 : A_1 \sim_{R'} A_2 \\ \Gamma, x : A_1/R; \Delta \vdash \gamma_2 : B_1 \sim_{R'} B_2 \\ B_3 = B_2\{x \triangleright_{R'} \mathbf{sym}\, \gamma_1/x\} \\ \Gamma \vdash \Pi^\rho x\!:\!A_1/R \to B_1 : \star/R' \\ \Gamma \vdash \Pi^\rho x\!:\!A_1/R \to B_2 : \star/R' \\ \Gamma \vdash \Pi^\rho x\!:\!A_2/R \to B_3 : \star/R' \\ R \leq R'\end{array}}{\Gamma; \Delta \vdash \Pi^{R,\rho} x\!:\!\gamma_1.\gamma_2 : (\Pi^\rho x\!:\!A_1/R \to B_1) \sim_{R'} (\Pi^\rho x\!:\!A_2/R \to B_3)} \quad \textsc{An\_PiCong}$$

$$\frac{\begin{array}{l}\Gamma;\Delta\vdash\gamma_1 : A_1 \sim_R A_2\\ \Gamma, x : A_1/R;\Delta\vdash\gamma_2 : b_1 \sim_{R'} b_2\\ b_3 = b_2\{x \triangleright_{R'} \mathbf{sym}\,\gamma_1/x\}\\ \Gamma\vdash A_1 : \star/R\\ \Gamma\vdash A_2 : \star/R\\ (\rho = +)\vee(x \notin \mathsf{fv}\ |b_1|_{R'})\\ (\rho = +)\vee(x \notin \mathsf{fv}\ |b_3|_{R'})\\ \Gamma\vdash(\lambda^\rho x\!:\!A_1/R.b_2) : B/R'\\ R\leq R'\end{array}}{\Gamma;\Delta\vdash(\lambda^{R,\rho}x\!:\!\gamma_1.\gamma_2) : (\lambda^\rho x\!:\!A_1/R.b_1) \sim_{R'} (\lambda^\rho x\!:\!A_2/R.b_3)}\quad\textsc{An\_AbsCong}$$

$$\frac{\begin{array}{l}\Gamma;\Delta\vdash\gamma_1 : a_1 \sim_{R'} b_1\\ \Gamma;\Delta\vdash\gamma_2 : a_2 \sim_R b_2\\ \Gamma\vdash a_1\ \ {a_2}^{R,\rho} : A/R'\\ \Gamma\vdash b_1\ \ {b_2}^{R,\rho} : B/R'\\ \Gamma;\widetilde{\Gamma}\vdash\gamma_3 : A \sim_{R'} B\end{array}}{\Gamma;\Delta\vdash\gamma_1\ \ \gamma_2^{R,\rho} : a_1\ \ {a_2}^{R,\rho} \sim_{R'} b_1\ \ {b_2}^{R,\rho}}\quad\textsc{An\_AppCong}$$

$$\frac{\Gamma;\Delta\vdash\gamma : \Pi^\rho x\!:\!A_1/R \to B_1 \sim_{R'} \Pi^\rho x\!:\!A_2/R \to B_2}{\Gamma;\Delta\vdash\mathbf{piFst}\,\gamma : A_1 \sim_R A_2}\quad\textsc{An\_PiFst}$$

$$\frac{\begin{array}{l}\Gamma;\Delta\vdash\gamma_1 : \Pi^\rho x\!:\!A_1/R \to B_1 \sim_{R'} \Pi^\rho x\!:\!A_2/R \to B_2\\ \Gamma;\Delta\vdash\gamma_2 : a_1 \sim_R a_2\\ \Gamma\vdash a_1 : A_1/R\\ \Gamma\vdash a_2 : A_2/R\end{array}}{\Gamma;\Delta\vdash\gamma_1@\gamma_2 : B_1\{a_1/x\} \sim_{R'} B_2\{a_2/x\}}\quad\textsc{An\_PiSnd}$$

$$\frac{\begin{array}{l}\Gamma;\Delta\vdash\gamma_1 : a_1 \sim_{A_1/R} b_1 \sim a_2 \sim_{A_2/R} b_2\\ \Gamma, c : a_1 \sim_{A_1/R} b_1;\Delta\vdash\gamma_3 : B_1 \sim_{R'} B_2\\ B_3 = B_2\{c \triangleright_{R'} \mathbf{sym}\,\gamma_1/c\}\\ \Gamma\vdash\forall c\!:\!a_1 \sim_{A_1/R} b_1.B_1 : \star/R'\\ \Gamma\vdash\forall c\!:\!a_2 \sim_{A_2/R} b_2.B_3 : \star/R'\\ \Gamma\vdash\forall c\!:\!a_1 \sim_{A_1/R} b_1.B_2 : \star/R'\end{array}}{\Gamma;\Delta\vdash(\forall c\!:\!\gamma_1.\gamma_3) : (\forall c\!:\!a_1 \sim_{A_1/R} b_1.B_1) \sim_R (\forall c\!:\!a_2 \sim_{A_2/R} b_2.B_3)}\quad\textsc{An\_CPiCong}$$

$$\frac{\begin{array}{l}\Gamma;\Delta\vdash\gamma_1 : b_0 \sim_{A_1/R} b_1 \sim b_2 \sim_{A_2/R} b_3\\ \Gamma, c : b_0 \sim_{A_1/R} b_1;\Delta\vdash\gamma_3 : a_1 \sim_{R'} a_2\\ a_3 = a_2\{c \triangleright_{R'} \mathbf{sym}\,\gamma_1/c\}\\ \Gamma\vdash(\Lambda c\!:\!b_0 \sim_{A_1/R} b_1.a_1) : \forall c\!:\!b_0 \sim_{A_1/R} b_1.B_1/R'\\ \Gamma\vdash(\Lambda c\!:\!b_0 \sim_{A_1/R} b_1.a_2) : B/R'\\ \Gamma\vdash(\Lambda c\!:\!b_2 \sim_{A_2/R} b_3.a_3) : \forall c\!:\!b_2 \sim_{A_2/R} b_3.B_2/R'\\ \Gamma;\widetilde{\Gamma}\vdash\gamma_4 : \forall c\!:\!b_0 \sim_{A_1/R} b_1.B_1 \sim_{R'} \forall c\!:\!\phi_2.B_2\end{array}}{\Gamma;\Delta\vdash(\lambda c\!:\!\gamma_1.\gamma_3@\gamma_4) : (\Lambda c\!:\!b_0 \sim_{A_1/R} b_1.a_1) \sim_{R'} (\Lambda c\!:\!b_2 \sim_{A_2/R} b_3.a_3)}\quad\textsc{An\_CAbsCong}$$

$$\frac{\begin{array}{l}\Gamma;\Delta\vdash\gamma_1 : a_1 \sim_R b_1\\ \Gamma;\widetilde{\Gamma}\vdash\gamma_2 : a_2 \sim_{R'} b_2\\ \Gamma;\widetilde{\Gamma}\vdash\gamma_3 : a_3 \sim_{R'} b_3\\ \Gamma\vdash a_1[\gamma_2] : A/R\\ \Gamma\vdash b_1[\gamma_3] : B/R\\ \Gamma;\widetilde{\Gamma}\vdash\gamma_4 : A \sim_R B\end{array}}{\Gamma;\Delta\vdash\gamma_1(\gamma_2,\gamma_3) : a_1[\gamma_2] \sim_R b_1[\gamma_3]}\quad\textsc{An\_CAppCong}$$

$$\frac{\begin{array}{l}\Gamma; \Delta \vdash \gamma_1 : (\forall c_1 \!:\! a \sim_{A/R} a'.B_1) \sim_{R_0} (\forall c_2 \!:\! b \sim_{B/R'} b'.B_2) \\ \Gamma; \widetilde{\Gamma} \vdash \gamma_2 : a \sim_R a' \\ \Gamma; \widetilde{\Gamma} \vdash \gamma_3 : b \sim_{R'} b'\end{array}}{\Gamma; \Delta \vdash \gamma_1 @(\gamma_2 \sim \gamma_3) : B_1\{\gamma_2/c_1\} \sim_{R_0} B_2\{\gamma_3/c_2\}} \quad \textsc{An\_CPiSnd}$$

$$\frac{\begin{array}{l}\Gamma; \Delta \vdash \gamma_1 : a \sim_{R_1} a' \\ \Gamma; \Delta \vdash \gamma_2 : a \sim_{A/R_1} a' \sim b \sim_{B/R_1} b'\end{array}}{\Gamma; \Delta \vdash \gamma_1 \triangleright_{R_1} \gamma_2 : b \sim_{R_1} b'} \quad \textsc{An\_Cast}$$

$$\frac{\Gamma; \Delta \vdash \gamma : (a \sim_{A/R} a') \sim (b \sim_{B/R} b')}{\Gamma; \Delta \vdash \mathbf{isoSnd}\, \gamma : A \sim_R B} \quad \textsc{An\_IsoSnd}$$

$$\frac{\begin{array}{l}\Gamma; \Delta \vdash \gamma : a \sim_{R_1} b \\ R_1 \leq R_2\end{array}}{\Gamma; \Delta \vdash \mathbf{sub}\, \gamma : a \sim_{R_2} b} \quad \textsc{An\_Sub}$$

$\boxed{\vdash \Gamma}$ context wellformedness

$$\frac{}{\vdash \varnothing} \quad \textsc{An\_Empty}$$

$$\frac{\begin{array}{l}\vdash \Gamma \\ \Gamma \vdash A : \star/R \\ x \notin \mathsf{dom}\,\Gamma\end{array}}{\vdash \Gamma, x : A/R} \quad \textsc{An\_ConsTm}$$

$$\frac{\begin{array}{l}\vdash \Gamma \\ \Gamma \vdash \phi \;\; \mathsf{ok} \\ c \notin \mathsf{dom}\,\Gamma\end{array}}{\vdash \Gamma, c : \phi} \quad \textsc{An\_ConsCo}$$

$\boxed{\vdash \Sigma}$ signature wellformedness

$$\frac{}{\vdash \varnothing} \quad \textsc{An\_Sig\_Empty}$$

$$\frac{\begin{array}{l}\vdash \Sigma \\ \varnothing \vdash A : \star/R \\ \varnothing \vdash a : A/R \\ F \notin \mathsf{dom}\,\Sigma\end{array}}{\vdash \Sigma \cup \{F \sim a : A/R\}} \quad \textsc{An\_Sig\_ConsAx}$$

$\boxed{\Gamma \vdash a \leadsto b/R}$ single-step, weak head reduction to values for annotated language

$$\frac{\Gamma \vdash a \leadsto a'/R_1}{\Gamma \vdash a\ b^{R,\rho} \leadsto a'\ b^{R,\rho}/R_1} \quad \textsc{An\_AppLeft}$$

$$\frac{\mathsf{Value}_R\ (\lambda^\rho x \!:\! A/R.w)}{\Gamma \vdash (\lambda^\rho x \!:\! A/R.w)\ a^{R,\rho} \leadsto w\{a/x\}/R} \quad \textsc{An\_AppAbs}$$

$$\frac{\Gamma \vdash a \leadsto a'/R}{\Gamma \vdash a[\gamma] \leadsto a'[\gamma]/R} \quad \textsc{An\_CAppLeft}$$

$$\frac{}{\Gamma \vdash (\Lambda c \!:\! \phi.b)[\gamma] \leadsto b\{\gamma/c\}/R} \quad \textsc{An\_CAppCAbs}$$

$$\frac{\begin{array}{l}\Gamma \vdash A : \star/R \\ \Gamma, x : A/R \vdash b \leadsto b'/R_1\end{array}}{\Gamma \vdash (\lambda^- x \!:\! A/R.b) \leadsto (\lambda^- x \!:\! A/R.b')/R_1} \quad \textsc{An\_AbsTerm}$$

$$\frac{F \sim a : A/R \in \Sigma_1}{\Gamma \vdash F \rightsquigarrow a/R} \quad \textsc{An\_Axiom}$$

$$\frac{\Gamma \vdash a \rightsquigarrow a'/R}{\Gamma \vdash a \triangleright_{R_1} \gamma \rightsquigarrow a' \triangleright_{R_1} \gamma/R} \quad \textsc{An\_ConvTerm}$$

$$\frac{\mathsf{Value}_R\ v}{\Gamma \vdash (v \triangleright_{R_2} \gamma_1) \triangleright_{R_2} \gamma_2 \rightsquigarrow v \triangleright_{R_2} (\gamma_1 ; \gamma_2)/R} \quad \textsc{An\_Combine}$$

$$\frac{\begin{array}{l}\mathsf{Value}_R\ v \\ \Gamma; \widetilde{\Gamma} \vdash \gamma : \Pi^\rho x_1 \!:\! A_1/R \to B_1 \sim_{R'} \Pi^\rho x_2 \!:\! A_2/R \to B_2 \\ b' = b \triangleright_{R'} \mathbf{sym}\,(\mathbf{piFst}\,\gamma) \\ \gamma' = \gamma @ (b' \mid = \mid_{(\mathbf{piFst}\,\gamma)} b)\end{array}}{\Gamma \vdash (v \triangleright_{R'} \gamma)\ b^{R,\rho} \rightsquigarrow ((v\ b'^{R,\rho}) \triangleright_{R'} \gamma')/R} \quad \textsc{An\_Push}$$

$$\frac{\begin{array}{l}\mathsf{Value}_R\ v \\ \Gamma; \widetilde{\Gamma} \vdash \gamma : \forall c_1 \!:\! a_1 \sim_{B_1/R} b_1 . A_1 \sim_{R'} \forall c_2 \!:\! a_2 \sim_{B_2/R} b_2 . A_2 \\ \gamma_1' = \gamma_1 \triangleright_{R'} \mathbf{sym}\,(\mathbf{cpiFst}\,\gamma) \\ \gamma' = \gamma @ (\gamma_1' \sim \gamma_1)\end{array}}{\Gamma \vdash (v \triangleright_{R'} \gamma)[\gamma_1] \rightsquigarrow ((v[\gamma_1']) \triangleright_{R'} \gamma')/R} \quad \textsc{An\_CPush}$$

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Definition rules:        158 good    0 bad
Definition rule clauses: 473 good    0 bad
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