

$tnvar, x, y, f, m, n$	variables
$covar, c$	coercion variables
$datacon, K$	
$const, T, F$	
$index, i$	indices

$role, R$	$::=$ $ $ Nom $ $ Rep $ $ $R_1 \cap R_2$ S	Role
$relflag, \rho$	$::=$ $ $ $+$ $ $ $-$	relevance flag
$constraint, \phi$	$::=$ $ $ $a \sim_{A/R} b$ $ $ (ϕ) S $ $ $\phi\{b/x\}$ S $ $ $ \phi $ S $ $ $a \sim_R b$ S	props
tm, a, b, v, w, A, B	$::=$ $ $ \star $ $ x $ $ $\lambda^\rho x:A/R.b$ bind x in b $ $ $\lambda^{R,\rho} x.b$ bind x in b $ $ $a \ b^{R,\rho}$ $ $ F $ $ $\Pi^\rho x:A/R \rightarrow B$ bind x in B $ $ $a \triangleright_R \gamma$ $ $ $\forall c:\phi.B$ bind c in B $ $ $\Lambda c:\phi.b$ bind c in b $ $ $\Lambda c.b$ bind c in b $ $ $a[\gamma]$ $ $ \square $ $ ifPath $R \ a' \ a \ b_1 \ b_2$ $ $ K $ $ match a with brs $ $ sub $R \ a$ $ $ $a\{b/x\}$ S $ $ a S $ $ $a\{\gamma/c\}$ S $ $ a S $ $ (a) S $ $ a S $ $ $ a _R$ S $ $ Int S $ $ Bool S $ $ Nat S $ $ Vec S $ $ 0 S $ $ S S	<p>types and kinds</p> <p>parsing precedence is hard</p>

		True	S	
		Fix	S	
		Age	S	
		$a \rightarrow b$	S	
		$\phi \Rightarrow A$	S	
		$a \ b$	S	
		$\lambda x. a$	S	
		$\lambda x : A. a$	S	
		$\forall x : A/R \rightarrow B$	S	
		if ϕ then a else b	S	
brs	::=			case branches
		none		
		$K \Rightarrow a; brs$		
		$brs\{a/x\}$	S	
		$brs\{\gamma/c\}$	S	
		(brs)	S	
co, γ	::=			explicit coercions
		•		
		c		
		red $a \ b$		
		refl a		
		$(a \models_{\gamma} b)$		
		sym γ		
		$\gamma_1; \gamma_2$		
		sub γ		
		$\Pi^{R,\rho} x : \gamma_1. \gamma_2$	bind x in γ_2	
		$\lambda^{R,\rho} x : \gamma_1. \gamma_2$	bind x in γ_2	
		$\gamma_1 \ \gamma_2^{R,\rho}$		
		piFst γ		
		cpiFst γ		
		isoSnd γ		
		$\gamma_1 @ \gamma_2$		
		$\forall c : \gamma_1. \gamma_3$	bind c in γ_3	
		$\lambda c : \gamma_1. \gamma_3 @ \gamma_4$	bind c in γ_3	
		$\gamma(\gamma_1, \gamma_2)$		
		$\gamma @ (\gamma_1 \sim \gamma_2)$		
		$\gamma_1 \triangleright_R \gamma_2$		
		$\gamma_1 \sim_A \gamma_2$		
		conv $\phi_1 \sim_{\gamma} \phi_2$		
		eta a		
		left $\gamma \ \gamma'$		
		right $\gamma \ \gamma'$		
		(γ)	S	
		γ	S	

		$\gamma\{a/x\}$	S	
$role_context, \Omega$::=			$role_contexts$
		\emptyset		
		$\Omega, x : R$		
		(Ω)	M	
		Ω	M	
sig_sort	::=			signature classifier
		$: A/R$		
		$\sim a : A/R$		
$sort$::=			binding classifier
		Tm $A R$		
		Co ϕ		
$context, \Gamma$::=			contexts
		\emptyset		
		$\Gamma, x : A/R$		
		$\Gamma, c : \phi$		
		$\Gamma\{b/x\}$	M	
		$\Gamma\{\gamma/c\}$	M	
		Γ, Γ'	M	
		$ \Gamma $	M	
		(Γ)	M	
		Γ	M	
sig, Σ	::=			signatures
		\emptyset		
		$\Sigma \cup \{F sig_sort\}$		
		Σ_0	M	
		Σ_1	M	
		$ \Sigma $	M	
$available_props, \Delta$::=			
		\emptyset		
		Δ, c		
		$\tilde{\Gamma}$	M	
		(Δ)	M	
$terminals$::=			
		\leftrightarrow		
		\Leftrightarrow		
		\longrightarrow		
		min		
		\equiv		
		\forall		

	\in \notin \Leftarrow \Rightarrow \Rightarrow^* \rightarrow Λ \square \vdash \dashv \models \vDash \neq \triangleright \mathbf{ok} $-$ \rightsquigarrow \rightsquigarrow^* \rightsquigarrow \emptyset \circ \mathbf{fv} \mathbf{dom} \sim \succ $ $ \bullet \mathbf{fst} \mathbf{snd} $ \Rightarrow $ $\vdash=$ $\mathbf{refl_2}$ $++$
<i>formula, ψ</i>	$::=$ $\textit{judgement}$ $x : A/R \in \Gamma$ $x : R \in \Omega$ $c : \phi \in \Gamma$ $F \textit{ sig_sort} \in \Sigma$ $K : T \Gamma \in \Sigma$ $x \in \Delta$ $c \in \Delta$ $c \mathbf{not\ relevant} \in \gamma$ $x \notin \mathbf{fva}$

	$ \begin{array}{ l} x \notin \text{dom } \Gamma \\ \text{uniq}(\Omega) \\ c \notin \text{dom } \Gamma \\ T \notin \text{dom } \Sigma \\ F \notin \text{dom } \Sigma \\ a = b \\ \phi_1 = \phi_2 \\ \Gamma_1 = \Gamma_2 \\ \gamma_1 = \gamma_2 \\ \neg\psi \\ \psi_1 \wedge \psi_2 \\ \psi_1 \vee \psi_2 \\ \psi_1 \Rightarrow \psi_2 \\ (\psi) \\ \psi \\ c : (a : A \sim b : B) \in \Gamma \end{array} $	suppress lc hypothesis generated by Ott
<i>JSubRole</i>	$ \begin{array}{ l} R_1 \leq R_2 \end{array} $	Subroling judgement
<i>JPath</i>	$ \begin{array}{ l} \text{Path}_R a = F \end{array} $	Type headed by constant (partial function)
<i>JValue</i>	$ \begin{array}{ l} \text{Value}_R A \end{array} $	values
<i>JValueType</i>	$ \begin{array}{ l} \text{ValueType}_R A \end{array} $	Types with head forms (erased language)
<i>Jconsistent</i>	$ \begin{array}{ l} \text{consistent}_R ab \end{array} $	(erased) types do not differ in their heads
<i>Jerased</i>	$ \begin{array}{ l} \Omega \models a : R \end{array} $	
<i>JChk</i>	$ \begin{array}{ l} (\rho = +) \vee (x \notin \text{fv } A) \end{array} $	irrelevant argument check
<i>Jpar</i>	$ \begin{array}{ l} \Omega \models a \Rightarrow_R b \\ \Omega \vdash a \Rightarrow_R^* b \\ \Omega \vdash a \Leftrightarrow_R b \end{array} $	parallel reduction (implicit language) multistep parallel reduction parallel reduction to a common term
<i>Jbeta</i>	$ \begin{array}{ l} \models a > b/R \\ \models a \rightsquigarrow b/R \end{array} $	primitive reductions on erased terms single-step head reduction for implicit language

		$\models a \rightsquigarrow^* b/R$	multistep reduction
<i>Jett</i>	$::=$	<ul style="list-style-type: none"> $\Gamma \models \phi \text{ ok}$ $\Gamma \models a : A/R$ $\Gamma; \Delta \models \phi_1 \equiv \phi_2$ $\Gamma; \Delta \models a \equiv b : A/R$ $\models \Gamma$ 	Prop wellformedness typing prop equality definitional equality context wellformedness
<i>Jsig</i>	$::=$	$\models \Sigma$	signature wellformedness
<i>Jann</i>	$::=$	<ul style="list-style-type: none"> $\Gamma \vdash \phi \text{ ok}$ $\Gamma \vdash a : A/R$ $\Gamma; \Delta \vdash \gamma : \phi_1 \sim \phi_2$ $\Gamma; \Delta \vdash \gamma : A \sim_R B$ $\vdash \Gamma$ $\vdash \Sigma$ 	prop wellformedness typing coercion between props coercion between types context wellformedness signature wellformedness
<i>Jred</i>	$::=$	$\Gamma \vdash a \rightsquigarrow b/R$	single-step, weak head reduction to values for annotated lang
<i>judgement</i>	$::=$	<ul style="list-style-type: none"> <i>JSubRole</i> <i>JPath</i> <i>JValue</i> <i>JValueType</i> <i>Jconsistent</i> <i>Jerased</i> <i>JChk</i> <i>Jpar</i> <i>Jbeta</i> <i>Jett</i> <i>Jsig</i> <i>Jann</i> <i>Jred</i> 	
<i>user_syntax</i>	$::=$	<ul style="list-style-type: none"> <i>tmvar</i> <i>covar</i> <i>datacon</i> <i>const</i> <i>index</i> <i>role</i> <i>relflag</i> <i>constraint</i> 	

tm
 brs
 co
 $role_context$
 sig_sort
 $sort$
 $context$
 sig
 $available_props$
 $terminals$
 $formula$

$R_1 \leq R_2$ Subroling judgement

$$\begin{array}{c}
\overline{\mathbf{Nom} \leq R} \quad \text{NOMBOT} \\
\overline{R \leq \mathbf{Rep}} \quad \text{REPTOP} \\
\overline{R \leq R} \quad \text{REFL} \\
\frac{R_1 \leq R_2 \quad R_2 \leq R_3}{R_1 \leq R_3} \quad \text{TRANS}
\end{array}$$

$\text{Path}_R a = F$ Type headed by constant (partial function)

$$\begin{array}{c}
\frac{F \sim a : A/R_1 \in \Sigma_0 \quad \neg(R_1 \leq R)}{\text{Path}_R F = F} \quad \text{PATH_CONST} \\
\frac{\text{Path}_R a = F}{\text{Path}_R (a \ b'^{R_1, \rho}) = F} \quad \text{PATH_APP} \\
\frac{\text{Path}_R a = F}{\text{Path}_R (a[\bullet]) = F} \quad \text{PATH_CAPP}
\end{array}$$

$\text{Value}_R A$ values

$$\begin{array}{c}
\overline{\text{Value}_R \star} \quad \text{VALUE_STAR} \\
\overline{\text{Value}_R \Pi^\rho x : A/R_1 \rightarrow B} \quad \text{VALUE_PI} \\
\overline{\text{Value}_R \forall c : \phi. B} \quad \text{VALUE_CPI} \\
\overline{\text{Value}_R \lambda^+ x : A/R_1. a} \quad \text{VALUE_ABSR} \\
\overline{\text{Value}_R \lambda^{R_1, +} x. a} \quad \text{VALUE_UABSR} \\
\frac{\text{Value}_R a}{\text{Value}_R \lambda^{R_1, -} x. a} \quad \text{VALUE_UABSI} \\
\overline{\text{Value}_R \Lambda c : \phi. a} \quad \text{VALUE_CABS} \\
\overline{\text{Value}_R \Lambda c. a} \quad \text{VALUE_UCABS}
\end{array}$$

$$\frac{\text{Path}_R a = F}{\text{Value}_R a} \quad \text{VALUE_PATH}$$

$\boxed{\text{ValueType}_R A}$ Types with head forms (erased language)

$$\frac{}{\text{ValueType}_R \star} \quad \text{VALUE_TYPE_STAR}$$

$$\frac{}{\text{ValueType}_R \Pi^\rho x : A / R_1 \rightarrow B} \quad \text{VALUE_TYPE_PI}$$

$$\frac{}{\text{ValueType}_R \forall c : \phi. B} \quad \text{VALUE_TYPE_CPI}$$

$$\frac{\frac{\text{Path}_R A = F}{\text{Value}_R A}}{\text{ValueType}_R A} \quad \text{VALUE_TYPE_PATH}$$

$\boxed{\text{consistent}_R ab}$ (erased) types do not differ in their heads

$$\frac{}{\text{consistent}_R \star \star} \quad \text{CONSISTENT_A_STAR}$$

$$\frac{}{\text{consistent}_{R'} (\Pi^\rho x_1 : A_1 / R \rightarrow B_1) (\Pi^\rho x_2 : A_2 / R \rightarrow B_2)} \quad \text{CONSISTENT_A_PI}$$

$$\frac{}{\text{consistent}_R (\forall c_1 : \phi_1. A_1) (\forall c_2 : \phi_2. A_2)} \quad \text{CONSISTENT_A_CPI}$$

$$\frac{\frac{\text{Path}_R a_1 = F}{\text{Path}_R a_2 = F}}{\text{consistent}_R a_1 a_2} \quad \text{CONSISTENT_A_PATH}$$

$$\frac{\neg \text{ValueType}_R b}{\text{consistent}_R ab} \quad \text{CONSISTENT_A_STEP_R}$$

$$\frac{\neg \text{ValueType}_R a}{\text{consistent}_R ab} \quad \text{CONSISTENT_A_STEP_L}$$

$\boxed{\Omega \models a : R}$

$$\frac{\text{uniq}(\Omega)}{\Omega \models \square : R} \quad \text{ERASED_A_BULLET}$$

$$\frac{\text{uniq}(\Omega)}{\Omega \models \star : R} \quad \text{ERASED_A_STAR}$$

$$\frac{\frac{\text{uniq}(\Omega)}{x : R \in \Omega}}{R \leq R_1} \quad \text{ERASED_A_VAR}$$

$$\frac{\Omega, x : R_1 \models a : R}{\Omega \models (\lambda^{R_1, \rho} x. a) : R} \quad \text{ERASED_A_ABS}$$

$$\frac{\frac{\Omega \models a : R}{\Omega \models b : R_1}}{\Omega \models (a \ b^{R_1, \rho}) : R} \quad \text{ERASED_A_APP}$$

$$\frac{\frac{\Omega \models A : R_1}{\Omega, x : R_1 \models B : R}}{\Omega \models (\Pi^\rho x : A / R_1 \rightarrow B) : R} \quad \text{ERASED_A_PI}$$

$$\frac{\begin{array}{c} \Omega \models a : R_1 \\ \Omega \models b : R_1 \\ \Omega \models A : R_1 \\ \Omega \models B : R \end{array}}{\Omega \models (\forall c : a \sim_{A/R_1} b.B) : R} \text{ERASED_A_CPI}$$

$$\frac{\Omega \models b : R}{\Omega \models (\Lambda c.b) : R} \text{ERASED_A_CAbs}$$

$$\frac{\Omega \models a : R}{\Omega \models (a[\bullet]) : R} \text{ERASED_A_CApP}$$

$$\frac{\begin{array}{c} \text{uniq}(\Omega) \\ F \sim a : A/R \in \Sigma_0 \end{array}}{\Omega \models F : R_1} \text{ERASED_A_FAM}$$

$$\frac{\begin{array}{c} F \sim a_0 : A/R' \in \Sigma_0 \\ \Omega \models a : R \\ \Omega \models b_1 : R_1 \\ \Omega \models b_2 : R_1 \end{array}}{\Omega \models (\text{ifPath } R \ F \ a \ b_1 \ b_2) : R_1} \text{ERASED_A_PATTERN}$$

$$\boxed{(\rho = +) \vee (x \notin \text{fv } A)} \quad \text{irrelevant argument check}$$

$$\overline{(+ = +) \vee (x \notin \text{fv } A)} \quad \text{RHO_REL}$$

$$\frac{x \notin \text{fv } A}{(- = +) \vee (x \notin \text{fv } A)} \quad \text{RHO_IRRREL}$$

$$\boxed{\Omega \models a \Rightarrow_R b} \quad \text{parallel reduction (implicit language)}$$

$$\frac{\Omega \models a : R}{\Omega \models a \Rightarrow_R a} \quad \text{PAR_REFL}$$

$$\frac{\begin{array}{c} \Omega \models a \Rightarrow_R (\lambda^{R_1, \rho} x. a') \\ \Omega \models b \Rightarrow_{R_1} b' \end{array}}{\Omega \models a \ b^{R_1, \rho} \Rightarrow_R a' \{b'/x\}} \quad \text{PAR_BETA}$$

$$\frac{\begin{array}{c} \Omega \models a \Rightarrow_R a' \\ \Omega \models b \Rightarrow_{R_1} b' \end{array}}{\Omega \models a \ b^{R_1, \rho} \Rightarrow_R a' \ b'^{R_1, \rho}} \quad \text{PAR_APP}$$

$$\frac{\Omega \models a \Rightarrow_R (\Lambda c. a')}{\Omega \models a[\bullet] \Rightarrow_R a' \{\bullet/c\}} \quad \text{PAR_CBETA}$$

$$\frac{\Omega \models a \Rightarrow_R a'}{\Omega \models a[\bullet] \Rightarrow_R a'[\bullet]} \quad \text{PAR_CApP}$$

$$\frac{\Omega, x : R_1 \models a \Rightarrow_R a'}{\Omega \models \lambda^{R_1, \rho} x. a \Rightarrow_R \lambda^{R_1, \rho} x. a'} \quad \text{PAR_ABS}$$

$$\frac{\begin{array}{c} \Omega \models A \Rightarrow_{R_1} A' \\ \Omega, x : R_1 \models B \Rightarrow_R B' \end{array}}{\Omega \models \Pi^\rho x : A/R_1 \rightarrow B \Rightarrow_R \Pi^\rho x : A'/R_1 \rightarrow B'} \quad \text{PAR_PI}$$

$$\frac{\Omega \models a \Rightarrow_R a'}{\Omega \models \Lambda c. a \Rightarrow_R \Lambda c. a'} \quad \text{PAR_CAbs}$$

$$\begin{array}{c}
\Omega \models A \Rightarrow_{R_1} A' \\
\Omega \models a \Rightarrow_{R_1} a' \\
\Omega \models b \Rightarrow_{R_1} b' \\
\Omega \models B \Rightarrow_R B' \\
\hline
\Omega \models \forall c : a \sim_{A/R_1} b.B \Rightarrow_R \forall c : a' \sim_{A'/R_1} b'.B' \quad \text{PAR_CPI}
\end{array}$$

$$\begin{array}{c}
F \sim a : A/R_1 \in \Sigma_0 \\
R_1 \leq R \\
\text{uniq}(\Omega) \\
\hline
\Omega \models F \Rightarrow_R a \quad \text{PAR_AXIOM}
\end{array}$$

$$\begin{array}{c}
F \sim a_0 : A/R' \in \Sigma_0 \\
\Omega \models a \Rightarrow_R a' \\
\Omega \models b_1 \Rightarrow_{R_0} b'_1 \\
\Omega \models b_2 \Rightarrow_{R_0} b'_2 \\
\hline
\Omega \models \mathbf{ifPath} \, R \, F \, a \, b_1 \, b_2 \Rightarrow_{R_0} \mathbf{ifPath} \, R \, F \, a' \, b'_1 \, b'_2 \quad \text{PAR_PATTERN}
\end{array}$$

$$\begin{array}{c}
\Omega \models a \Rightarrow_R a' \\
\Omega \models b_1 \Rightarrow_{R_0} b'_1 \\
\Omega \models b_2 \Rightarrow_{R_0} b'_2 \\
\text{Path}_R \, a' = F \\
\hline
\Omega \models \mathbf{ifPath} \, R \, F \, a \, b_1 \, b_2 \Rightarrow_{R_0} b'_1 \quad \text{PAR_PATTERNTRUE}
\end{array}$$

$$\begin{array}{c}
F \sim a_0 : A/R' \in \Sigma_0 \\
\Omega \models a \Rightarrow_R a' \\
\Omega \models b_1 \Rightarrow_{R_0} b'_1 \\
\Omega \models b_2 \Rightarrow_{R_0} b'_2 \\
\text{Value}_R \, a' \\
\neg(\text{Path}_R \, a' = F) \\
\hline
\Omega \models \mathbf{ifPath} \, R \, F \, a \, b_1 \, b_2 \Rightarrow_{R_0} b'_2 \quad \text{PAR_PATTERNFALSE}
\end{array}$$

$\boxed{\Omega \vdash a \Rightarrow_R^* b}$ multistep parallel reduction

$$\begin{array}{c}
\overline{\Omega \vdash a \Rightarrow_R^* a} \quad \text{MP_REFL} \\
\Omega \models a \Rightarrow_R b \\
\Omega \vdash b \Rightarrow_R^* a' \\
\hline
\Omega \vdash a \Rightarrow_R^* a' \quad \text{MP_STEP}
\end{array}$$

$\boxed{\Omega \vdash a \Leftrightarrow_R b}$ parallel reduction to a common term

$$\begin{array}{c}
\Omega \vdash a_1 \Rightarrow_R^* b \\
\Omega \vdash a_2 \Rightarrow_R^* b \\
\hline
\Omega \vdash a_1 \Leftrightarrow_R a_2 \quad \text{JOIN}
\end{array}$$

$\boxed{\models a > b/R}$ primitive reductions on erased terms

$$\begin{array}{c}
\text{Value}_{R_1} (\lambda^{R,\rho} x.v) \\
\hline
\models (\lambda^{R,\rho} x.v) \, b^{R,\rho} > v\{b/x\}/R_1 \quad \text{BETA_APPABS}
\end{array}$$

$$\begin{array}{c}
\hline
\models (\Lambda c.a')[\bullet] > a'\{\bullet/c\}/R \quad \text{BETA_CAPPCABS}
\end{array}$$

$$\begin{array}{c}
F \sim a : A/R \in \Sigma_0 \\
R \leq R_1 \\
\hline
\models F > a/R_1 \quad \text{BETA_AXIOM}
\end{array}$$

$$\frac{\text{Path}_R a = F}{\models \mathbf{ifPath} R F a b_1 b_2 > b_1/R_0} \quad \text{BETA_PATTERNTRUE}$$

$$\frac{\begin{array}{c} F \sim a_0 : A/R' \in \Sigma_0 \\ \text{Value}_R a \\ \neg(\text{Path}_R a = F) \end{array}}{\models \mathbf{ifPath} R F a b_1 b_2 > b_2/R_0} \quad \text{BETA_PATTERNFALSE}$$

$\boxed{\models a \rightsquigarrow b/R}$ single-step head reduction for implicit language

$$\frac{\models a \rightsquigarrow a'/R_1}{\models \lambda^{R,-} x. a \rightsquigarrow \lambda^{R,-} x. a'/R_1} \quad \text{E_ABSTERM}$$

$$\frac{\models a \rightsquigarrow a'/R_1}{\models a \ b^{R,\rho} \rightsquigarrow a' \ b^{R,\rho}/R_1} \quad \text{E_APPLEFT}$$

$$\frac{\models a \rightsquigarrow a'/R}{\models a[\bullet] \rightsquigarrow a'[\bullet]/R} \quad \text{E_CAPPLEFT}$$

$$\frac{\models a \rightsquigarrow a'/R}{\models \mathbf{ifPath} R F a b_1 b_2 \rightsquigarrow \mathbf{ifPath} R F a' b_1 b_2/R_0} \quad \text{E_PATTERN}$$

$$\frac{\models a > b/R}{\models a \rightsquigarrow b/R} \quad \text{E_PRIM}$$

$\boxed{\models a \rightsquigarrow^* b/R}$ multistep reduction

$$\overline{\models a \rightsquigarrow^* a/R} \quad \text{EQUAL}$$

$$\frac{\begin{array}{c} \models a \rightsquigarrow b/R \\ \models b \rightsquigarrow^* a'/R \end{array}}{\models a \rightsquigarrow^* a'/R} \quad \text{STEP}$$

$\boxed{\Gamma \models \phi \text{ ok}}$ Prop wellformedness

$$\frac{\begin{array}{c} \Gamma \models a : A/R \\ \Gamma \models b : A/R \\ \Gamma \models A : \star/R \end{array}}{\Gamma \models a \sim_{A/R} b \text{ ok}} \quad \text{E_WFF}$$

$\boxed{\Gamma \models a : A/R}$ typing

$$\frac{\begin{array}{c} R_1 \leq R_2 \\ \Gamma \models a : A/R_1 \end{array}}{\Gamma \models a : A/R_2} \quad \text{E_SUBROLE}$$

$$\frac{\models \Gamma}{\Gamma \models \star : \star/R} \quad \text{E_STAR}$$

$$\frac{\begin{array}{c} \models \Gamma \\ x : A/R \in \Gamma \end{array}}{\Gamma \models x : A/R} \quad \text{E_VAR}$$

$$\frac{\begin{array}{c} \Gamma, x : A/R \models B : \star/R' \\ \Gamma \models A : \star/R \end{array}}{\Gamma \models \Pi^{\rho} x : A/R \rightarrow B : \star/R'} \quad \text{E_PI}$$

$$\frac{\begin{array}{c} \Gamma, x : A/R \models a : B/R' \\ \Gamma \models A : \star/R \\ (\rho = +) \vee (x \notin \text{fv } a) \end{array}}{\Gamma \models \lambda^{R, \rho} x. a : (\Pi^{\rho} x : A/R \rightarrow B)/R'} \quad \text{E_ABS}$$

$$\frac{\begin{array}{c} \Gamma \models b : \Pi^+ x : A/R \rightarrow B/R' \\ \Gamma \models a : A/R \end{array}}{\Gamma \models b \ a^{R, +} : B\{a/x\}/R'} \quad \text{E_APP}$$

$$\frac{\begin{array}{c} \Gamma \models b : \Pi^- x : A/R \rightarrow B/R' \\ \Gamma \models a : A/R \end{array}}{\Gamma \models b \ \Box^{R, -} : B\{a/x\}/R'} \quad \text{E_IAPP}$$

$$\frac{\begin{array}{c} \Gamma \models a : A/R \\ \Gamma; \tilde{\Gamma} \models A \equiv B : \star/\mathbf{Rep} \\ \Gamma \models B : \star/R \end{array}}{\Gamma \models a : B/R} \quad \text{E_CONV}$$

$$\frac{\begin{array}{c} \Gamma, c : \phi \models B : \star/R \\ \Gamma \models \phi \ \text{ok} \end{array}}{\Gamma \models \forall c : \phi. B : \star/R} \quad \text{E_CPI}$$

$$\frac{\begin{array}{c} \Gamma, c : \phi \models a : B/R \\ \Gamma \models \phi \ \text{ok} \end{array}}{\Gamma \models \Lambda c. a : \forall c : \phi. B/R} \quad \text{E_CABS}$$

$$\frac{\begin{array}{c} \Gamma \models a_1 : \forall c : (a \sim_{A/R} b). B_1/R' \\ \Gamma; \tilde{\Gamma} \models a \equiv b : A/R \end{array}}{\Gamma \models a_1[\bullet] : B_1\{\bullet/c\}/R'} \quad \text{E_CAPP}$$

$$\frac{\begin{array}{c} \models \Gamma \\ F \sim a : A/R \in \Sigma_0 \\ \emptyset \models A : \star/R_1 \end{array}}{\Gamma \models F : A/R_1} \quad \text{E_FAM}$$

$$\frac{\begin{array}{c} F \sim a_0 : A_0/R' \in \Sigma_0 \\ \Gamma \models a : A/R \\ \Gamma \models b_1 : B/R_0 \\ \Gamma \models b_2 : B/R_0 \end{array}}{\Gamma \models \mathbf{ifPath } R \ F \ a \ b_1 \ b_2 : B/R_0} \quad \text{E_PAT}$$

$$\boxed{\Gamma; \Delta \models \phi_1 \equiv \phi_2} \quad \text{prop equality}$$

$$\frac{\begin{array}{c} \Gamma; \Delta \models A_1 \equiv A_2 : A/R \\ \Gamma; \Delta \models B_1 \equiv B_2 : A/R \end{array}}{\Gamma; \Delta \models A_1 \sim_{A/R} B_1 \equiv A_2 \sim_{A/R} B_2} \quad \text{E_PROPCONG}$$

$$\frac{\begin{array}{c} \Gamma; \Delta \models A \equiv B : \star/R \\ \Gamma \models A_1 \sim_{A/R} A_2 \ \text{ok} \\ \Gamma \models A_1 \sim_{B/R} A_2 \ \text{ok} \end{array}}{\Gamma; \Delta \models A_1 \sim_{A/R} A_2 \equiv A_1 \sim_{B/R} A_2} \quad \text{E_ISOCONV}$$

$$\frac{\Gamma; \Delta \models \forall c : (a_1 \sim_{A/R_1} a_2). B_1 \equiv \forall c : (b_1 \sim_{B/R_2} b_2). B_2 : \star/R'}{\Gamma; \Delta \models a_1 \sim_{A/R_1} a_2 \equiv b_1 \sim_{B/R_2} b_2} \quad \text{E_CPIFST}$$

$$\boxed{\Gamma; \Delta \models a \equiv b : A/R} \quad \text{definitional equality}$$

$$\begin{array}{c}
\frac{\begin{array}{c} \vdash \Gamma \\ c : (a \sim_{A/R} b) \in \Gamma \\ c \in \Delta \end{array}}{\Gamma; \Delta \vdash a \equiv b : A/R} \quad \text{E_ASSN} \\
\\
\frac{\Gamma \vdash a : A/R}{\Gamma; \Delta \vdash a \equiv a : A/R} \quad \text{E_REFL} \\
\\
\frac{\Gamma; \Delta \vdash b \equiv a : A/R}{\Gamma; \Delta \vdash a \equiv b : A/R} \quad \text{E_SYM} \\
\\
\frac{\begin{array}{c} \Gamma; \Delta \vdash a \equiv a_1 : A/R \\ \Gamma; \Delta \vdash a_1 \equiv b : A/R \end{array}}{\Gamma; \Delta \vdash a \equiv b : A/R} \quad \text{E_TRANS} \\
\\
\frac{\begin{array}{c} \Gamma; \Delta \vdash a \equiv b : A/R_1 \\ R_1 \leq R_2 \end{array}}{\Gamma; \Delta \vdash a \equiv b : A/R_2} \quad \text{E_SUB} \\
\\
\frac{\begin{array}{c} \Gamma \vdash a_1 : B/R \\ \Gamma \vdash a_2 : B/R \\ \vdash a_1 > a_2/R \end{array}}{\Gamma; \Delta \vdash a_1 \equiv a_2 : B/R} \quad \text{E_BETA} \\
\\
\frac{\begin{array}{c} \Gamma; \Delta \vdash A_1 \equiv A_2 : \star/R \\ \Gamma, x : A_1/R; \Delta \vdash B_1 \equiv B_2 : \star/R' \\ \Gamma \vdash A_1 : \star/R \\ \Gamma \vdash \Pi^\rho x : A_1/R \rightarrow B_1 : \star/R' \\ \Gamma \vdash \Pi^\rho x : A_2/R \rightarrow B_2 : \star/R' \end{array}}{\Gamma; \Delta \vdash (\Pi^\rho x : A_1/R \rightarrow B_1) \equiv (\Pi^\rho x : A_2/R \rightarrow B_2) : \star/R'} \quad \text{E_PICONG} \\
\\
\frac{\begin{array}{c} \Gamma, x : A_1/R; \Delta \vdash b_1 \equiv b_2 : B/R' \\ \Gamma \vdash A_1 : \star/R \\ (\rho = +) \vee (x \notin \text{fv } b_1) \\ (\rho = +) \vee (x \notin \text{fv } b_2) \end{array}}{\Gamma; \Delta \vdash (\lambda^{R,\rho} x. b_1) \equiv (\lambda^{R,\rho} x. b_2) : (\Pi^\rho x : A_1/R \rightarrow B)/R'} \quad \text{E_ABSCONG} \\
\\
\frac{\begin{array}{c} \Gamma; \Delta \vdash a_1 \equiv b_1 : (\Pi^+ x : A/R \rightarrow B)/R' \\ \Gamma; \Delta \vdash a_2 \equiv b_2 : A/R \end{array}}{\Gamma; \Delta \vdash a_1 \ a_2^{R,+} \equiv b_1 \ b_2^{R,+} : (B\{a_2/x\})/R'} \quad \text{E_APPCONG} \\
\\
\frac{\begin{array}{c} \Gamma; \Delta \vdash a_1 \equiv b_1 : (\Pi^- x : A/R \rightarrow B)/R' \\ \Gamma \vdash a : A/R \end{array}}{\Gamma; \Delta \vdash a_1 \ \Box^{R,-} \equiv b_1 \ \Box^{R,-} : (B\{a/x\})/R'} \quad \text{E_IAPPCONG} \\
\\
\frac{\Gamma; \Delta \vdash \Pi^\rho x : A_1/R \rightarrow B_1 \equiv \Pi^\rho x : A_2/R \rightarrow B_2 : \star/R'}{\Gamma; \Delta \vdash A_1 \equiv A_2 : \star/R} \quad \text{E_PIFST} \\
\\
\frac{\begin{array}{c} \Gamma; \Delta \vdash \Pi^\rho x : A_1/R \rightarrow B_1 \equiv \Pi^\rho x : A_2/R \rightarrow B_2 : \star/R' \\ \Gamma; \Delta \vdash a_1 \equiv a_2 : A_1/R \end{array}}{\Gamma; \Delta \vdash B_1\{a_1/x\} \equiv B_2\{a_2/x\} : \star/R'} \quad \text{E_PISND}
\end{array}$$

$$\begin{array}{c}
\frac{\begin{array}{l}
\Gamma; \Delta \vdash a_1 \sim_{A_1/R} b_1 \equiv a_2 \sim_{A_2/R} b_2 \\
\Gamma, c : a_1 \sim_{A_1/R} b_1; \Delta \vdash A \equiv B : \star/R' \\
\Gamma \vdash a_1 \sim_{A_1/R} b_1 \text{ ok} \\
\Gamma \vdash \forall c : a_1 \sim_{A_1/R} b_1. A : \star/R' \\
\Gamma \vdash \forall c : a_2 \sim_{A_2/R} b_2. B : \star/R'
\end{array}}{\Gamma; \Delta \vdash \forall c : a_1 \sim_{A_1/R} b_1. A \equiv \forall c : a_2 \sim_{A_2/R} b_2. B : \star/R'} \text{E_CPiCONG} \\
\\
\frac{\begin{array}{l}
\Gamma, c : \phi_1; \Delta \vdash a \equiv b : B/R \\
\Gamma \vdash \phi_1 \text{ ok}
\end{array}}{\Gamma; \Delta \vdash (\Lambda c. a) \equiv (\Lambda c. b) : \forall c : \phi_1. B/R} \text{E_CAbsCONG} \\
\\
\frac{\begin{array}{l}
\Gamma; \Delta \vdash a_1 \equiv b_1 : (\forall c : (a \sim_{A/R} b). B)/R' \\
\Gamma; \tilde{\Gamma} \vdash a \equiv b : A/R
\end{array}}{\Gamma; \Delta \vdash a_1[\bullet] \equiv b_1[\bullet] : (B\{\bullet/c\})/R'} \text{E_CApPCONG} \\
\\
\frac{\begin{array}{l}
\Gamma; \Delta \vdash \forall c : (a_1 \sim_{A/R} a_2). B_1 \equiv \forall c : (a'_1 \sim_{A'/R'} a'_2). B_2 : \star/R_0 \\
\Gamma; \tilde{\Gamma} \vdash a_1 \equiv a_2 : A/R \\
\Gamma; \tilde{\Gamma} \vdash a'_1 \equiv a'_2 : A'/R'
\end{array}}{\Gamma; \Delta \vdash B_1\{\bullet/c\} \equiv B_2\{\bullet/c\} : \star/R_0} \text{E_CPiSND} \\
\\
\frac{\begin{array}{l}
\Gamma; \Delta \vdash a \equiv b : A/R \\
\Gamma; \Delta \vdash a \sim_{A/R} b \equiv a' \sim_{A'/R'} b'
\end{array}}{\Gamma; \Delta \vdash a' \equiv b' : A'/R'} \text{E_CAST} \\
\\
\frac{\begin{array}{l}
\Gamma; \Delta \vdash a \equiv b : A/R \\
\Gamma; \tilde{\Gamma} \vdash A \equiv B : \star/\mathbf{Rep} \\
\Gamma \vdash B : \star/R
\end{array}}{\Gamma; \Delta \vdash a \equiv b : B/R} \text{E_EqCONV} \\
\\
\frac{\Gamma; \Delta \vdash a \sim_{A/R_1} b \equiv a' \sim_{A'/R_1} b'}{\Gamma; \Delta \vdash A \equiv A' : \star/R_1} \text{E_ISOsND} \\
\\
\frac{\begin{array}{l}
F \sim a_0 : A_0/R' \in \Sigma_0 \\
\Gamma; \Delta \vdash a \equiv a' : A/R \\
\Gamma; \Delta \vdash b_1 \equiv b'_1 : B/R_0 \\
\Gamma; \Delta \vdash b_2 \equiv b'_2 : B/R_0
\end{array}}{\Gamma; \Delta \vdash \mathbf{ifPath} R F a b_1 b_2 \equiv \mathbf{ifPath} R F a' b'_1 b'_2 : B/R_0} \text{E_PATCONG} \\
\\
\frac{\begin{array}{l}
\text{Path}_{R'} a = F \\
\text{Path}_{R'} a' = F \\
\Gamma \vdash a : \Pi^+ x : A/R_1 \rightarrow B/R' \\
\Gamma \vdash b : A/R_1 \\
\Gamma \vdash a' : \Pi^+ x : A/R_1 \rightarrow B/R' \\
\Gamma \vdash b' : A/R_1 \\
\Gamma; \Delta \vdash a \ b^{R_1,+} \equiv a' \ b'^{R_1,+} : B\{b/x\}/R' \\
\Gamma; \tilde{\Gamma} \vdash B\{b/x\} \equiv B\{b'/x\} : \star/R_0
\end{array}}{\Gamma; \Delta \vdash a \equiv a' : \Pi^+ x : A/R_1 \rightarrow B/R'} \text{E_LEFTREL}
\end{array}$$

$$\begin{array}{c}
\text{Path}_{R'} a = F \\
\text{Path}_{R'} a' = F \\
\Gamma \vdash a : \Pi^- x : A/R_1 \rightarrow B/R' \\
\Gamma \vdash b : A/R_1 \\
\Gamma \vdash a' : \Pi^- x : A/R_1 \rightarrow B/R' \\
\Gamma \vdash b' : A/R_1 \\
\Gamma; \Delta \vdash a \square^{R_1, -} \equiv a' \square^{R_1, -} : B\{b/x\}/R' \\
\Gamma; \tilde{\Gamma} \vdash B\{b/x\} \equiv B\{b'/x\} : \star/R_0 \\
\hline
\Gamma; \Delta \vdash a \equiv a' : \Pi^- x : A/R_1 \rightarrow B/R' \quad \text{E_LEFTIRREL}
\end{array}$$

$$\begin{array}{c}
\text{Path}_{R'} a = F \\
\text{Path}_{R'} a' = F \\
\Gamma \vdash a : \Pi^+ x : A/R_1 \rightarrow B/R' \\
\Gamma \vdash b : A/R_1 \\
\Gamma \vdash a' : \Pi^+ x : A/R_1 \rightarrow B/R' \\
\Gamma \vdash b' : A/R_1 \\
\Gamma; \Delta \vdash a b^{R_1, +} \equiv a' b'^{R_1, +} : B\{b/x\}/R' \\
\Gamma; \tilde{\Gamma} \vdash B\{b/x\} \equiv B\{b'/x\} : \star/R_0 \\
\hline
\Gamma; \Delta \vdash b \equiv b' : A/R_1 \quad \text{E_RIGHT}
\end{array}$$

$$\begin{array}{c}
\text{Path}_{R'} a = F \\
\text{Path}_{R'} a' = F \\
\Gamma \vdash a : \forall c : (a_1 \sim_{A/R_1} a_2). B/R' \\
\Gamma \vdash a' : \forall c : (a_1 \sim_{A/R_1} a_2). B/R' \\
\Gamma; \tilde{\Gamma} \vdash a_1 \equiv a_2 : A/R_1 \\
\Gamma; \Delta \vdash a[\bullet] \equiv a'[\bullet] : B\{\bullet/c\}/R' \\
\hline
\Gamma; \Delta \vdash a \equiv a' : \forall c : (a_1 \sim_{A/R_1} a_2). B/R' \quad \text{E_CLEFT}
\end{array}$$

$\boxed{\vdash \Gamma}$ context wellformedness

$$\begin{array}{c}
\overline{\vdash \emptyset} \quad \text{E_EMPTY} \\
\\
\begin{array}{c}
\vdash \Gamma \\
\Gamma \vdash A : \star/R \\
x \notin \text{dom } \Gamma \\
\hline
\vdash \Gamma, x : A/R \quad \text{E_CONSTM}
\end{array} \\
\\
\begin{array}{c}
\vdash \Gamma \\
\Gamma \vdash \phi \text{ ok} \\
c \notin \text{dom } \Gamma \\
\hline
\vdash \Gamma, c : \phi \quad \text{E_CONSCo}
\end{array}
\end{array}$$

$\boxed{\vdash \Sigma}$ signature wellformedness

$$\begin{array}{c}
\overline{\vdash \emptyset} \quad \text{SIG_EMPTY} \\
\\
\begin{array}{c}
\vdash \Sigma \\
\emptyset \vdash a : A/R' \\
F \notin \text{dom } \Sigma \\
\hline
\vdash \Sigma \cup \{F \sim a : A/R'\} \quad \text{SIG_CONSAx}
\end{array}
\end{array}$$

$\boxed{\Gamma \vdash \phi \text{ ok}}$ prop wellformedness

$$\frac{\begin{array}{c} \Gamma \vdash a : A/R \\ \Gamma \vdash b : B/R \\ |A|_R = |B|_R \end{array}}{\Gamma \vdash a \sim_{A/R} b \text{ ok}} \text{ AN_WFF}$$

$\boxed{\Gamma \vdash a : A/R}$ typing

$$\frac{\vdash \Gamma}{\Gamma \vdash \star : \star/R} \text{ AN_STAR}$$

$$\frac{\begin{array}{c} \vdash \Gamma \\ x : A/R \in \Gamma \end{array}}{\Gamma \vdash x : A/R} \text{ AN_VAR}$$

$$\frac{\begin{array}{c} \Gamma, x : A/R \vdash B : \star/R' \\ \Gamma \vdash A : \star/R \end{array}}{\Gamma \vdash \Pi^\rho x : A/R \rightarrow B : \star/R'} \text{ AN_PI}$$

$$\frac{\begin{array}{c} \Gamma \vdash A : \star/R \\ \Gamma, x : A/R \vdash a : B/R' \\ (\rho = +) \vee (x \notin \text{fv } |a|_{R'}) \\ R \leq R' \end{array}}{\Gamma \vdash \lambda^\rho x : A/R. a : (\Pi^\rho x : A/R \rightarrow B)/R'} \text{ AN_ABS}$$

$$\frac{\begin{array}{c} \Gamma \vdash b : (\Pi^\rho x : A/R \rightarrow B)/R' \\ \Gamma \vdash a : A/R \end{array}}{\Gamma \vdash b \ a^{R,\rho} : (B\{a/x\})/R'} \text{ AN_APP}$$

$$\frac{\begin{array}{c} \Gamma \vdash a : A/R \\ \Gamma; \tilde{\Gamma} \vdash \gamma : A \sim_R B \\ \Gamma \vdash B : \star/R \end{array}}{\Gamma \vdash a \triangleright_R \gamma : B/R} \text{ AN_CONV}$$

$$\frac{\begin{array}{c} \Gamma \vdash \phi \text{ ok} \\ \Gamma, c : \phi \vdash B : \star/R \end{array}}{\Gamma \vdash \forall c : \phi. B : \star/R} \text{ AN_CPI}$$

$$\frac{\begin{array}{c} \Gamma \vdash \phi \text{ ok} \\ \Gamma, c : \phi \vdash a : B/R \end{array}}{\Gamma \vdash \Lambda c : \phi. a : (\forall c : \phi. B)/R} \text{ AN_CABS}$$

$$\frac{\begin{array}{c} \Gamma \vdash a_1 : (\forall c : a \sim_{A_1/R} b. B)/R' \\ \Gamma; \tilde{\Gamma} \vdash \gamma : a \sim_R b \end{array}}{\Gamma \vdash a_1[\gamma] : B\{\gamma/c\}/R'} \text{ AN_CAPP}$$

$$\frac{\begin{array}{c} \vdash \Gamma \\ F \sim a : A/R \in \Sigma_1 \\ \emptyset \vdash A : \star/R_1 \end{array}}{\Gamma \vdash F : A/R_1} \text{ AN_FAM}$$

$$\frac{\begin{array}{c} R_1 \leq R_2 \\ \Gamma \vdash a : A/R_1 \end{array}}{\Gamma \vdash \mathbf{sub}_{R_1} a : A/R_2} \text{ AN_SUBROLE}$$

$\boxed{\Gamma; \Delta \vdash \gamma : \phi_1 \sim \phi_2}$ coercion between props

$$\begin{array}{c}
\frac{\begin{array}{c} \Gamma; \Delta \vdash \gamma_1 : A_1 \sim_R A_2 \\ \Gamma; \Delta \vdash \gamma_2 : B_1 \sim_R B_2 \\ \Gamma \vdash A_1 \sim_{A/R} B_1 \text{ ok} \\ \Gamma \vdash A_2 \sim_{A/R} B_2 \text{ ok} \end{array}}{\Gamma; \Delta \vdash (\gamma_1 \sim_A \gamma_2) : (A_1 \sim_{A/R} B_1) \sim (A_2 \sim_{A/R} B_2)} \text{AN_PROP_CONG} \\
\\
\frac{\Gamma; \Delta \vdash \gamma : \forall c : \phi_1. A_2 \sim_R \forall c : \phi_2. B_2}{\Gamma; \Delta \vdash \mathbf{cpiFst} \gamma : \phi_1 \sim \phi_2} \text{AN_CPIFST} \\
\\
\frac{\Gamma; \Delta \vdash \gamma : \phi_1 \sim \phi_2}{\Gamma; \Delta \vdash \mathbf{sym} \gamma : \phi_2 \sim \phi_1} \text{AN_ISOSYM} \\
\\
\frac{\begin{array}{c} \Gamma; \Delta \vdash \gamma : A \sim_R B \\ \Gamma \vdash a_1 \sim_{A/R} a_2 \text{ ok} \\ \Gamma \vdash a'_1 \sim_{B/R} a'_2 \text{ ok} \\ |a_1|_R = |a'_1|_R \\ |a_2|_R = |a'_2|_R \end{array}}{\Gamma; \Delta \vdash \mathbf{conv} (a_1 \sim_{A/R} a_2) \sim_\gamma (a'_1 \sim_{B/R} a'_2) : (a_1 \sim_{A/R} a_2) \sim (a'_1 \sim_{B/R} a'_2)} \text{AN_ISOCONV} \\
\\
\boxed{\Gamma; \Delta \vdash \gamma : A \sim_R B} \quad \text{coercion between types} \\
\\
\frac{\begin{array}{c} \vdash \Gamma \\ c : a \sim_{A/R} b \in \Gamma \\ c \in \Delta \end{array}}{\Gamma; \Delta \vdash c : a \sim_R b} \text{AN_ASSN} \\
\\
\frac{\Gamma \vdash a : A/R}{\Gamma; \Delta \vdash \mathbf{refl} a : a \sim_R a} \text{AN_REFL} \\
\\
\frac{\begin{array}{c} \Gamma \vdash a : A/R \\ \Gamma \vdash b : B/R \\ |a|_R = |b|_R \\ \Gamma; \tilde{\Gamma} \vdash \gamma : A \sim_R B \end{array}}{\Gamma; \Delta \vdash (a \models_\gamma b) : a \sim_R b} \text{AN_ERASEEQ} \\
\\
\frac{\begin{array}{c} \Gamma \vdash b : B/R \\ \Gamma \vdash a : A/R \\ \Gamma; \tilde{\Gamma} \vdash \gamma_1 : B \sim_R A \\ \Gamma; \Delta \vdash \gamma : b \sim_R a \end{array}}{\Gamma; \Delta \vdash \mathbf{sym} \gamma : a \sim_R b} \text{AN_SYM} \\
\\
\frac{\begin{array}{c} \Gamma; \Delta \vdash \gamma_1 : a \sim_R a_1 \\ \Gamma; \Delta \vdash \gamma_2 : a_1 \sim_R b \\ \Gamma \vdash a : A/R \\ \Gamma \vdash a_1 : A_1/R \\ \Gamma; \tilde{\Gamma} \vdash \gamma_3 : A \sim_R A_1 \end{array}}{\Gamma; \Delta \vdash (\gamma_1; \gamma_2) : a \sim_R b} \text{AN_TRANS} \\
\\
\frac{\begin{array}{c} \Gamma \vdash a_1 : B_0/R \\ \Gamma \vdash a_2 : B_1/R \\ |B_0|_R = |B_1|_R \\ \models |a_1|_R > |a_2|_R/R \end{array}}{\Gamma; \Delta \vdash \mathbf{red} a_1 a_2 : a_1 \sim_R a_2} \text{AN_BETA}
\end{array}$$

$$\begin{array}{c}
\Gamma; \Delta \vdash \gamma_1 : A_1 \sim_{R'} A_2 \\
\Gamma, x : A_1/R; \Delta \vdash \gamma_2 : B_1 \sim_{R'} B_2 \\
B_3 = B_2\{x \triangleright_{R'} \mathbf{sym} \gamma_1/x\} \\
\Gamma \vdash \Pi^\rho x : A_1/R \rightarrow B_1 : \star/R' \\
\Gamma \vdash \Pi^\rho x : A_1/R \rightarrow B_2 : \star/R' \\
\Gamma \vdash \Pi^\rho x : A_2/R \rightarrow B_3 : \star/R' \\
R \leq R' \\
\hline
\Gamma; \Delta \vdash \Pi^{R,\rho} x : \gamma_1.\gamma_2 : (\Pi^\rho x : A_1/R \rightarrow B_1) \sim_{R'} (\Pi^\rho x : A_2/R \rightarrow B_3) \quad \text{AN_PiCong}
\end{array}$$

$$\begin{array}{c}
\Gamma; \Delta \vdash \gamma_1 : A_1 \sim_R A_2 \\
\Gamma, x : A_1/R; \Delta \vdash \gamma_2 : b_1 \sim_{R'} b_2 \\
b_3 = b_2\{x \triangleright_{R'} \mathbf{sym} \gamma_1/x\} \\
\Gamma \vdash A_1 : \star/R \\
\Gamma \vdash A_2 : \star/R \\
(\rho = +) \vee (x \notin \mathbf{fv} |b_1|_{R'}) \\
(\rho = +) \vee (x \notin \mathbf{fv} |b_3|_{R'}) \\
\Gamma \vdash (\lambda^\rho x : A_1/R.b_2) : B/R' \\
R \leq R' \\
\hline
\Gamma; \Delta \vdash (\lambda^{R,\rho} x : \gamma_1.\gamma_2) : (\lambda^\rho x : A_1/R.b_1) \sim_{R'} (\lambda^\rho x : A_2/R.b_3) \quad \text{AN_AbsCong}
\end{array}$$

$$\begin{array}{c}
\Gamma; \Delta \vdash \gamma_1 : a_1 \sim_{R'} b_1 \\
\Gamma; \Delta \vdash \gamma_2 : a_2 \sim_R b_2 \\
\Gamma \vdash a_1 \ a_2^{R,\rho} : A/R' \\
\Gamma \vdash b_1 \ b_2^{R,\rho} : B/R' \\
\Gamma; \tilde{\Gamma} \vdash \gamma_3 : A \sim_{R'} B \\
\hline
\Gamma; \Delta \vdash \gamma_1 \ \gamma_2^{R,\rho} : a_1 \ a_2^{R,\rho} \sim_{R'} b_1 \ b_2^{R,\rho} \quad \text{AN_AppCong}
\end{array}$$

$$\begin{array}{c}
\Gamma; \Delta \vdash \gamma : \Pi^\rho x : A_1/R \rightarrow B_1 \sim_{R'} \Pi^\rho x : A_2/R \rightarrow B_2 \\
\hline
\Gamma; \Delta \vdash \mathbf{piFst} \gamma : A_1 \sim_R A_2 \quad \text{AN_PiFst}
\end{array}$$

$$\begin{array}{c}
\Gamma; \Delta \vdash \gamma_1 : \Pi^\rho x : A_1/R \rightarrow B_1 \sim_{R'} \Pi^\rho x : A_2/R \rightarrow B_2 \\
\Gamma; \Delta \vdash \gamma_2 : a_1 \sim_R a_2 \\
\Gamma \vdash a_1 : A_1/R \\
\Gamma \vdash a_2 : A_2/R \\
\hline
\Gamma; \Delta \vdash \gamma_1 @ \gamma_2 : B_1\{a_1/x\} \sim_{R'} B_2\{a_2/x\} \quad \text{AN_PiSnd}
\end{array}$$

$$\begin{array}{c}
\Gamma; \Delta \vdash \gamma_1 : a_1 \sim_{A_1/R} b_1 \sim_{A_2/R} b_2 \\
\Gamma, c : a_1 \sim_{A_1/R} b_1; \Delta \vdash \gamma_3 : B_1 \sim_{R'} B_2 \\
B_3 = B_2\{c \triangleright_{R'} \mathbf{sym} \gamma_1/c\} \\
\Gamma \vdash \forall c : a_1 \sim_{A_1/R} b_1.B_1 : \star/R' \\
\Gamma \vdash \forall c : a_2 \sim_{A_2/R} b_2.B_3 : \star/R' \\
\Gamma \vdash \forall c : a_1 \sim_{A_1/R} b_1.B_2 : \star/R' \\
\hline
\Gamma; \Delta \vdash (\forall c : \gamma_1.\gamma_3) : (\forall c : a_1 \sim_{A_1/R} b_1.B_1) \sim_R (\forall c : a_2 \sim_{A_2/R} b_2.B_3) \quad \text{AN_CPiCong}
\end{array}$$

$$\begin{array}{c}
\Gamma; \Delta \vdash \gamma_1 : b_0 \sim_{A_1/R} b_1 \sim_{A_2/R} b_3 \\
\Gamma, c : b_0 \sim_{A_1/R} b_1; \Delta \vdash \gamma_3 : a_1 \sim_{R'} a_2 \\
a_3 = a_2\{c \triangleright_{R'} \mathbf{sym} \gamma_1/c\} \\
\Gamma \vdash (\Lambda c : b_0 \sim_{A_1/R} b_1.a_1) : \forall c : b_0 \sim_{A_1/R} b_1.B_1/R' \\
\Gamma \vdash (\Lambda c : b_0 \sim_{A_1/R} b_1.a_2) : B/R' \\
\Gamma \vdash (\Lambda c : b_2 \sim_{A_2/R} b_3.a_3) : \forall c : b_2 \sim_{A_2/R} b_3.B_2/R' \\
\Gamma; \tilde{\Gamma} \vdash \gamma_4 : \forall c : b_0 \sim_{A_1/R} b_1.B_1 \sim_{R'} \forall c : \phi_2.B_2 \\
\hline
\Gamma; \Delta \vdash (\lambda c : \gamma_1.\gamma_3 @ \gamma_4) : (\Lambda c : b_0 \sim_{A_1/R} b_1.a_1) \sim_{R'} (\Lambda c : b_2 \sim_{A_2/R} b_3.a_3) \quad \text{AN_CAbsCong}
\end{array}$$

$$\begin{array}{c}
\Gamma; \Delta \vdash \gamma_1 : a_1 \sim_R b_1 \\
\Gamma; \tilde{\Gamma} \vdash \gamma_2 : a_2 \sim_{R'} b_2 \\
\Gamma; \tilde{\Gamma} \vdash \gamma_3 : a_3 \sim_{R'} b_3 \\
\Gamma \vdash a_1[\gamma_2] : A/R \\
\Gamma \vdash b_1[\gamma_3] : B/R \\
\Gamma; \tilde{\Gamma} \vdash \gamma_4 : A \sim_R B \\
\hline
\Gamma; \Delta \vdash \gamma_1(\gamma_2, \gamma_3) : a_1[\gamma_2] \sim_R b_1[\gamma_3] \quad \text{AN_CAPP_CONG} \\
\\
\Gamma; \Delta \vdash \gamma_1 : (\forall c_1 : a \sim_{A/R} a'. B_1) \sim_{R_0} (\forall c_2 : b \sim_{B/R'} b'. B_2) \\
\Gamma; \tilde{\Gamma} \vdash \gamma_2 : a \sim_R a' \\
\Gamma; \tilde{\Gamma} \vdash \gamma_3 : b \sim_{R'} b' \\
\hline
\Gamma; \Delta \vdash \gamma_1 @ (\gamma_2 \sim \gamma_3) : B_1\{\gamma_2/c_1\} \sim_{R_0} B_2\{\gamma_3/c_2\} \quad \text{AN_CPI_SND} \\
\\
\Gamma; \Delta \vdash \gamma_1 : a \sim_{R_1} a' \\
\Gamma; \Delta \vdash \gamma_2 : a \sim_{A/R_1} a' \sim b \sim_{B/R_1} b' \\
\hline
\Gamma; \Delta \vdash \gamma_1 \triangleright_{R_1} \gamma_2 : b \sim_{R_1} b' \quad \text{AN_CAST} \\
\\
\Gamma; \Delta \vdash \gamma : (a \sim_{A/R} a') \sim (b \sim_{B/R} b') \\
\hline
\Gamma; \Delta \vdash \mathbf{isoSnd} \gamma : A \sim_R B \quad \text{AN_ISO_SND} \\
\\
\Gamma; \Delta \vdash \gamma : a \sim_{R_1} b \\
R_1 \leq R_2 \\
\hline
\Gamma; \Delta \vdash \mathbf{sub} \gamma : a \sim_{R_2} b \quad \text{AN_SUB}
\end{array}$$

$\boxed{\vdash \Gamma}$ context wellformedness

$$\begin{array}{c}
\overline{\vdash \emptyset} \quad \text{AN_EMPTY} \\
\\
\vdash \Gamma \\
\Gamma \vdash A : \star/R \\
x \notin \text{dom } \Gamma \\
\hline
\vdash \Gamma, x : A/R \quad \text{AN_CONSTM} \\
\\
\vdash \Gamma \\
\Gamma \vdash \phi \text{ ok} \\
c \notin \text{dom } \Gamma \\
\hline
\vdash \Gamma, c : \phi \quad \text{AN_CONSCo}
\end{array}$$

$\boxed{\vdash \Sigma}$ signature wellformedness

$$\begin{array}{c}
\overline{\vdash \emptyset} \quad \text{AN_SIG_EMPTY} \\
\\
\vdash \Sigma \\
\emptyset \vdash A : \star/R \\
\emptyset \vdash a : A/R \\
F \notin \text{dom } \Sigma \\
\hline
\vdash \Sigma \cup \{F \sim a : A/R\} \quad \text{AN_SIG_CONSAx}
\end{array}$$

$\boxed{\Gamma \vdash a \rightsquigarrow b/R}$ single-step, weak head reduction to values for annotated language

$$\begin{array}{c}
\Gamma \vdash a \rightsquigarrow a'/R_1 \\
\hline
\Gamma \vdash a \ b^{R,\rho} \rightsquigarrow a' \ b^{R,\rho}/R_1 \quad \text{AN_APPLEFT} \\
\\
\text{Value}_R (\lambda^\rho x : A/R.w) \\
\hline
\Gamma \vdash (\lambda^\rho x : A/R.w) \ a^{R,\rho} \rightsquigarrow w\{a/x\}/R \quad \text{AN_APPABS}
\end{array}$$

$$\begin{array}{c}
\frac{\Gamma \vdash a \rightsquigarrow a'/R}{\Gamma \vdash a[\gamma] \rightsquigarrow a'[\gamma]/R} \quad \text{AN_CAPPLEFT} \\
\\
\frac{}{\Gamma \vdash (\Lambda c:\phi.b)[\gamma] \rightsquigarrow b\{\gamma/c\}/R} \quad \text{AN_CAPPCABS} \\
\\
\frac{\Gamma \vdash A : \star/R \quad \Gamma, x : A/R \vdash b \rightsquigarrow b'/R_1}{\Gamma \vdash (\lambda^- x:A/R.b) \rightsquigarrow (\lambda^- x:A/R.b')/R_1} \quad \text{AN_ABSTERM} \\
\\
\frac{F \sim a : A/R \in \Sigma_1}{\Gamma \vdash F \rightsquigarrow a/R} \quad \text{AN_AXIOM} \\
\\
\frac{\Gamma \vdash a \rightsquigarrow a'/R}{\Gamma \vdash a \triangleright_{R_1} \gamma \rightsquigarrow a' \triangleright_{R_1} \gamma/R} \quad \text{AN_CONVTERM} \\
\\
\frac{\text{Value}_R v}{\Gamma \vdash (v \triangleright_{R_2} \gamma_1) \triangleright_{R_2} \gamma_2 \rightsquigarrow v \triangleright_{R_2} (\gamma_1; \gamma_2)/R} \quad \text{AN_COMBINE} \\
\\
\frac{\begin{array}{l} \text{Value}_R v \\ \Gamma; \tilde{\Gamma} \vdash \gamma : \Pi^\rho x_1:A_1/R \rightarrow B_1 \sim_{R'} \Pi^\rho x_2:A_2/R \rightarrow B_2 \\ b' = b \triangleright_{R'} \mathbf{sym}(\mathbf{piFst} \gamma) \\ \gamma' = \gamma @ (b' \models_{(\mathbf{piFst} \gamma)} b) \end{array}}{\Gamma \vdash (v \triangleright_{R'} \gamma) \ b^{R,\rho} \rightsquigarrow ((v \ b'^{R,\rho}) \triangleright_{R'} \gamma')/R} \quad \text{AN_PUSH} \\
\\
\frac{\begin{array}{l} \text{Value}_R v \\ \Gamma; \tilde{\Gamma} \vdash \gamma : \forall c_1:a_1 \sim_{B_1/R} b_1.A_1 \sim_{R'} \forall c_2:a_2 \sim_{B_2/R} b_2.A_2 \\ \gamma'_1 = \gamma_1 \triangleright_{R'} \mathbf{sym}(\mathbf{cpiFst} \gamma) \\ \gamma' = \gamma @ (\gamma'_1 \sim \gamma_1) \end{array}}{\Gamma \vdash (v \triangleright_{R'} \gamma)[\gamma_1] \rightsquigarrow ((v[\gamma'_1]) \triangleright_{R'} \gamma')/R} \quad \text{AN_CPUSH}
\end{array}$$

Definition rules: 162 good 0 bad

Definition rule clauses: 517 good 0 bad