

$tnvar, x, y, f, m, n$	variables
$covar, c$	coercion variables
$datacon, K$	
$const, T, F$	
$index, i$	indices

$relflag, \rho$	$::=$ $ $ $+$ $ $ $-$ $ $ $app\_rho \nu$ $ $ $(\rho)$	relevance flag
$appflag, \nu$	$::=$ $ $ $R$ $ $ $\rho$	applicative flag
$role, R$	$::=$ $ $ <b>Nom</b> $ $ <b>Rep</b> $ $ $R_1 \cap R_2$ S $ $ <b>param</b> $R_1 R_2$ S $ $ $app\_role \nu$ S $ $ $(R)$ S	Role
$constraint, \phi$	$::=$ $ $ $a \sim_{A/R} b$ $ $ $(\phi)$ S $ $ $\phi\{b/x\}$ S $ $ $ \phi $ S $ $ $a \sim_R b$ S	props
$tm, a, b, p, v, w, A, B, C$	$::=$ $ $ $\star$ $ $ $x$ $ $ $\lambda^\rho x:A.b$ bind $x$ in $b$ $ $ $\lambda^\rho x.b$ bind $x$ in $b$ $ $ $a \ b^\nu$ $ $ $\Pi^\rho x:A \rightarrow B$ bind $x$ in $B$ $ $ $\Lambda c:\phi.b$ bind $c$ in $b$ $ $ $\Lambda c.b$ bind $c$ in $b$ $ $ $a[\gamma]$ $ $ $\forall c:\phi.B$ bind $c$ in $B$ $ $ $a \triangleright_R \gamma$ $ $ $F$ $ $ $\square$ $ $ $\text{case}_R a \text{ of } F \rightarrow b_1 \parallel - \rightarrow b_2$ $ $ $K$ $ $ <b>match</b> $a$ <b>with</b> $brs$ $ $ <b>sub</b> $R a$ $ $ $a\{b/x\}$ S $ $ $a\{\gamma/c\}$ S $ $ $a$ S $ $ $a$ S	types and kinds

		$(a)$	S	
		$a$	S	parsing precedence is hard
		$ a _R$	S	
		<b>Int</b>	S	
		<b>Bool</b>	S	
		<b>Nat</b>	S	
		<b>Vec</b>	S	
		0	S	
		S	S	
		<b>True</b>	S	
		<b>Fix</b>	S	
		<b>Age</b>	S	
		$a \rightarrow b$	S	
		$\phi \Rightarrow A$	S	
		$a \ b$	S	
		$\lambda x. a$	S	
		$\lambda x : A. a$	S	
		$\forall x : A \rightarrow B$	S	
		<b>if</b> $\phi$ <b>then</b> $a$ <b>else</b> $b$	S	
$brs$	$::=$			case branches
		<b>none</b>		
		$K \Rightarrow a; brs$		
		$brs\{a/x\}$	S	
		$brs\{\gamma/c\}$	S	
		$(brs)$	S	
$co, \gamma$	$::=$			explicit coercions
		$\bullet$		
		$c$		
		<b>red</b> $a \ b$		
		<b>refl</b> $a$		
		$(a \models_\gamma b)$		
		<b>sym</b> $\gamma$		
		$\gamma_1; \gamma_2$		
		<b>sub</b> $\gamma$		
		$\Pi^{R,\rho} x : \gamma_1. \gamma_2$	bind $x$ in $\gamma_2$	
		$\lambda^{R,\rho} x : \gamma_1. \gamma_2$	bind $x$ in $\gamma_2$	
		$\gamma_1 \ \gamma_2^{R,\rho}$		
		<b>piFst</b> $\gamma$		
		<b>cpiFst</b> $\gamma$		
		<b>isoSnd</b> $\gamma$		
		$\gamma_1 @ \gamma_2$		
		$\forall c : \gamma_1. \gamma_3$	bind $c$ in $\gamma_3$	
		$\lambda c : \gamma_1. \gamma_3 @ \gamma_4$	bind $c$ in $\gamma_3$	
		$\gamma(\gamma_1, \gamma_2)$		

		$\gamma @ (\gamma_1 \sim \gamma_2)$	
		$\gamma_1 \triangleright_R \gamma_2$	
		$\gamma_1 \sim_A \gamma_2$	
		<b>conv</b> $\phi_1 \sim_\gamma \phi_2$	
		<b>eta</b> $a$	
		<b>left</b> $\gamma \gamma'$	
		<b>right</b> $\gamma \gamma'$	
		$(\gamma)$	S
		$\gamma$	S
		$\gamma\{a/x\}$	S
$role\_context, \Omega$	::=		$role\_contexts$
		$\emptyset$	
		$x : R$	
		$\Omega, x : R$	
		$\Omega, \Omega'$	M
		$\Gamma_{\text{Nom}}$	
		$(\Omega)$	M
		$\Omega$	M
$roles, Rs$	::=		
		<b>nilR</b>	
		$R, Rs$	
$sig\_sort$	::=		signature classifier
		$: A @ Rs$	
		$[p] \sim a : A / R @ Rs$	
$sort$	::=		binding classifier
		<b>Tm</b> $A$	
		<b>Co</b> $\phi$	
$context, \Gamma$	::=		contexts
		$\emptyset$	
		$\Gamma, x : A$	
		$\Gamma, c : \phi$	
		$\Gamma\{b/x\}$	M
		$\Gamma\{\gamma/c\}$	M
		$\Gamma, \Gamma'$	M
		$ \Gamma $	M
		$(\Gamma)$	M
		$\Gamma$	M
$sig, \Sigma$	::=		signatures
		$\emptyset$	
		$\Sigma \cup \{F sig\_sort\}$	
		$\Sigma_0$	M

		$\Sigma_1$	M
		$ \Sigma $	M
$available\_props, \Delta$	$::=$		
		$\emptyset$	
		$\Delta, c$	
		$\tilde{\Gamma}$	M
		$(\Delta)$	M
$terminals$	$::=$		
		$\leftrightarrow$	
		$\Leftrightarrow$	
		$\longrightarrow$	
		<b>min</b>	
		$\equiv$	
		$\forall$	
		$\in$	
		$\notin$	
		$\Leftarrow$	
		$\Rightarrow$	
		$\Rightarrow^*$	
		$\rightarrow$	
		$\Lambda$	
		$\square$	
		$\vdash$	
		$\dashv$	
		$\models$	
		$\models$	
		$\neq$	
		$\triangleright$	
		<b>ok</b>	
		$-$	
		$\rightsquigarrow$	
		$\rightsquigarrow^*$	
		$\rightsquigarrow$	
		$\emptyset$	
		$\circ$	
		<b>fv</b>	
		<b>dom</b>	
		$\sim$	
		$\succ$	
		$ $	
		$\bullet$	
		<b>fst</b>	
		<b>snd</b>	

	$ \begin{array}{ l} \text{as} \\   \Rightarrow   \\ \vdash = \\ \text{refl}_2 \\ ++ \\ \end{array} $	
<i>formula, <math>\psi</math></i>	$ \begin{array}{ l} ::= \\ \text{judgement} \\ x : A \in \Gamma \\ x : R \in \Omega \\ c : \phi \in \Gamma \\ F \text{ sig\_sort} \in \Sigma \\ x \in \Delta \\ c \in \Delta \\ c \text{ not relevant} \in \gamma \\ x \notin \text{fva} \\ x \notin \text{dom } \Gamma \\ \text{uniq}\Gamma \\ \text{uniq}(\Omega) \\ c \notin \text{dom } \Gamma \\ T \notin \text{dom } \Sigma \\ F \notin \text{dom } \Sigma \\ R_1 = R_2 \\ a = b \\ \phi_1 = \phi_2 \\ \Gamma_1 = \Gamma_2 \\ \gamma_1 = \gamma_2 \\ \neg\psi \\ \psi_1 \wedge \psi_2 \\ \psi_1 \vee \psi_2 \\ \psi_1 \Rightarrow \psi_2 \\ (\psi) \\ \psi \\ c : (a : A \sim b : B) \in \Gamma \\ \{y/x\}B = B_1 \\ \{c_1/c_2\}B = B_1 \end{array} $	<p>suppress lc hypothesis generated by Ott</p>
<i>JSubRole</i>	$ \begin{array}{ l} ::= \\   \quad R_1 \leq R_2 \end{array} $	Subroling judgement
<i>JPath</i>	$ \begin{array}{ l} ::= \\   \quad \text{Path}_R a = F@Rs \end{array} $	Type headed by constant (partial function)
<i>JPat</i>	$ \begin{array}{ l} ::= \\   \quad \Gamma \models a : A \text{ pat}/R \end{array} $	Pattern judgment

$JCaseSyntax$	$::=$   $\Gamma \models \text{case } a : A \text{ of } b : B \Rightarrow C \mid C'$	Case Syntax judgment
$JIrrelVarCheck$	$::=$   $\text{IrrelevantVar } a \cap \text{fv } b = \emptyset$	Irrelevant Variable Check
$JPatCtx$	$::=$   $\models p : A \text{ patctx} = \Omega \mid \Gamma$	Contexts associated to a pattern
$JMatchSubst$	$::=$   $\text{match}_R a_1 \text{ with } a_2 \rightarrow b_1 = b_2$	match and substitute
$JMatchApply$	$::=$   $\text{match } a \text{ applyto } b = b'$	match and apply arguments
$JValue$	$::=$   $\text{Value}_R A$	values
$JValueType$	$::=$   $\text{ValueType}_R A$	Types with head forms (erased language)
$Jconsistent$	$::=$   $\text{consistent}_R a \ b$	(erased) types do not differ in their heads
$Jroleing$	$::=$   $\Omega \models a : R$	Roleing judgment
$JChk$	$::=$   $(\rho = +) \vee (x \notin \text{fv } A)$	irrelevant argument check
$Jpar$	$::=$   $\Omega \models a \Rightarrow_R b$   $\Omega \models a \Rightarrow_R^* b$   $\Omega \models a \Leftrightarrow_R b$	parallel reduction (implicit language) multistep parallel reduction parallel reduction to a common term
$Jbeta$	$::=$   $\models a > b / R$   $\models a \rightsquigarrow b / R$   $\models a \rightsquigarrow^* b / R$	primitive reductions on erased terms single-step head reduction for implicit language multistep reduction
$Jett$	$::=$   $\Gamma \models \phi \text{ ok}$   $\Gamma \models a : A$   $\Gamma; \Delta \models \phi_1 \equiv \phi_2$   $\Gamma; \Delta \models a \equiv b : A / R$   $\models \Gamma$	Prop wellformedness typing prop equality definitional equality context wellformedness

$Jsig$	$::=$	$\models \Sigma$	signature wellformedness
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$judgement$	$::=$	
		$JSubRole$
		$JPath$
		$JPat$
		$JCaseSyntax$
		$JIrrelVarCheck$
		$JPatCtx$
		$JMatchSubst$
		$JMatchApply$
		$JValue$
		$JValueType$
		$Jconsistent$
		$Jroleing$
		$JChk$
		$Jpar$
		$Jbeta$
		$Jett$
		$Jsig$

$user\_syntax$	$::=$	
		$tmvar$
		$covar$
		$datacon$
		$const$
		$index$
		$relflag$
		$appflag$
		$role$
		$constraint$
		$tm$
		$brs$
		$co$
		$role\_context$
		$roles$
		$sig\_sort$
		$sort$
		$context$
		$sig$
		$available\_props$
		$terminals$
		$formula$

$\boxed{R_1 \leq R_2}$	Subroling judgement
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$\overline{\mathbf{Nom} \leq R}$	NOMBOT
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$\overline{R \leq \mathbf{Rep}}$	REPTOP
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$$\frac{}{\overline{R \leq R}} \text{REFL}$$

$$\frac{R_1 \leq R_2 \quad R_2 \leq R_3}{R_1 \leq R_3} \text{TRANS}$$

$\boxed{\text{Path}_R \ a = F@Rs}$  Type headed by constant (partial function)

$$\frac{F : A@Rs \in \Sigma_0}{\text{Path}_R \ F = F@Rs} \text{PATH\_ABSCONST}$$

$$\frac{F [p] \sim a : A/R_1@Rs \in \Sigma_0 \quad \neg(R_1 \leq R)}{\text{Path}_R \ F = F@Rs} \text{PATH\_CONST}$$

$$\frac{\text{Path}_R \ a = F@R_1, Rs \quad \text{app\_role} \nu = R_1}{\text{Path}_R \ (a \ b''') = F@Rs} \text{PATH\_APP}$$

$$\frac{\text{Path}_R \ a = F@Rs}{\text{Path}_R \ (a[\bullet]) = F@Rs} \text{PATH\_CAPP}$$

$\boxed{\Gamma \models a : A \text{ pat}/R}$  Pattern judgment

$$\frac{F : A@Rs \in \Sigma_0}{\emptyset \models F : A \text{ pat}/R} \text{PAT\_ABSCONST}$$

$$\frac{F [p] \sim a : A/R_1@Rs \in \Sigma_0 \quad \neg(R_1 \leq R)}{\emptyset \models F : A \text{ pat}/R} \text{PAT\_CONST}$$

$$\frac{\Gamma \models a : \Pi^\rho y : A_1 \rightarrow B_1 \text{ pat}/R \quad \{y/x\}B = B_1}{\Gamma, x : A_1 \models (a \ x^\rho) : B \text{ pat}/R} \text{PAT\_APP}$$

$$\frac{\Gamma \models a : \forall c_1 : \phi. B_1 \text{ pat}/R \quad \{c_1/c\}B = B_1}{\Gamma, c : \phi \models (a[c]) : B \text{ pat}/R} \text{PAT\_CAPP}$$

$\boxed{\Gamma \models \text{case } a : A \text{ of } b : B \Rightarrow C | C'}$  Case Syntax judgment

$$\frac{\text{uniq}\Gamma}{\Gamma \models \text{case } a : A \text{ of } b : A \Rightarrow \forall c : (a \sim_{A/R} b). C | C'} \text{CASESYNTAX\_BASE}$$

$$\frac{\Gamma, x : A \models \text{case } a : A_1 \text{ of } b \ x^+ : B \Rightarrow C | C'}{\Gamma \models \text{case } a : A_1 \text{ of } b : \Pi^+ x : A \rightarrow B \Rightarrow \Pi^+ x : A \rightarrow C | C'} \text{CASESYNTAX\_PIREL}$$

$$\frac{\Gamma, x : A \models \text{case } a : A_1 \text{ of } b \ x^- : B \Rightarrow C | C'}{\Gamma \models \text{case } a : A_1 \text{ of } b : \Pi^- x : A \rightarrow B \Rightarrow \Pi^- x : A \rightarrow C | C'} \text{CASESYNTAX\_PIIRREL}$$

$$\frac{\Gamma, c : \phi \models \text{case } a : A \text{ of } b[c] : B \Rightarrow C | C'}{\Gamma \models \text{case } a : A \text{ of } b : \forall c : \phi. B \Rightarrow \forall c : \phi. C | C'} \text{CASESYNTAX\_CPI}$$

$\boxed{\text{IrrelevantVar } a \cap \text{fv } b = \emptyset}$  Irrelevant Variable Check

$$\overline{\text{IrrelevantVar } F \cap \text{fv } b = \emptyset} \text{IRRELVARCHECK\_CONST}$$

$$\begin{array}{c}
\frac{\text{IrrelevantVar } a \cap \text{fv } b = \emptyset}{\text{IrrelevantVar}(a \ x^+) \cap \text{fv } b = \emptyset} \quad \text{IRRELVARCHECK\_APP} \\
\frac{\text{IrrelevantVar } a \cap \text{fv } b = \emptyset}{x \notin \text{fv } b} \quad \text{IRRELVARCHECK\_IAPP} \\
\frac{\text{IrrelevantVar } a \cap \text{fv } b = \emptyset}{\text{IrrelevantVar}(a[c]) \cap \text{fv } b = \emptyset} \quad \text{IRRELVARCHECK\_CAPP}
\end{array}$$

$\boxed{\models p : A \text{ patctx} = \Omega \mid \Gamma}$  Contexts associated to a pattern

$$\begin{array}{c}
\frac{}{\models F : A \text{ patctx} = \emptyset \mid \emptyset} \quad \text{PATCTX\_CONST} \\
\frac{\models p : \Pi^+ x : A' \rightarrow A \text{ patctx} = \Omega \mid \Gamma}{\models p \ x : A \text{ patctx} = \Omega, x : R \mid \Gamma, x : A'} \quad \text{PATCTX\_PIREL} \\
\frac{\models p : \Pi^- x : A' \rightarrow A \text{ patctx} = \Omega \mid \Gamma}{\models p \ \square : A \text{ patctx} = \Omega \mid \Gamma, x : A'} \quad \text{PATCTX\_PIIRR} \\
\frac{\models p : \forall c : \phi. A \text{ patctx} = \Omega \mid \Gamma}{\models p[\bullet] : A \text{ patctx} = \Omega \mid \Gamma, c : \phi} \quad \text{PATCTX\_CPI}
\end{array}$$

$\boxed{\text{match}_R \ a_1 \text{ with } a_2 \rightarrow b_1 = b_2}$  match and substitute

$$\begin{array}{c}
\frac{F : A @ Rs \in \Sigma_0}{\text{match}_R \ F \text{ with } F \rightarrow b = b} \quad \text{MATCHSUBST\_ABSCONST} \\
\frac{F[p] \sim a : A / R_1 @ Rs \in \Sigma_0}{\neg(R_1 \leq R)} \quad \text{MATCHSUBST\_CONST} \\
\frac{\text{match}_R \ a_1 \text{ with } a_2 \rightarrow b_1 = b_2}{\text{match}_R \ (a_1 \ a^{R'}) \text{ with } (a_2 \ x^+) \rightarrow b_1 = (b_2\{a/x\})} \quad \text{MATCHSUBST\_APPRELR} \\
\frac{\text{match}_R \ a_1 \text{ with } a_2 \rightarrow b_1 = b_2}{\text{match}_R \ (a_1 \ a^+) \text{ with } (a_2 \ x^+) \rightarrow b_1 = (b_2\{a/x\})} \quad \text{MATCHSUBST\_APPREL} \\
\frac{\text{match}_R \ a_1 \text{ with } a_2 \rightarrow b_1 = b_2}{\text{match}_R \ (a_1 \ \square^-) \text{ with } (a_2 \ x^-) \rightarrow b_1 = (b_2\{\square/x\})} \quad \text{MATCHSUBST\_APPIRREL} \\
\frac{\text{match}_R \ a_1 \text{ with } a_2 \rightarrow b_1 = b_2}{\text{match}_R \ (a_1[\bullet]) \text{ with } (a_2[c]) \rightarrow b_1 = (b_2\{\bullet/c\})} \quad \text{MATCHSUBST\_CAPP}
\end{array}$$

$\boxed{\text{match } a \text{ applyto } b = b'}$  match and apply arguments

$$\begin{array}{c}
\frac{F : A @ Rs \in \Sigma_0}{\text{match } F \text{ applyto } b = b} \quad \text{MATCHAPPLY\_CONST} \\
\frac{\text{match } a \text{ applyto } b = b'}{\text{match } a \ a'' \text{ applyto } b = b' \ a'^{(app.rhov)}} \quad \text{MATCHAPPLY\_APP} \\
\frac{\text{match } a \text{ applyto } b = b'}{\text{match } a[\gamma] \text{ applyto } b = b'[\gamma]} \quad \text{MATCHAPPLY\_CAPP}
\end{array}$$

$\boxed{\text{Value}_R \ A}$  values

$$\frac{}{\text{Value}_R \ \star} \quad \text{VALUE\_STAR}$$

$$\begin{array}{c}
\frac{}{\text{Value}_R \Pi^\rho x : A \rightarrow B} \text{VALUE\_PI} \\
\frac{}{\text{Value}_R \forall c : \phi. B} \text{VALUE\_CPI} \\
\frac{}{\text{Value}_R \lambda^+ x : A. a} \text{VALUE\_ABSREL} \\
\frac{}{\text{Value}_R \lambda^+ x. a} \text{VALUE\_UABSREL} \\
\frac{\text{Value}_R a}{\text{Value}_R \lambda^- x. a} \text{VALUE\_UABSIRREL} \\
\frac{}{\text{Value}_R \Lambda c : \phi. a} \text{VALUE\_CABS} \\
\frac{}{\text{Value}_R \Lambda c. a} \text{VALUE\_UCABS} \\
\frac{\text{Path}_R a = F @ R s}{\text{Value}_R a} \text{VALUE\_PATH}
\end{array}$$

$\boxed{\text{ValueType}_R A}$  Types with head forms (erased language)

$$\begin{array}{c}
\frac{}{\text{ValueType}_R \star} \text{VALUE\_TYPE\_STAR} \\
\frac{}{\text{ValueType}_R \Pi^\rho x : A \rightarrow B} \text{VALUE\_TYPE\_PI} \\
\frac{}{\text{ValueType}_R \forall c : \phi. B} \text{VALUE\_TYPE\_CPI} \\
\frac{\text{Path}_R a = F @ R s}{\text{ValueType}_R a} \text{VALUE\_TYPE\_PATH}
\end{array}$$

$\boxed{\text{consistent}_R a \ b}$  (erased) types do not differ in their heads

$$\begin{array}{c}
\frac{}{\text{consistent}_R \star \star} \text{CONSISTENT\_A\_STAR} \\
\frac{}{\text{consistent}_{R'} (\Pi^\rho x_1 : A_1 \rightarrow B_1) (\Pi^\rho x_2 : A_2 \rightarrow B_2)} \text{CONSISTENT\_A\_PI} \\
\frac{}{\text{consistent}_R (\forall c_1 : \phi_1. A_1) (\forall c_2 : \phi_2. A_2)} \text{CONSISTENT\_A\_CPI} \\
\frac{\text{Path}_R a_1 = F @ R s \quad \text{Path}_R a_2 = F @ R s}{\text{consistent}_R a_1 \ a_2} \text{CONSISTENT\_A\_PATH} \\
\frac{\neg \text{ValueType}_R b}{\text{consistent}_R a \ b} \text{CONSISTENT\_A\_STEP\_R} \\
\frac{\neg \text{ValueType}_R a}{\text{consistent}_R a \ b} \text{CONSISTENT\_A\_STEP\_L}
\end{array}$$

$\boxed{\Omega \models a : R}$  Roleing judgment

$$\begin{array}{c}
\frac{\text{uniq}(\Omega)}{\Omega \models \square : R} \text{ROLE\_A\_BULLET} \\
\frac{\text{uniq}(\Omega)}{\Omega \models \star : R} \text{ROLE\_A\_STAR}
\end{array}$$

$$\frac{\begin{array}{c} \text{uniq}(\Omega) \\ x : R \in \Omega \\ R \leq R_1 \end{array}}{\Omega \models x : R_1} \quad \text{ROLE\_A\_VAR}$$

$$\frac{\Omega, x : \mathbf{Nom} \models a : R}{\Omega \models (\lambda^\rho x. a) : R} \quad \text{ROLE\_A\_ABS}$$

$$\frac{\begin{array}{c} \Omega \models a : R \\ \Omega \models b : \text{app\_role}\nu \end{array}}{\Omega \models (a \ b^\nu) : R} \quad \text{ROLE\_A\_APP}$$

$$\frac{\begin{array}{c} \Omega \models A : R \\ \Omega, x : \mathbf{Nom} \models B : R \end{array}}{\Omega \models (\Pi^\rho x : A \rightarrow B) : R} \quad \text{ROLE\_A\_PI}$$

$$\frac{\begin{array}{c} \Omega \models a : R_1 \\ \Omega \models b : R_1 \\ \Omega \models A : R_0 \\ \Omega \models B : R \end{array}}{\Omega \models (\forall c : a \sim_{A/R_1} b. B) : R} \quad \text{ROLE\_A\_CPI}$$

$$\frac{\Omega \models b : R}{\Omega \models (\Lambda c. b) : R} \quad \text{ROLE\_A\_CABS}$$

$$\frac{\Omega \models a : R}{\Omega \models (a[\bullet]) : R} \quad \text{ROLE\_A\_CAPP}$$

$$\frac{\begin{array}{c} \text{uniq}(\Omega) \\ F : A@Rs \in \Sigma_0 \end{array}}{\Omega \models F : R} \quad \text{ROLE\_A\_CONST}$$

$$\frac{\begin{array}{c} \text{uniq}(\Omega) \\ F[p] \sim a : A/R@Rs \in \Sigma_0 \end{array}}{\Omega \models F : R_1} \quad \text{ROLE\_A\_FAM}$$

$$\frac{\begin{array}{c} \Omega \models a : R \\ \Omega, \Gamma_{\mathbf{Nom}} \models b_1 : R_1 \\ \Omega \models b_2 : R_1 \end{array}}{\Omega \models \text{case}_R a \text{ of } F \rightarrow b_1 \parallel - \rightarrow b_2 : R_1} \quad \text{ROLE\_A\_PATTERN}$$

$$\boxed{(\rho = +) \vee (x \notin \text{fv } A)} \quad \text{irrelevant argument check}$$

$$\overline{(+ = +) \vee (x \notin \text{fv } A)} \quad \text{RHO\_REL}$$

$$\frac{x \notin \text{fv } A}{(- = +) \vee (x \notin \text{fv } A)} \quad \text{RHO\_IRRREL}$$

$$\boxed{\Omega \models a \Rightarrow_R b} \quad \text{parallel reduction (implicit language)}$$

$$\frac{\Omega \models a : R}{\Omega \models a \Rightarrow_R a} \quad \text{PAR\_REFL}$$

$$\frac{\begin{array}{c} \Omega \models a \Rightarrow_R (\lambda^\rho x. a') \\ \Omega \models b \Rightarrow_{\text{app\_role}\nu} b' \end{array}}{\Omega \models a \ b^\nu \Rightarrow_R a' \{b'/x\}} \quad \text{PAR\_BETA}$$

$$\begin{array}{c}
\frac{\Omega \models a \Rightarrow_R a' \quad \Omega \models b \Rightarrow_{app.role} b'}{\Omega \models a \ b^\nu \Rightarrow_R a' \ b'^\nu} \text{PAR\_APP} \\
\\
\frac{\Omega \models a \Rightarrow_R (\Lambda c. a')}{\Omega \models a[\bullet] \Rightarrow_R a' \{ \bullet / c \}} \text{PAR\_CBETA} \\
\\
\frac{\Omega \models a \Rightarrow_R a'}{\Omega \models a[\bullet] \Rightarrow_R a'[\bullet]} \text{PAR\_CAPP} \\
\\
\frac{\Omega, x : \mathbf{Nom} \models a \Rightarrow_R a'}{\Omega \models \lambda^\rho x. a \Rightarrow_R \lambda^\rho x. a'} \text{PAR\_ABS} \\
\\
\frac{\Omega \models A \Rightarrow_R A' \quad \Omega, x : \mathbf{Nom} \models B \Rightarrow_R B'}{\Omega \models \Pi^\rho x : A \rightarrow B \Rightarrow_R \Pi^\rho x : A' \rightarrow B'} \text{PAR\_PI} \\
\\
\frac{\Omega \models a \Rightarrow_R a'}{\Omega \models \Lambda c. a \Rightarrow_R \Lambda c. a'} \text{PAR\_CABS} \\
\\
\frac{\Omega \models A \Rightarrow_{R_0} A' \quad \Omega \models a \Rightarrow_{R_1} a' \quad \Omega \models b \Rightarrow_{R_1} b' \quad \Omega \models B \Rightarrow_R B'}{\Omega \models \forall c : a \sim_{A/R_1} b. B \Rightarrow_R \forall c : a' \sim_{A'/R_1} b'. B'} \text{PAR\_CPI} \\
\\
\frac{\text{Path}_{R_1} a = F@Rs \quad F[p] \sim b : A/R_1@Rs \in \Sigma_0 \quad \Omega \models a \Rightarrow_R a' \quad \text{match}_{R_2} a' \text{ with } p \rightarrow b = b' \quad R_1 \leq R \quad \text{uniq}(\Omega)}{\Omega \models a \Rightarrow_R b'} \text{PAR\_AXIOM} \\
\\
\frac{\Omega \models a \Rightarrow_R a' \quad \Omega \models b_1 \Rightarrow_{R_0} b'_1 \quad \Omega \models b_2 \Rightarrow_{R_0} b'_2}{\Omega \models (\text{case}_R a \text{ of } F \rightarrow b_1 \parallel - \rightarrow b_2) \Rightarrow_{R_0} (\text{case}_R a' \text{ of } F \rightarrow b'_1 \parallel - \rightarrow b'_2)} \text{PAR\_PATTERN} \\
\\
\frac{\Omega \models a \Rightarrow_R a' \quad \Omega \models b_1 \Rightarrow_{R_0} b'_1 \quad \text{Path}_{R_1} a' = F@Rs \quad \text{match } a' \text{ applyto } b'_1 = b}{\Omega \models (\text{case}_R a \text{ of } F \rightarrow b_1 \parallel - \rightarrow b_2) \Rightarrow_{R_0} b[\bullet]} \text{PAR\_PATTERNTRUE} \\
\\
\frac{\Omega \models a \Rightarrow_R a' \quad \Omega \models b_2 \Rightarrow_{R_0} b'_2 \quad \text{Value}_R a' \quad \neg(\text{Path}_{R_1} a' = F@Rs)}{\Omega \models (\text{case}_R a \text{ of } F \rightarrow b_1 \parallel - \rightarrow b_2) \Rightarrow_{R_0} b'_2} \text{PAR\_PATTERNFALSE} \\
\\
\boxed{\Omega \models a \Rightarrow_R^* b} \quad \text{multistep parallel reduction} \\
\\
\frac{}{\Omega \models a \Rightarrow_R^* a} \text{MP\_REFL}
\end{array}$$

$$\frac{\Omega \models a \Rightarrow_R b \quad \Omega \models b \Rightarrow_R^* a'}{\Omega \models a \Rightarrow_R^* a'} \quad \text{MP\_STEP}$$

$\boxed{\Omega \models a \Leftrightarrow_R b}$  parallel reduction to a common term

$$\frac{\Omega \models a_1 \Rightarrow_R^* b \quad \Omega \models a_2 \Rightarrow_R^* b}{\Omega \models a_1 \Leftrightarrow_R a_2} \quad \text{JOIN}$$

$\boxed{\models a > b/R}$  primitive reductions on erased terms

$$\frac{\text{Value}_{R_1} (\lambda^\rho x.v)}{\models (\lambda^\rho x.v) \ b^\nu > v\{b/x\}/R_1} \quad \text{BETA\_APPAbs}$$

$$\frac{}{\models (\Lambda c.a')[\bullet] > a'\{\bullet/c\}/R} \quad \text{BETA\_CAPPCAbs}$$

$$\frac{\begin{array}{l} \text{Path}_{R_1} a = F@Rs \\ F[p] \sim b : A/R_1@Rs \in \Sigma_0 \\ \text{match}_{R_2} a \text{ with } p \rightarrow b = b' \\ R_1 \leq R \end{array}}{\models a > b'/R} \quad \text{BETA\_AXIOM}$$

$$\frac{\begin{array}{l} \text{Path}_{R_1} a = F@Rs \\ \text{match } a \text{ applyto } b_1 = b'_1 \end{array}}{\models \text{case}_R a \text{ of } F \rightarrow b_1 \parallel \_ \rightarrow b_2 > b'_1[\bullet]/R_0} \quad \text{BETA\_PATTERNTRUE}$$

$$\frac{\begin{array}{l} \text{Value}_R a \\ \neg(\text{Path}_{R_1} a = F@Rs) \end{array}}{\models \text{case}_R a \text{ of } F \rightarrow b_1 \parallel \_ \rightarrow b_2 > b_2/R_0} \quad \text{BETA\_PATTERNFALSE}$$

$\boxed{\models a \rightsquigarrow b/R}$  single-step head reduction for implicit language

$$\frac{\models a \rightsquigarrow a'/R_1}{\models \lambda^- x.a \rightsquigarrow \lambda^- x.a'/R_1} \quad \text{E\_ABSTERM}$$

$$\frac{\models a \rightsquigarrow a'/R_1}{\models a \ b^\nu \rightsquigarrow a' \ b^\nu/R_1} \quad \text{E\_APPLEFT}$$

$$\frac{\models a \rightsquigarrow a'/R}{\models a[\bullet] \rightsquigarrow a'[\bullet]/R} \quad \text{E\_CAPPLEFT}$$

$$\frac{\models a \rightsquigarrow a'/R}{\models \text{case}_R a \text{ of } F \rightarrow b_1 \parallel \_ \rightarrow b_2 \rightsquigarrow \text{case}_R a' \text{ of } F \rightarrow b_1 \parallel \_ \rightarrow b_2/R_0} \quad \text{E\_PATTERN}$$

$$\frac{\models a > b/R}{\models a \rightsquigarrow b/R} \quad \text{E\_PRIM}$$

$\boxed{\models a \rightsquigarrow^* b/R}$  multistep reduction

$$\frac{}{\models a \rightsquigarrow^* a/R} \quad \text{EQUAL}$$

$$\frac{\begin{array}{l} \models a \rightsquigarrow b/R \\ \models b \rightsquigarrow^* a'/R \end{array}}{\models a \rightsquigarrow^* a'/R} \quad \text{STEP}$$

$\boxed{\Gamma \models \phi \text{ ok}}$  Prop wellformedness

$$\frac{\begin{array}{c} \Gamma \models a : A \\ \Gamma \models b : A \\ \Gamma \models A : \star \end{array}}{\Gamma \models a \sim_{A/R} b \text{ ok}} \quad \text{E\_WFF}$$

$\boxed{\Gamma \models a : A}$     typing

$$\frac{\begin{array}{c} \vdash \Gamma \\ \hline \Gamma \vdash \star : \star \end{array}}{\quad} \quad \text{E\_STAR}$$

$$\frac{\begin{array}{c} \vdash \Gamma \\ x : A \in \Gamma \\ \hline \Gamma \vdash x : A \end{array}}{\quad} \quad \text{E\_VAR}$$

$$\frac{\begin{array}{c} \Gamma, x : A \vdash B : \star \\ \Gamma \vdash A : \star \\ \hline \Gamma \vdash \Pi^\rho x : A \rightarrow B : \star \end{array}}{\quad} \quad \text{E\_PI}$$

$$\frac{\begin{array}{c} \Gamma, x : A \vdash a : B \\ \Gamma \vdash A : \star \\ (\rho = +) \vee (x \notin \text{fv } a) \\ \hline \Gamma \vdash \lambda^\rho x. a : (\Pi^\rho x : A \rightarrow B) \end{array}}{\quad} \quad \text{E\_ABS}$$

$$\frac{\begin{array}{c} \Gamma \vdash b : \Pi^+ x : A \rightarrow B \\ \Gamma \vdash a : A \\ \hline \Gamma \vdash b \ a^+ : B\{a/x\} \end{array}}{\quad} \quad \text{E\_APP}$$

$$\frac{\begin{array}{c} \Gamma \vdash b : \Pi^+ x : A \rightarrow B \\ \Gamma \vdash a : A \\ \text{Path}_{R'} \ b = F @ R, Rs \\ \hline \Gamma \vdash b \ a^R : B\{a/x\} \end{array}}{\quad} \quad \text{E\_TAPP}$$

$$\frac{\begin{array}{c} \Gamma \vdash b : \Pi^- x : A \rightarrow B \\ \Gamma \vdash a : A \\ \hline \Gamma \vdash b \ \Box^- : B\{a/x\} \end{array}}{\quad} \quad \text{E\_IAPP}$$

$$\frac{\begin{array}{c} \Gamma \vdash a : A \\ \Gamma; \tilde{\Gamma} \vdash A \equiv B : \star / \mathbf{Rep} \\ \Gamma \vdash B : \star \\ \hline \Gamma \vdash a : B \end{array}}{\quad} \quad \text{E\_CONV}$$

$$\frac{\begin{array}{c} \Gamma, c : \phi \vdash B : \star \\ \Gamma \vdash \phi \text{ ok} \\ \hline \Gamma \vdash \forall c : \phi. B : \star \end{array}}{\quad} \quad \text{E\_CPI}$$

$$\frac{\begin{array}{c} \Gamma, c : \phi \vdash a : B \\ \Gamma \vdash \phi \text{ ok} \\ \hline \Gamma \vdash \Lambda c. a : \forall c : \phi. B \end{array}}{\quad} \quad \text{E\_CABS}$$

$$\frac{\begin{array}{c} \Gamma \vdash a_1 : \forall c : (a \sim_{A/R} b). B_1 \\ \Gamma; \tilde{\Gamma} \vdash a \equiv b : A/R \\ \hline \Gamma \vdash a_1[\bullet] : B_1\{\bullet/c\} \end{array}}{\quad} \quad \text{E\_CAPP}$$

$$\frac{\begin{array}{c} \vdash \Gamma \\ F : A @ Rs \in \Sigma_0 \\ \emptyset \vdash A : \star \\ \hline \Gamma \vdash F : A \end{array}}{\quad} \quad \text{E\_CONST}$$

$$\begin{array}{c}
\frac{\begin{array}{l} \vdash \Gamma \\ F[p] \sim a : A/R_1 @ R_s \in \Sigma_0 \\ \emptyset \vdash A : \star \end{array}}{\Gamma \vdash F : A} \quad \text{E\_FAM} \\
\\
\frac{\begin{array}{l} \Gamma \vdash a : A \\ \Gamma \vdash F : A_1 \\ \Gamma \vdash b_1 : B \\ \Gamma \vdash b_2 : C \\ \Gamma \vdash \text{case } a : A \text{ of } F : A_1 \Rightarrow B | C \end{array}}{\Gamma \vdash \text{case}_R a \text{ of } F \rightarrow b_1 || - \rightarrow b_2 : C} \quad \text{E\_CASE} \\
\\
\boxed{\Gamma; \Delta \vdash \phi_1 \equiv \phi_2} \quad \text{prop equality} \\
\\
\frac{\begin{array}{l} \Gamma; \Delta \vdash A_1 \equiv A_2 : A/R \\ \Gamma; \Delta \vdash B_1 \equiv B_2 : A/R \end{array}}{\Gamma; \Delta \vdash A_1 \sim_{A/R} B_1 \equiv A_2 \sim_{A/R} B_2} \quad \text{E\_PROP\_CONG} \\
\\
\frac{\begin{array}{l} \Gamma; \Delta \vdash A \equiv B : \star/R_0 \\ \Gamma \vdash A_1 \sim_{A/R} A_2 \text{ ok} \\ \Gamma \vdash A_1 \sim_{B/R} A_2 \text{ ok} \end{array}}{\Gamma; \Delta \vdash A_1 \sim_{A/R} A_2 \equiv A_1 \sim_{B/R} A_2} \quad \text{E\_ISO\_CONV} \\
\\
\frac{\Gamma; \Delta \vdash \forall c : (a_1 \sim_{A/R_1} a_2). B_1 \equiv \forall c : (b_1 \sim_{B/R_2} b_2). B_2 : \star/R'}{\Gamma; \Delta \vdash a_1 \sim_{A/R_1} a_2 \equiv b_1 \sim_{B/R_2} b_2} \quad \text{E\_CPIFST} \\
\\
\boxed{\Gamma; \Delta \vdash a \equiv b : A/R} \quad \text{definitional equality} \\
\\
\frac{\begin{array}{l} \vdash \Gamma \\ c : (a \sim_{A/R} b) \in \Gamma \\ c \in \Delta \end{array}}{\Gamma; \Delta \vdash a \equiv b : A/R} \quad \text{E\_ASSN} \\
\\
\frac{\Gamma \vdash a : A}{\Gamma; \Delta \vdash a \equiv a : A/\mathbf{Nom}} \quad \text{E\_REFL} \\
\\
\frac{\Gamma; \Delta \vdash b \equiv a : A/R}{\Gamma; \Delta \vdash a \equiv b : A/R} \quad \text{E\_SYM} \\
\\
\frac{\begin{array}{l} \Gamma; \Delta \vdash a \equiv a_1 : A/R \\ \Gamma; \Delta \vdash a_1 \equiv b : A/R \end{array}}{\Gamma; \Delta \vdash a \equiv b : A/R} \quad \text{E\_TRANS} \\
\\
\frac{\begin{array}{l} \Gamma; \Delta \vdash a \equiv b : A/R_1 \\ R_1 \leq R_2 \end{array}}{\Gamma; \Delta \vdash a \equiv b : A/R_2} \quad \text{E\_SUB} \\
\\
\frac{\begin{array}{l} \Gamma \vdash a_1 : B \\ \Gamma \vdash a_2 : B \\ \vdash a_1 > a_2/R \end{array}}{\Gamma; \Delta \vdash a_1 \equiv a_2 : B/R} \quad \text{E\_BETA} \\
\\
\frac{\begin{array}{l} \Gamma; \Delta \vdash A_1 \equiv A_2 : \star/R' \\ \Gamma, x : A_1; \Delta \vdash B_1 \equiv B_2 : \star/R' \\ \Gamma \vdash A_1 : \star \\ \Gamma \vdash \Pi^\rho x : A_1 \rightarrow B_1 : \star \\ \Gamma \vdash \Pi^\rho x : A_2 \rightarrow B_2 : \star \end{array}}{\Gamma; \Delta \vdash (\Pi^\rho x : A_1 \rightarrow B_1) \equiv (\Pi^\rho x : A_2 \rightarrow B_2) : \star/R'} \quad \text{E\_PI\_CONG}
\end{array}$$



$$\begin{array}{c}
\frac{\Gamma, x : A_1; \Delta \models b_1 \equiv b_2 : B/R' \quad \Gamma \models A_1 : \star \quad (\rho = +) \vee (x \notin \text{fv } b_1) \quad (\rho = +) \vee (x \notin \text{fv } b_2)}{\Gamma; \Delta \models (\lambda^\rho x. b_1) \equiv (\lambda^\rho x. b_2) : (\Pi^\rho x : A_1 \rightarrow B)/R'} \quad \text{E\_ABSCONG} \\
\\
\frac{\Gamma; \Delta \models a_1 \equiv b_1 : (\Pi^+ x : A \rightarrow B)/R' \quad \Gamma; \Delta \models a_2 \equiv b_2 : A/\mathbf{Nom}}{\Gamma; \Delta \models a_1 \ a_2^+ \equiv b_1 \ b_2^+ : (B\{a_2/x\})/R'} \quad \text{E\_APPCONG} \\
\\
\frac{\Gamma; \Delta \models a_1 \equiv b_1 : (\Pi^+ x : A \rightarrow B)/R' \quad \text{Path}_{R'} \ a_1 = F @ R, Rs \quad \Gamma; \Delta \models a_2 \equiv b_2 : A/\mathbf{param} \ R \ R'}{\Gamma; \Delta \models a_1 \ a_2^R \equiv b_1 \ b_2^R : (B\{a_2/x\})/R'} \quad \text{E\_TAPPCONG} \\
\\
\frac{\Gamma; \Delta \models a_1 \equiv b_1 : (\Pi^- x : A \rightarrow B)/R' \quad \Gamma \models a : A}{\Gamma; \Delta \models a_1 \ \Box^- \equiv b_1 \ \Box^- : (B\{a/x\})/R'} \quad \text{E\_IAPPCONG} \\
\\
\frac{\Gamma; \Delta \models \Pi^\rho x : A_1 \rightarrow B_1 \equiv \Pi^\rho x : A_2 \rightarrow B_2 : \star/R'}{\Gamma; \Delta \models A_1 \equiv A_2 : \star/R'} \quad \text{E\_PIFST} \\
\\
\frac{\Gamma; \Delta \models \Pi^\rho x : A_1 \rightarrow B_1 \equiv \Pi^\rho x : A_2 \rightarrow B_2 : \star/R' \quad \Gamma; \Delta \models a_1 \equiv a_2 : A_1/R'}{\Gamma; \Delta \models B_1\{a_1/x\} \equiv B_2\{a_2/x\} : \star/R'} \quad \text{E\_PISND} \\
\\
\frac{\Gamma; \Delta \models a_1 \sim_{A_1/R} b_1 \equiv a_2 \sim_{A_2/R} b_2 \quad \Gamma, c : a_1 \sim_{A_1/R} b_1; \Delta \models A \equiv B : \star/R' \quad \Gamma \models a_1 \sim_{A_1/R} b_1 \ \text{ok} \quad \Gamma \models \forall c : a_1 \sim_{A_1/R} b_1. A : \star \quad \Gamma \models \forall c : a_2 \sim_{A_2/R} b_2. B : \star}{\Gamma; \Delta \models \forall c : a_1 \sim_{A_1/R} b_1. A \equiv \forall c : a_2 \sim_{A_2/R} b_2. B : \star/R'} \quad \text{E\_CPICONG} \\
\\
\frac{\Gamma, c : \phi_1; \Delta \models a \equiv b : B/R \quad \Gamma \models \phi_1 \ \text{ok}}{\Gamma; \Delta \models (\Lambda c. a) \equiv (\Lambda c. b) : \forall c : \phi_1. B/R} \quad \text{E\_CABSCONG} \\
\\
\frac{\Gamma; \Delta \models a_1 \equiv b_1 : (\forall c : (a \sim_{A/R} b). B)/R' \quad \Gamma; \tilde{\Gamma} \models a \equiv b : A/\mathbf{param} \ R \ R'}{\Gamma; \Delta \models a_1[\bullet] \equiv b_1[\bullet] : (B\{\bullet/c\})/R'} \quad \text{E\_CAPPCCONG} \\
\\
\frac{\Gamma; \Delta \models \forall c : (a_1 \sim_{A/R} a_2). B_1 \equiv \forall c : (a'_1 \sim_{A'/R'} a'_2). B_2 : \star/R_0 \quad \Gamma; \tilde{\Gamma} \models a_1 \equiv a_2 : A/\mathbf{param} \ R \ R_0 \quad \Gamma; \tilde{\Gamma} \models a'_1 \equiv a'_2 : A'/\mathbf{param} \ R' \ R_0}{\Gamma; \Delta \models B_1\{\bullet/c\} \equiv B_2\{\bullet/c\} : \star/R_0} \quad \text{E\_CPISND} \\
\\
\frac{\Gamma; \Delta \models a \equiv b : A/R \quad \Gamma; \Delta \models a \sim_{A/R} b \equiv a' \sim_{A'/R'} b'}{\Gamma; \Delta \models a' \equiv b' : A'/R'} \quad \text{E\_CAST} \\
\\
\frac{\Gamma; \Delta \models a \equiv b : A/R \quad \Gamma; \tilde{\Gamma} \models A \equiv B : \star/\mathbf{Rep} \quad \Gamma \models B : \star}{\Gamma; \Delta \models a \equiv b : B/R} \quad \text{E\_EQCONV}
\end{array}$$

$$\begin{array}{c}
\frac{\Gamma; \Delta \models a \sim_{A/R_1} b \equiv a' \sim_{A'/R_1} b'}{\Gamma; \Delta \models A \equiv A' : \star / \mathbf{Rep}} \quad \text{E\_ISO\_SND} \\
\\
\frac{\begin{array}{c} \Gamma; \Delta \models a \equiv a' : A/R \\ \Gamma; \Delta \models b_1 \equiv b'_1 : B/R_0 \\ \Gamma; \Delta \models b_2 \equiv b'_2 : B/R_0 \end{array}}{\Gamma; \Delta \models \text{case}_R a \text{ of } F \rightarrow b_1 \parallel - \rightarrow b_2 \equiv \text{case}_R a' \text{ of } F \rightarrow b'_1 \parallel - \rightarrow b'_2 : B/R_0} \quad \text{E\_PAT\_CONG} \\
\\
\frac{\begin{array}{c} \text{Path}_{R'} a = F @ R, Rs \\ \text{Path}_{R'} a' = F @ R, Rs \\ \Gamma \models a : \Pi^+ x : A \rightarrow B \\ \Gamma \models b : A \\ \Gamma \models a' : \Pi^+ x : A \rightarrow B \\ \Gamma \models b' : A \\ \Gamma; \Delta \models a \ b^{R_1} \equiv a' \ b'^{R_1} : B\{b/x\}/R' \\ \Gamma; \tilde{\Gamma} \models B\{b/x\} \equiv B\{b'/x\} : \star / R' \end{array}}{\Gamma; \Delta \models a \equiv a' : \Pi^+ x : A \rightarrow B/R'} \quad \text{E\_LEFT\_REL} \\
\\
\frac{\begin{array}{c} \text{Path}_{R'} a = F @ R, Rs \\ \text{Path}_{R'} a' = F @ R, Rs \\ \Gamma \models a : \Pi^- x : A \rightarrow B \\ \Gamma \models b : A \\ \Gamma \models a' : \Pi^- x : A \rightarrow B \\ \Gamma \models b' : A \\ \Gamma; \Delta \models a \ \square^- \equiv a' \ \square^- : B\{b/x\}/R' \\ \Gamma; \tilde{\Gamma} \models B\{b/x\} \equiv B\{b'/x\} : \star / R_0 \end{array}}{\Gamma; \Delta \models a \equiv a' : \Pi^- x : A \rightarrow B/R'} \quad \text{E\_LEFT\_IRREL} \\
\\
\frac{\begin{array}{c} \text{Path}_{R'} a = F @ R, Rs \\ \text{Path}_{R'} a' = F @ R, Rs \\ \Gamma \models a : \Pi^+ x : A \rightarrow B \\ \Gamma \models b : A \\ \Gamma \models a' : \Pi^+ x : A \rightarrow B \\ \Gamma \models b' : A \\ \Gamma; \Delta \models a \ b^+ \equiv a' \ b'^+ : B\{b/x\}/R' \\ \Gamma; \tilde{\Gamma} \models B\{b/x\} \equiv B\{b'/x\} : \star / R_0 \end{array}}{\Gamma; \Delta \models b \equiv b' : A / \mathbf{param} \ R_1 \ R'} \quad \text{E\_RIGHT} \\
\\
\frac{\begin{array}{c} \text{Path}_{R'} a = F @ R, Rs \\ \text{Path}_{R'} a' = F @ R, Rs \\ \Gamma \models a : \forall c : (a_1 \sim_{A/R_1} a_2). B \\ \Gamma \models a' : \forall c : (a_1 \sim_{A/R_1} a_2). B \\ \Gamma; \tilde{\Gamma} \models a_1 \equiv a_2 : A/R' \\ \Gamma; \Delta \models a[\bullet] \equiv a'[\bullet] : B\{\bullet/c\}/R' \end{array}}{\Gamma; \Delta \models a \equiv a' : \forall c : (a_1 \sim_{A/R_1} a_2). B/R'} \quad \text{E\_CLEFT}
\end{array}$$

$\boxed{\models \Gamma}$  context wellformedness

$$\begin{array}{c}
\frac{}{\models \emptyset} \quad \text{E\_EMPTY} \\
\\
\frac{\begin{array}{c} \models \Gamma \\ \Gamma \models A : \star \\ x \notin \text{dom } \Gamma \end{array}}{\models \Gamma, x : A} \quad \text{E\_CONSTM}
\end{array}$$

$$\frac{\begin{array}{l} \models \Gamma \\ \Gamma \models \phi \text{ ok} \\ c \notin \text{dom } \Gamma \end{array}}{\models \Gamma, c : \phi} \quad \text{E\_CONSCo}$$

$\boxed{\models \Sigma}$  signature wellformedness

$$\frac{}{\models \emptyset} \quad \text{SIG\_EMPTY}$$

$$\frac{\begin{array}{l} \models \Sigma \\ \emptyset \models A : \star \\ F \notin \text{dom } \Sigma \end{array}}{\models \Sigma \cup \{F : A @ R_s\}} \quad \text{SIG\_CONSCONST}$$

$$\frac{\begin{array}{l} \models \Sigma \\ F \notin \text{dom } \Sigma \\ \models p : A \text{ patctx} = \Omega \mid \Gamma \\ \Gamma \models a : A \\ \Omega \models a : \mathbf{Rep} \end{array}}{\models \Sigma \cup \{F[p] \sim a : A / R @ R_s\}} \quad \text{SIG\_CONSAx}$$

Definition rules: 143 good 0 bad

Definition rule clauses: 403 good 0 bad