$tmvar$, x, y, f, m, n	variables
$covar$, c	coercion variables
$datacon$, K	
$const$, T, F	
$index$, i	indices

role, R	$::=$			Role
	$\mid$	$\mathbf{Nom}$		
	$\mid$	$\mathbf{Rep}$		
	$\mid$	$R_1 \cap R_2$	S	
	$\mid$	$\mathbf{param}\, R_1\, R_2$	S	
	$\mid$	(R)	S	
relflag, ρ	$::=$			relevance flag
	$\mid$	$+$		
	$\mid$	$-$		
constraint, ϕ	$::=$			props
	$\mid$	$a \sim_{A/R} b$		
	$\mid$	(ϕ)	S	
	$\mid$	$\phi\{b/x\}$	S	
	$\mid$	$\lvert\phi\rvert$	S	
	$\mid$	$a \sim_R b$	S	
tm, a, b, v, w, A, B	$::=$			types and kinds
	$\mid$	$\star$		
	$\mid$	x		
	$\mid$	$\lambda^{\rho} x\!:\!A/R.b$	bind x in b	
	$\mid$	$\lambda^{R,\rho} x.b$	bind x in b	
	$\mid$	$a\ b^{R,\rho}$		
	$\mid$	F		
	$\mid$	$\Pi^{\rho} x\!:\!A/R \to B$	bind x in B	
	$\mid$	$a \triangleright_R \gamma$		
	$\mid$	$\forall c\!:\!\phi.B$	bind c in B	
	$\mid$	$\Lambda c\!:\!\phi.b$	bind c in b	
	$\mid$	$\Lambda c.b$	bind c in b	
	$\mid$	$a[\gamma]$		
	$\mid$	$\square$		
	$\mid$	$case_R\ a\ of\ a' \to b_1 \Vert_{\_} \to b_2$		
	$\mid$	K		
	$\mid$	$\mathbf{match}\, a\, \mathbf{with}\, brs$		
	$\mid$	$\mathbf{sub}\, R\, a$		
	$\mid$	$a\{b/x\}$	S	
	$\mid$	a	S	
	$\mid$	$a\{\gamma/c\}$	S	
	$\mid$	a	S	
	$\mid$	(a)	S	
	$\mid$	a	S	parsing precedence is hard
	$\mid$	$\lvert a\rvert_R$	S	
	$\mid$	$\mathbf{Int}$	S	
	$\mid$	$\mathbf{Bool}$	S	
	$\mid$	$\mathbf{Nat}$	S	
	$\mid$	$\mathbf{Vec}$	S	

	$\vert$	0	S	
	$\vert$	S	S	
	$\vert$	**True**	S	
	$\vert$	**Fix**	S	
	$\vert$	**Age**	S	
	$\vert$	$a \to b$	S	
	$\vert$	$a/R \to b$	S	
	$\vert$	$\phi \Rightarrow A$	S	
	$\vert$	$a\ \ b$	S	
	$\vert$	$\lambda x.a$	S	
	$\vert$	$\lambda x\!:\!A.a$	S	
	$\vert$	$\forall\, x : A/R \to B$	S	
	$\vert$	**if** ϕ **then** a **else** b	S	
brs	$::=$			case branches
	$\vert$	**none**		
	$\vert$	$K \Rightarrow a; brs$		
	$\vert$	$brs\{a/x\}$	S	
	$\vert$	$brs\{\gamma/c\}$	S	
	$\vert$	(brs)	S	
$co,\ \gamma$	$::=$			explicit coercions
	$\vert$	$\bullet$		
	$\vert$	c		
	$\vert$	**red** a b		
	$\vert$	**refl** a		
	$\vert$	$(a \mid=\mid_{\gamma}\ b)$		
	$\vert$	**sym** γ		
	$\vert$	$\gamma_1; \gamma_2$		
	$\vert$	**sub** γ		
	$\vert$	$\Pi^{R,\rho} x\!:\!\gamma_1.\gamma_2$	bind x in γ_2	
	$\vert$	$\lambda^{R,\rho} x\!:\!\gamma_1.\gamma_2$	bind x in γ_2	
	$\vert$	$\gamma_1\ \ \gamma_2^{R,\rho}$		
	$\vert$	**piFst** γ		
	$\vert$	**cpiFst** γ		
	$\vert$	**isoSnd** γ		
	$\vert$	$\gamma_1 @ \gamma_2$		
	$\vert$	$\forall c\!:\!\gamma_1.\gamma_3$	bind c in γ_3	
	$\vert$	$\lambda c\!:\!\gamma_1.\gamma_3 @ \gamma_4$	bind c in γ_3	
	$\vert$	$\gamma(\gamma_1, \gamma_2)$		
	$\vert$	$\gamma @ (\gamma_1 \sim \gamma_2)$		
	$\vert$	$\gamma_1 \triangleright_R \gamma_2$		
	$\vert$	$\gamma_1 \sim_A \gamma_2$		
	$\vert$	**conv** $\phi_1 \sim_{\gamma} \phi_2$		
	$\vert$	**eta** a		
	$\vert$	**left** γ γ'		

	$\mid$	$\textbf{right}\,\gamma\,\gamma'$		
	$\mid$	(γ)	S	
	$\mid$	γ	S	
	$\mid$	$\gamma\{a/x\}$	S	
$\mathit{role\_context},\ \Omega$	$::=$			$\mathrm{role}_c\mathit{ontexts}$
	$\mid$	$\varnothing$		
	$\mid$	$\Omega, x : R$		
	$\mid$	(Ω)	M	
	$\mid$	Ω	M	
$\mathit{sig\_sort}$	$::=$			signature classifier
	$\mid$	$: A$		
	$\mid$	$\sim a : A/R$		
sort	$::=$			binding classifier
	$\mid$	$\textbf{Tm}\,A\,R$		
	$\mid$	$\textbf{Co}\,\phi$		
$\mathit{context},\ \Gamma$	$::=$			contexts
	$\mid$	$\varnothing$		
	$\mid$	$\Gamma, x : A/R$		
	$\mid$	$\Gamma, c : \phi$		
	$\mid$	$\Gamma\{b/x\}$	M	
	$\mid$	$\Gamma\{\gamma/c\}$	M	
	$\mid$	Γ, Γ'	M	
	$\mid$	$\lvert\Gamma\rvert$	M	
	$\mid$	(Γ)	M	
	$\mid$	Γ	M	
$\mathit{sig},\ \Sigma$	$::=$			signatures
	$\mid$	$\varnothing$		
	$\mid$	$\Sigma \cup \{F\,\mathit{sig\_sort}\}$		
	$\mid$	Σ_0	M	
	$\mid$	Σ_1	M	
	$\mid$	$\lvert\Sigma\rvert$	M	
$\mathit{available\_props},\ \Delta$	$::=$			
	$\mid$	$\varnothing$		
	$\mid$	Δ, c		
	$\mid$	$\widetilde{\Gamma}$	M	
	$\mid$	(Δ)	M	
$\mathit{terminals}$	$::=$			
	$\mid$	$\leftrightarrow$		
	$\mid$	$\Leftrightarrow$		
	$\mid$	$\longrightarrow$		

$\mid$ min
$\mid$ $\equiv$
$\mid$ $\forall$
$\mid$ $\in$
$\mid$ $\notin$
$\mid$ $\Leftarrow$
$\mid$ $\Rightarrow$
$\mid$ $\Rightarrow^*$
$\mid$ $\rightarrow$
$\mid$ Λ
$\mid$
$\mid$ $\square$
$\mid$ $\vdash$
$\mid$ $\dashv$
$\mid$ $\models$
$\mid$ $\models$
$\mid$ $\neq$
$\mid$ $\triangleright$
$\mid$ ok
$\mid$ $\_$
$\mid$ $\leadsto$
$\mid$ $\leadsto^*$
$\mid$ $\leadsto$
$\mid$ $\varnothing$
$\mid$ $\circ$
$\mid$ fv
$\mid$ dom
$\mid$ $\sim$
$\mid$ $\asymp$
$\mid$ $\mid$
$\mid$ $\bullet$
$\mid$ $\mathbf{fst}$
$\mid$ $\mathbf{snd}$
$\mid$ $\mid\Rightarrow\mid$
$\mid$ $\vdash_=$
$\mid$ $\mathbf{refl_2}$
$\mid$ $++$

$\mathit{formula},\ \psi$ ::=
$\mid$ $\mathit{judgement}$
$\mid$ $x : A/R \in \Gamma$
$\mid$ $x : R \in \Omega$
$\mid$ $c : \phi \in \Gamma$
$\mid$ $F\ \mathit{sig\_sort} \in \Sigma$
$\mid$ $K : T\,\Gamma \in \Sigma$
$\mid$ $x \in \Delta$

	|	$c \in \Delta$	
	|	$c\ \textbf{not relevant} \in \gamma$	
	|	$x \notin \mathsf{fv}\, a$	
	|	$x \notin \mathsf{dom}\, \Gamma$	
	|	$uniq(\Omega)$	
	|	$c \notin \mathsf{dom}\, \Gamma$	
	|	$T \notin \mathsf{dom}\, \Sigma$	
	|	$F \notin \mathsf{dom}\, \Sigma$	
	|	$a = b$	
	|	$\phi_1 = \phi_2$	
	|	$\Gamma_1 = \Gamma_2$	
	|	$\gamma_1 = \gamma_2$	
	|	$\neg\psi$	
	|	$\psi_1 \wedge \psi_2$	
	|	$\psi_1 \vee \psi_2$	
	|	$\psi_1 \Rightarrow \psi_2$	
	|	(ψ)	
	|	ψ	
	|	$c : (a : A \sim b : B) \in \Gamma$	
	|		suppress lc hypothesis generated by Ott
JSubRole	::=		
	|	$R_1 \leq R_2$	Subroling judgement
JPath	::=		
	|	$\mathsf{Path}_R\ a = F$	Type headed by constant (partial function)
JValue	::=		
	|	$\mathsf{Value}_R\ A$	values
JValueType	::=		
	|	$\mathsf{ValueType}_R\ A$	Types with head forms (erased language)
Jconsistent	::=		
	|	$\mathsf{consistent}_R\ ab$	(erased) types do not differ in their heads
Jroleing	::=		
	|	$\Omega \vDash a : R$	
JChk	::=		
	|	$(\rho = +) \vee (x \notin \mathsf{fv}\ A)$	irrelevant argument check
Jpar	::=		
	|	$\Omega \vDash a \Rightarrow_R b$	parallel reduction (implicit language)
	|	$\Omega \vdash a \Rightarrow^*_R b$	multistep parallel reduction
	|	$\Omega \vdash a \Leftrightarrow_R b$	parallel reduction to a common term

Jbeta	::=		
	|	$\vDash a > b / R$	primitive reductions on erased terms
	|	$\vDash a \leadsto b / R$	single-step head reduction for implicit language
	|	$\vDash a \leadsto^* b / R$	multistep reduction
Jett	::=		
	|	$\Gamma \vDash \phi \ \mathsf{ok}$	Prop wellformedness
	|	$\Gamma \vDash a : A / R$	typing
	|	$\Gamma; \Delta \vDash \phi_1 \equiv \phi_2$	prop equality
	|	$\Gamma; \Delta \vDash a \equiv b : A / R$	definitional equality
	|	$\vDash \Gamma$	context wellformedness
Jsig	::=		
	|	$\vDash \Sigma$	signature wellformedness
Jann	::=		
	|	$\Gamma \vdash \phi \ \mathsf{ok}$	prop wellformedness
	|	$\Gamma \vdash a : A / R$	typing
	|	$\Gamma; \Delta \vdash \gamma : \phi_1 \sim \phi_2$	coercion between props
	|	$\Gamma; \Delta \vdash \gamma : A \sim_R B$	coercion between types
	|	$\vdash \Gamma$	context wellformedness
	|	$\vdash \Sigma$	signature wellformedness
Jred	::=		
	|	$\Gamma \vdash a \leadsto b / R$	single-step, weak head reduction to values for annotated lang
judgement	::=		
	|	*JSubRole*	
	|	*JPath*	
	|	*JValue*	
	|	*JValueType*	
	|	*Jconsistent*	
	|	*Jroleing*	
	|	*JChk*	
	|	*Jpar*	
	|	*Jbeta*	
	|	*Jett*	
	|	*Jsig*	
	|	*Jann*	
	|	*Jred*	
user_syntax	::=		
	|	*tmvar*	
	|	*covar*	
	|	*datacon*	
	|	*const*	
	|	*index*	

| *role*
| *relflag*
| *constraint*
| *tm*
| *brs*
| *co*
| *role_context*
| *sig_sort*
| *sort*
| *context*
| *sig*
| *available_props*
| *terminals*
| *formula*

$\boxed{R_1 \leq R_2}$ Subroling judgement

$$\frac{}{\mathbf{Nom} \leq R} \quad \textsc{NomBot}$$

$$\frac{}{R \leq \mathbf{Rep}} \quad \textsc{RepTop}$$

$$\frac{}{R \leq R} \quad \textsc{Refl}$$

$$\frac{\begin{array}{c} R_1 \leq R_2 \\ R_2 \leq R_3 \end{array}}{R_1 \leq R_3} \quad \textsc{Trans}$$

$\boxed{\mathsf{Path}_R\ a = F}$ Type headed by constant (partial function)

$$\frac{F\ : A\ \in\ \Sigma_0}{\mathsf{Path}_R\ F = F} \quad \textsc{Path\_AbsConst}$$

$$\frac{\begin{array}{l} F\ \sim a : A/R_1\ \in\ \Sigma_0 \\ \neg(R_1 \leq R) \end{array}}{\mathsf{Path}_R\ F = F} \quad \textsc{Path\_Const}$$

$$\frac{\mathsf{Path}_R\ a = F}{\mathsf{Path}_R\ (a\ b'^{R_1,\rho}) = F} \quad \textsc{Path\_App}$$

$$\frac{\mathsf{Path}_R\ a = F}{\mathsf{Path}_R\ (a[\bullet]) = F} \quad \textsc{Path\_CApp}$$

$\boxed{\mathsf{Value}_R\ A}$ values

$$\frac{}{\mathsf{Value}_R\ \star} \quad \textsc{Value\_Star}$$

$$\frac{}{\mathsf{Value}_R\ \Pi^\rho x\!:\!A/R_1 \to B} \quad \textsc{Value\_Pi}$$

$$\frac{}{\mathsf{Value}_R\ \forall c\!:\!\phi.B} \quad \textsc{Value\_CPi}$$

$$\frac{}{\mathsf{Value}_R\ \lambda^+ x\!:\!A/R_1.a} \quad \textsc{Value\_AbsRel}$$

$$\frac{}{\mathsf{Value}_R\ \lambda^{R_1,+} x.a} \quad \textsc{Value\_UAbsRel}$$

$$\frac{\mathsf{Value}_R\ a}{\mathsf{Value}_R\ \lambda^{R_1,-}x.a} \quad \textsc{Value\_UAbsIrrel}$$

$$\frac{}{\mathsf{Value}_R\ \Lambda c\!:\!\phi.a} \quad \textsc{Value\_CAbs}$$

$$\frac{}{\mathsf{Value}_R\ \Lambda c.a} \quad \textsc{Value\_UCAbs}$$

$$\frac{\mathsf{Path}_R\ a = F}{\mathsf{Value}_R\ a} \quad \textsc{Value\_Path}$$

$\boxed{\mathsf{ValueType}_R\ A}$ Types with head forms (erased language)

$$\frac{}{\mathsf{ValueType}_R\ \star} \quad \textsc{value\_type\_Star}$$

$$\frac{}{\mathsf{ValueType}_R\ \Pi^\rho x\!:\!A/R_1 \to B} \quad \textsc{value\_type\_Pi}$$

$$\frac{}{\mathsf{ValueType}_R\ \forall c\!:\!\phi.B} \quad \textsc{value\_type\_CPi}$$

$$\frac{\mathsf{Path}_R\ a = F}{\mathsf{ValueType}_R\ a} \quad \textsc{value\_type\_Path}$$

$\boxed{\mathsf{consistent}_R\ ab}$ (erased) types do not differ in their heads

$$\frac{}{\mathsf{consistent}_R\ \star\star} \quad \textsc{consistent\_a\_Star}$$

$$\frac{}{\mathsf{consistent}_{R'}\ (\Pi^\rho x_1\!:\!A_1/R \to B_1)(\Pi^\rho x_2\!:\!A_2/R \to B_2)} \quad \textsc{consistent\_a\_Pi}$$

$$\frac{}{\mathsf{consistent}_R\ (\forall c_1\!:\!\phi_1.A_1)(\forall c_2\!:\!\phi_2.A_2)} \quad \textsc{consistent\_a\_CPi}$$

$$\frac{\begin{array}{c}\mathsf{Path}_R\ a_1 = F\\ \mathsf{Path}_R\ a_2 = F\end{array}}{\mathsf{consistent}_R\ a_1 a_2} \quad \textsc{consistent\_a\_Path}$$

$$\frac{\neg\mathsf{ValueType}_R\ b}{\mathsf{consistent}_R\ ab} \quad \textsc{consistent\_a\_Step\_R}$$

$$\frac{\neg\mathsf{ValueType}_R\ a}{\mathsf{consistent}_R\ ab} \quad \textsc{consistent\_a\_Step\_L}$$

$\boxed{\Omega \vDash a : R}$

$$\frac{uniq(\Omega)}{\Omega \vDash \Box : R} \quad \textsc{role\_a\_Bullet}$$

$$\frac{uniq(\Omega)}{\Omega \vDash \star : R} \quad \textsc{role\_a\_Star}$$

$$\frac{\begin{array}{l}uniq(\Omega)\\ x : R \in \Omega\\ R \leq R_1\end{array}}{\Omega \vDash x : R_1} \quad \textsc{role\_a\_Var}$$

$$\frac{\Omega, x : R_1 \vDash a : R}{\Omega \vDash (\lambda^{R_1,\rho}x.a) : R} \quad \textsc{role\_a\_Abs}$$

$$\frac{\Omega \vDash a : R \quad \Omega \vDash b : (\mathbf{param}\, R_1\ R)}{\Omega \vDash (a\ b^{R_1,\rho}) : R} \quad \textsc{role\_a\_App}$$

$$\frac{\Omega \vDash A : R \quad \Omega, x : R_1 \vDash B : R}{\Omega \vDash (\Pi^\rho x\!:\!A/R_1 \to B) : R} \quad \textsc{role\_a\_Pi}$$

$$\frac{\Omega \vDash a : R_1 \quad \Omega \vDash b : R_1 \quad \Omega \vDash A : R_0 \quad \Omega \vDash B : R}{\Omega \vDash (\forall c\!:\!a \sim_{A/R_1} b.B) : R} \quad \textsc{role\_a\_CPi}$$

$$\frac{\Omega \vDash b : R}{\Omega \vDash (\Lambda c.b) : R} \quad \textsc{role\_a\_CAbs}$$

$$\frac{\Omega \vDash a : R}{\Omega \vDash (a[\bullet]) : R} \quad \textsc{role\_a\_CApp}$$

$$\frac{uniq(\Omega) \quad F\ : A\ \in\ \Sigma_0}{\Omega \vDash F : R} \quad \textsc{role\_a\_Const}$$

$$\frac{uniq(\Omega) \quad F\ \sim a : A/R\ \in\ \Sigma_0}{\Omega \vDash F : R_1} \quad \textsc{role\_a\_Fam}$$

$$\frac{F\ sig\_sort\ \in\ \Sigma_0 \quad \Omega \vDash a : R \quad \Omega \vDash b_1 : R_1 \quad \Omega \vDash b_2 : R_1}{\Omega \vDash (case_R\ a\ of\ F \to b_1 \|_{\_} \to b_2) : R_1} \quad \textsc{role\_a\_Pattern}$$

$\boxed{(\rho = +) \vee (x \notin \mathsf{fv}\ A)}$ irrelevant argument check

$$\frac{}{(+ = +) \vee (x \notin \mathsf{fv}\ A)} \quad \textsc{Rho\_Rel}$$

$$\frac{x\ \notin\ \mathsf{fv} A}{(- = +) \vee (x \notin \mathsf{fv}\ A)} \quad \textsc{Rho\_IrrRel}$$

$\boxed{\Omega \vDash a \Rightarrow_R b}$ parallel reduction (implicit language)

$$\frac{\Omega \vDash a : R}{\Omega \vDash a \Rightarrow_R a} \quad \textsc{Par\_Refl}$$

$$\frac{\Omega \vDash a \Rightarrow_R (\lambda^{R_1,\rho} x.a') \quad \Omega \vDash b \Rightarrow_{\mathbf{param}\, R_1\ R} b'}{\Omega \vDash a\ b^{R_1,\rho} \Rightarrow_R a'\{b'/x\}} \quad \textsc{Par\_Beta}$$

$$\frac{\Omega \vDash a \Rightarrow_R a' \quad \Omega \vDash b \Rightarrow_{\mathbf{param}\, R_1\ R} b'}{\Omega \vDash a\ b^{R_1,\rho} \Rightarrow_R a'\ b'^{R_1,\rho}} \quad \textsc{Par\_App}$$

$$\frac{\Omega \vDash a \Rightarrow_R (\Lambda c.a')}{\Omega \vDash a[\bullet] \Rightarrow_R a'\{\bullet/c\}} \quad \textsc{Par\_CBeta}$$

$$\frac{\Omega \vDash a \Rightarrow_R a'}{\Omega \vDash a[\bullet] \Rightarrow_R a'[\bullet]} \quad \text{PAR\_CAPP}$$

$$\frac{\Omega, x : R_1 \vDash a \Rightarrow_R a'}{\Omega \vDash \lambda^{R_1,\rho} x.a \Rightarrow_R \lambda^{R_1,\rho} x.a'} \quad \text{PAR\_ABS}$$

$$\frac{\begin{array}{c}\Omega \vDash A \Rightarrow_R A' \\ \Omega, x : R_1 \vDash B \Rightarrow_R B'\end{array}}{\Omega \vDash \Pi^\rho x\!:\!A/R_1 \to B \Rightarrow_R \Pi^\rho x\!:\!A'/R_1 \to B'} \quad \text{PAR\_PI}$$

$$\frac{\Omega \vDash a \Rightarrow_R a'}{\Omega \vDash \Lambda c.a \Rightarrow_R \Lambda c.a'} \quad \text{PAR\_CABS}$$

$$\frac{\begin{array}{c}\Omega \vDash A \Rightarrow_{R_0} A' \\ \Omega \vDash a \Rightarrow_{R_1} a' \\ \Omega \vDash b \Rightarrow_{R_1} b' \\ \Omega \vDash B \Rightarrow_R B'\end{array}}{\Omega \vDash \forall c\!:\!a \sim_{A/R_1} b.B \Rightarrow_R \forall c\!:\!a' \sim_{A'/R_1} b'.B'} \quad \text{PAR\_CPI}$$

$$\frac{\begin{array}{l}F \sim a : A/R_1 \in \Sigma_0 \\ R_1 \leq R \\ uniq(\Omega)\end{array}}{\Omega \vDash F \Rightarrow_R a} \quad \text{PAR\_AXIOM}$$

$$\frac{\begin{array}{l}F\ sig\_sort \in \Sigma_0 \\ \Omega \vDash a \Rightarrow_R a' \\ \Omega \vDash b_1 \Rightarrow_{R_0} b_1' \\ \Omega \vDash b_2 \Rightarrow_{R_0} b_2'\end{array}}{\Omega \vDash case_R\ a\ of\ F \to b_1 \|\_ \to b_2 \Rightarrow_{R_0} case_R\ a'\ of\ F \to b_1' \|\_ \to b_2'} \quad \text{PAR\_PATTERN}$$

$$\frac{\begin{array}{l}\Omega \vDash a \Rightarrow_R a' \\ \Omega \vDash b_1 \Rightarrow_{R_0} b_1' \\ \Omega \vDash b_2 \Rightarrow_{R_0} b_2' \\ \mathsf{Path}_R\ a' = F\end{array}}{\Omega \vDash case_R\ a\ of\ F \to b_1 \|\_ \to b_2 \Rightarrow_{R_0} b_1'} \quad \text{PAR\_PATTERNTRUE}$$

$$\frac{\begin{array}{l}F\ sig\_sort \in \Sigma_0 \\ \Omega \vDash a \Rightarrow_R a' \\ \Omega \vDash b_1 \Rightarrow_{R_0} b_1' \\ \Omega \vDash b_2 \Rightarrow_{R_0} b_2' \\ \mathsf{Value}_R\ a' \\ \neg(\mathsf{Path}_R\ a' = F)\end{array}}{\Omega \vDash case_R\ a\ of\ F \to b_1 \|\_ \to b_2 \Rightarrow_{R_0} b_2'} \quad \text{PAR\_PATTERNFALSE}$$

$\boxed{\Omega \vdash a \Rightarrow_R^* b}$ multistep parallel reduction

$$\frac{}{\Omega \vdash a \Rightarrow_R^* a} \quad \text{MP\_REFL}$$

$$\frac{\begin{array}{c}\Omega \vDash a \Rightarrow_R b \\ \Omega \vdash b \Rightarrow_R^* a'\end{array}}{\Omega \vdash a \Rightarrow_R^* a'} \quad \text{MP\_STEP}$$

$\boxed{\Omega \vdash a \Leftrightarrow_R b}$ parallel reduction to a common term

$$\frac{\begin{array}{c}\Omega \vdash a_1 \Rightarrow_R^* b \\ \Omega \vdash a_2 \Rightarrow_R^* b\end{array}}{\Omega \vdash a_1 \Leftrightarrow_R a_2} \quad \text{JOIN}$$

$\boxed{\vDash a > b/R}$ primitive reductions on erased terms

$$\frac{\mathsf{Value}_{R_1}\ (\lambda^{R,\rho}x.v)}{\vDash (\lambda^{R,\rho}x.v)\ b^{R,\rho} > v\{b/x\}/R_1}\quad \text{B}\textsc{eta\_AppAbs}$$

$$\frac{}{\vDash (\Lambda c.a')[\bullet] > a'\{\bullet/c\}/R}\quad \text{B}\textsc{eta\_CAppCAbs}$$

$$\frac{\begin{array}{l} F\ \sim a : A/R\ \in\ \Sigma_0 \\ R \leq R_1 \end{array}}{\vDash F > a/R_1}\quad \text{B}\textsc{eta\_Axiom}$$

$$\frac{\mathsf{Path}_R\ a = F}{\vDash case_R\ a\ of\ F \to b_1 \| \_ \to b_2 > b_1/R_0}\quad \text{B}\textsc{eta\_PatternTrue}$$

$$\frac{\begin{array}{l} F\ sig\_sort\ \in\ \Sigma_0 \\ \mathsf{Value}_R\ a \\ \neg(\mathsf{Path}_R\ a = F) \end{array}}{\vDash case_R\ a\ of\ F \to b_1 \| \_ \to b_2 > b_2/R_0}\quad \text{B}\textsc{eta\_PatternFalse}$$

$\boxed{\vDash a \rightsquigarrow b/R}$ single-step head reduction for implicit language

$$\frac{\vDash a \rightsquigarrow a'/R_1}{\vDash \lambda^{R,-}x.a \rightsquigarrow \lambda^{R,-}x.a'/R_1}\quad \text{E}\textsc{\_AbsTerm}$$

$$\frac{\vDash a \rightsquigarrow a'/R_1}{\vDash a\ b^{R,\rho} \rightsquigarrow a'\ b^{R,\rho}/R_1}\quad \text{E}\textsc{\_AppLeft}$$

$$\frac{\vDash a \rightsquigarrow a'/R}{\vDash a[\bullet] \rightsquigarrow a'[\bullet]/R}\quad \text{E}\textsc{\_CAppLeft}$$

$$\frac{\vDash a \rightsquigarrow a'/R}{\vDash case_R\ a\ of\ F \to b_1 \| \_ \to b_2 \rightsquigarrow case_R\ a'\ of\ F \to b_1 \| \_ \to b_2/R_0}\quad \text{E}\textsc{\_Pattern}$$

$$\frac{\vDash a > b/R}{\vDash a \rightsquigarrow b/R}\quad \text{E}\textsc{\_Prim}$$

$\boxed{\vDash a \rightsquigarrow^* b/R}$ multistep reduction

$$\frac{}{\vDash a \rightsquigarrow^* a/R}\quad \text{E}\textsc{qual}$$

$$\frac{\begin{array}{l} \vDash a \rightsquigarrow b/R \\ \vDash b \rightsquigarrow^* a'/R \end{array}}{\vDash a \rightsquigarrow^* a'/R}\quad \text{S}\textsc{tep}$$

$\boxed{\Gamma \vDash \phi\ \mathsf{ok}}$ Prop wellformedness

$$\frac{\begin{array}{l} \Gamma \vDash a : A/R \\ \Gamma \vDash b : A/R \\ \Gamma \vDash A : \star/R_0 \end{array}}{\Gamma \vDash a \sim_{A/R} b\ \ \mathsf{ok}}\quad \text{E}\textsc{\_Wff}$$

$\boxed{\Gamma \vDash a : A/R}$ typing

$$\frac{\begin{array}{l} R_1 \leq R_2 \\ \Gamma \vDash a : A/R_1 \end{array}}{\Gamma \vDash a : A/R_2}\quad \text{E}\textsc{\_SubRole}$$

$$\frac{\vDash \Gamma}{\Gamma \vDash \star : \star / R} \quad \text{E\_Star}$$

$$\frac{\begin{array}{l} \vDash \Gamma \\ x : A/R \in \Gamma \end{array}}{\Gamma \vDash x : A/R} \quad \text{E\_Var}$$

$$\frac{\begin{array}{l} \Gamma, x : A/R \vDash B : \star/R' \\ \Gamma \vDash A : \star/R' \end{array}}{\Gamma \vDash \Pi^{\rho} x{:}A/R \to B : \star/R'} \quad \text{E\_Pi}$$

$$\frac{\begin{array}{l} \Gamma, x : A/R \vDash a : B/R' \\ \Gamma \vDash A : \star/R_0 \\ (\rho = +) \vee (x \notin \mathsf{fv}\ a) \end{array}}{\Gamma \vDash \lambda^{R,\rho} x.a : (\Pi^{\rho} x{:}A/R \to B)/R'} \quad \text{E\_Abs}$$

$$\frac{\begin{array}{l} \Gamma \vDash b : \Pi^{+} x{:}A/R \to B/R' \\ \Gamma \vDash a : A/\mathbf{param}\, R\, R' \end{array}}{\Gamma \vDash b\ \ a^{R,+} : B\{a/x\}/R'} \quad \text{E\_App}$$

$$\frac{\begin{array}{l} \Gamma \vDash b : \Pi^{-} x{:}A/R \to B/R' \\ \Gamma \vDash a : A/\mathbf{param}\, R\, R' \end{array}}{\Gamma \vDash b\ \Box^{R,-} : B\{a/x\}/R'} \quad \text{E\_IApp}$$

$$\frac{\begin{array}{l} \Gamma \vDash a : A/R \\ \Gamma; \widetilde{\Gamma} \vDash A \equiv B : \star/\mathbf{Rep} \\ \Gamma \vDash B : \star/R_0 \end{array}}{\Gamma \vDash a : B/R} \quad \text{E\_Conv}$$

$$\frac{\begin{array}{l} \Gamma, c : \phi \vDash B : \star/R \\ \Gamma \vDash \phi\ \ \mathsf{ok} \end{array}}{\Gamma \vDash \forall c{:}\phi.B : \star/R} \quad \text{E\_CPi}$$

$$\frac{\begin{array}{l} \Gamma, c : \phi \vDash a : B/R \\ \Gamma \vDash \phi\ \ \mathsf{ok} \end{array}}{\Gamma \vDash \Lambda c.a : \forall c{:}\phi.B/R} \quad \text{E\_CAbs}$$

$$\frac{\begin{array}{l} \Gamma \vDash a_1 : \forall c{:}(a \sim_{A/R} b).B_1/R' \\ \Gamma; \widetilde{\Gamma} \vDash a \equiv b : A/\mathbf{param}\, R\, R' \end{array}}{\Gamma \vDash a_1[\bullet] : B_1\{\bullet/c\}/R'} \quad \text{E\_CApp}$$

$$\frac{\begin{array}{l} \vDash \Gamma \\ F\ : A\ \in\ \Sigma_0 \\ \varnothing \vDash A : \star/R_0 \end{array}}{\Gamma \vDash F : A/R_1} \quad \text{E\_Const}$$

$$\frac{\begin{array}{l} \vDash \Gamma \\ F\ \sim a : A/R\ \in\ \Sigma_0 \\ \varnothing \vDash A : \star/R_0 \end{array}}{\Gamma \vDash F : A/R_1} \quad \text{E\_Fam}$$

$$\frac{\begin{array}{l} F\ sig\_sort\ \in\ \Sigma_0 \\ \Gamma \vDash a : A/R \\ \Gamma \vDash b_1 : B/R_0 \\ \Gamma \vDash b_2 : B/R_0 \end{array}}{\Gamma \vDash case_R\ \ a\ \ of\ \ F \to b_1 \|_{\_} \to b_2 : B/R_0} \quad \text{E\_Pat}$$

$\boxed{\Gamma; \Delta \vDash \phi_1 \equiv \phi_2}$ prop equality

$$\frac{\begin{array}{c}\Gamma; \Delta \vDash A_1 \equiv A_2 : A/R \\ \Gamma; \Delta \vDash B_1 \equiv B_2 : A/R\end{array}}{\Gamma; \Delta \vDash A_1 \sim_{A/R} B_1 \equiv A_2 \sim_{A/R} B_2} \quad \text{E\_PropCong}$$

$$\frac{\begin{array}{c}\Gamma; \Delta \vDash A \equiv B : \star/R_0 \\ \Gamma \vDash A_1 \sim_{A/R} A_2 \ \mathsf{ok} \\ \Gamma \vDash A_1 \sim_{B/R} A_2 \ \mathsf{ok}\end{array}}{\Gamma; \Delta \vDash A_1 \sim_{A/R} A_2 \equiv A_1 \sim_{B/R} A_2} \quad \text{E\_IsoConv}$$

$$\frac{\Gamma; \Delta \vDash \forall c{:}(a_1 \sim_{A/R_1} a_2).B_1 \equiv \forall c{:}(b_1 \sim_{B/R_2} b_2).B_2 : \star/R'}{\Gamma; \Delta \vDash a_1 \sim_{A/R_1} a_2 \equiv b_1 \sim_{B/R_2} b_2} \quad \text{E\_CPiFst}$$

$\boxed{\Gamma; \Delta \vDash a \equiv b : A/R}$ definitional equality

$$\frac{\begin{array}{l}\vDash \Gamma \\ c : (a \sim_{A/R} b) \in \Gamma \\ c \in \Delta\end{array}}{\Gamma; \Delta \vDash a \equiv b : A/R} \quad \text{E\_Assn}$$

$$\frac{\Gamma \vDash a : A/R}{\Gamma; \Delta \vDash a \equiv a : A/R} \quad \text{E\_Refl}$$

$$\frac{\Gamma; \Delta \vDash b \equiv a : A/R}{\Gamma; \Delta \vDash a \equiv b : A/R} \quad \text{E\_Sym}$$

$$\frac{\begin{array}{c}\Gamma; \Delta \vDash a \equiv a_1 : A/R \\ \Gamma; \Delta \vDash a_1 \equiv b : A/R\end{array}}{\Gamma; \Delta \vDash a \equiv b : A/R} \quad \text{E\_Trans}$$

$$\frac{\begin{array}{l}\Gamma; \Delta \vDash a \equiv b : A/R_1 \\ R_1 \leq R_2\end{array}}{\Gamma; \Delta \vDash a \equiv b : A/R_2} \quad \text{E\_Sub}$$

$$\frac{\begin{array}{l}\Gamma \vDash a_1 : B/R \\ \Gamma \vDash a_2 : B/R \\ \vDash a_1 > a_2/R\end{array}}{\Gamma; \Delta \vDash a_1 \equiv a_2 : B/R} \quad \text{E\_Beta}$$

$$\frac{\begin{array}{l}\Gamma; \Delta \vDash A_1 \equiv A_2 : \star/R' \\ \Gamma, x : A_1/R; \Delta \vDash B_1 \equiv B_2 : \star/R' \\ \Gamma \vDash A_1 : \star/R' \\ \Gamma \vDash \Pi^\rho x{:}A_1/R \to B_1 : \star/R' \\ \Gamma \vDash \Pi^\rho x{:}A_2/R \to B_2 : \star/R'\end{array}}{\Gamma; \Delta \vDash (\Pi^\rho x{:}A_1/R \to B_1) \equiv (\Pi^\rho x{:}A_2/R \to B_2) : \star/R'} \quad \text{E\_PiCong}$$

$$\frac{\begin{array}{l}\Gamma, x : A_1/R; \Delta \vDash b_1 \equiv b_2 : B/R' \\ \Gamma \vDash A_1 : \star/R_0 \\ (\rho = +) \vee (x \notin \mathsf{fv}\ b_1) \\ (\rho = +) \vee (x \notin \mathsf{fv}\ b_2)\end{array}}{\Gamma; \Delta \vDash (\lambda^{R,\rho} x.b_1) \equiv (\lambda^{R,\rho} x.b_2) : (\Pi^\rho x{:}A_1/R \to B)/R'} \quad \text{E\_AbsCong}$$

$$\frac{\begin{array}{c}\Gamma; \Delta \vDash a_1 \equiv b_1 : (\Pi^+ x{:}A/R \to B)/R' \\ \Gamma; \Delta \vDash a_2 \equiv b_2 : A/\mathbf{param}\, R\, R'\end{array}}{\Gamma; \Delta \vDash a_1\ {a_2}^{R,+} \equiv b_1\ {b_2}^{R,+} : (B\{a_2/x\})/R'} \quad \text{E\_AppCong}$$

$$\frac{\begin{array}{l}\Gamma; \Delta \vDash a_1 \equiv b_1 : (\Pi^{-} x\!:\!A/R \to B)/R' \\ \Gamma \vDash a : A/\mathbf{param}\, R\, R'\end{array}}{\Gamma; \Delta \vDash a_1\ \square^{R,-} \equiv b_1\ \square^{R,-} : (B\{a/x\})/R'} \quad \text{E\_IAppCong}$$

$$\frac{\Gamma; \Delta \vDash \Pi^{\rho} x\!:\!A_1/R \to B_1 \equiv \Pi^{\rho} x\!:\!A_2/R \to B_2 : \star/R'}{\Gamma; \Delta \vDash A_1 \equiv A_2 : \star/R'} \quad \text{E\_PiFst}$$

$$\frac{\begin{array}{l}\Gamma; \Delta \vDash \Pi^{\rho} x\!:\!A_1/R \to B_1 \equiv \Pi^{\rho} x\!:\!A_2/R \to B_2 : \star/R' \\ \Gamma; \Delta \vDash a_1 \equiv a_2 : A_1/\mathbf{param}\, R\, R'\end{array}}{\Gamma; \Delta \vDash B_1\{a_1/x\} \equiv B_2\{a_2/x\} : \star/R'} \quad \text{E\_PiSnd}$$

$$\frac{\begin{array}{l}\Gamma; \Delta \vDash a_1 \sim_{A_1/R} b_1 \equiv a_2 \sim_{A_2/R} b_2 \\ \Gamma, c : a_1 \sim_{A_1/R} b_1; \Delta \vDash A \equiv B : \star/R' \\ \Gamma \vDash a_1 \sim_{A_1/R} b_1\ \ \mathsf{ok} \\ \Gamma \vDash \forall c\!:\!a_1 \sim_{A_1/R} b_1.A : \star/R' \\ \Gamma \vDash \forall c\!:\!a_2 \sim_{A_2/R} b_2.B : \star/R'\end{array}}{\Gamma; \Delta \vDash \forall c\!:\!a_1 \sim_{A_1/R} b_1.A \equiv \forall c\!:\!a_2 \sim_{A_2/R} b_2.B : \star/R'} \quad \text{E\_CPiCong}$$

$$\frac{\begin{array}{l}\Gamma, c : \phi_1; \Delta \vDash a \equiv b : B/R \\ \Gamma \vDash \phi_1\ \ \mathsf{ok}\end{array}}{\Gamma; \Delta \vDash (\Lambda c.a) \equiv (\Lambda c.b) : \forall c\!:\!\phi_1.B/R} \quad \text{E\_CAbsCong}$$

$$\frac{\begin{array}{l}\Gamma; \Delta \vDash a_1 \equiv b_1 : (\forall c\!:\!(a \sim_{A/R} b).B)/R' \\ \Gamma; \widetilde{\Gamma} \vDash a \equiv b : A/\mathbf{param}\, R\, R'\end{array}}{\Gamma; \Delta \vDash a_1[\bullet] \equiv b_1[\bullet] : (B\{\bullet/c\})/R'} \quad \text{E\_CAppCong}$$

$$\frac{\begin{array}{l}\Gamma; \Delta \vDash \forall c\!:\!(a_1 \sim_{A/R} a_2).B_1 \equiv \forall c\!:\!(a_1' \sim_{A'/R'} a_2').B_2 : \star/R_0 \\ \Gamma; \widetilde{\Gamma} \vDash a_1 \equiv a_2 : A/\mathbf{param}\, R\, R_0 \\ \Gamma; \widetilde{\Gamma} \vDash a_1' \equiv a_2' : A'/\mathbf{param}\, R'\, R_0\end{array}}{\Gamma; \Delta \vDash B_1\{\bullet/c\} \equiv B_2\{\bullet/c\} : \star/R_0} \quad \text{E\_CPiSnd}$$

$$\frac{\begin{array}{l}\Gamma; \Delta \vDash a \equiv b : A/R \\ \Gamma; \Delta \vDash a \sim_{A/R} b \equiv a' \sim_{A'/R'} b'\end{array}}{\Gamma; \Delta \vDash a' \equiv b' : A'/R'} \quad \text{E\_Cast}$$

$$\frac{\begin{array}{l}\Gamma; \Delta \vDash a \equiv b : A/R \\ \Gamma; \widetilde{\Gamma} \vDash A \equiv B : \star/\mathbf{Rep} \\ \Gamma \vDash B : \star/R_0\end{array}}{\Gamma; \Delta \vDash a \equiv b : B/R} \quad \text{E\_EqConv}$$

$$\frac{\Gamma; \Delta \vDash a \sim_{A/R_1} b \equiv a' \sim_{A'/R_1} b'}{\Gamma; \Delta \vDash A \equiv A' : \star/\mathbf{Rep}} \quad \text{E\_IsoSnd}$$

$$\frac{\begin{array}{l}F\ \mathit{sig\_sort} \in \Sigma_0 \\ \Gamma; \Delta \vDash a \equiv a' : A/R \\ \Gamma; \Delta \vDash b_1 \equiv b_1' : B/R_0 \\ \Gamma; \Delta \vDash b_2 \equiv b_2' : B/R_0\end{array}}{\Gamma; \Delta \vDash \mathit{case}_R\ a\ \mathit{of}\ F \to b_1 \|_{\_} \to b_2 \equiv \mathit{case}_R\ a'\ \mathit{of}\ F \to b_1' \|_{\_} \to b_2' : B/R_0} \quad \text{E\_PatCong}$$

$$\frac{\begin{array}{l}\mathsf{Path}_{R'}\ a = F\\ \mathsf{Path}_{R'}\ a' = F\\ \Gamma \vDash a : \Pi^{+}x\!:\!A/R_1 \to B/R'\\ \Gamma \vDash b : A/\mathbf{param}\, R_1\, R'\\ \Gamma \vDash a' : \Pi^{+}x\!:\!A/R_1 \to B/R'\\ \Gamma \vDash b' : A/\mathbf{param}\, R_1\, R'\\ \Gamma; \Delta \vDash a\ \ b^{R_1,+} \equiv a'\ \ b'^{R_1,+} : B\{b/x\}/R'\\ \Gamma; \widetilde{\Gamma} \vDash B\{b/x\} \equiv B\{b'/x\} : \star/R_0\end{array}}{\Gamma; \Delta \vDash a \equiv a' : \Pi^{+}x\!:\!A/R_1 \to B/R'}\ \textsc{E\_LeftRel}$$

$$\frac{\begin{array}{l}\mathsf{Path}_{R'}\ a = F\\ \mathsf{Path}_{R'}\ a' = F\\ \Gamma \vDash a : \Pi^{-}x\!:\!A/R_1 \to B/R'\\ \Gamma \vDash b : A/\mathbf{param}\, R_1\, R'\\ \Gamma \vDash a' : \Pi^{-}x\!:\!A/R_1 \to B/R'\\ \Gamma \vDash b' : A/\mathbf{param}\, R_1\, R'\\ \Gamma; \Delta \vDash a\ \ \square^{R_1,-} \equiv a'\ \ \square^{R_1,-} : B\{b/x\}/R'\\ \Gamma; \widetilde{\Gamma} \vDash B\{b/x\} \equiv B\{b'/x\} : \star/R_0\end{array}}{\Gamma; \Delta \vDash a \equiv a' : \Pi^{-}x\!:\!A/R_1 \to B/R'}\ \textsc{E\_LeftIrrel}$$

$$\frac{\begin{array}{l}\mathsf{Path}_{R'}\ a = F\\ \mathsf{Path}_{R'}\ a' = F\\ \Gamma \vDash a : \Pi^{+}x\!:\!A/R_1 \to B/R'\\ \Gamma \vDash b : A/\mathbf{param}\, R_1\, R'\\ \Gamma \vDash a' : \Pi^{+}x\!:\!A/R_1 \to B/R'\\ \Gamma \vDash b' : A/\mathbf{param}\, R_1\, R'\\ \Gamma; \Delta \vDash a\ \ b^{R_1,+} \equiv a'\ \ b'^{R_1,+} : B\{b/x\}/R'\\ \Gamma; \widetilde{\Gamma} \vDash B\{b/x\} \equiv B\{b'/x\} : \star/R_0\end{array}}{\Gamma; \Delta \vDash b \equiv b' : A/\mathbf{param}\, R_1\, R'}\ \textsc{E\_Right}$$

$$\frac{\begin{array}{l}\mathsf{Path}_{R'}\ a = F\\ \mathsf{Path}_{R'}\ a' = F\\ \Gamma \vDash a : \forall c\!:\!(a_1 \sim_{A/R_1} a_2).B/R'\\ \Gamma \vDash a' : \forall c\!:\!(a_1 \sim_{A/R_1} a_2).B/R'\\ \Gamma; \widetilde{\Gamma} \vDash a_1 \equiv a_2 : A/\mathbf{param}\, R_1\, R'\\ \Gamma; \Delta \vDash a[\bullet] \equiv a'[\bullet] : B\{\bullet/c\}/R'\end{array}}{\Gamma; \Delta \vDash a \equiv a' : \forall c\!:\!(a_1 \sim_{A/R_1} a_2).B/R'}\ \textsc{E\_CLeft}$$

$\boxed{\vDash \Gamma}$ context wellformedness

$$\frac{}{\vDash \varnothing}\ \textsc{E\_Empty}$$

$$\frac{\begin{array}{l}\vDash \Gamma\\ \Gamma \vDash A : \star/R'\\ x \notin \mathsf{dom}\,\Gamma\end{array}}{\vDash \Gamma, x : A/R}\ \textsc{E\_ConsTm}$$

$$\frac{\begin{array}{l}\vDash \Gamma\\ \Gamma \vDash \phi\ \ \mathsf{ok}\\ c \notin \mathsf{dom}\,\Gamma\end{array}}{\vDash \Gamma, c : \phi}\ \textsc{E\_ConsCo}$$

$\boxed{\vDash \Sigma}$ signature wellformedness

$$\frac{}{\vDash \varnothing} \quad \textsc{Sig\_Empty}$$

$$\frac{\begin{array}{l}\vDash \Sigma \\ \varnothing \vDash A : \star / R' \\ F \ \notin \ \mathsf{dom}\, \Sigma\end{array}}{\vDash \Sigma \cup \{F : A\}} \quad \textsc{Sig\_ConsConst}$$

$$\frac{\begin{array}{l}\vDash \Sigma \\ \varnothing \vDash a : A / R' \\ F \ \notin \ \mathsf{dom}\, \Sigma\end{array}}{\vDash \Sigma \cup \{F \sim a : A / R'\}} \quad \textsc{Sig\_ConsAx}$$

$\boxed{\Gamma \vdash \phi \ \mathsf{ok}}$ prop wellformedness

$$\frac{\begin{array}{l}\Gamma \vdash a : A / R \\ \Gamma \vdash b : B / R \\ |A|_R = |B|_R\end{array}}{\Gamma \vdash a \sim_{A/R} b \ \mathsf{ok}} \quad \textsc{An\_Wff}$$

$\boxed{\Gamma \vdash a : A / R}$ typing

$$\frac{\vdash \Gamma}{\Gamma \vdash \star : \star / R} \quad \textsc{An\_Star}$$

$$\frac{\begin{array}{l}\vdash \Gamma \\ x : A / R \in \Gamma\end{array}}{\Gamma \vdash x : A / R} \quad \textsc{An\_Var}$$

$$\frac{\begin{array}{l}\Gamma, x : A / R \vdash B : \star / R' \\ \Gamma \vdash A : \star / R\end{array}}{\Gamma \vdash \Pi^\rho x\!:\!A / R \to B : \star / R'} \quad \textsc{An\_Pi}$$

$$\frac{\begin{array}{l}\Gamma \vdash A : \star / R \\ \Gamma, x : A / R \vdash a : B / R' \\ (\rho = +) \vee (x \notin \mathsf{fv}\ |a|_{R'}) \\ R \leq R'\end{array}}{\Gamma \vdash \lambda^\rho x\!:\!A / R.a : (\Pi^\rho x\!:\!A / R \to B) / R'} \quad \textsc{An\_Abs}$$

$$\frac{\begin{array}{l}\Gamma \vdash b : (\Pi^\rho x\!:\!A / R \to B) / R' \\ \Gamma \vdash a : A / R\end{array}}{\Gamma \vdash b \ a^{R,\rho} : (B\{a/x\}) / R'} \quad \textsc{An\_App}$$

$$\frac{\begin{array}{l}\Gamma \vdash a : A / R \\ \Gamma; \widetilde{\Gamma} \vdash \gamma : A \sim_R B \\ \Gamma \vdash B : \star / R\end{array}}{\Gamma \vdash a \triangleright_R \gamma : B / R} \quad \textsc{An\_Conv}$$

$$\frac{\begin{array}{l}\Gamma \vdash \phi \ \mathsf{ok} \\ \Gamma, c : \phi \vdash B : \star / R\end{array}}{\Gamma \vdash \forall c\!:\!\phi.B : \star / R} \quad \textsc{An\_CPi}$$

$$\frac{\begin{array}{l}\Gamma \vdash \phi \ \mathsf{ok} \\ \Gamma, c : \phi \vdash a : B / R\end{array}}{\Gamma \vdash \Lambda c\!:\!\phi.a : (\forall c\!:\!\phi.B) / R} \quad \textsc{An\_CAbs}$$

$$\frac{\begin{array}{l}\Gamma \vdash a_1 : (\forall c{:}a \sim_{A_1/R} b.B)/R' \\ \Gamma; \widetilde{\Gamma} \vdash \gamma : a \sim_R b\end{array}}{\Gamma \vdash a_1[\gamma] : B\{\gamma/c\}/R'} \quad \textsc{An\_CApp}$$

$$\frac{\begin{array}{l}\vdash \Gamma \\ F \sim a : A/R \in \Sigma_1 \\ \varnothing \vdash A : \star/R_0\end{array}}{\Gamma \vdash F : A/R_1} \quad \textsc{An\_Fam}$$

$$\frac{\begin{array}{l}R_1 \leq R_2 \\ \Gamma \vdash a : A/R_1\end{array}}{\Gamma \vdash \mathbf{sub}\, R_1\, a : A/R_2} \quad \textsc{An\_SubRole}$$

$\boxed{\Gamma; \Delta \vdash \gamma : \phi_1 \sim \phi_2}$ coercion between props

$$\frac{\begin{array}{l}\Gamma; \Delta \vdash \gamma_1 : A_1 \sim_R A_2 \\ \Gamma; \Delta \vdash \gamma_2 : B_1 \sim_R B_2 \\ \Gamma \vdash A_1 \sim_{A/R} B_1 \ \mathsf{ok} \\ \Gamma \vdash A_2 \sim_{A/R} B_2 \ \mathsf{ok}\end{array}}{\Gamma; \Delta \vdash (\gamma_1 \sim_A \gamma_2) : (A_1 \sim_{A/R} B_1) \sim (A_2 \sim_{A/R} B_2)} \quad \textsc{An\_PropCong}$$

$$\frac{\Gamma; \Delta \vdash \gamma : \forall c{:}\phi_1.A_2 \sim_R \forall c{:}\phi_2.B_2}{\Gamma; \Delta \vdash \mathbf{cpiFst}\, \gamma : \phi_1 \sim \phi_2} \quad \textsc{An\_CPiFst}$$

$$\frac{\Gamma; \Delta \vdash \gamma : \phi_1 \sim \phi_2}{\Gamma; \Delta \vdash \mathbf{sym}\, \gamma : \phi_2 \sim \phi_1} \quad \textsc{An\_IsoSym}$$

$$\frac{\begin{array}{l}\Gamma; \Delta \vdash \gamma : A \sim_R B \\ \Gamma \vdash a_1 \sim_{A/R} a_2 \ \mathsf{ok} \\ \Gamma \vdash a_1' \sim_{B/R} a_2' \ \mathsf{ok} \\ |a_1|_R = |a_1'|_R \\ |a_2|_R = |a_2'|_R\end{array}}{\Gamma; \Delta \vdash \mathbf{conv}\ (a_1 \sim_{A/R} a_2) \sim_\gamma (a_1' \sim_{B/R} a_2') : (a_1 \sim_{A/R} a_2) \sim (a_1' \sim_{B/R} a_2')} \quad \textsc{An\_IsoConv}$$

$\boxed{\Gamma; \Delta \vdash \gamma : A \sim_R B}$ coercion between types

$$\frac{\begin{array}{l}\vdash \Gamma \\ c : a \sim_{A/R} b \in \Gamma \\ c \in \Delta\end{array}}{\Gamma; \Delta \vdash c : a \sim_R b} \quad \textsc{An\_Assn}$$

$$\frac{\Gamma \vdash a : A/R}{\Gamma; \Delta \vdash \mathbf{refl}\, a : a \sim_R a} \quad \textsc{An\_Refl}$$

$$\frac{\begin{array}{l}\Gamma \vdash a : A/R \\ \Gamma \vdash b : B/R \\ |a|_R = |b|_R \\ \Gamma; \widetilde{\Gamma} \vdash \gamma : A \sim_R B\end{array}}{\Gamma; \Delta \vdash (a \mid=\mid_\gamma b) : a \sim_R b} \quad \textsc{An\_EraseEq}$$

$$\frac{\begin{array}{l}\Gamma \vdash b : B/R \\ \Gamma \vdash a : A/R \\ \Gamma; \widetilde{\Gamma} \vdash \gamma_1 : B \sim_R A \\ \Gamma; \Delta \vdash \gamma : b \sim_R a\end{array}}{\Gamma; \Delta \vdash \mathbf{sym}\, \gamma : a \sim_R b} \quad \textsc{An\_Sym}$$

$$\frac{\begin{array}{l}\Gamma;\Delta \vdash \gamma_1 : a \sim_R a_1 \\ \Gamma;\Delta \vdash \gamma_2 : a_1 \sim_R b \\ \Gamma \vdash a : A/R \\ \Gamma \vdash a_1 : A_1/R \\ \Gamma;\widetilde{\Gamma} \vdash \gamma_3 : A \sim_R A_1\end{array}}{\Gamma;\Delta \vdash (\gamma_1;\gamma_2) : a \sim_R b} \quad \textsc{An\_Trans}$$

$$\frac{\begin{array}{l}\Gamma \vdash a_1 : B_0/R \\ \Gamma \vdash a_2 : B_1/R \\ |B_0|_R = |B_1|_R \\ \vDash |a_1|_R > |a_2|_R/R\end{array}}{\Gamma;\Delta \vdash \mathbf{red}\, a_1\ a_2 : a_1 \sim_R a_2} \quad \textsc{An\_Beta}$$

$$\frac{\begin{array}{l}\Gamma;\Delta \vdash \gamma_1 : A_1 \sim_{R'} A_2 \\ \Gamma, x : A_1/R;\Delta \vdash \gamma_2 : B_1 \sim_{R'} B_2 \\ B_3 = B_2\{x \triangleright_{R'} \mathbf{sym}\, \gamma_1/x\} \\ \Gamma \vdash \Pi^\rho x\!:\!A_1/R \to B_1 : \star/R' \\ \Gamma \vdash \Pi^\rho x\!:\!A_1/R \to B_2 : \star/R' \\ \Gamma \vdash \Pi^\rho x\!:\!A_2/R \to B_3 : \star/R' \\ R \leq R'\end{array}}{\Gamma;\Delta \vdash \Pi^{R,\rho} x\!:\!\gamma_1.\gamma_2 : (\Pi^\rho x\!:\!A_1/R \to B_1) \sim_{R'} (\Pi^\rho x\!:\!A_2/R \to B_3)} \quad \textsc{An\_PiCong}$$

$$\frac{\begin{array}{l}\Gamma;\Delta \vdash \gamma_1 : A_1 \sim_R A_2 \\ \Gamma, x : A_1/R;\Delta \vdash \gamma_2 : b_1 \sim_{R'} b_2 \\ b_3 = b_2\{x \triangleright_{R'} \mathbf{sym}\, \gamma_1/x\} \\ \Gamma \vdash A_1 : \star/R \\ \Gamma \vdash A_2 : \star/R \\ (\rho = +) \vee (x \notin \mathsf{fv}\ |b_1|_{R'}) \\ (\rho = +) \vee (x \notin \mathsf{fv}\ |b_3|_{R'}) \\ \Gamma \vdash (\lambda^\rho x\!:\!A_1/R.b_2) : B/R' \\ R \leq R'\end{array}}{\Gamma;\Delta \vdash (\lambda^{R,\rho} x\!:\!\gamma_1.\gamma_2) : (\lambda^\rho x\!:\!A_1/R.b_1) \sim_{R'} (\lambda^\rho x\!:\!A_2/R.b_3)} \quad \textsc{An\_AbsCong}$$

$$\frac{\begin{array}{l}\Gamma;\Delta \vdash \gamma_1 : a_1 \sim_{R'} b_1 \\ \Gamma;\Delta \vdash \gamma_2 : a_2 \sim_R b_2 \\ \Gamma \vdash a_1\ {a_2}^{R,\rho} : A/R' \\ \Gamma \vdash b_1\ {b_2}^{R,\rho} : B/R' \\ \Gamma;\widetilde{\Gamma} \vdash \gamma_3 : A \sim_{R'} B\end{array}}{\Gamma;\Delta \vdash \gamma_1\ \gamma_2^{R,\rho} : a_1\ {a_2}^{R,\rho} \sim_{R'} b_1\ {b_2}^{R,\rho}} \quad \textsc{An\_AppCong}$$

$$\frac{\Gamma;\Delta \vdash \gamma : \Pi^\rho x\!:\!A_1/R \to B_1 \sim_{R'} \Pi^\rho x\!:\!A_2/R \to B_2}{\Gamma;\Delta \vdash \mathbf{piFst}\, \gamma : A_1 \sim_R A_2} \quad \textsc{An\_PiFst}$$

$$\frac{\begin{array}{l}\Gamma;\Delta \vdash \gamma_1 : \Pi^\rho x\!:\!A_1/R \to B_1 \sim_{R'} \Pi^\rho x\!:\!A_2/R \to B_2 \\ \Gamma;\Delta \vdash \gamma_2 : a_1 \sim_R a_2 \\ \Gamma \vdash a_1 : A_1/R \\ \Gamma \vdash a_2 : A_2/R\end{array}}{\Gamma;\Delta \vdash \gamma_1 @ \gamma_2 : B_1\{a_1/x\} \sim_{R'} B_2\{a_2/x\}} \quad \textsc{An\_PiSnd}$$

$$\frac{\begin{array}{l}\Gamma;\Delta \vdash \gamma_1 : a_1 \sim_{A_1/R} b_1 \sim a_2 \sim_{A_2/R} b_2\\ \Gamma, c : a_1 \sim_{A_1/R} b_1;\Delta \vdash \gamma_3 : B_1 \sim_{R'} B_2\\ B_3 = B_2\{c \triangleright_{R'} \mathbf{sym}\,\gamma_1/c\}\\ \Gamma \vdash \forall c\!:\!a_1 \sim_{A_1/R} b_1.B_1 : \star/R'\\ \Gamma \vdash \forall c\!:\!a_2 \sim_{A_2/R} b_2.B_3 : \star/R'\\ \Gamma \vdash \forall c\!:\!a_1 \sim_{A_1/R} b_1.B_2 : \star/R'\end{array}}{\Gamma;\Delta \vdash (\forall c\!:\!\gamma_1.\gamma_3) : (\forall c\!:\!a_1 \sim_{A_1/R} b_1.B_1) \sim_R (\forall c\!:\!a_2 \sim_{A_2/R} b_2.B_3)} \quad \textsc{An\_CPiCong}$$

$$\frac{\begin{array}{l}\Gamma;\Delta \vdash \gamma_1 : b_0 \sim_{A_1/R} b_1 \sim b_2 \sim_{A_2/R} b_3\\ \Gamma, c : b_0 \sim_{A_1/R} b_1;\Delta \vdash \gamma_3 : a_1 \sim_{R'} a_2\\ a_3 = a_2\{c \triangleright_{R'} \mathbf{sym}\,\gamma_1/c\}\\ \Gamma \vdash (\Lambda c\!:\!b_0 \sim_{A_1/R} b_1.a_1) : \forall c\!:\!b_0 \sim_{A_1/R} b_1.B_1/R'\\ \Gamma \vdash (\Lambda c\!:\!b_0 \sim_{A_1/R} b_1.a_2) : B/R'\\ \Gamma \vdash (\Lambda c\!:\!b_2 \sim_{A_2/R} b_3.a_3) : \forall c\!:\!b_2 \sim_{A_2/R} b_3.B_2/R'\\ \Gamma;\widetilde{\Gamma} \vdash \gamma_4 : \forall c\!:\!b_0 \sim_{A_1/R} b_1.B_1 \sim_{R'} \forall c\!:\!\phi_2.B_2\end{array}}{\Gamma;\Delta \vdash (\lambda c\!:\!\gamma_1.\gamma_3@\gamma_4) : (\Lambda c\!:\!b_0 \sim_{A_1/R} b_1.a_1) \sim_{R'} (\Lambda c\!:\!b_2 \sim_{A_2/R} b_3.a_3)} \quad \textsc{An\_CAbsCong}$$

$$\frac{\begin{array}{l}\Gamma;\Delta \vdash \gamma_1 : a_1 \sim_R b_1\\ \Gamma;\widetilde{\Gamma} \vdash \gamma_2 : a_2 \sim_{R'} b_2\\ \Gamma;\widetilde{\Gamma} \vdash \gamma_3 : a_3 \sim_{R'} b_3\\ \Gamma \vdash a_1[\gamma_2] : A/R\\ \Gamma \vdash b_1[\gamma_3] : B/R\\ \Gamma;\widetilde{\Gamma} \vdash \gamma_4 : A \sim_R B\end{array}}{\Gamma;\Delta \vdash \gamma_1(\gamma_2,\gamma_3) : a_1[\gamma_2] \sim_R b_1[\gamma_3]} \quad \textsc{An\_CAppCong}$$

$$\frac{\begin{array}{l}\Gamma;\Delta \vdash \gamma_1 : (\forall c_1\!:\!a \sim_{A/R} a'.B_1) \sim_{R_0} (\forall c_2\!:\!b \sim_{B/R'} b'.B_2)\\ \Gamma;\widetilde{\Gamma} \vdash \gamma_2 : a \sim_R a'\\ \Gamma;\widetilde{\Gamma} \vdash \gamma_3 : b \sim_{R'} b'\end{array}}{\Gamma;\Delta \vdash \gamma_1@(\gamma_2 \sim \gamma_3) : B_1\{\gamma_2/c_1\} \sim_{R_0} B_2\{\gamma_3/c_2\}} \quad \textsc{An\_CPiSnd}$$

$$\frac{\begin{array}{l}\Gamma;\Delta \vdash \gamma_1 : a \sim_{R_1} a'\\ \Gamma;\Delta \vdash \gamma_2 : a \sim_{A/R_1} a' \sim b \sim_{B/R_1} b'\end{array}}{\Gamma;\Delta \vdash \gamma_1 \triangleright_{R_1} \gamma_2 : b \sim_{R_1} b'} \quad \textsc{An\_Cast}$$

$$\frac{\Gamma;\Delta \vdash \gamma : (a \sim_{A/R} a') \sim (b \sim_{B/R} b')}{\Gamma;\Delta \vdash \mathbf{isoSnd}\,\gamma : A \sim_R B} \quad \textsc{An\_IsoSnd}$$

$$\frac{\begin{array}{l}\Gamma;\Delta \vdash \gamma : a \sim_{R_1} b\\ R_1 \leq R_2\end{array}}{\Gamma;\Delta \vdash \mathbf{sub}\,\gamma : a \sim_{R_2} b} \quad \textsc{An\_Sub}$$

$\boxed{\vdash \Gamma}$ context wellformedness

$$\frac{}{\vdash \varnothing} \quad \textsc{An\_Empty}$$

$$\frac{\begin{array}{l}\vdash \Gamma\\ \Gamma \vdash A : \star/R\\ x \notin \mathsf{dom}\,\Gamma\end{array}}{\vdash \Gamma, x : A/R} \quad \textsc{An\_ConsTm}$$

$$\frac{\begin{array}{l}\vdash \Gamma\\ \Gamma \vdash \phi\ \mathsf{ok}\\ c \notin \mathsf{dom}\,\Gamma\end{array}}{\vdash \Gamma, c : \phi} \quad \textsc{An\_ConsCo}$$

$\boxed{\vdash \Sigma}$ signature wellformedness

$$\frac{}{\vdash \varnothing} \quad \text{AN\_SIG\_EMPTY}$$

$$\frac{\begin{array}{l}\vdash \Sigma \\ \varnothing \vdash A : \star/R \\ \varnothing \vdash a : A/R \\ F \notin \mathsf{dom}\,\Sigma\end{array}}{\vdash \Sigma \cup \{F \sim a : A/R\}} \quad \text{AN\_SIG\_CONSAX}$$

$\boxed{\Gamma \vdash a \leadsto b/R}$ single-step, weak head reduction to values for annotated language

$$\frac{\Gamma \vdash a \leadsto a'/R_1}{\Gamma \vdash a\ b^{R,\rho} \leadsto a'\ b^{R,\rho}/R_1} \quad \text{AN\_APPLEFT}$$

$$\frac{\mathsf{Value}_R\ (\lambda^\rho x{:}A/R.w)}{\Gamma \vdash (\lambda^\rho x{:}A/R.w)\ a^{R,\rho} \leadsto w\{a/x\}/R} \quad \text{AN\_APPABS}$$

$$\frac{\Gamma \vdash a \leadsto a'/R}{\Gamma \vdash a[\gamma] \leadsto a'[\gamma]/R} \quad \text{AN\_CAPPLEFT}$$

$$\frac{}{\Gamma \vdash (\Lambda c{:}\phi.b)[\gamma] \leadsto b\{\gamma/c\}/R} \quad \text{AN\_CAPPCABS}$$

$$\frac{\begin{array}{l}\Gamma \vdash A : \star/R \\ \Gamma, x : A/R \vdash b \leadsto b'/R_1\end{array}}{\Gamma \vdash (\lambda^- x{:}A/R.b) \leadsto (\lambda^- x{:}A/R.b')/R_1} \quad \text{AN\_ABSTERM}$$

$$\frac{F \sim a : A/R \in \Sigma_1}{\Gamma \vdash F \leadsto a/R} \quad \text{AN\_AXIOM}$$

$$\frac{\Gamma \vdash a \leadsto a'/R}{\Gamma \vdash a \triangleright_{R_1} \gamma \leadsto a' \triangleright_{R_1} \gamma/R} \quad \text{AN\_CONVTERM}$$

$$\frac{\mathsf{Value}_R\ v}{\Gamma \vdash (v \triangleright_{R_2} \gamma_1) \triangleright_{R_2} \gamma_2 \leadsto v \triangleright_{R_2} (\gamma_1; \gamma_2)/R} \quad \text{AN\_COMBINE}$$

$$\frac{\begin{array}{l}\mathsf{Value}_R\ v \\ \Gamma; \widetilde{\Gamma} \vdash \gamma : \Pi^\rho x_1{:}A_1/R \to B_1 \sim_{R'} \Pi^\rho x_2{:}A_2/R \to B_2 \\ b' = b \triangleright_{R'} \mathbf{sym}\,(\mathbf{piFst}\,\gamma) \\ \gamma' = \gamma@(b' \mid=\mid_{(\mathbf{piFst}\,\gamma)} b)\end{array}}{\Gamma \vdash (v \triangleright_{R'} \gamma)\ b^{R,\rho} \leadsto ((v\ b'^{R,\rho}) \triangleright_{R'} \gamma')/R} \quad \text{AN\_PUSH}$$

$$\frac{\begin{array}{l}\mathsf{Value}_R\ v \\ \Gamma; \widetilde{\Gamma} \vdash \gamma : \forall c_1{:}a_1 \sim_{B_1/R} b_1.A_1 \sim_{R'} \forall c_2{:}a_2 \sim_{B_2/R} b_2.A_2 \\ \gamma_1' = \gamma_1 \triangleright_{R'} \mathbf{sym}\,(\mathbf{cpiFst}\,\gamma) \\ \gamma' = \gamma@(\gamma_1' \sim \gamma_1)\end{array}}{\Gamma \vdash (v \triangleright_{R'} \gamma)[\gamma_1] \leadsto ((v[\gamma_1']) \triangleright_{R'} \gamma')/R} \quad \text{AN\_CPUSH}$$

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Definition rules:         166 good    0 bad
Definition rule clauses: 529 good    0 bad
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