Network Value Calculations For The River Barrier Problem

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11/9/2015

1 Introduction

River networks contain barriers, like dams and culverts, that make it difficult for fish to swim from one habitat to another. These barriers decrease the overall value of the river network because they restrict the movement of fish withing the system. As conservationists begin to work towards removing barriers from the river network it is important for them to take into account the increase in network value per dollar spent so that barriers are removed in order to open up the most habitat for the fish. This document will outline the algorithms for calculating the total network value for a given river network.

The river network is modeled as a rooted bidirected tree. Each node in the tree represents a habitat in the river network. The bigger the node value the larger the habitat. Each edge has an associated probability which represents the probability that a fish could swim from one habitat to the other (see figure 1). If the probability between 2 nodes is 1 in both directions this represents the case when fish can swim freely between both habitats, which occurs only if

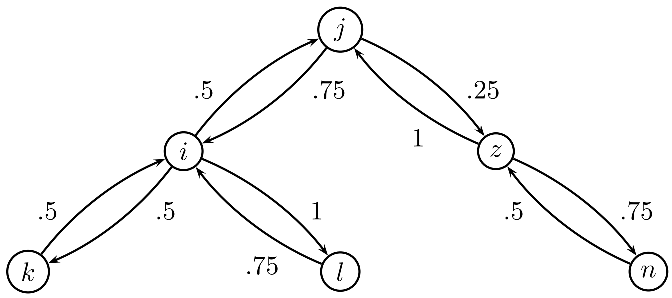

Figure 1: Example of a bidirected tree representing a river network. Each edge is labeled with the probability of moving from the node on one side to the node on the other.

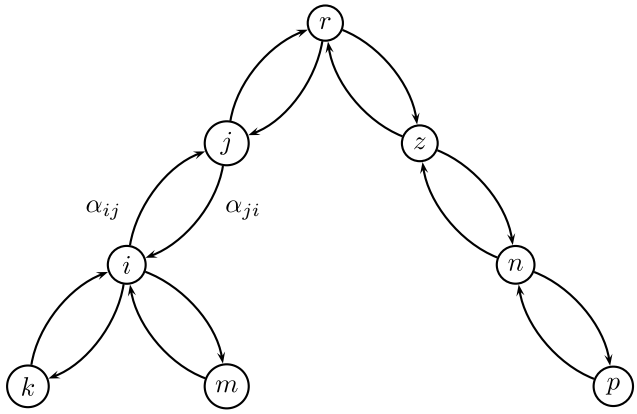

Figure 2: Example river network with α messages between i and j labeled

the river forks. Otherwise the probability will be less than 1, representing the case when two habitats are divided by a barrier (such as a dam or culvert). Note that even though we treat the tree as being rooted, the node that is selected as the root is arbitrary. No matter which node is treated as the root, the total network value will be the same.

In order to compute the total network value the calculation is broken up into 3 pieces:

- α_{ij} Value: The sum of all paths from nodes in the subtree rooted at i that end at i, where j is the parent.
- β_{ij} Value: The sum of all paths starting at i and ending at a node in the subtree rooted at i, where j is the parent.
- γ_{ij} Value: The sum of all paths between any 2 nodes in the subtree rooted at i, where j is the parent.

For each value we must calculate the value each node passes to it's parent and what value it will receive from it's parent. Therefore for each node we will calculate α_{ij} (alpha to parent of i), α_{ji} (alpha from parent of i), β_{ij}, β_{ji}, γ_{ij}, and γ_{ji}.

We do not calculate these values for the root because using the message passing model there is one set of messages per edge, so we have n-1 messages and n nodes. We are mapping the edge values onto the nodes for the calculation, so we choose to exclude 1 node, and for convenience we exclude the node.

Figure 1 depicts a tree rooted at r. Nodes i and j are labeled and the edges connecting i and j are labeled to show the direction of α_{ij} and α_{ji}, the α value from i to its parent and the value of α from i's parent to i respectively. Message passing for β_{ij} and γ_{ij} is the same.

2 α Calculations

α_{ij} finds the value of all paths starting in the subtree rooted at i and ending at i, where neighbor j is the only neighbor of i not in the subtree.

$$\alpha_{ij} = v_i + \sum_{k \in N(i) \setminus j} \alpha_{ki} p_{ki}$$

3 β Calculations

β_{ij} finds the value of all paths starting at i and ending in the subtree rooted at i, where neighbor j is the only neighbor of i not in the subtree.

$$\beta_{ij} = v_i + \sum_{k \in N(i) \setminus j} p_{ik} \beta_{ki}$$

4 γ Calculations

γ_{ij} finds all of the paths between any pair of nodes in the subtree rooted at i, including those that start and end at i, where neighbor j is the only neighbor of i not in the subtree.

$$\gamma_{ij} = \sum_{k \in N(i) \setminus j} \gamma_{ki} + \sum_{k,l \in N(i) \setminus j, k \neq l} \alpha_{ki} p_{ki} p_{il} \beta_{li} + v_i * \beta_{ij} + v_i * \alpha_{ij} - v_i^2$$

The first term finds all γ_{ki} values for all children of i, which takes care of all paths that start and end in the tree rooted at one of i's children. The second term finds all paths that start in a subtree rooted by one child of i and end in a subtree rooted by a different child of i (see figure 3). The third term finds all paths starting at i and ending in the subtree of i (including the path that starts and ends at i). The fourth term finds all paths that start in the subtree of i and end at i (including the path that starts and ends at i). The last term subtracts the value of the path starting and ending at i since we counted it twice, once in the third and once in the fourth term.

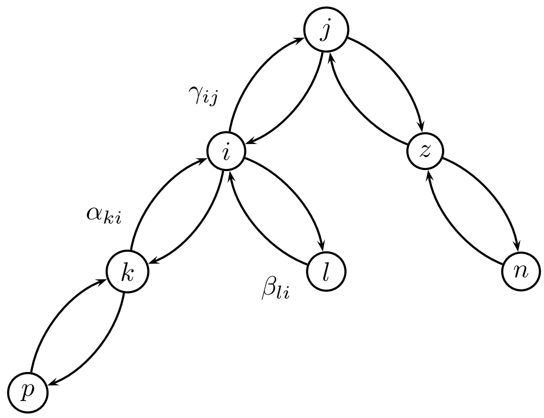

Figure 3: Depiction of the second term of the γ_{ij} calculation

5 Total Network Value Calculation

The total value of the network is a sum of all paths between all pairs of nodes in the network.

$$NetworkValue = \gamma_{ij} + \alpha_{ij} p_{ij} \beta_{ji} + \alpha_{ji} p_{ji} \beta_{ij} + \gamma_{ji}$$

The first and last terms sum the value of all paths within the subtrees of i and j. The second and third terms find the value of all paths starting in one subtree and ending in the other. This accounts for all possible paths between any 2 nodes in the tree.