

## \* Description of Model

Shape

$$S = \sum_{i=1}^m a_i S_i$$

Texture

$$T = \sum_{i=1}^m b_i T_i$$

In terms  
of Vector

$$\vec{S} = \vec{s} + \sum_{i=1}^{m-1} \alpha_i \cdot s_i$$

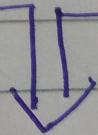
$m \Rightarrow$  no. of exemplar faces

$I_{model}(x, y)$

$$= (I_{s, \text{mod}}(x, y), I_{g, \text{mod}}(x, y), \\ I_{b, \text{mod}}(x, y))^T$$

$$E_I = \sum_{x, y} \| I_{\text{input}}(x, y) - I_{\text{model}}(x, y) \|^2$$

Eqn. to be reduced / differentiated



$$E = \frac{1}{\sigma^2_N} E_I + \sum_{j=1}^{m-1} \frac{\alpha_j^2}{\sigma^2_{s,j}} +$$

$$\sum_{j=1}^{m-1} \frac{\beta_j^2}{\sigma^2_{r,j}} + \sum_j \frac{(\rho_j - \bar{\rho}_j)^2}{\sigma^2_{f,j}}$$

$$\Rightarrow E_I = \sum_{\text{tiny}} \| I_{\text{input}} - (\vec{s} + \alpha \vec{s}) \|^2$$

Using Frobenius Norm eqn.,

$$= \sum_{\text{tiny}} \| (I_{\text{input}} - \vec{s}) + (-\vec{\alpha s}) \|^2$$

Similar to,  $(\vec{A} + \alpha \vec{B})$

$$\text{where } \vec{A} = I_{\text{input}} - \vec{s}$$

$$\vec{B} = -\vec{\alpha s}$$

Using Frobenius Norm Eqn.

where,

$$\left\| \vec{A} + \alpha \vec{B} \right\|$$

$$= \sqrt{\sum_{i=1}^m \sum_{j=1}^n |d_{ij}|^2}$$

where  $d_{ij} \in (\vec{A} + \alpha \vec{B})$

Putting this in our eqn.,

$$E_2 = \sum_{x,y} \left( \sqrt{\sum_{i=1}^m \sum_{j=1}^n |d_{ij}|^2} \right)^2$$

$$= \sum_{x,y} \left[ \sum_{i=1}^m \sum_{j=1}^n |d_{ij}|^2 \right]$$

$n \Rightarrow 3$  (for R,G,B)

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Opening this eqn,

$$\Rightarrow \left[ A(a_{11} + d_{11}b_{11})^2 + (a_{12} + d_{12}b_{12})^2 \right] \\ \dots \dots + (a_{1n} + d_{1n}b_{1n})^2 \\ + \left[ (a_{21} + d_{21}b_{21})^2 + \dots \dots \right. \\ \left. + (a_{2n} + d_{2n}b_{2n})^2 \right] \\ \vdots \\ + \left[ (a_{m1} + d_{m1}b_{m1})^2 \dots \dots \right. \\ \left. + (a_{mn} + d_{mn}b_{mn})^2 \right]$$

$\frac{\partial E}{\partial d}$

$$\sum_{\text{deg}} 2 \left[ \sum_{i=1}^m \sum_{j=1}^n a_{ij} b_{ij} + \sum_{i=1}^m d_i \sum_{j=1}^n b_{ij} \right]$$

$$\boxed{\frac{\partial E}{\partial x} = \frac{1}{\sigma_N^2} \frac{\partial E_2}{\partial x} + \frac{2}{\sigma_{S,j}^2} \sum_{j=1}^{m-1} \frac{d_j}{\sigma_{S,j}^2}}$$