* Description of Model

Shape

$$S = \sum_{i=1}^{m} a_i S_i$$

Texture

$$T = \sum_{i=1}^{m} b_i T_i$$

In terms of Vector

$$S = \vec{S} + \sum_{i=1}^{m-1} \alpha_i . s_i$$

$m \Rightarrow$ no. of exemplar faces

$$I_{model}(x,y) = \left(I_{r,mod}(x,y),\ I_{g,mod}(x,y),\ I_{b,mod}(x,y) \right)^T$$

$$E_I = \sum_{x,y} \left\| I_{input}(x,y) - I_{model}(x,y) \right\|^2$$

Eqn. to be reduced / differentiated

$$E = \frac{1}{\sigma_N^2} E_I + \sum_{j=1}^{m-1} \frac{\alpha_j^2}{\sigma_{S,j}^2} + \sum_{j=1}^{m-1} \frac{\beta_j^2}{\sigma_{T,j}^2} + \sum_j \frac{(\rho_j - \bar{\rho}_j)^2}{\sigma_{\rho,j}^2}$$

$$\Rightarrow E_I = \sum_{x,y} \left\| I_{input} - \left(\vec{\bar{S}} + \vec{\alpha} \vec{S}\right) \right\|^2$$

~~Using Frobenius Norm Eqn.,~~

$$= \sum_{x,y} \left\| \left(I_{input} - \bar{S}\right) + \left(-\vec{\alpha} \vec{S}\right) \right\|^2$$

Similar to, $\left(\vec{A} + \alpha \vec{B}\right)$

where $\vec{A} = I_{input} - \bar{S}$

$\vec{B} = \cancel{\alpha} - \vec{S}$

Using Frobenius Norm Eqn.

where,

$$\|\vec{A} + \alpha\vec{B}\|$$

$$= \sqrt{\sum_{i=1}^{m}\sum_{j=1}^{n} |d_{ij}|^2}$$

where $d_{ij} \in (\vec{A} + \alpha\vec{B})$

Putting this in our eqn.,

$$E_\Sigma = \sum_{x,y}\left(\sqrt{\sum_{i=1}^{m}\sum_{j=1}^{n} |d_{ij}|^2}\right)^2$$

$$= \sum_{x,y}\left[\sum_{i=1}^{m}\sum_{j=1}^{n} |d_{ij}|^2\right]$$

$n \Rightarrow 3$ (for R,G,B)

Using Frobenius Norm Eqn.

where,

$$\| \vec{A} + \alpha \vec{B} \|$$

$$= \sqrt{\sum_{i=1}^{m} \sum_{j=1}^{n} |d_{ij}|^2}$$

where $d_{ij} \in (\vec{A} + \alpha \vec{B})$

Putting this in our eqn.,

$$E_2 = \sum_{x,y} \left(\sqrt{\sum_{i=1}^{m} \sum_{j=1}^{n} |d_{ij}|^2} \right)^2$$

$$= \sum_{x,y} \left[\sum_{i=1}^{m} \sum_{j=1}^{n} |d_{ij}|^2 \right]$$

$n \Rightarrow 3$ (for R,G,B)

opening this eqn,

$$\Rightarrow \left[(a_{11} + \alpha_{11} b_{11})^2 + (a_{12} + \alpha_{12} b_{12})^2 \right.$$
$$\left. \ldots + (a_{1n} + \alpha_{1n} b_{1n})^2 \right]$$

$$+ \left[(a_{21} + \alpha_{21} b_{21})^2 + \ldots \right.$$
$$\left. + (a_{2n} + \alpha_{2n} b_{2n})^2 \right]$$

$$\vdots$$

$$+ \left[(a_{m1} + \alpha_{m1} b_{m1})^2 \ldots \right.$$
$$\left. + (a_{mn} + \alpha_{mn} b_{mn})^2 \right]$$

$$\frac{\partial f}{\partial \alpha}$$

$$\frac{\partial}{\partial \alpha_j} \sum 2 \left[\sum_{i=1}^{m} \sum_{j=1}^{n} a_{ij} b_{ij} + \sum_{i=1}^{m} \alpha_i \sum_{j=1} b_{ij} \right]$$

$$\boxed{\frac{\partial E}{\partial \alpha} = \frac{1}{\sigma_N^2} \frac{\partial E_s}{\partial \alpha} + \frac{2}{\sigma^2_{\cancel{s,j}}} \sum_{j=1}^{m-1} \frac{\alpha_j}{\sigma^2_{s,j}}}$$