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CSE - R.

MA 6201 - LINEAR ALGEBRA.

ASSIGNMENT - II.

1) solve the following system by LU decomposition method.

$x+y+z=3$; $2x-y+3z=16$; $3x+y-z=-3$.

soln

Given :-

$$x+y+z=3$$

$$2x-y+3z=16$$

$$3x+y-z=-3.$$

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & -1 & -3 \\ 3 & 1 & -1 \end{bmatrix} ; X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ \& } B = \begin{bmatrix} 3 \\ 11 \\ -3 \end{bmatrix}.$$

such that $AX = B$.

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & -1 & -3 \\ 3 & 1 & -1 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 1 & 1 \\ 0 & -3 & 1 \\ 0 & -2 & -4 \end{bmatrix} \begin{matrix} R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - 3R_1 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 1 & 1 \\ 0 & -3 & 1 \\ 0 & 0 & -2 \end{bmatrix} R_3 \to 3R_3 + 2R_2 .$$

Therefore $L = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 1 & 1 \end{bmatrix}$ and $U = \begin{bmatrix} 1 & 1 & 1 \\ 0 & -3 & 1 \\ 0 & 0 & -2 \end{bmatrix}$.

Next to solve $LY = B$ when $Y = \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix}$.

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} = \begin{bmatrix} 3 \\ 11 \\ -3 \end{bmatrix}.$$

$$\boxed{x_1 = 3}$$

→ $2x_1 + y_1 = 11$

$6 + y_1 = 11$

$-y_1 = 11 - 6$

$\boxed{y_1 = 5}$

→ $3x_1 + y_1 + z_1 = -3$

$9 + 5 + z_1 = -3$

$\boxed{z_1 = -17}$

Next to solve $Ux = y$.

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & -3 & 1 \\ 0 & 0 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 11 \\ -3 \end{bmatrix}$$

→ $\boxed{z = +3/2}$

→ $-3y + 3/2 = 11$.

$-6y + 3 = 22$.

$-6y = 19$

$\boxed{y = -19/6}$

→ $x + y + z = 3$

$x - \frac{19}{6} + \frac{3}{2} = 3$.

$x - \frac{19+9}{6} = 3$

$x - \frac{10}{6} = 3$

$6x - 10 = 18$.

$6x = 28$.

$x = 28/3$

$\boxed{x = 14/3}$

The solution is $x = 14/3, y = -19/6$ & $z = 3/2$.

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2). Using Gauss Jordan method solve $10x - 2y + 3z = 23$; $2x + 10y - 5z = -33$; $3x - 4y + 10z = 41$.

Soln.

Given :-

$$10x - 2y + 3z = 23$$
$$2x + 10y - 5z = -33$$
$$3x - 4y + 10z = 41.$$

The augmented matrix is

$$\begin{bmatrix} 10 & -2 & 3 & 23 \\ 2 & 10 & -5 & -33 \\ 3 & -4 & 10 & 41 \end{bmatrix}.$$

$$\sim \begin{bmatrix} 1 & -1/5 & 3/10 & 23/10 \\ 2 & 10 & -5 & -33 \\ 3 & -4 & 10 & 41 \end{bmatrix} \quad R_1 \rightarrow R_1/10$$

$$\sim \begin{bmatrix} 1 & -1/5 & 3/10 & 23/10 \\ 0 & 52/5 & -28/5 & -188/5 \\ 0 & -17/5 & 91/10 & 341/10 \end{bmatrix} \quad \begin{array}{l} R_2 \rightarrow R_2 - 2R_1 \\ R_3 \rightarrow R_3 - 3R_1. \end{array}$$

$$\sim \begin{bmatrix} 1 & -1/5 & 3/10 & 23/10 \\ 0 & 1 & -7/13 & -47/13 \\ 0 & -17/5 & 91/10 & 341/10 \end{bmatrix} \quad R_2 \rightarrow \frac{5R_2}{52}.$$

$$\sim \begin{bmatrix} 1 & 0 & 5/26 & 41/26 \\ 0 & 1 & -7/13 & -47/13. \\ 0 & 0 & -189/26 & 567/26 \end{bmatrix} \quad \begin{array}{l} R_1 \rightarrow R_1 + \frac{R_2}{5} \\ R_3 \rightarrow R_3 + \frac{17R_2}{5}. \end{array}$$

$$\sim \begin{bmatrix} 1 & 0 & 5/26 & 41/26 \\ 0 & 1 & -7/13 & -47/13 \\ 0 & 0 & 1 & 3 \end{bmatrix} \quad R_3 \rightarrow \frac{26 R_3}{189}.$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 3 \end{bmatrix} \quad \begin{array}{l} R_1 \rightarrow R_1 - \frac{5R_3}{26}. \\ R_2 \rightarrow R_2 + \frac{7R_2}{13}. \end{array}$$

$\therefore$ The solution is

$$x = 1 \; ; \; y = -2 \; ; \; z = 3 .$$

3) ~~to~~ Solve the following system by applying first two iterations Gauss Jacobi and continue using Gauss Seidel methods correct to 4 decimal places.

$3x - y + z = 1;\ 3x + 6y + 2z = 0;\ 3x + 3y + 7z = 4.$

Soln

Given :-

$3x - y + z = 1$

$3x + 6y + 2z = 0.$

$3x + 3y + 7z = 4.$

Therefore, $x = \frac{1}{3}(1 + y - z)$

$y = \frac{1}{6}(0 - 3x - 2z)$

$z = \frac{1}{7}(4 - 3x - 3y).$

Initial, $x_0 = y_0 = z_0 = 0.$

First Iterative By Gauss Jacobi Method :-

$x_1 = \frac{1}{3}(1 + y_0 - z_0) = \frac{1}{3}(1 + 0 - 0) = 0.3333.$

$y_1 = \frac{1}{6}(0 - 3x_0 - 2z_0) = \frac{1}{6}(0) = 0.$

$z_1 = \frac{1}{7}(4 - 3x_0 - 3y_0) = \frac{1}{7}(4) = 0.5714.$

$\therefore x_1 = 0.3333;\ y_1 = 0;\ z_1 = 0.5714.$

Second Iterative by Gauss Jacobi method:-

$x_2 = \frac{1}{3}(1 + 0 - 0.5714) = 0.1429.$

$y_2 = \frac{1}{6}(0 - 3(0.3333) - 2(0.5714)] = -0.3571$

$z_2 = \frac{1}{7}[4 - 3(0.3333) - 3(0)] = 0.4286.$

$\therefore x_2 = 0.1429;\ y_2 = -0.3571;\ z_1 = 0.4286.$

Gauss Seidal method:-

Third Iteration

$x_3 = \frac{1}{3}(1-0.3571-0.4286) = 0.0714$

$y_3 = \frac{1}{6}(-3(0.0714) - 2(0.4286) = -0.1786$

$z_3 = \frac{1}{7}(4-3(0.0714)-3(-0.1786) = 0.6174$

Fourth Iteration:-

$x_4 = \frac{1}{3}(1-0.1786-0.6174) = 0.0680$.

$y_4 = \frac{1}{6}(-3(0.068) - 2(0.6174) = -0.2398$

$z_4 = \frac{1}{7}(4-3(0.0680-3(-0.2398) = 0.6455$

Fifth Iteration:-

$x_5 = 0.0384$

$y_5 = -0.2342$.

$z_5 = 0.6553$

Sixth Iteration:-

$x_6 = 0.0368$

$y_6 = -0.2368$

$z_6 = 0.6572$.

Seventh Iteration:-

$x_7 = 0.0353$

$y_7 = -0.2367$

$z_7 = 0.6577$.

8th Iteration:

$x_8 = 0.0352$

$y_8 = -0.2368$

$z_8 = 0.6578$

9th Iteration

$x_9 = 0.0351.$

$y_9 = -0.2368$

$z_9 = 0.6578$

10th Iteration

$x_{10} = 0.0351$

$y_{10} = -0.2368$

$z_{10} = 0.6578.$

$\therefore$ The solution

$x = 0.0351; y = -0.2368; z = 0.6578.$

4). Solve by Gauss Elimination Method

$2x + 6y + 10z = 0$; $x + 3y + 3z = 2$; $3x + 14y + 28z = -8$.

Soln

Given

$2x + 6y + 10z = 0$

$x + 3y + 3z = 2$.

$3x + 14y + 28z = -8$.

The augmented matrices is

$$\begin{bmatrix} 2 & 6 & 10 & 0 \\ 1 & 3 & 3 & 2 \\ 3 & 14 & 28 & -8 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 3 & 5 & 0 \\ 1 & 3 & 3 & 2 \\ 3 & 14 & 28 & -8 \end{bmatrix} \quad R_1 \to R_1/2.$$

$$\sim \begin{bmatrix} 1 & 3 & 5 & 0 \\ 0 & 0 & -2 & 2 \\ 0 & 5 & 13 & -8 \end{bmatrix} \quad \begin{array}{l} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - 3R_1. \end{array}$$

$$\sim \begin{bmatrix} 1 & 3 & 5 & 0 \\ 0 & 5 & 13 & -8 \\ 0 & 0 & -2 & 2 \end{bmatrix} \quad R_2 \leftrightarrow R_3.$$

$$\sim \begin{bmatrix} 1 & 3 & 5 & 0 \\ 0 & 1 & 13/5 & -8/5 \\ 0 & 0 & -2 & 2 \end{bmatrix} \quad R_2 \to R_2/5.$$

$$\sim \begin{bmatrix} 1 & 0 & -14/5 & 24/5 \\ 0 & 1 & 13/5 & -8/5 \\ 0 & 0 & -2 & 2 \end{bmatrix} \quad R_1 \to R_1 - 3R_2.$$

$$\sim \begin{bmatrix} 1 & 0 & -14/5 & 24/5 \\ 0 & 1 & 13/5 & -8/5 \\ 0 & 0 & 1 & -1 \end{bmatrix} \quad R_3 \to R_3/-2.$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -1 \end{bmatrix} \quad R_1 \to R_1 + \frac{14}{5} R_3.$$

Therefore the solution is

$x = 2,\ y = 1\ \&\ z = -1.$

5) Determine whether $V_2(R)$ is an inner product space or not with an inner product defined by

$\langle x, y\rangle = x_1y_1 + x_2y_1 - x_1y_2 + 4x_2y_2$

where $x = (x_1, x_2)$ and $y = (y_1, y_2)$.

soln

Given:

$\langle x, y\rangle = x_1y_1 + x_2y_1 - x_1y_2 + 4x_2y_2$.

where $x = (x_1, x_2)$ and $y = (y_1, y_2)$.

Let $z = (z_1, z_2)$.

a) $\langle x + z, y\rangle = \langle (x_1, x_2) + (z_1, z_2), (y_1, y_2)\rangle$

$= \langle ((x_1 + z_1), (x_2 + z_2)), (y_1, y_2)\rangle$

$= (x_1 + z_1)y_1 + (x_2 + z_2)y_1 - (x_1 + z_1)y_2 + 4(x_2 + z_2)y_2$

$= x_1y_1 + y_1z_1 + x_2y_1 + y_1z_2 - x_1y_2 - y_2z_1 + 4x_2y_2 + 4y_2z_2$

$= (x_1y_1 + x_2y_1 - x_1y_2 + 4x_2y_2) + (z_1y_1 + z_2y_1 - z_1y_2 + 4z_2y_2)$

$= \langle x, y\rangle + \langle z, y\rangle$.

b). $\langle cx, y\rangle = \langle c(x_1, x_2), (y_1, y_2)\rangle$

$= \langle (cx_1, cx_2), (y_1, y_2)\rangle$

$= cx_1y_1 + cx_2y_1 - cx_1y_2 + 4cx_2y_2$.

$= c(x_1y_1 + x_2y_1 - x_1y_2 + 4x_2y_2)$

$= c\langle x, y\rangle$.

c) $\langle x, x\rangle = x^2 + \cancel{x_2x_1} - \cancel{x_1x_2} + 4x^2$.

$= x_1^2 + 4x^2$.

d). $\overline{\langle x, y\rangle} = \overline{x_1y_1 + x_2y_1 - x_1y_2 + 4x_2y_2}$

$= \overline{x_1y_1} + \overline{x_2y_1} - \overline{x_1y_2} + 4\overline{x_2y_2}$

$= x_1y_1 + x_2y_1 - x_1y_2 + 4x_2y_2$.

$\neq \langle y, x\rangle$.

Therefore, the condition $\overline{\langle x, y\rangle} \neq \langle y, x\rangle$

The given $V_2(R)$ is not an inner product space.

6) Let $V = P_2(R)$ be the set of all polynomials of degree less than or equal to 2 with real coefficients and with the ~~linear equation~~ inner product

$$\langle f(x), g(x)\rangle = \int_0^1 f(t)\, g(t)\, dt.$$ Using Gram-Schmidt orthogonalization process construct an orthogonal basis from the given set $S = \{1, x, x^2\}$ of V.

Soln Basis for $P_2(R)$, $S = \{1, x, x^2\}$; $w_1 = 1$; $w_2 = x$; $w_3 = x^2$

To find $S' = \{v_1, v_2, v_3\}$.

Take $v_1 = w_1 = 1$.

$$v_2 = w_2 - \frac{\langle w_2, v_1\rangle}{\|v_1\|^2} v_1$$

$$\|v_1\|^2 = \langle v_1, v_2\rangle = \int_0^1 1.1\,dt = \left[\frac{t}{2}\right]_0^1 = 1/2.$$

$$\therefore v_2 = x - 1/2$$

$$\boxed{v_2 = \frac{2x-1}{2}}$$

$$v_3 = w_3 - \frac{\langle w_3, v_1\rangle}{\|v_1\|^2} v_1 - \frac{\langle w_3, v_2\rangle}{\|v_2\|^2} v_2.$$

$$\langle w_3, v_1\rangle = \int_0^1 t^2.\,1\,dt = \left[\frac{t^3}{3}\right]_0^1 = 1/3.$$

$$\langle w_3, v_2\rangle = \int_0^1 t^2.\frac{(2t-1)}{2}\,dt = \frac{1}{2}\int_0^1 (2t^3 - t^2)\,dt$$

$$= \frac{1}{2}\left[\frac{2t^4}{4} - \frac{t^3}{3}\right]_0^1$$

$$= \frac{1}{2}\left[\frac{1}{2} - \frac{1}{3}\right]$$

$$\langle w_3, v_2\rangle = \frac{1}{12}.$$

$$\|v_2\|^2 = \langle v_2, v_2\rangle = \int_0^1 \frac{(2t-1)^2}{4}\,dt = \frac{1}{4}\int_0^1 (4t^2 - 4t + 1)\,dt$$

$$= \frac{1}{4}\left[\frac{4t^3}{3} + \frac{4t^2}{2} + t\right]_0^1 = \frac{1}{4}\left[\frac{4}{3} - 2 + 1\right]$$

$$= \frac{1}{4}\left[\frac{4-6+3}{3}\right] = \frac{1}{12}$$

$$\boxed{\|v_2\|^2 = 1/12}$$

$$\therefore V_3 = x^2 - \left(\frac{1}{3}\right)1 - \left(\frac{1}{12}\right)\left(\frac{12}{1}\right)\left(\frac{2x-1}{2}\right)$$

$$= \frac{3x^2-1}{3} - \frac{2x-1}{2} = \frac{6x^2-2-6x-3}{6} = \frac{6x^2-6x-5}{6}.$$

$$\boxed{V_3 = \frac{6x^2-6x-5}{6}}$$

The set is $\left\{1, \frac{2x-1}{2}, \frac{6x^2-6x-5}{6}\right\}$ is an orthogonal set

For V_1: $\frac{1}{\|V_1\|} V_1 = \frac{1}{1}\cdot 1 = 1$

For V_2: $\frac{1}{\|V_2\|} V_2 = \cancel{\frac{1}{\sqrt{1/12}}}\, 2\sqrt{12}\,\frac{(2x-1)}{2} = \overset{2}{\cancel{4}}\sqrt{3}\,\frac{(2x-1)}{\cancel{2}}$

$$= 2\sqrt{3}(2x-1).$$

For V_3: $\frac{1}{\|V_3\|} V_3 = \sqrt{\frac{180}{181}}\,\frac{(6x^2-6x-5)}{6} = \frac{\cancel{6}\sqrt{5}}{181}\,\frac{(6x^2-6x-5)}{\cancel{6}}$

$$\|V_3\|^2 = \int_0^1 \left[\frac{6t^2-6t-5}{6}\right]^2 dt.$$

$$= \frac{1}{180}\left[t(36t^4-90t^3-40t^2+150t+125)\right]_0^1$$

$$= \frac{1}{180}\left[36-90-40+150+125\right]$$

$$= \frac{181}{180}.$$

$\therefore \left\{1, 2\sqrt{3}(2x-1), \frac{\sqrt{5}}{181}(6x^2-6x-5)\right\}$

form a orthonormal basis.

7) using least square approximation determine the best fit for the data:

$\{(-3,9), (-2,6), (0,2), (1,1)\}$.

Soln.

$$A = \begin{pmatrix} t_1 & 1 \\ t_2 & 1 \\ \vdots & \vdots \\ t_n & 1 \end{pmatrix} = \begin{pmatrix} -3 & 1 \\ -2 & 1 \\ 0 & 1 \\ 1 & 1 \end{pmatrix} ; \quad y = \begin{pmatrix} 9 \\ 6 \\ 2 \\ 1 \end{pmatrix} ; \quad x_0 = \begin{pmatrix} c \\ d \end{pmatrix}.$$

To find: c and d where $y = ct + d$ is the least square approximation.

WKT $\quad x_0 = (A^* A)^{-1} A^* y$.

$$\bar{A} = \begin{pmatrix} -3 & 1 \\ -2 & 1 \\ 0 & 1 \\ 1 & 1 \end{pmatrix} \qquad A^* = (\bar{A})^T = \begin{pmatrix} -3 & -2 & 0 & 1 \\ 1 & 1 & 1 & 1 \end{pmatrix}$$

$$A^* A = \begin{pmatrix} -3 & -2 & 0 & 1 \\ 1 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} -3 & 1 \\ -2 & 1 \\ 0 & 1 \\ 1 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} (9)+(4)+(1) & -3-2+1 \\ -3-2+1 & 1+1+1+1 \end{pmatrix}$$

$$A^* A = \begin{pmatrix} 14 & -4 \\ -4 & 4 \end{pmatrix}$$

$$(A^* A)^{-1} = \frac{1}{|A^* A|} \operatorname{adj}(A^* A). \quad \left[\because A^{-1} = \frac{1}{|A|}\operatorname{adj}(A)\right]$$

$$\operatorname{adj}(A^* A) = \begin{pmatrix} 4 & +4 \\ +4 & 14 \end{pmatrix} \qquad |A^* A| = 40$$

$$(A^* A)^{-1} = \frac{1}{40} \begin{pmatrix} 4 & 4 \\ 4 & 14 \end{pmatrix}.$$

Therefore,

$$\begin{pmatrix} c \\ d \end{pmatrix} = x_0 = \frac{1}{40}\begin{pmatrix} 4 & 4 \\ 4 & 14 \end{pmatrix}\begin{pmatrix} -3 & -2 & 0 & 1 \\ 1 & 1 & 1 & 1 \end{pmatrix}\begin{pmatrix} 9 \\ 6 \\ 2 \\ 1 \end{pmatrix}.$$

$$= \frac{1}{40}\begin{pmatrix} -8 & -4 & 4 & 8 \\ 2 & 6 & 14 & 18 \end{pmatrix}\begin{pmatrix} 9 \\ 6 \\ 2 \\ 1 \end{pmatrix}.$$

$$= \frac{1}{40}\begin{pmatrix} -80 \\ 100 \end{pmatrix}.$$

$$\Rightarrow \begin{pmatrix} c \\ d \end{pmatrix} = \begin{pmatrix} -20 \\ 25 \end{pmatrix}.$$

$c = -20$; $d = 25$.

$\therefore$ The line $y = ct + d = -20t + 25$ is the square line.

Error $E = \|Ax_0 - y\|^2$.

$$Ax_0 = \begin{pmatrix} -3 & 1 \\ -2 & 1 \\ 0 & 1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} -20 \\ 25 \end{pmatrix} = \begin{pmatrix} 340 \\ 260 \\ 100 \\ 20 \end{pmatrix}.$$

$$Ax_0 - y = \begin{pmatrix} 340 - 9 \\ 260 - 6 \\ 100 - 2 \\ 20 - 1 \end{pmatrix} = \begin{pmatrix} 331 \\ 254 \\ 98 \\ 19 \end{pmatrix}.$$

$$\|Ax_0 - y\|^2 = (331)^2 + (254)^2 + (98)^2 + (19)^2$$

Error, $E = 184042$.

8). Obtain Cholesky decomposition for matrix

$$A = \begin{pmatrix} 1 & 2 \\ 2 & 13 \end{pmatrix}.$$

Soln

Given $A = \begin{bmatrix} 1 & 2 \\ 2 & 13 \end{bmatrix}.$

$$L_{11} = \sqrt{a_{11}} = 1.$$

$$L_{21} = \frac{a_{21}}{L_{11}} = \frac{2}{1} = 2.$$

$$L_{22} = \sqrt{a_{22} - L_{21}^2} = \sqrt{13-4} = 3.$$

$$\therefore L = \begin{bmatrix} L_{11} & 0 \\ L_{21} & L_{22} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 2 & 3 \end{bmatrix}$$

$$L^T = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}.$$

WKT

$$LL^T = A. \quad \text{—— } (1)$$

$$\begin{bmatrix} 1 & 0 \\ 2 & 3 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 2 & 4+9 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 \\ 2 & 13 \end{bmatrix}$$

$$= A.$$

$\therefore$ Eq (1) is verified.

9). Let $S = \{v_1, v_2, \dots v_n\}$ be an orthogonal set of non zero vector in an inner product space V. Then prove that S is linearly independent.

Soln. Let $S = \{v_1, v_2, \dots v_n\}$ be as set of nonzero vector in R^n. and S is an orthogonal set.

To show that S is linearly independent

consider the linear combination

$$c_1 v_1 + c_2 v_2 + \dots + c_n v_n = 0.$$

to show that $c_1 = c_2 = c_3 = \dots = c_n = 0$.

compute the dot product of v_i & the above linear combination for each $i = 1, 2, 3, \dots n$.

$$0 = v_i \cdot 0.$$
$$= v_i \cdot (c_1 v_1 + c_2 v_2 + \dots + c_n v_n).$$
$$= c_1 v_i \cdot v_1 + c_2 v_i \cdot v_2 + \dots + c_k v_i \cdot v_n$$

As S is an orthogonal set,

$$v_i \cdot v_j = 0 \text{ if } i \neq j.$$

Hence all terms but the i-th one are zero,

Therefore $0 = c_i v_i \cdot v_i$
$$= c_i \, \|v_i\|^2.$$

Since v_i is a non zero vector, its length $\|v_i\|$ is non zero.

It follow that $c_i = 0$.

As this computation, every $i = 1, 2 \dots n$,

$$c_1 = c_2 = c_3 = \dots = c_n = 0.$$

Hence the set S is linearly independent

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10) The three vectors $v_1 = (1,2,1)$, $v_2 = (2,1,-4)$ and $v_3 = (3,-2,1)$ are mutually orthogonal. Express the vectors $v = (7,1,9)$ as linear combination of v_1, v_2, v_3.

Soln Given v_1, v_2, v_3 are mutually orthogonal.

$\langle v_1, v_2 \rangle = 0$; $\langle v_2, v_3 \rangle = 0$; $\langle v_1, v_3 \rangle = 0$

Given v is a linear combination of vector v_1, v_2 & v_3

$\therefore v = c_1 v_1 + c_2 \cdot v_2 + c_3 v_3$.

$(7,1,9) = c_1(1,2,1) + c_2(2,1,-4) + c_3(3,-2,1)$.

multiply both sides by $\vec{v_1}$.

$$\vec{v} \cdot \vec{v_1} = (c\vec{v_1} + c_2\vec{v_2} + c_3\vec{v_3})\vec{v_1}$$

$$\vec{v} \cdot \vec{v_1} = c_1 \vec{v_1} \cdot \vec{v_1} + c_2 \vec{v_2} \cdot \vec{v_1} + c_3 \vec{v_3} \cdot \vec{v_1}$$

$$c_1 = \frac{\vec{v} \cdot \vec{v_1}}{|\vec{v_1}|^2} \; ; \; c_2 = \frac{\vec{v} \cdot \vec{v_2}}{|\vec{v_2}|^2} \; ; \; c_3 = \frac{\vec{v} \cdot \vec{v_3}}{|\vec{v_3}|^2}$$

$$c_1 = \frac{(7,1,9) \cdot (1,2,1)}{1+4+1} = \frac{18}{6} = 3$$

$$c_2 = \frac{(7,1,9) \cdot (2,1,-4)}{4+1+16} = \frac{-21}{21} = -1.$$

$$c_3 = \frac{(7,1,9) \cdot (3,-2,1)}{9+4+1} = \frac{28}{14} = 2.$$

$\therefore v = c_1 v_1 + c_2 v_2 + c_3 v_3$

$$\boxed{v = 3v_1 - 1v_2 + 2v_3}$$

$$\boxed{(7,1,9) = 3(1,2,1) - (2,1,-4) + 2(3,-2,1)}$$