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RESEARCH NOTES AND COMMENTARIES

RANK FRICTION: AN ORDINAL APPROACH TO PERSISTENT PROFITABILITY

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We present an ordinal method for studying persistence in firm profitability. The method is based on the degree of stability in a ranked performance distribution over time. The method gives a numerical index of rank friction (Rf) that can be applied to any ranked data over any period of time. Rf is nonparametric and can be used to test theoretical assumptions in strategic management. We illustrate the method in an empirical study of 40 years of profit data in 12 industries. Copyright © 2010 John Wiley & Sons, Ltd.

INTRODUCTION

Strategic management is concerned with the persistence of firm performance (Rumelt, Schendel, and Teece, 1994). Empirical studies show that above-normal profit rates do not converge to the industry mean as fast as expected in a competitive market economy (Wiggins and Ruefli, 2002; Jacobsen, 1988; Waring, 1996; Goddard and Wilson, 1996; Geroski and Jacquemin, 1988). These results are usually based on cardinal analysis involving the estimation of an autoregressive model of return on assets (ROA) of the form $\mathrm{ROA}_{i,t} = \alpha + \beta\mathrm{ROA}_{i,t-1} + \gamma\mathrm{ROA}_{i,t-2} + \ldots + \varepsilon_{i,t}$, where $\mathrm{ROA}_{i,t}$ is firm i's return on assets in period t and $\varepsilon_{i,t}$ is a random disturbance term.

Autoregressive methods make full use of cardinal performance data. However, cardinal data are not directly comparable across time periods or industries, and they require assumptions about the true form of the unobserved profit distribution (Ruefli and Wilson, 1987; Collins and Ruefli, 1992; Greene, 2008). By contrast, ranked data allow identically scaled comparisons across industries, time periods and performance measures, and are monotonic to any underlying performance distribution.

Some researchers have used ordinal methods as a complement to cardinal analysis. For example, Mueller (1977, 1986) used quantiles to illustrate persistence in leading performance groups, McGahan (1999) used percentiles to identify positive and negative persistence, and Powell (2003) and

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Powell and Lloyd (2005) examined distributions of first-ranked firms. In a *McKinsey Quarterly* paper, Huyett and Viguerie (2005: 47) defined an industry's 'topple rate' as the persistence of an industry's leading firms in the top revenue quintile (see also Hagel, Seely Brown, and Davison, 2009). Ruefli and Wiggins (2003) used nonparametric methods to compare industry, corporate, and business-segment effects, and Ruefli and Wilson (1987) and Collins and Ruefli (1992) analyzed data from *Fortune* 500 lists using an entropy measure of ordinal performance.

There are theoretical reasons for using ranked data in strategic management research. The concept of competitive advantage is inherently ordinal, implying returns above the industry norm (Peteraf, 1993) or returns superior to those of a competitive reference class (Ghemawat *et al*., 1999). Behavioral decision research shows that decision makers do not maximize cardinal payoffs but payoffs relative to a salient reference group (Kahneman and Tversky, 1979; Moore and Kim, 2003). Social comparison theory shows that decision makers are motivated by the hierarchical position of salient competitors (Festinger, 1954), and neural evidence suggests that ranked hierarchies are hardwired into the brain's reward circuitry (Fliessbach *et al*., 2007). General managers attend to competitors as strategic reference points (Fiegenbaum, Hart, and Schendel, 1996; Porac and Rosa, 1996), and experimental evidence shows that in strategic moves such as market entry and strategic alliance, decision makers evaluate impacts on the firm's rankings—for example, whether the move allows the firm to be ranked first in its industry, or in the *Fortune* 500 (Garcia, Tor, and Gonalez, 2006). Anecdotally, such behavior is illustrated in GE's goal of ranking first or second in every market, and in the emphasis placed by business schools on rankings in *Business Week* and *Financial Times*.

Ordinal analysis is not a substitute for cardinal methods. On the principle that more data is better, researchers should generally prefer cardinal to ranked data (McGahan and Porter, 2005). However, if cardinal data are unavailable or incomparable across time or industries, or if theoretical considerations suggest rank-based motivations, researchers can supplement their analysis using the methods described here.

RANK FRICTION

Consider a hypothetical industry with the same four firms competing for 20 years. If these firms exchange ranks at random on a measure such as ROA, then the industry has no ROA persistence: ranks are freely exchanged, no firm has competitive advantage, and rank friction is zero. If firms persist more or less in the same ranks from year to year, then some firms have sustained advantages and the industry has positive rank friction. If firms invert their ranks each year—that is, there is more rank shuffling than expected if ranks were exchanged at random—then the industry has negative rank friction (decreasing returns to ROA). This is the intuition behind the method presented here. In this section we develop an index of rank friction *(Rf)* that varies from $Rf = 1$ (perfect friction) to $Rf = -1/2$ (perfect negative friction), with random rank shuffling at $Rf = 0$ (frictionless competition).

Consider the general case of an industry with n firms without entry or exit. In frictionless competition, the rank ordering is chosen randomly from $n!$ possible orderings, and for a given firm the probability of taking any rank is $1/n$. In the four-firm example, the rank ordering is chosen randomly from $4! = 24$ rank orderings, and for a given firm the probability of taking any rank is 1/4.

A useful measure of industry rank shuffling is Spearman's footrule, defined as $D = \sum_{r=1}^{n} \left| r_{t_1} - r_{t_2} \right|$, that is, the sum of the absolute values of the rank differences for all firms (Spearman, 1906; Diaconis and Graham, 1977). For example, if the rank orderings for four firms in two periods are (1, 2, 3, 4) and (3, 2, 4, 1), Spearman's footrule yields $D = |1-3| + |2-2| + |3-4| + |4-1| = 6$.

Spearman's D has useful statistical properties for measuring persistence. The minimum value ($D = 0$) equates with perfect friction, that is, all ranks remain the same. The maximum value converges to $D = n^2 \big/ 2$ as n becomes large, and equates with perfect negative friction, that is, the top-ranked firms move to the bottom of the distribution and vice versa.[1] Under random rank shuffling, D converges to a normal random variable as n becomes large, with expected value $D =$

[1] Rank inversion (the first-ranked firm goes to rank n, the second-ranked firm goes to $n - 1$, etc.) is a special case of this.

$n^2/3$ and variance $2n^3/45$ (Diaconis and Graham, 1977; Ury and Kleinecke, 1979). Hence, as n becomes large, the ratio of the expected value of D under randomness to the maximum value of D converges to 2/3. In an industry with four firms and no entry or exit, Spearman's footrule yields $D_{\min} = 0$, $D_{\max} = 8$, and under random rank shuffling $E(D) = 5$ and $\text{Var}(D) = 4^1/_3$.[2]

Spearman's D is a measure of rank shuffling, or nonpersistence. To convert D to a measure of persistence, we perform the following translation:

$$Rf = 1 - \frac{D}{E(D)}, \text{ where :} \tag{1}$$

Rf = Rank friction

D = Actual rank shuffling (Spearman's footrule)

$E(D)$ = Expected rank shuffling under randomness

Since $\text{E}(D) = \frac{n^2 - 1}{3}$ in closed competition, Formula (1) can be expressed as a function of D and industry size (n):

$$Rf = 1 - \frac{3D}{n^2 - 1} \tag{2}$$

Rf is a measure of rank persistence. Its maximum value of 1 implies perfect rank friction, with all firms staying in the same ranks. Its minimum value converges asymptotically to $-1/2$, which implies that firms have inverted their ranks (perfect negative rank friction). If ranks are exchanged randomly, *Rf* converges asymptotically to 0 (frictionless competition).

For hypothesis testing, it has been shown that the distribution of D is sufficiently normal for $n \geq 15$ that $\frac{D - E[D]}{\sigma[D]}$ can be treated as a standard normal random variable (Ury and Kleinecke, 1979). As n becomes large, the quantity $z = \sqrt{\frac{5n}{2}} Rf$ is standard normal and can be used to test the null hypothesis of frictionless competition ($Rf = 0$). For small values of n, exact distributions have been computed for D (Ury and Kleinecke, 1979; Franklin, 1988), and can be used for testing *Rf*. Table 1 shows the exact cumulative distribution functions of D and *Rf* for $n \leq 15$, with p values for significance testing. In the four-firm example with $D = 6$, Table 1 shows that $Rf = -0.20$, indicating a small degree of negative persistence, which for $n = 4$ does not differ significantly from random rank shuffling ($p = 1 - 0.4583 = 0.5417$).

Rank friction gives researchers a more complete picture of persistence than ordinal methods based on first-ranked firms (Powell, 2003), percentiles (Huyett and Viguerie, 2005), or entropy measures developed in the field of information statistics (Ruefli and Wilson, 1987). Unlike these methods, *Rf* can be computed for each period and for all ranks. For comparison to entropy measures, it is possible to form a transition matrix showing the proportions of transitions from rank i in one period to j in the next for the entire period of observation. In this matrix, perfect rank friction ($Rf = 1$) gives unit proportions on the principal diagonal and zeroes elsewhere, rank inversion ($Rf = -1/2$) gives unit proportions on the secondary diagonal and zeroes elsewhere, and frictionless competition ($Rf = 0$) gives an expected value of $1/n$ in all cells.[3]

[2] The value $D_{\max} = \frac{n^2}{2}$ is exact for all closed competitions with an even number of firms. If the number of firms is odd, the exact value is $D_{\max} = \frac{n^2 - 1}{2}$. The exact expected value under random rank shuffling is $\text{E}(D) = \frac{n^2 - 1}{3}$ for all n, and the exact variance is $\text{Var}(D) = \frac{(n + 1)(2n^2 + 7)}{45}$.

[3] Entropy measures were developed in the field of information statistics (see Shannon, 1948; Theil, 1967; Ruefli and Wilson, 1987). If $p_{i,j}$ represents the historical proportion of transitions for n firms from rank i in one period to j in the next, one such measure is:

$$E = \frac{-\sum_{i,j=1}^{n} p_{i,j} \ln p_{i,j}}{n \ln n}$$

E is an index of industry rank shuffling (nonpersistence) normalized to the range 0–1 by the term $n \ln n$. Under perfect positive or negative rank persistence, the numerator in (2) is zero, and $E = 0$. In frictionless competition (random rank shuffling), the numerator sums to $n \ln n$, and $E = 1$ (E is always nonnegative). E is an alternative index of rank shuffling for the entire period of observation, and its statistical significance can be tested using variations on the χ^2 statistical test, such as the G^2 test (Hays, 1988; Agresti, 2007) used by Wiggins and Ruefli (2002). Unlike *Rf*, E and G^2 do not distinguish between positive and negative rank persistence, and are concerned with transition proportions established over long periods. *Rf* varies with the degree and direction of rank changes and can be measured for a single period.

Table 1. Exact distributions for D and Rf when $n \leq 15$

	n = 2			n = 7			n = 9			n = 10			n = 11	
D	*Rf*	P	*D*	*Rf*	P	*D*	*Rf*	P	*D*	*Rf*	P	*D*	*Rf*	P
0	1.000	0.5000	0	1.000	0.0002	0	1.000	0.0000	0	1.000	0.0000	0	1.000	0.0000
2	−1.000	1.0000	2	0.875	0.0014	2	0.925	0.0000	2	0.939	0.0000	2	0.950	0.0000
			4	0.750	0.0063	4	0.850	0.0001	4	0.879	0.0000	4	0.900	0.0000
	n = 3		6	0.625	0.0214	6	0.775	0.0006	6	0.818	0.0000	6	0.850	0.0000
D	*Rf*	P	8	0.500	0.0585	8	0.700	0.0020	8	0.758	0.0003	8	0.800	0.0000
0	1.000	0.1667	10	0.375	0.1312	10	0.625	0.0059	10	0.697	0.0009	10	0.750	0.0001
2	0.250	0.5000	12	0.250	0.2484	12	0.550	0.0148	12	0.636	0.0026	12	0.700	0.0004
4	−0.500	1.0000	14	0.125	0.3960	14	0.475	0.0325	14	0.576	0.0064	14	0.650	0.0011
			16	0.000	0.5714	16	0.400	0.0637	16	0.515	0.0141	16	0.600	0.0026
	n = 4		18	−0.125	0.7365	18	0.325	0.1123	18	0.455	0.0280	18	0.550	0.0056
D	*Rf*	P	20	−0.250	0.8786	20	0.250	0.1825	20	0.394	0.0510	20	0.500	0.0113
0	1.000	0.0417	22	−0.375	0.9500	22	0.175	0.2742	22	0.333	0.0862	22	0.450	0.0211
2	0.600	0.1667	24	−0.500	1.0000	24	0.100	0.3859	24	0.273	0.1364	24	0.400	0.0368
4	0.200	0.4583				26	0.025	0.5078	26	0.212	0.2032	26	0.350	0.0606
6	−0.200	0.8333		**n = 8**		28	−0.050	0.6332	28	0.152	0.2863	28	0.300	0.0946
8	−0.600	1.0000	*D*	*Rf*	P	30	−0.125	0.7451	30	0.091	0.3830	30	0.250	0.1403
			0	1.000	0.0000	32	−0.200	0.8430	32	0.030	0.4884	32	0.200	0.1990
	n = 5		2	0.905	0.0002	34	−0.275	0.9136	34	−0.030	0.5957	34	0.150	0.2700
D	*Rf*	P	4	0.810	0.0010	36	−0.350	0.9635	36	−0.091	0.6986	36	0.100	0.3522
0	1.000	0.0083	6	0.714	0.0039	38	−0.425	0.9857	38	−0.152	0.7903	38	0.050	0.4420
2	0.750	0.0417	8	0.619	0.0120	40	−0.500	1.0000	40	−0.212	0.8655	40	0.100	0.5362
4	0.500	0.1417	10	0.524	0.0310				42	−0.273	0.9230	42	−0.050	0.6295
6	0.250	0.3417	12	0.429	0.0687				44	−0.333	0.9606	44	−0.100	0.7180
8	0.000	0.6333	14	0.333	0.1320				46	−0.394	0.9832	46	−0.150	0.7955
10	−0.250	0.8333	16	0.238	0.2237				48	−0.455	0.9960	48	−0.200	0.8617
12	−0.500	1.0000	18	0.143	0.3440				50	−0.515	1.0000	50	−0.250	0.9120
			20	0.048	0.4856							52	−0.300	0.9498
	n = 6		22	−0.048	0.6317							54	−0.350	0.9740
D	*Rf*	P	24	−0.143	0.7670							56	−0.400	0.9895
0	1.000	0.0014	26	−0.238	0.8714							58	−0.450	0.9960
2	0.829	0.0083	28	−0.333	0.9420							60	−0.500	1.0000
4	0.657	0.0333	30	−0.429	0.9857									
6	0.486	0.0972	32	−0.524	1.0000									
8	0.314	0.2264												
10	0.143	0.4167												
12	−0.029	0.6222												
14	−0.200	0.8111												
16	−0.371	0.9500												
18	−0.543	1.0000												

Note: (1) The numbers in the P column show the probability of observing values less than or equal to those in the D or Rf column.

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Table 1. continued

	n = 12			n = 13			n = 14			n = 14			n = 15	
D	*Rf*	P	*D*	*Rf*	P	*D*	*Rf*	P	*D*	*Rf*	P	*D*	*Rf*	P
0	1.00	0.0000	0	1.000	0.0000	0	1.000	0.0000	86	−0.323	0.9776	66	0.116	0.2736
2	0.96	0.0000	2	0.964	0.0000	2	0.969	0.0000	88	−0.354	0.9871	68	0.089	0.3258
4	0.92	0.0000	4	0.929	0.0000	4	0.938	0.0000	90	−0.385	0.9932	70	0.063	0.3819
6	0.87	0.0000	6	0.893	0.0000	6	0.908	0.0000	92	−0.415	0.9968	72	0.036	0.4408
8	0.83	0.0000	8	0.857	0.0000	8	0.877	0.0000	94	−0.446	0.9987	74	0.009	0.5013
10	0.79	0.0000	10	0.821	0.0000	10	0.846	0.0000	96	−0.477	0.9997	76	−0.018	0.5623
12	0.75	0.0000	12	0.786	0.0000	12	0.815	0.0000	98	−0.508	1.0000	78	−0.045	0.6222
14	0.71	0.0001	14	0.750	0.0000	14	0.785	0.0000				80	−0.071	0.6799
16	0.66	0.0004	16	0.714	0.0000	16	0.754	0.0000		n= 15		82	−0.098	0.7340
18	0.62	0.0009	18	0.679	0.0001	18	0.723	0.0000	*D*	*Rf*	P	84	−0.125	0.7838
20	0.58	0.0021	20	0.643	0.0003	20	0.692	0.0000	0	1.000	0.0000	86	−0.152	0.8282
22	0.54	0.0042	22	0.607	0.0007	22	0.662	0.0001	2	0.973	0.0000	88	−0.179	0.8670
24	0.50	0.0080	24	0.571	0.0014	24	0.631	0.0002	4	0.946	0.0000	90	−0.205	0.8998
26	0.45	0.0143	26	0.536	0.0028	26	0.600	0.0005	6	0.920	0.0000	92	−0.232	0.9269
28	0.41	0.0244	28	0.500	0.0051	28	0.569	0.0009	8	0.893	0.0000	94	−0.259	0.9480
30	0.37	0.0395	30	0.464	0.0090	30	0.538	0.0017	10	0.866	0.0000	96	−0.286	0.9650
32	0.33	0.0611	32	0.429	0.0150	32	0.508	0.0030	12	0.839	0.0000	98	−0.313	0.9772
34	0.29	0.0907	34	0.393	0.0240	34	0.477	0.0052	14	0.813	0.0000	100	−0.339	0.9861
36	0.24	0.1295	36	0.357	0.0370	36	0.446	0.0086	16	0.786	0.0000	102	−0.366	0.9919
38	0.20	0.1782	38	0.321	0.0550	38	0.415	0.0137	18	0.759	0.0000	104	−0.393	0.9957
40	0.16	0.2369	40	0.286	0.0791	40	0.385	0.0210	20	0.732	0.0000	106	−0.420	0.9979
42	0.12	0.3049	42	0.250	0.1102	42	0.354	0.0314	22	0.705	0.0000	108	−0.446	0.9992
44	0.08	0.3807	44	0.214	0.1490	44	0.323	0.0454	24	0.679	0.0000	110	−0.473	0.9997
46	0.03	0.4619	46	0.179	0.1958	46	0.292	0.0640	26	0.652	0.0000	112	−0.500	1.0000
48	−0.01	0.5455	48	0.143	0.2506	48	0.262	0.0878	28	0.625	0.0001			
50	−0.05	0.6281	50	0.107	0.3127	50	0.231	0.1174	30	0.598	0.0003			
52	−0.09	0.7063	52	0.071	0.3808	52	0.200	0.1535	32	0.571	0.0005			
54	−0.13	0.7771	54	0.036	0.4531	54	0.169	0.1960	34	0.545	0.0010			
56	−0.17	0.8383	56	0.000	0.5277	56	0.138	0.2451	36	0.518	0.0017			
58	−0.22	0.8888	58	−0.036	0.6018	58	0.108	0.3001	38	0.491	0.0028			
60	−0.26	0.9281	60	−0.071	0.6735	60	0.077	0.3602	40	0.464	0.0046			
62	−0.30	0.9569	62	−0.107	0.7400	62	0.046	0.4243	42	0.438	0.0073			
64	−0.34	0.9766	64	−0.143	0.7999	64	0.015	0.4908	44	0.411	0.0113			
66	−0.38	0.9886	66	−0.179	0.8515	66	−0.015	0.5580	46	0.384	0.0169			
68	−0.43	0.9953	68	−0.214	0.8946	68	−0.046	0.6242	48	0.357	0.0246			
70	−0.47	0.9989	70	−0.250	0.9284	70	−0.077	0.6876	50	0.330	0.0350			
72	−0.51	1.0000	72	−0.286	0.9545	72	−0.108	0.7466	52	0.304	0.0486			
			74	−0.321	0.9725	74	−0.138	0.7999	54	0.277	0.0660			
			76	−0.357	0.9850	76	−0.169	0.8467	56	0.250	0.0877			
			78	−0.393	0.9925	78	−0.200	0.8863	58	0.223	0.1144			
			80	−0.429	0.9971	80	−0.231	0.9188	60	0.196	0.1462			
			82	−0.464	0.9989	82	−0.262	0.9443	62	0.170	0.1834			
			84	−0.500	1.0000	84	−0.292	0.9636	64	0.143	0.2259			

Notes: (1) The numbers in the P column show the probability of observing values less than or equal to those in the *D* or *Rf* column.
(2) For n>15, *D* and *Rf* can be tested using the standard normal test given in the text.

EMPIRICAL ANALYSIS

To illustrate the method, we calculated *Rf* for ROA for 12 U.S. manufacturing industries. The data were taken from *Fortune* 500 lists published from 1955 through 1994. After 1994, *Fortune* included service firms in the list, and these years were excluded to avoid biasing the industry sample sizes. The sample comprises all firms in *Fortune* 500 manufacturing industries with at least 10 firms in all 40 years. The average number of firms per industry ranged from 12 (mining and crude oil) to 56 (foods), and industry sizes varied annually due to entry and exit—for example, in the food industry 145 firms appeared in the *Fortune* 500 lists at least once.

Because industry sizes fluctuate over time, the formula $E(D) = \frac{n^2 - 1}{3}$ does not give an exact value for rank shuffling under frictionless competition. There are three ways to calculate $E(D)$ with entry and exit. The first is to simulate random rank shuffling using actual industry sizes for each year, which gives an exact value for $E(D)$. The second is to estimate $E(D)$ using average industry sizes. It can be shown that substituting n_{mean} for n in $\frac{n^2 - 1}{3}$ gives a systematic overestimate of $E(D)$, and that the error is increasing in entry and exit (the average overestimate for industry sizes in the *Fortune* data is about 5%). The third method involves using the rate of entry and exit to improve this estimate. In an industry with constant size n, in which the total number of firms that compete at least once during T periods is N, N—n firms are no longer in the industry and the average annual rate of entry and exit is $e = \frac{N - n}{T - 1}$.[4] For example, in the food industry, the average rate of entry and exit is $e = \frac{145 - 56.4}{40 - 1} = 2.27$ firms per year. Using e, it can be shown that a close approximation to $E(D)$ is:[5]

$$E(D) = \frac{(n - \frac{1}{2}e)^2 - 1}{3}, \text{ where :} \qquad (3)$$

$E(D)$ = Expected rank shuffling under randomness

D = Actual rank shuffling (Spearman's footrule)

n = Mean industry size

$e = \frac{N - n}{T - 1}$

Formula (3) enables researchers to compute *Rf* for industries of any size without the need to simulate random rank shuffling. It gives a slight overestimate of *Rf* if entry and exit are large, but produces good results in the ranges usually observed in industries.[6] In the *Fortune* data, Formula (3) gave exact values of *Rf* in seven industries, an error of 0.01 in four industries, and an error of 0.03 in one industry.

Table 2 shows the results for all industries using simulated values of $E(D)$ and *Rf*.[7] The results show that rank friction ranged from 0.73 (pharmaceuticals) to 0.43 (aerospace), with a mean of 0.55 for all industries and a standard deviation of 0.08. In all industries the null hypothesis of frictionless competition ($Rf = 0$) was rejected at $p < 0.001$. Table 3 compares the simulated values with those obtained using the approximation for $E(D)$ in Formula (3). Using the approximation (Rf^*), rank friction ranged from 0.72 (pharmaceuticals) to 0.43 (aerospace), with a mean of 0.55 and standard deviation of 0.08. The overall mean error of Formula (3) in computing *Rf* was 0.2 percent.

We also computed a transition matrix for each industry showing the proportion of transitions from each rank to every other rank. Using the entropy index discussed earlier, these proportions can be tested against the null hypothesis of random rank

[4] This assumes that firms do not reenter after exiting, an assumption supported in the industry data.

[5] In an industry of constant size n, where entry and exit occur at a constant rate e, random rank shuffling yields $E(D) = \frac{(n - e)^2 - 1}{3}$. This is a lower bound on $E(D)$, and $E(D) = \frac{n^2 - 1}{3}$ is an upper bound. Formula (3) is the midpoint of these values.

[6] The approximation gives good estimates when $\frac{\sigma_n}{n} < 0.15$, where n is the mean number of firms in the industry during the time period and σ_n is the standard deviation of n.

[7] In computing D from one period to the next, we considered only firms that competed in both periods. For example, if firm A exited after period 1, or if firm B entered in period 2, neither firm contributed to actual or random rank shuffling between periods 1 and 2. An alternative method is to treat entry and exit as a climb or fall from below the last rank. The two methods give similar results for the industry sizes studied here, but researchers may prefer one or the other method for theoretical reasons. For example, the latter method is preferable if membership in the competition is determined by the performance measure being studied—for example, membership in the *Fortune* 500 is not determined by ROA but by total revenues—thus, the latter method may be preferable for computing *Rf* for total revenues.

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Table 2. *Rf* values in 12 industries

Industry	Number of firms		Mean from 1955 to 1994			
	1 Mean (n)	2 Total (N)	3 D	4 $E(D)$	5 Rf	6 ROA
Aerospace	16.60	44	50.10	88.08	0.43	0.047
Chemicals	36.25	105	173.03	418.94	0.59	0.061
Electronics	37.68	129	180.08	445.59	0.60	0.063
Foods	56.40	145	362.13	1015.31	0.64	0.056
Forest and paper products	29.43	80	131.41	272.57	0.52	0.057
Industrial and farm equipment	35.70	115	170.56	397.86	0.57	0.052
Metals	36.05	99	202.69	427.74	0.53	0.038
Mining, crude-oil prodn	11.65	56	21.51	40.43	0.47	0.058
Motor vehicles and parts	22.73	62	79.67	166.59	0.52	0.042
Petroleum refining	31.73	74	152.79	318.08	0.52	0.054
Pharmaceuticals	16.28	31	23.79	86.95	0.73	0.114
Textiles	13.98	37	28.56	62.16	0.54	0.042
Mean	**28.70**	**81.42**	**131.36**	**311.69**	**0.55**	**0.057**
Stand dev	**13.01**	**37.29**	**99.05**	**270.42**	**0.08**	**0.020**
Min	**11.65**	**31**	**21.51**	**40.43**	**0.43**	**0.038**
Max	**56.40**	**145**	**362.13**	**1015.31**	**0.73**	**0.114**

Table 3. Approximating $E(D)$ and Rf with entry and exit

Industry	**Number of firms**		***E*(*D*) and *Rf* values**						
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
	Mean (*n*)	**Total (*N*)**	***E*(*D*)**	$\frac{n^2-1}{3}$	$\frac{(n-e)^2-1}{3}$	$\frac{(n-\frac{1}{2}e)^2-1}{3}$	***Rf****	***Rf***	***Rf**** **error**
Aerospace	16.60	44	88.08	91.52	83.91	87.67	0.43	0.43	−0.62%
Chemicals	36.25	105	418.94	437.69	396.12	416.65	0.58	0.59	−0.39%
Electronics	37.68	129	445.59	472.8	415.81	443.85	0.59	0.60	−0.27%
Foods	56.40	145	1015.31	1059.99	976.29	1017.71	0.64	0.64	0.13%
Forest and paper prods	29.43	80	272.57	288.28	263.40	275.70	0.52	0.52	1.06%
Industrial and farm equip	35.70	115	397.86	424.5	377.48	400.64	0.57	0.57	0.52%
Metals	36.05	99	427.74	432.87	394.94	413.69	0.51	0.53	−3.06%
Mining, crude-oil prodn	11.65	56	40.43	44.91	36.51	40.60	0.47	0.47	0.46%
Motor vehicles and parts	22.73	62	166.59	171.81	156.89	164.26	0.52	0.52	−1.30%
Petroleum refining	31.73	74	318.08	335.16	312.62	323.79	0.53	0.52	1.63%
Pharmaceuticals	16.28	31	86.95	87.96	83.91	85.92	0.72	0.73	−0.45%
Textiles	13.98	37	62.16	64.77	59.38	62.05	0.54	0.54	−0.16%
Mean	**28.70**	**81.42**	**311.69**	**326.02**	**296.44**	**311.04**	**0.55**	**0.55**	**−0.20%**
Stand dev	**13.01**	**37.29**	**270.42**	**282.53**	**258.95**	**270.59**	**0.08**	**0.08**	**1**.19%
Min	**11.65**	**31**	**40.43**	**44.91**	**36.51**	**40.60**	**0.43**	**0.43**	**−3.06%**
Max	**56.40**	**145**	**1015.31**	**1059.99**	**976.29**	**1017.71**	**0.72**	**0.73**	**1.63%**

Explanation of columns:
Column 3: Simulated values of $E(D)$ using actual industry sizes for each year for each industry.
Column 4: $E(D)$ calculated by substituting the industry mean number of firms for n (an upper bound on $E(D)$).
Column 5: $E(D)$ calculated by using the average rate of entry and exit e (a lower bound on $E(D)$).
Column 6: The midpoint of Columns 4 and 5: applying Formula (3) in the text.
Column 7: Rf^* is the value of *Rf* derived from the approximation in Column 6.
Column 8: The actual value of *Rf* derived from simulation (taken from Table 2).
Column 9: The percentage overestimate or underestimate of *Rf* using the approximation in Column 6.

		To rank									
	Rank	1	2	3	4	5	6	7	8	9	10
	1	22	14	1	0	0	0	1	0	0	1
	2	7	12	8	2	1	3	1	0	0	0
	3	4	4	12	6	3	4	2	0	0	0
	4	1	1	2	9	7	6	1	1	0	2
From rank	5	0	0	3	7	5	5	0	2	4	0
	6	0	1	0	3	5	7	7	1	2	4
	7	0	1	1	1	4	2	8	4	4	2
	8	0	0	0	4	2	2	5	5	5	3
	9	1	0	1	0	1	0	2	7	5	2
	10	0	1	1	0	0	0	0	2	4	3

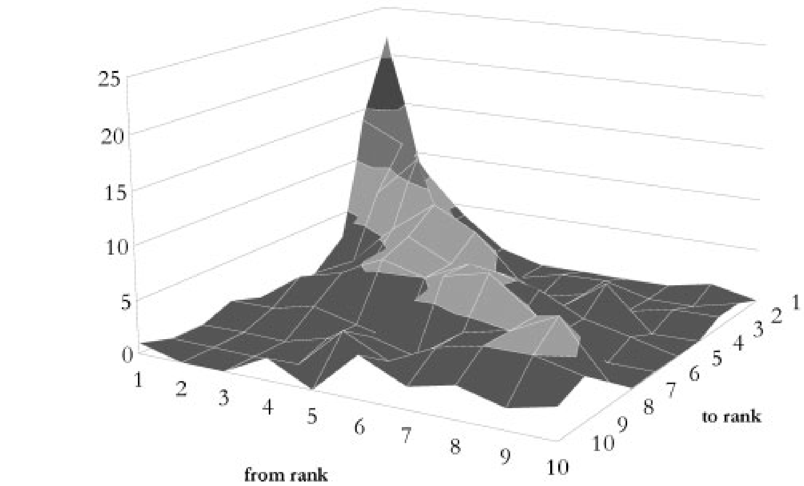

Figure 1. Rank-transition matrix and graph for the chemicals industry.

transitions. For illustration, the transition matrix and graph for the first 10 ranks of the chemicals industry are shown in Figure 1. The value of entropy index E (see Footnote 3) for 40 years of competition in the chemicals industry is 0.61, similar to findings reported by Ruefli and Wilson (1987) and rejecting the hypothesis of frictionless competition ($E = 1$) at $p < 0.001$. Entropy analysis for all industries is available from the authors.

We also compared rank analysis in the chemicals industry to results using an autoregressive model of profit persistence. A simple version is the process $\mathrm{ROA}_{i,t} = \alpha + \beta \mathrm{ROA}_{i,t-1} + \varepsilon_{i,t}$, where $\mathrm{ROA}_{i,t}$ is firm i's return on assets in period t and $\varepsilon_{i,t}$ are independent standard normal random variables (Jacobsen, 1988). Applying this model to the chemicals industry, ordinary least squares analysis yields $\beta = 0.78$. In general, autoregressive analysis is sensitive to outlying cardinal values, whereas the rank method gives equal weighting to rank changes throughout the distribution, providing a complementary measure of persistence.

APPLICATIONS

Rank friction has numerous applications in strategic management research, some of which are discussed below.

Theory testing

Profit statistics do not, in general, allow us to make inferences about unobserved management or firm behavior (McGahan and Porter, 2005). However, Rf can be used to test statistical models of sustained competitive advantage. For example, models of resource accumulation involving asset

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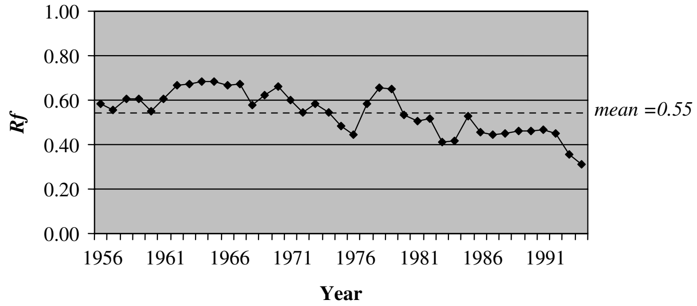

Figure 2. Mean *Rf* values for 12 manufacturing industries: 1955 to 1994.

mass efficiencies (Dierickx and Cool, 1989) imply increasing persistence at the top ranks, whereas a random walk (Denrell, 2004) implies a symmetrical pattern of rank friction at higher and lower ranks. These models can be tested using the methods presented in the following section.

It is also possible to test parameter values proposed in models of competitive advantage. For example, the random walk proposed by Denrell (2004) posited that the stock $R_{i,t}$ of firm i's strategic resources in an industry follows a modified random walk $R_{i,t} = bR_{i,t-1} + \varepsilon_{i,t}$.[8] If resource stocks follow this modified random walk, then *Rf* quickly converges to a unique value independent of the initial distribution of resources, depending on the parameter b. If $b = 0$, then $Rf = 0$; if $b = 1$, then $Rf = 1$. Denrell simulated 20 years of competition assuming b values between 0.70 and 1.00. In the *Fortune* data, the range of *Rf* values (0.43–0.73) corresponds to b values between 0.70 (aerospace) and 0.93 (pharmaceuticals), implying a half-life of competitive advantage between two and 10 years.

The value of *Rf* does not depend on industry size or ROA—that is, rank friction can be high in a large or low-performing industry.[9] However, the data in Table 2 show a large positive correlation between ROA and rank friction ($r = 0.75$; $p < 0.01$) and a nonsignificant correlation between industry size and rank friction. This suggests a close association between rank friction and the market power of individual firms, independent of industry size.

[8] In this model, $\varepsilon_{i,t}$ are independently and identically standard normally distributed random variables for $t = 1, 2, \ldots$, and b can be interpreted as a measure of sustainability of advantages.

[9] Industry size has a negligible influence on *Rf* for $n \geq 10$.

Analysis by rank

Spearman's footrule (D) is not the only measure of rank shuffling, and it correlates highly with alternative measures such as Spearman's rho and Kendall's tau (Kendall, 1938, 1970; Durbin and Stuart, 1951; Ury and Kleinecke, 1979). Although D is seldom used in empirical research (for an exception, see Kim *et al*., 2004), it has the advantage of allowing analysis at different ranks. For example, D can be used to determine whether persistence is abnormally large at the first rank as compared with other ranks. Specifically, let R_k be the number of firms that rise from a rank below k to a rank k or better, and let F_k be the number of firms falling from a rank k or better to a rank below k. In a closed competition, R_k and F_k are identical for all $1 \leq k \leq n-1$, and $D = \sum_{k=1}^{n-1} R_k + \sum_{k=1}^{n-1} F_k$. It can be shown that under random rank shuffling the expected number of firms moving between ranks k and $k+1$ is $E(R_k) = E(F_k) = -\frac{1}{n}k^2 + k$. For example, with four firms the expected number of movements between ranks 2 and 3 is $-1/4(2)^2 + 2 = 1$. These expected values can be used to compare differences in persistence at higher, middle, and lower ranks, and these differences can be compared across industries.

Rank friction over time

From an ordinal perspective, the hypothesis of hypercompetition posits that rank friction is declining over time (D'Aveni, 1994; McNamara, Vaaler, and Devers, 2003; Wiggins and Ruefli, 2005). *Rf* can be used to analyze temporal changes in persistence, and Figure 2 shows average *Rf* values for ROA in the 12 industries from 1955 to 1994. The data show a broad decline in rank friction since the late 1970s, with 11 of the 12 lowest *Rf*

Table 4. *Rf* values in eight nonbusiness competitions

	Time period		Number of ranked competitors		Rank friction
	From	Until	Mean (n)	Total (N)	*Rf*
American League Baseball	1941	1980	9.8	14	0.45
National League Baseball	1949	1988	10.35	12	0.36
Australia's Victorian Football League	1932	1971	11.95	12	0.36
Bundesliga (soccer)	1968	2007	18.05	47	0.29
Ligue 1 (soccer)	1968	2007	19.65	51	0.23
UK Premiere League (soccer)	1968	2007	21.23	52	0.30
Primera Divisiόn (soccer)	1968	2007	18.9	47	0.32
Serie A (soccer)	1968	2007	17.1	53	0.31
Mean			**15.88**	**36.00**	**0.33**
Stand dev			**4.49**	**19.45**	**0.06**
Min			**9.80**	**12**	**0.23**
Max			**21.23**	**53**	**0.45**

values appearing since 1980. To test whether the decline is statistically significant, *Rf* values can be compared for any time periods using nonparametric methods such as Wilcoxon's rank-sum test or the (equivalent) Mann-Whitney U test (Howell, 2001). For example, the inverted mean rank of *Rf* values pre-1980 was 26.88, and for post-1980 was 9.00; Wilcoxon ($W = 135$) and Mann-Whitney ($U = 15$) tests were significant at $p < 0.0001$, giving general support to the hypothesis of hypercompetition for this time period and set of industries.

These results align with those of Wiggins and Ruefli (2005), who reported an increasing rate of exit from the upper ROA strata of industries after 1980. Wiggins and Ruefli also found a general decline in post-1980 ROA at all performance levels, with 87 percent of ROA variance explained by time alone. *Rf* offers a method of comparing intra-industry profit rates independent of these systemic effects.

Comparing performance measures

Rf can be computed for any performance measure or domain of competition for which competitive performance can be ranked. Using the *Fortune* data, we computed *Rf* values for return on sales (mean = 0.60), return on equity (mean = 0.50), total assets (mean = 0.87), total sales (mean = 0.86) and total profits (mean = 0.71). As expected, measures of firm size produce larger *Rf* values than measures of profitability.

Rf can be used to compare industry persistence with persistence in nonbusiness domains such as professional baseball, football, or rankings of business schools (see Powell, 2003). Table 4 shows average *Rf* values in eight sports competitions over a 40-year period. On average, these domains yielded lower *Rf* values (mean = 0.33) than those found in industry competition (mean = 0.55).

CONCLUSION

Strategic management research is concerned with the persistence of firm performance. Rank friction offers a flexible, nonparametric method for analyzing persistence. Ordinal methods are not a substitute for cardinal analysis, and in most cases researchers will prefer to use all of the available data. However, as a complement to cardinal data, the method described here requires fewer distributional assumptions; enables comparisons of persistence across time periods, ranks, industries, and nonindustrial forms of competition; and gives a more complete picture of persistence than existing ordinal methods. In some cases, rank friction may be preferred to cardinal methods, particularly if cardinal data are unavailable or incomparable over time or industries, or if theoretical considerations such as social comparison imply ordinal motivations for strategic behavior.

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