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THINKING ABOUT U: THEORIZING AND TESTING U- AND INVERTED U-SHAPED RELATIONSHIPS IN STRATEGY RESEARCH

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Research summary: *U- and inverted U-shaped relationships are increasingly explored in strategy research, with 11 percent of all articles published in Strategic Management Journal (SMJ) in 2008–2012 investigating such quadratic relationships. Moreover, a movement towards introducing moderation to quadratic relationships has emerged. By reviewing 110 articles published in SMJ from 1980 to 2012, we identify several critical issues in theorizing and testing of these relationships for which current practice falls short. These include insufficient causal argumentation, incorrect testing, mixing up two different types of moderation, and not realizing that the curve can flip completely. For these and other issues, a guideline is provided which, when followed, may bring clarity to theoretical motivation and rigor to empirical testing.*

Managerial summary: *Too much can be as bad as too little. Many relationships in strategic management follow an inverted U-shaped pattern, where moderate levels of a strategy lead to optimal performance. To gain deeper insights into the conventional wisdom that too much of a good thing can be harmful to performance, we discuss how such relationships can be better theorized and tested based on a review of articles exploring U-shaped relationships in Strategic Management Journal during 1980–2012. We identify several critical issues that require close attention and provide a guideline to further develop and validate this important managerial intuition.*

INTRODUCTION

As the field of strategic management has progressed, developing and testing hypotheses that go beyond simple linear relationships has been high on the agenda of many strategy scholars. Most common in the field are quadratic relationships, such as the inverted U-shaped relationship between diversification and performance (Palich, Cardinal, and Miller, 2000), the inverted U-shaped relationship between product market competition and innovation (Aghion *et al.,* 2005), and the U-shaped relationship between social responsibility and financial performance (Barnett and Salomon, 2006). Moreover, researchers in recent years have started adding moderation to quadratic relationships (e.g., Fernhaber and Patel, 2012; Oriani and Sobrero, 2008). With this added complexity come many new demands on researchers in terms of theoretical development, hypothesizing, and empirical testing.

The present study provides the first review of current practice in strategy research with regard to the full procedure from theorizing to testing (moderated) U-shaped relationships.[1] By reviewing a set of 110 relevant articles published in *Strategic*

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[1] Unless stated otherwise, we use the term "U-shaped relationships" and "U-shapes" to refer to both U-shaped and inverted U-shaped relationships, and to quadratic relationships in general.

Management Journal (*SMJ*) over a period of 33 years, we identify many issues that warrant attention in future research. Among others, we find serious underdevelopment of U-shaped relationships, where researchers fail to explicate the latent mechanisms underlying such relationships. This lack of theory development subsequently hinders convincing extension to moderation of U-shaped relationships. We also find that few articles use adequate procedures to test for the presence of U-shapes, raising the concern that the reported positive findings may be spurious. Moreover, disconnection between theory, hypotheses, and testing for moderated relationships is widespread, leading to incomplete hypothesis development and misleading result interpretation. At the core of this issue lies the fact that moderation in U-shaped relationships can be separated into two types: the curve can shift left or right and its shape can flatten or steepen. Finally, we explicate the theoretical and empirical nature of a novel phenomenon called "shape-flip," which leads to the relationship changing from an inverted U-shape to a U-shape, or vice versa.

In order to further explore U-shaped relationships in the field of strategic management, we must have more respect for the additional intricacies of such relationships compared to simple linear relationships. This paper contributes to this important research agenda in several ways. First, we provide to our knowledge the first systematic review of theorizing and testing of U-shaped relationships in strategic management. Second, we offer a general framework to develop firmly a U-shaped relationship by combining latent theoretical mechanisms underlying the U-shaped relationship. Third, it is shown that two types of moderation, turning point shift and flattening or steepening of the curve, are conceptually and empirically distinct and should be treated as such. Finally, we delve into the phenomenon of shape-flip, which opens up interesting research opportunities and offers important implications for both theory and practice.

This paper continues as follows. We first describe the process of selecting articles for review. We continue with theorization and testing for basic U-shaped relationships. We proceed by discussing how to theorize and test for the two distinct moderation types. Thereafter, we explore the phenomenon of shape-flip and discuss its implications. We conclude by providing a summary of our findings and recommendations.

ARTICLE SELECTION

We first composed a list of keywords pertaining to U-shaped relationships by reading the general methods literature on curvilinearity (e.g., Aiken and West, 1991; Dalal and Zickar, 2012; Dawson, 2014; Ganzach, 1997; Hirschberg and Lye, 2005). We then conducted a full-text search for keywords such as "inverted U," "U-shape," "curvilinear," and "quadratic" in all articles, including research notes, published in *SMJ* from its establishment in 1980–2012.[2] This led to 490 articles that were then manually checked, resulting in 110 articles that hypothesize *and* test U-shaped relationships, of which 30 additionally test for moderation.

Figure 1 illustrates the investigation of U-shaped relationships in *SMJ* over time, showing a clear upward trend, with 13 percent of all articles in 2012 theorizing and testing for U-shaped relationships, of which 40 percent also contain moderation hypotheses. In total, the 110 articles give rise to 163 main U-shape hypotheses and 50 relevant moderation hypotheses.

THEORIZING AND TESTING U

The theory behind U

A U-shaped relationship exists if the dependent variable Y first decreases with the independent variable X at a decreasing rate to reach a minimum, after which Y increases at an increasing rate as X continues to rise. An inverted U-shaped relationship exists if Y first increases with X at a decreasing rate to reach a maximum, after which Y decreases at an increasing rate. Throughout this paper, the point at which the curve attains its maximum or minimum is termed "turning point." Since 114 out of the 163 main hypotheses (70%) across *SMJ* articles propose an inverted U-shaped relationship, the majority of our discussion will focus on the inverted U-shape. All arguments have a straightforward analogue to the U-shape.

Typically, an inverted U-shaped relationship may be conceptualized as two latent functions jointly making up the inverted U-shape. While the latent

[2] The full search contained the following keywords: inverted U, u-shape, u-shaped, u-curved, u-curve, curvilinear, nonmonotonic, non-monotonic, quadratic, non-linear, nonlinear, parabola, and parabolic.

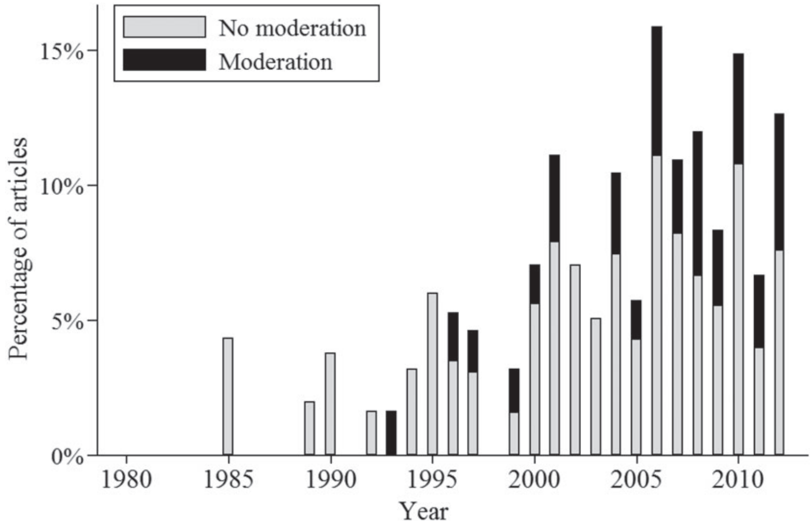
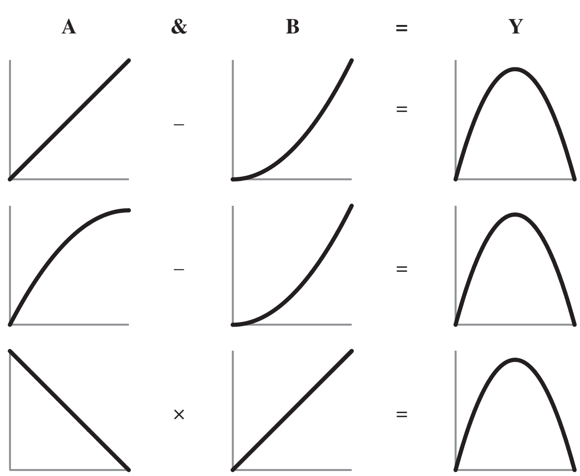


Figure 1. Relative trends of U-shaped relationships in *SMJ*, 1980–2012. Numbers are with respect to the total number of *SMJ* articles (including research notes) published in each year

functions are commonly not observable, they may be combined either additively or multiplicatively to explain an inverted U-shaped relationship that reveals the "net effect" of X on Y. Table 1 provides an illustration of different combinations to construct an inverted U-shaped relationship.

Table 1. Additive and multiplicative combinations of latent mechanisms resulting in an inverted U-shaped relationship



The general framework laid out in Table 1 may aid in hypothesis development by explicitly and jointly considering two countervailing forces. Most common are additive benefit/cost arguments, with some form of performance as the dependent variable. Researchers build the argument that benefits to the independent variable are linearly increasing, whereas costs tend to escalate rapidly with the independent variable, resulting in a convex or exponential cost curve. Subtracting these costs from the benefits gives rise to an inverted U-shaped relationship between the independent variable and the performance outcome. For example, Jones (2003) argues that while there are many benefits of high product developments rates, with rising product development rates also come escalating diseconomies such as cannibalization, scope limitations, and increased complexity. After a certain point, these costs start to dominate the linearly increasing benefits of product development rates. An inverted U-shaped relationship between product development rates and firm performance is therefore predicted. This theoretical development fits well with the first combination in the general framework laid out in Table 1.

Another way to theorize an inverted U-shaped relationship is by combining the legitimation and competition effects of density on a firm's entry or exit in a market or region. Conceptually, this is similar to the benefit/cost arguments above, with the alteration that the benefit line (now, the legitimation effect) takes on a concave or logarithmic shape. The net effect of the legitimation and competition effects is then an inverted U-shaped relationship between density and firms' entry. Chang and Park (2005), for example, argue that agglomeration in a specific region not only offers benefits such as legitimacy and knowledge spillover, but also intensifies competition, drives up business costs, and leads to groupthink. While the positive force increases at a decreasing rate and eventually levels off, the negative force rises quickly with agglomeration. As a result, the likelihood that a firm enters a specific region will initially increase, but then decrease with regional agglomeration. This type of argument matches closely with the second combination in Table 1.

One may also construct an inverted U-curve by interacting two latent linear functions, one positive and one negative in the independent variable. Though less commonly used in *SMJ*, a typical case is the combined effect of ability or opportunity and motivation on an outcome, often a strategic choice. One example is how Ang (2008) develops an inverted U-shaped relationship between competitive intensity and collaboration by combining a negative linear opportunity function and a positive linear motivation function. Firms facing low levels of competitive intensity tend to hold distinctive resources that bring them more opportunities to collaborate as these resources make them more attractive to potential business partners. However, these firms also have little motivation to collaborate to reduce competitive intensity, as it is already low for them. Conversely, firms facing high levels of competitive intensity have high motivation to collaborate in order to reduce competitive intensity but lack distinctive resources to attract partners. Interacting the two functions, an inverted U emerges such that collaboration is expected to be highest at moderate levels of competitive intensity. This general argument is illustrated by the third combination in Table 1.

We find that 59 out of the 163 main hypotheses (36%) have insufficient theoretical underpinnings, where the latent mechanisms are not touched upon in the construction of the U-shape. Typically, vague and general arguments such as "anything carried to the extreme can be harmful" or "one gets stuck in the middle" are used, thus failing to provide strong theoretical grounds for the quadratic relationship. Furthermore, the type of combination (additive or multiplicative) is seldom made explicit in hypothesis development. In addition, we find that 16 articles (15%) theorize only one half of the curve, yet proceed to hypothesize and test for the whole curve. This is typically done by citing the diminishing returns argument, which effectively argues for the left half of an inverted U-shape. However, diminishing returns do not dictate that firms should overshoot a certain strategy such that their performance would decline.

Several lessons can be drawn to push more on the theory when hypothesizing a quadratic relationship. First and foremost, we must "burrow deeply into microprocesses" to articulate the latent mechanisms (Sutton and Staw, 1995: 378). Simply stating that "too much of a good thing can be harmful" or that one can get "stuck in the middle" fails to capture the causal drivers of the observed relationship. Such arguments tell only *what* is expected rather than *why* this relationship comes into existence, yet the question of why—a chain of arguments that explain why we should expect a certain pattern in our data—is at the core of theory development (Sutton and Staw, 1995; Whetten, 1989). The graphs in Table 1 may be built upon to consider two or more countervailing forces in the argumentation and justification of why a certain force should dominate at different levels of the independent variable. Moreover, this lays a solid foundation for further theorization such as adding moderation to the curvilinear relationship, which we discuss further into this paper. Although a simple graph for theoretical development can be especially illuminating to the reader, we find that only three out of the 110 articles (3%) illustrate their theoretical foundations with graphs during hypothesis development (Chang and Park, 2005: 598; Folta and O'Brien, 2004: 125; Henderson, Miller, and Hambrick, 2006: 450).

Next, two types of thought experiment may be useful to bring further clarity and transparency to theoretical development (Weick, 1989). While the first type of thought experiment considers a cross section of firms with a wide range of X-values and argues why firms with moderate levels of X outperform or underperform those with low or high levels of X, the second illustrates how Y is expected to change if a given firm gradually pushes a strategy characterized by a low X to a strategy high on X. These may be termed between-theorization and within-theorization, respectively. If the theory is solid and based on the same latent drivers or mechanisms, the two thought experiments should lead to the same prediction, because they ultimately invoke the same counterfactual in theoretical reasoning: what value of Y would we observe should X take on a different value? Nevertheless, it is

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desirable for researchers to articulate which type of thought experiment they are doing and to maintain consistency across different hypotheses in the same article. Among the 163 main hypotheses in our sample, 31 (19%) are based on between-theorization, a majority of 110 (67%) resort to within-theorization, and 21 (13%) provide ambiguous theory development. One article (Deephouse, 1999) stands out by providing both clear between-theorization and clear within-theorization for an inverted U-shaped relationship between strategic similarity and firm performance. Among 39 articles that have multiple main hypotheses, six (15%) switch between the two types of thought experiment, which may cause confusion.

Then, it is crucial to motivate the *whole* U-shaped curve in the theoretical treatment. In particular, when the dependent variable is some form of firm performance, researchers face this difficult question: Why would a significant fraction of firms, not just a few outliers, undershoot or overshoot the optimal level of a strategy such that their performance is far below the theoretical optimum? This critical question is seldom addressed in the literature, though a few exceptions exist (e.g., Jiang, Tan, and Thursby, 2011; Laursen and Salter, 2006). A simple answer may be that some firms are foolish, irrational or even emotional (March, 2006), showing symptoms like escalating commitment (Ross and Staw, 1986) and threat rigidity (Staw, Sandelands, and Dutton, 1981). Another possible explanation of non-maximizing behavior is bounded rationality: firms may intend to maximize, but are unable to respond optimally because they have limited cognitive capabilities (Simon, 1959), because their preferences or goals are ambiguous and inconsistent (March, 1981), or because they cope with an uncertain environment that is further complicated by the simultaneously adapting behavior of other firms (March, 2006). Relating to our proposed way of theorizing based on latent mechanisms, this implies that a considerable proportion of firms in the population may be unable to locate their exact benefit and cost curves, thus missing the optimum in their strategic choice.

However, an inverted U-shaped relationship need not imply that (some) firms must be irrational or limitedly rational. Rather, firms may be perfectly rational, each operating around a local optimum where various benefits and costs associated with a strategic choice are balanced, and a concatenation of these apices forms an inverted U-shape.[3] Various sources of firm heterogeneity—such as an organization's specific resource and institutional space and the imprint effect of founding conditions (see Carroll, 1993, for a review)—set the firm on a particular optimization curve where it adapts incrementally. Each individual apex may be viewed as comprising a set of tightly linked and mutually reinforcing routines, which are difficult to reconfigure once they are developed and have become engrained in the organization of the firm (Gavetti and Levinthal, 2000). Even if the organizational recipes for success become known, switching to a different optimization trajectory with a presumably higher apex may be prohibitively expensive or risky because the firm is constrained by early strategic decisions and subsequent historical development (Teece, Pisano, and Shuen, 1997). Such path-dependent and incremental adaptation, instead of spontaneous optimization, can determine the set of activities that the firm carries out and can lead firms into equilibria that may be far from the global maximum (Levitt and March, 1988; March, 2006).

The test in search of U

To provide evidence for a U-shaped relationship, researchers commonly regress the dependent variable Y on the independent variable X and its square[4]:

$$Y = \beta_0 + \beta_1 X + \beta_2 X^2 \tag{1}$$

Including the first-order X in the regression equation is essential (Aiken and West, 1991: 63), as leaving it out is tantamount to assuming that the turning point is at X = 0 — a very strong assumption to make, *a priori*. A significant and negative β_2 indicates an inverted U-shaped relationship and a significant and positive β_2 a U-shaped relationship.

Though necessary, a significant β_2 coefficient alone is not sufficient to establish a quadratic relationship. For this, Lind and Mehlum (2010) propose a three-step procedure. First, *β_2 needs to be significant and of the expected sign*, as discussed above. Second, *the slope must be sufficiently steep at both*

[3] We thank an anonymous reviewer for directing us to this interesting insight.

[4] For expositional simplicity, we do not include control variables, as they affect neither theorization nor the mathematical properties of the estimated X–Y relationship such as calculation of the turning point.

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ends of the data range. Suppose X_L is at the low end of the X-range, and X_H at the high end. A formal test for an inverted U-shaped relationship is to show that the slope at X_L, which is $\beta_1 + 2\beta_2 X_L$, is positive and significant, *and* the slope at X_H, which is $\beta_1 + 2\beta_2 X_H$, is negative and significant. It is critical that *both* slope tests are significant: if only one is significant, the true relationship might be merely one half of a U-shape that can be more parsimoniously fitted by Y being a logarithmic or exponential function of X. Third, *the turning point needs to be located well within the data range*. Taking the first derivative of Equation 1 and setting it to zero yields the turning point at $-\beta_1/2\beta_2$.[5] This condition can be tested by estimating the 95 percent confidence interval of the turning point: if this confidence interval is within the data range, one can be reasonably sure that there exists a U-shaped curve. If its lower or upper bound is outside the X-range, then maybe only one half of the curve is revealed by the data.

Among the 110 *SMJ* articles in our sample, Fernhaber and Patel (2012) is the only study that formally tests for a quadratic relationship by following the above three steps. We recommend following all three steps of Lind and Mehlum (2010) and, following Hirschberg and Lye (2005), using the Fieller method (Fieller, 1954) instead of the usual Delta method (Rao, 1973) in constructing the confidence interval of the turning point in Step 3 to account for finite sample bias and to correct for biases caused by departure from normality.

We find that the turning point is a pervasive source of error. Despite its economic and statistical importance, the turning point is not reported for 104 out of the 163 main hypotheses (64%). Based on reported coefficients and descriptive statistics, we additionally find that for 40 relationships (25%), the turning point is dangerously close to the limits of the data range, and in a few cases even beyond. For these 40 relationships, it is likely that the seemingly quadratic relationship is an artifact of functional misspecification as illustrated in a simulation study by Shaver (2007: 279–283). Note that, as we could not formally assess this issue for some articles, this is a conservative estimate.[6]

Besides the Lind and Mehlum procedure, more can be done to improve empirical rigor and result interpretation. Among the 110 *SMJ* articles, 46 (42%) graph the X–Y relationship over the relevant range of X, while holding other variables at the sample means, medians, or other meaningful values. It is advisable to include such graphs as a routine to demonstrate that the curve takes the expected shape and that the turning point lies well within the data range. In fact, we find that articles that provide a graphical illustration of their results are less likely to have a turning point close to, or outside, the data range: 24 percent of articles that use graphical illustrations have such turning points, compared to 34 percent of articles that do not. It thus appears that plotting of results guides researchers in assessing whether they indeed have a U-shaped relationship and whether additional considerations in result interpretation are required.

Furthermore, several robustness checks can be conducted to confirm that the observed relationship is indeed quadratic. For example, adding a cubic term (X^3) to Equation 1 tests whether the relationship is perhaps S-shaped rather than U-shaped. Seven articles (6%) report that they performed this robustness check, finding that the cubic term did not improve model fit, and thus provide stronger support for a quadratic relationship. Another robustness check is to split the data based on the empirically determined turning point, and to check whether two linear regressions give slopes that are consistent with the predicted shape of the curve. For example, for an inverted U, the regression on the subsample with X-values below the turning point should indicate a positive relationship between X and Y, while the regression on the subsample above the turning point should indicate a negative relationship. One article (Qian *et al.,* 2010: 1027) conducts this type of robustness check, though they split the data based

[5] With formal proofs and numerical methods, we find that the expression of the turning point is exactly the same as the linear case for various models, including binary logit or probit, tobit, ordered logit or probit, Poisson, negative binomial, and various hazard models. Proof for binary logit is shown in Appendix 1, and other proofs are available upon request. However, for models that fit multiple discrete choices with more than one equation (e.g., the multinomial models and the generalized ordered logit model), obtaining an analytical solution for the turning point is algebraically involved, and we advise using numerical methods to locate the turning point.

[6] In case of standardized estimates, it is often unclear whether the Friedrich (1982) approach (standardize "component" variables only) or a blanket approach (standardize all variables including higher order terms) is followed, preventing calculation of the turning point. Similarly, the range of X (minimum and maximum) is often not reported, causing problems because for a highly skewed distribution of X, a turning point beyond two or three standard deviations from the mean may be still within the relevant X-range. We only flag cases when the calculated turning point is beyond mean $\pm$ five standard deviations.

on the median rather than the turning point. Furthermore, to more directly guard against the possibility that a few extreme observations are driving the results, researchers may either exclude various outliers from the sample or censor the data by winsorizing, and then re-estimate the model to see whether the main findings remain. Ten articles (9%) include this type of robustness check, such as Barnett and Salomon (2006: 1110), McCann and Vroom (2010: 294), and Souder, Simsek, and Johnson (2012: 36).

Other, more flexible robustness checks include categorical dummies that indicate different segments of the X-values and smoothing splines that model nonlinearity semi- or nonparametrically (Ai and Chen, 2003; Imbs and Wacziarg, 2003). Such methods allow researchers to explore the shape of the relationship by imposing as little structure on the functional form as possible, before proceeding to test their hypotheses in a confirmatory manner with a restrictive quadratic specification. One excellent example outside strategy research is from Aghion *et al.* (2005), who first establish the existence of an approximately inverted U-shaped relationship between competition (X) and innovation (Y) using nonparametric splines, and then continue to confirm the inverted U-shape with the usual regression analysis. Within our sample, we find only two articles that perform similar checks, yet after testing hypotheses with a quadratic specification (Laursen and Salter, 2006: 142–143; Lenox, Rockart, and Lewin, 2010: 135). Therefore, strategy research has much to gain by implementing these flexible methods to reveal first the presence of a U-shaped relationship in the data, before fitting the data with a convenient quadratic function.

Passing the Lind and Mehlum procedure and various robustness checks, however, is not sufficient to establish that X has a *causal* quadratic effect on Y. Our reading of *SMJ* articles suggests that more serious effort is needed to strengthen empirical identification, especially in terms of dealing with endogeneity. In particular, when Y is some form of firm performance and X is a strategic choice, the empirical analysis may be plagued by endogeneity (Shaver, 1998). Suppose we hypothesize an inverted U-shaped relationship between X and Y. However, if Y also acts to reduce X and this reverse causation strengthens as X increases, then the downward sloping part of the inverted U may be spurious and we may incorrectly conclude that there is an inverted U while the true relationship is linear positive (committing Type I error). It does not take much imagination to devise a scenario where a true inverted U is not detected because endogeneity is not corrected for (committing Type II error). We find that in 66 articles (60%), at least one of the relevant independent variables is potentially endogenous (i.e. some strategic choice by the firm), but only four of them use instrumental variables to correct for endogeneity.

While dealing with endogeneity for a quadratic relationship does not require completely new solutions, researchers using quadratic specifications are at risk of running a particular form of “forbidden regression” (Angrist and Pischke, 2009; Wooldridge, 2002), whereby in the second stage of instrumental variable (IV) estimation the quadratic term of an endogenous explanatory variable (X^2) is replaced by the quadratic term of the fitted value (the square of X-predicted). This leads to inconsistent estimation because the linear projection of the square is *not* the square of the linear projection (Wooldridge, 2002: 236). The correct procedure is to instrument X and X^2 separately in the first stage, using additional instruments derived from the original instrumental variables. For example, if M is a good instrument for X, then one may use both M and M^2 to instrument X and X^2 (see Angrist and Pischke, 2009: 190–192, for an intuitive explanation). We check the four articles that use IV estimation and find that all of them might have run forbidden regression, such that their X and X^2 are not instrumented separately.

Like reverse causality, unobserved heterogeneity or omitted variable bias may lead to Type I and Type II errors in testing for a quadratic relationship. For example, if each firm optimizes around a local apex with an inverted U curve, a cross-sectional assembly of these firms may form a linear or even U-shaped relationship due to the presence of some omitted or unobserved variable. Such bias is typically remedied through the use of fixed or random effects estimation that models individual heterogeneity. It is desirable that the thought experiment in theoretical reasoning is in line with the estimation method and the data used in empirical analysis. While fixed effects estimation with panel data is best suited to test hypotheses based on within-theorization, cross-sectional data may be used to test hypotheses based on between-theorization as long as individual heterogeneity is sufficiently accounted for by a number of control variables. Out of 75 articles that have at least one hypothesis based on

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within-theorization, only 21 (28%) use fixed or random effects to control for unobserved heterogeneity. Overall, strategy researchers need to engage more seriously with empirical identification, perhaps by following Aghion *et al.* (2005), who use appropriate instruments to provide exogenous variation in their independent variable and include fixed effects to account for unobserved heterogeneity.

There also appears to be considerable confusion regarding whether X should be mean-centered or standardized to reduce multicollinearity. This is unfortunate and unneeded. Recent developments in the methods literature have unequivocally confirmed that mean-centering is "much ado about nothing" (Kromrey and Foster-Johnson, 1998): the results obtained with centered data and raw data are mathematically equivalent and mean-centering does not increase the power to detect quadratic or interaction effects (Dalal and Zickar, 2012; Echambadi and Hess, 2007). Standardization does very much the same except that all coefficients and standard errors, not just those of X as in the case of mean-centering, will change predictably and systematically (Aiken and West, 1991: 42). Moreover, these transformations complicate the computation of the turning point and may lead to confusion in result interpretation.[7] Indeed, of the 29 articles that do mean-centering or standardization, 86 percent do not report the turning point, whereas of the remaining 81 articles, 53 percent do not report the turning point.

THEORIZING AND TESTING MODERATION

Theorizing moderation of U-shaped relationships

Moderation occurs if a third variable, Z, affects the relationship between X and Y such that it changes for varying values of Z. In our sample, 30 articles (27%) explore such moderation of a U-shaped relationship, resulting in 50 relevant moderation hypotheses.

The latent forces driving the observed relationship again take center stage when one wants to theorize moderation: the moderator affects one or both of the latent mechanisms and, in turn, the observed U-shaped relationship. U-shaped relationships can be moderated by Z in two distinct ways: it can shift the turning point of the curve left or right, and it can flatten or steepen the curve.

A turning point shift occurs when the moderator affects the latent mechanisms in such a way that the turning point of the observed relationship changes location, though the shape of the curve may not change. Consider, for illustration, an additive combination of latent mechanisms: a convex cost curve is subtracted from a linear benefit function, resulting in an inverted U-shape. Suppose that only the linear benefit function is affected by Z:

$$A = a_0 + \left(a_1 + Z\right) X \quad (2)$$

The convex cost curve, unaffected by Z, can be expressed as:

$$B = b_0 + b_1 X + b_2 X^2 \quad (3)$$

where b_2 is positive to make it convex. Subtracting Equation 3 from Equation 2 then results in the observed relationship between X and Y, which is moderated by Z:

$$Y = A - B = \left(a_0 - b_0\right) + \left(a_1 - b_1 + Z\right) X - b_2 X^2 \quad (4)$$

Taking the first derivative with respect to X and setting it to zero, we obtain:

$$X^* = \frac{a_1 - b_1 + Z}{2b_2} \quad (5)$$

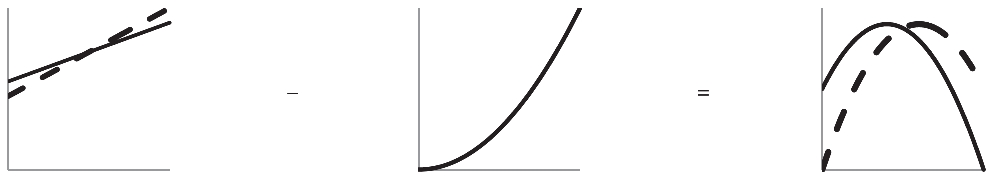
The turning point depends on the moderator: for each value of Z, a unique turning point exists. In Equation 5, X^* increases as Z increases, such that the turning point moves to the right. As there is no interaction between X^2 and Z in Equation 4, the curvature of the observed relationship does not change and thus no flattening or steepening occurs.

A flattening or steepening occurs when the moderator affects the latent mechanisms in such a way that the overall shape of the observed relationship changes, though the turning point of the relationship need not change. Consider again an additive combination of latent mechanisms, with the linear benefit

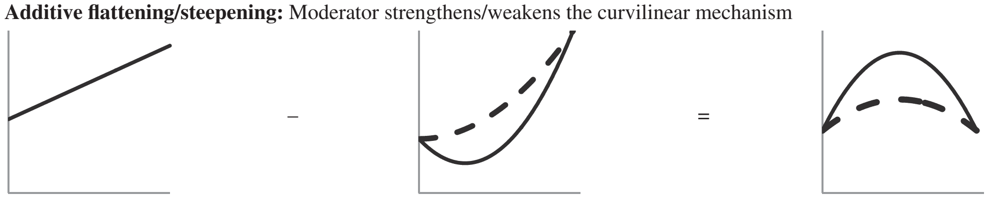
[7] In the case of mean-centering, the true turning point is obtained by adding the mean of X to the turning point calculated using the estimates of the transformed data. The case of standardization is much more complicated (see Rothaermel, Hitt, and Jobe, 2006 for an example).

Table 2. Illustrations of the two moderation types

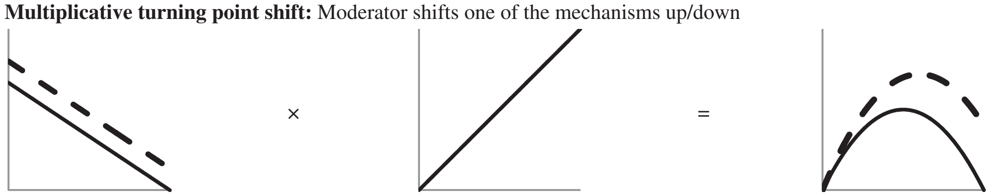
Additive turning point shift: Moderator strengthens/weakens the linear mechanism



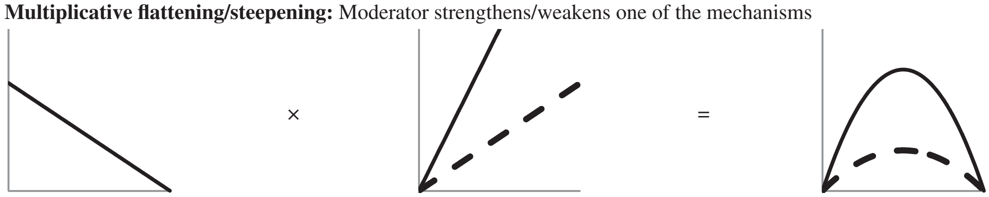
Additive flattening/steepening: Moderator strengthens/weakens the curvilinear mechanism



Multiplicative turning point shift: Moderator shifts one of the mechanisms up/down



Multiplicative flattening/steepening: Moderator strengthens/weakens one of the mechanisms



function now unaffected by the moderator:

$$A = a_0 + a_1 X \tag{6}$$

Suppose that now the curvilinearity of the convex cost curve is attenuated as Z increases:

$$B = b_0 + b_1 X + \left(b_2 - Z\right) X^2 \tag{7}$$

Subtracting Equation 7 from Equation 6 leads to the following observed relationship between X and Y:

$$Y = A - B = \left(a_0 - b_0\right) + \left(a_1 - b_1\right) X - \left(b_2 - Z\right) X^2 \tag{8}$$

Equation 8 contains a positive interaction between X^2 and Z, such that the inverted U-shaped X–Y relationship is flattened as Z increases. Moreover when $a_1 = b_1$, no turning point shift occurs, as the turning point for Equation 8 then does not depend on Z.[8]

Table 2 provides an overview of how each of these two moderation types may occur for both additive and multiplicative combinations of latent mechanisms. As discussed above, for additive combinations a turning point shift can be developed by arguing that the latent linear mechanism is strengthened or weakened by the moderator, while a flattening or steepening can be theorized by arguing that the curvilinearity of the latent mechanism is strengthened or weakened by the moderator. These are illustrated in the first two rows of Table 2. For multiplicative combinations of two latent linear

[8] In practice, a flattening or steepening often goes hand in hand with a turning point shift. This is because the turning point in the moderated relationship depends on the coefficients on both first-order terms (X and XZ) and second order terms (X^2 and X^2Z), while the curvature of the relationship is only determined by the coefficients on second-order terms. Theoretically and empirically, however, the two moderation types remain distinct.

functions, a turning point shift can be developed by arguing that the moderator moves the latent linear functions up or down without changing their slopes, while a flattening or steepening can be theorized by arguing that the moderator strengthens or weakens the latent linear functions. These are illustrated in the last two rows of Table 2 and are derived in Appendix 2.

The two moderation types are distinct and as a result should also be developed as such. In theory development, it needs to be explicit how the moderator, through its effects on the latent forces, manifests itself in the observed relationship. Folta and O'Brien (2004) clearly develop a turning point shift for the U-shaped relationship between uncertainty (X) and the probability of market entry (Y). The option value to defer is linearly increasing with uncertainty (equivalent to a cost line), whereas the option value to grow is a convex function of uncertainty (equivalent to a benefit curve). Combining the two option values leads to a U-shaped relationship. Then, the authors argue that irreversibility of the investment required for market entry (Z) steepens the cost line, making the option to defer increasingly more valuable for higher levels of uncertainty. As illustrated earlier, this can only lead to a shift in the turning point. Other examples of clear theoretical development for shifts in the turning point include Henderson *et al.* (2006), Oriani and Sobrero (2008), and Zhou and Wu (2010).

Ang (2008) is clear in developing a moderation hypothesis about steepening using a multiplicative combination of latent mechanisms. To start, a negative linear opportunity function interacts with a positive linear motivation function to create an inverted U-shaped relationship between competitive intensity (X) and collaboration (Y). Ang (2008) then argues that technological intensity, an industry-level contextual variable, moderates this relationship by steepening the opportunity line. As this steeper line interacts with the unaffected positive motivation line, the curvature of the inverted U-shaped relationship is accentuated, such that a steepening occurs. Other examples of good development of flattening or steepening include Li, Zhou, and Zajac (2009), Zhang and Rajagopalan (2010), and Mihalache *et al.* (2012).

Several observations arise from the 30 articles that theorize moderation of U-shaped relationships. Most seriously, we find that 12 articles (40%) do not theorize clearly how as well as why the moderator affects the latent forces, making readers wonder which of the two moderation types could be expected and even why moderation would occur in general. Typically, this is a direct result of lacking theoretical development for the basic U-shape. The challenges in developing moderation hypotheses are therefore compounded by a failure to explicate the latent mechanisms underlying U-shaped relationships. Although theoretical development can be greatly facilitated by a graphical representation of the moderator's effect on the latent mechanisms, only two articles do so in their theorization (Folta and O'Brien, 2004; Henderson *et al.,* 2006).

Next, there is an obvious disconnect between what is theorized and what is hypothesized, where the hypothesis wording fails to capture accurately the prediction that flows from theory. Of the 18 articles that are clear in their theory, 11 (61%) do not formulate a hypothesis that correctly captures the relevant moderation type. Adding the 12 articles lacking theory, this means that 23 articles (77%) contain a faulty link between theory and moderation hypotheses. This widespread disconnect between theory and hypotheses indicates that researchers may be unaware that two distinct moderation types exist. In fact, for 26 of the 50 moderation hypotheses (52%), it is unclear based on the hypothesis alone which type of moderation is expected to occur, with vague predictions such as Z "positively moderates" the U-shaped relationship between X and Y. These hypotheses are double barreled: do they indicate that the turning point will shift to the right or that the curve will become steeper? Both? Such hypotheses, rather than serving as "crucial bridges between theory and data" (Sutton and Staw, 1995: 376), are not a succinct summary of theory that could be put to the test, but introduce obscurity and confusion. Building on clear theoretical development of the two moderation types, hypotheses should be phrased separately for each.[9] Without building these "bridges" correctly, accurate testing becomes virtually impossible.

[9] Henderson *et al.* (2006) provide a good example of a clear shift hypothesis, as they posit that "peak firm-level performance will occur earlier in CEOs' tenures in a dynamic industry than in a stable one" (p. 451). Mihalache *et al.* (2012) hypothesize clearly a steepening by predicting that the inverted U-shaped relationship between offshoring and firm innovativeness "will be steeper in firms with high TMT informational diversity" (p. 1484).

Testing for moderation in U-shaped relationships

The following specification allows for isolated tests of each of the two moderation types:

$$Y = \beta_0 + \beta_1 X + \beta_2 X^2 + \beta_3 XZ + \beta_4 X^2 Z + \beta_5 Z \tag{9}$$

Compared to Equation 1, interactions between the moderator, Z, and X and its square are now introduced, as well as Z's main term (Aiken and West, 1991: 69). Just as the two moderation types are theoretically distinct, they can and should be tested separately. To illustrate how the moderator affects the turning point of the U-shaped relationship, we derive the turning point X^* of Equation 9 by setting the first derivative with respect to X to zero (see Appendix 3). Derivation of this turning point is the same across most regression models (see Footnote 5).

$$X^* = \frac{-\beta_1 - \beta_3 Z}{2\beta_2 + 2\beta_4 Z} \tag{10}$$

The turning point now depends on the moderator. To show how the turning point changes as Z changes, we take the derivative of this equation with respect to Z:

$$\frac{\delta X^*}{\delta Z} = \frac{\beta_1 \beta_4 - \beta_2 \beta_3}{2\left(\beta_2 + \beta_4 Z\right)^2} \tag{11}$$

As the denominator is strictly greater than zero, the direction of shift depends on the sign of the numerator: if $\beta_1\beta_4 - \beta_2\beta_3$ is positive, the turning point will move to the right as Z increases. If $\beta_1\beta_4 - \beta_2\beta_3$ is negative, the turning point will move to the left. It is critical to note that the direction of shift depends not only on β_3, but also on β_1, β_2, and β_4.

In order to test formally whether a shift in the turning point occurs, one has to assess whether Equation 11 as a whole is significantly different from zero. Given that this equation depends on the coefficients β_1, β_2, β_3 and β_4 as well as Z, testing for a shift in the turning point is *not* as simple as testing whether β_3 is significant, which is neither a necessary nor a sufficient condition for a turning point shift to occur. Moreover, as this equation depends on Z, one has to assign specific, meaningful Z-values to perform the test. This formal test is important because the direction of shift may look clear (e.g., $\beta_1\beta_4$ is much larger than $\beta_3\beta_2$) but the effect (the whole term in Equation 11) can be miniscule.[10]

Testing for flattening or steepening is straightforward when one uses the specification as shown in Equation 9. With such a specification, flattening or steepening does not depend on any other coefficient than β_4, nor on specific values of Z (see Appendix 4). Therefore, testing for flattening or steepening is equivalent to testing whether β_4 is significant. A flattening occurs for inverted U-shaped relationships when β_4 is positive and for U-shaped relationships when β_4 is negative. Conversely, a steepening occurs for inverted U-shaped relationships when β_4 is negative and for U-shaped relationships when β_4 is positive.

For alternative, nonlinear specifications such as Poisson or logit models, testing for flattening or steepening is less straightforward: in these models, a significant β_4 is neither necessary nor sufficient for flattening or steepening. This is due to the general difficulties when algebraically deriving interaction effects in such models (Hoetker, 2007) and the fact that flattening or steepening is at least partly an artifact of the chosen regression model (Greene, 2010).[11] For these nonlinear models, we propose that researchers numerically replicate the steps in Appendix 4 to establish whether a significant flattening or steepening has occurred. First, after model estimation, choose two or more meaningful values of the moderator and compute the corresponding turning points using Equation 10. Second, calculate the slopes at a given distance (denoted by "*a*" in our proofs) from each of the turning points. Repeat this step for different *a* distances. Then in the final step, preferably aided by graphical representation, draw the conclusion by checking whether and to what extent the series of slopes are becoming flatter or steeper as Z changes. This procedure is partially implemented in Kotha, Zheng, and George (2011), although they do not adjust for the fact that a unique turning point exists for every value of the moderator.

Several observations arise from our assessment of the empirical literature on moderated U-shapes.

[10] In *STATA*, the *nlcom* command can be used to test whether Equation 11 is indeed significantly different from zero at specific values of Z.

[11] To paraphrase Hoetker (2007: 338), for such "nonlinear" models, β_4 may reveal flattening or steepening when it does not exist, conceal flattening or steepening when it does exist, and even indicate flattening or steepening in the reverse direction of the actual situation.

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First and foremost, few articles recognize the fact that the two moderation types should be tested separately. For example, in one article only a turning point shift can occur based on its theory, yet a flattening is hypothesized and tested. In another article, both a turning point shift and steepening are evident in theory development, yet only the former is hypothesized. The article then precludes a test for steepening by excluding the X^2Z term in the regression analysis, leaving a fully developed moderation effect unexamined.

Second, we recommend using the full specification as laid out in Equation 9, even if only one moderation type is hypothesized. Doing so not only allows for formal elimination of the type that is not expected to occur, but also prevents potential bias in the estimated coefficients due to exclusion of either interaction term (Aiken and West, 1991: 93). Only one article discusses this issue (Oriani and Sobrero, 2008: 350), motivating the choice to exclude the interaction between X^2 and Z by stating that the coefficient for this interaction is not statistically different from zero in additional analyses. As they only hypothesize a shift in the turning point, their exclusion of the X^2Z term is appropriate. Out of 50 moderation hypotheses, 31 (62%) are tested using the full specification as in Equation 9, and five (10%) are tested by splitting the data based on the moderator and comparing results across subsamples—a valid alternative approach. For the remaining 14 hypotheses, 12 are concerned with a flattening, steepening, or are unclear which type of moderation is expected, yet they are tested by excluding the X^2Z term, thus implicitly making the choice to test only for a turning point shift.

Third, magnitudes of moderation effects must be assessed. For eight moderation hypotheses (16%), no effect sizes or magnitudes are discussed in any manner, be it graphically or verbally. Although graphical representation goes a long way in illustrating the effect of both moderation types, this never constitutes a formal test. More specifically, no article formally tests a shift in the turning point using our proposed method based on Equation 11. Often, only the sign and significance of the interaction between X and Z (i.e., β_3) is assessed to conclude whether a significant shift in the turning point is observed, which is neither necessary nor sufficient. Similarly, the β_4 coefficient in and of itself, while indicating whether a flattening or steepening occurs, says little about the magnitude of moderation, especially when nonlinear models are estimated.

Finally, we find six hypotheses across three articles (10%) that expect an asymmetric flattening or steepening of the curve, such that the two halves of the curve change steepness at different rates. While this is possible, as Z may in theory only affect the curvature of Y for some values of X, such hypotheses cannot be tested with the usual quadratic functional form as curves defined by specifications of Equation 9 *always* remain symmetric around the turning point. If an asymmetric flattening or steepening is hypothesized, then this needs to be tested through, for example, piecewise or segmented regression with moderation (Hansen, 2000). Three of these six hypotheses are tested using the specification in Equation 9, however, and the other three hypotheses are even tested without the X^2Z term, which means not even a symmetric flattening or steepening could have been tested.

SHAPE-FLIPPING CURVES: A RESEARCH OPPORTUNITY

When a significant flattening or steepening is found, the possibility that the curve changes shape to such an extent that it flips from an inverted U-shape to a U-shape or vice versa becomes a reality. Consider for illustration an inverted U-shaped relationship that is moderated by Z such that a flattening occurs as Z increases (β_4 is positive). As Z flattens the curve further, it will eventually turn the inverted U-shaped relationship into a U-shape. Figure 2 graphically shows this: the curve becomes a straight line when Z equals seven and turns into a U-shape when Z increases further. We term this phenomenon "shape-flip," as the shape of the curve flips from an inverted U-shape to a U-shape, or vice versa.

To determine the exact value of Z at which shape-flip occurs, consider again the equation with which the turning point of Equation 9 could be calculated (Equation 10). When the denominator of this equation approaches zero, the location of the turning point approaches infinity. By setting the denominator of Equation 10 to zero, the precise value of Z at which the shape-flip occurs is found:

$$Z^* = \frac{-\beta_2}{\beta_4} \tag{12}$$

At this value, the X–Y relationship is linear and, consequently, no turning point exists. Below and above the shape-flip Z-value, the curve takes on opposing shapes.

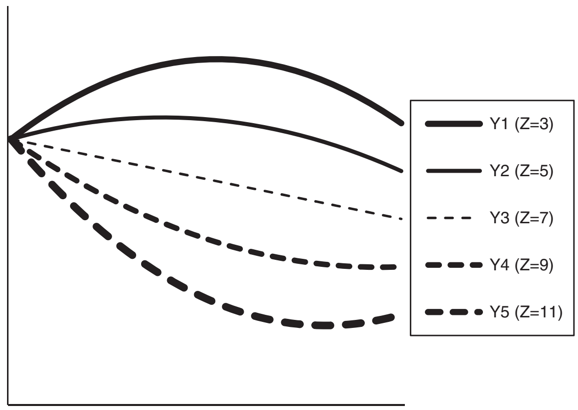

Figure 2. An illustration of shape-flip. $Y = 400 + 14.8X - 0.21X^2 - 2.4XZ + 0.03X^2Z$. X ranges from zero to 60. Coefficients are chosen for illustrative purposes

Though not theorized and interpreted as such during our sample period, shape-flip does occur in strategy research. For example, Uotila *et al.* (2009) find clear evidence of shape-flip for the relationship between relative exploration and firm performance that is moderated by industry technological dynamism: relative exploration has a U-shaped effect on firm performance when technological dynamism is low, but an inverted U-shaped effect when technological dynamism is high (p. 227). This shape-flip can be theorized by drawing upon two latent mechanisms. First, coordination costs of balancing exploration and exploitation are likely to be highest when the firm has to split its attention and resources more or less evenly between the two, whereas a focus on either incurs much lower coordination costs. This is because exploration and exploitation require substantially different structures, routines, processes, and cultures (March, 1991; O'Reilly and Tushman, 2008; Stettner and Lavie, 2014) and an integration of the two involves "trade-offs across time and space that are notoriously difficult to make" (March, 2006: 205). Therefore, coordination costs are a concave function of relative exploration. This cost curve is unlikely to vary with external conditions such as technological dynamism because the difficulties in balancing exploration and exploitation are fundamentally internal to the firm (March, 1991). Then, it follows that the shape-flip is caused by the moderator affecting the other latent mechanism: benefits to relative exploration.

In a stable environment such as one in which technological dynamism is low, firms do not face any significant risk of technological obsolescence and therefore should focus more on exploitation by "increasing efficiency and improving adaptation to current environments" (Uotila *et al.,* 2009: 222). In fact, returns from exploration have "greater variability, longer lead times, and lower average expectations" than returns from exploitation (March, 2006: 206), such that benefits decrease with relative exploration in a stable environment. As seen in Figure 3, the combination of decreasing benefits and concave costs of relative exploration in environments with low technological dynamism results in a U-shaped relationship between relative exploration and firm performance.

As technological dynamism increases, *sufficient* exploration becomes more important to reduce the risk of technological obsolescence and to reap the benefits from a growing abundance of technological opportunities (Uotila *et al.,* 2009). On the one hand, firms that place too much emphasis on exploitation in technologically dynamic environments "face a greater risk that their core technologies become rapidly obsolete" (Uotila *et al.,* 2009: 223). On the other hand, too much emphasis on exploration can be similarly detrimental: due to the highly competitive and fast evolving nature of dynamic environments (Zahra, 1996), swift exploitation is crucial to reap potential rents from exploratory effort before they are dissipated by aggressive market competition or reset to zero by incoming new technology (Eisenhardt, 1989). As pure exploitation and pure exploration are increasingly punished relative to some mixture of both, the benefit curve is progressively bent around the middle as technological dynamism rises, to such an extent that its curvature overtakes that of the cost line. As a result, this flips the observed relationship between relative exploration and firm performance from a U-shape to an inverted U (as seen in Figure 3).

Not only is the opportunity to theorize shape-flip missed in the literature, but the articles in our sample are also unaware of its repercussions for hypothesis testing. For main effect hypotheses, if one hypothesizes a U-shaped relationship between X and Y, but the shape flips to an inverted U when Z takes on a meaningful value, the main effect hypothesis no longer holds over the whole data range. We find that for 11 out of the 30 articles testing for moderation (37%), shape-flip occurs within the articles' range of Z, and for 8 of them

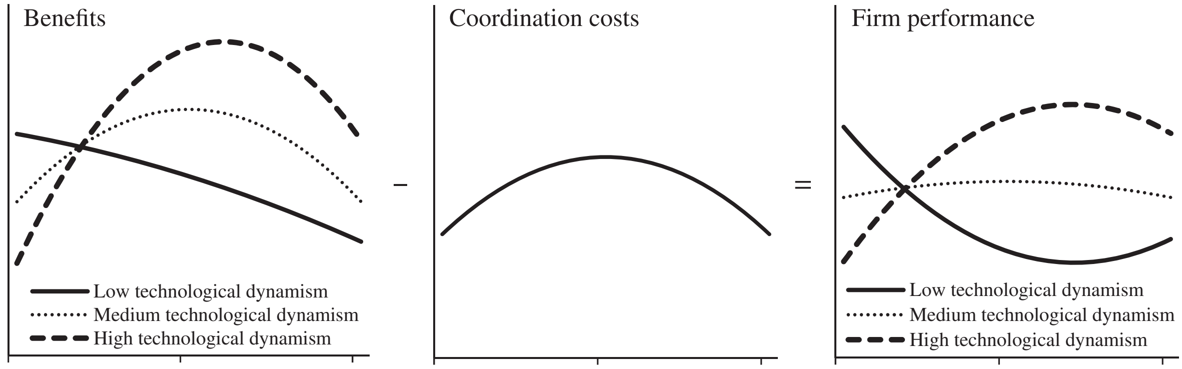

Figure 3. Shape-flip for the relationship between relative exploration (X-axis) and firm performance (Y-axis)

it does so well within the observed range of Z.[12] For these eight articles, shape-flip happens between the 20th and 80th percentile of Z or between Z minus and plus one standard deviation from the mean. All but one article fail to recognize these consequences. Mihalache *et al.* (2012: 1491) use a simple slope analysis to assess whether the main effect hypothesis holds for Z-values between one standard deviation below and above the mean. They find that the simple slopes of Y on X are not statistically significant when Z is at its mean plus one standard deviation, thus failing to support an inverted U-shape across the relevant range of Z. In fact, shape-flip occurs around this Z-value, such that the inverted U becomes linear. The remaining seven articles in which shape-flip occurs well within the range of Z do not discuss the consequences for the main effect hypothesis.

Shape-flip also has implications for testing moderation hypotheses. First, the turning point jumps to a radically different location after the shape-flip. In Figure 2, the turning point jumps to the far right after shape-flip, after which it shifts left again. Second, before and after shape-flip the curve changes its shape in an opposite manner. As shown in Figure 2, the curve first flattens, but steepens after shape-flip. Should we hypothesize a leftward shift of the turning point together with a flattening as Z increases, neither hypothesis would hold over the entire data range. Although some articles in which shape-flip occurs report "mixed results" or argue that "the findings are largely in line" with their moderation hypotheses, such statements do not capture the consequences of shape-flip. Therefore, any article that finds a significant flattening or steepening of the curve, be it hypothesized or not, should assess whether shape-flip occurs well within the range of the moderator. If this is indeed the case, the consequences for all hypotheses need to be addressed.

Nevertheless, shape-flip need not diminish contributions of a study in which it occurs well within the range of the moderator. Rather, this phenomenon provides interesting research opportunities if it is theoretically plausible and substantively interpretable, because the fundamental nature of the relationship between X and Y now depends on the moderator. Many theoretically interesting insights may arise. For example, whereas there perhaps first exists a maximum or optimal level of X, after the shape-flip there exists a minimum. This has far-reaching implications for the strategic value of both X and the moderator. Consider, for illustration, an organization that has been operating around the optimal level of X where its performance (Y) is maximized. If a crucial contingency (Z) then passes through the point of shape-flip, the same level of X may lead to the worst performance. Though no article in our sample expects shape-flip a priori, future work stands to gain from consideration of this phenomenon in theorizing and testing.

SUMMARY

To our knowledge, the present research is the first to review current practice and provide future directions for the entire process of theorizing, hypothesizing, and testing for U-shaped relationships in management research. We accomplish this through a review of 110 relevant articles in *SMJ* from

[12] For 16 articles, there is no risk of shape-flip. For the remaining three, we cannot compute the value at which shape-flip occurs as these articles report standardized estimates without providing the required information to back-transform the coefficients.

Table 3. Checklist for theorizing and testing (inverted) U-shaped relationships

Theory:
- Are the latent causal mechanisms clearly and separately developed?
- Is it clear how and why they jointly form an (inverted) U (i.e., in an additive or multiplicative manner)?
- Which type of thought experiment underlies the reasoning? Do both within- and between-theorization lead to the same prediction?
- Is the full curve developed, not just one half of the curve?

Specification and testing:
- Are both X and X^2 included in the specification?
- Is there a good fit between the theory and this specification?
- Are the following three conditions *all* met?
  - β_2 is significant with the expected sign (negative for inverted U, positive for U).
  - The slope of the curve is sufficiently steep at *both* ends of the X-range.
  - The turning point is located well within the data range. Both the minimum *and* maximum values of X are outside the confidence interval of the turning point.
- Do semi- or nonparametric analyses indicate an (inverted) U-shaped relationship?
- Can the following alternative specifications be ruled out?
  - Logarithmic transformation of X? Exponential transformation of X? Cubic specification?
- Are the main findings robust to excluding outliers or winsorizing X?
- How is empirical identification addressed? Is "forbidden regression" avoided?

Reporting:
- Is the turning point reported?
  - If transformations were applied (e.g., mean-centering or standardization), is the untransformed turning point reported?
- Is the X−Y relationship graphed over the relevant range of X?
- Are full descriptives (mean, standard deviation, maximum, minimum) reported for *all* variables, including the squared term of X?

1980 to 2012, of which 30 articles also introduce moderation.

We find that it is important to make the latent mechanisms that jointly comprise the U-shaped curve more explicit in theory development. We must have solid motivation for a U-shaped relationship and distinguish it clearly from a monotonic relationship (e.g., diminishing marginal returns or diminishing marginal rate of substitution). It is highly recommended to follow the three-step testing procedure of Lind and Mehlum (2010) before concluding that there truly exists a U-shaped curve over the data range. We must actively guard our research against false positive findings that according to Ioannidis (2011: 16) "have reached epidemic proportions" in many fields of social and natural sciences.

This solid theoretical and empirical foundation is crucial when researchers introduce moderation to these relationships, as two distinct types of moderation—a shift in the turning point and a flattening or steepening of the curve—have gone generally unrecognized in the literature. These two types of moderation come into existence as the moderator affects the underlying causal mechanisms in distinct ways. Many, if not almost all, articles fail to realize that these two types of moderation are theoretically and empirically distinct and as such their moderation hypotheses are not appropriately tested. In this paper, we have clarified how these two types of moderation hypotheses may be constructed and separately tested.

Finally, this is perhaps the first article in management to discuss explicitly the properties of the phenomenon that a quadratic relationship changes from an inverted U-shape to a U-shape or vice versa. We call this phenomenon "shape-flip" and explain its consequences for testing U-shaped relationships. Shape-flip has many implications for theory and practice, yet its potential has been overlooked in the recent literature.

A limitation of our study is that we only reviewed one journal. However, there is no reason to suspect that other journals in the field have different or better practice with respect to theorizing and testing U-shaped relationships. Furthermore, the provided recommendations are relevant to any management journal. Our large sample of articles over a long period of time and extensive consultation with the research methods literature further enhanced our ability to have a full overview of important issues and relevant solutions.

Table 4. Checklist for theorizing and testing moderation with (inverted) U

Theory:
- Is it clear how the latent causal mechanisms are affected by the moderator?
- As a result, is it clear which of the two types of moderation can be expected to occur?
- Are hypotheses phrased to be consistent with these expectations, and do they separately address the two types of moderation?

Specification and testing:
- Are both an interaction between the moderator and X and an interaction between the moderator and X^2 included?
- Is there a significant shift in the turning point? Is the direction of shift consistent with expectations?
- Is there a significant flattening or steepening of the curve? Is this consistent with expectations?

Shape-flip:
- Does shape-flip occur well within the range of the moderator?
  - How does this affect the main hypothesis?
  - How does this affect the moderation hypothesis?
  - What are the theoretical and practical implications?

Reporting:
- Are full descriptives (mean, standard deviation, maximum, minimum) reported for *all* variables, including the two interaction terms?
- Are the magnitudes of the two types of moderation reported or graphed?

Tables 3 and 4 provide a checklist that summarizes our recommendations and can serve as a guideline for theorizing and testing U-shaped relationships. Researchers who ask themselves these questions throughout their theorizing, hypothesizing, and testing will be able to introduce clarity to their theory and rigor to their testing. In addition, it is likely that by asking these questions, many interesting new research opportunities will arise in the increasingly popular investigation of U-shaped relationships.

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APPENDIX 1: DERIVING THE TURNING POINT AND THE DIRECTION OF ITS SHIFT FOR LOGIT MODELS

Let $\boldsymbol{\beta X} = \beta_0 + \beta_1 X + \beta_2 X^2 + \beta_3 XZ + \beta_4 X^2 Z + \beta_5 Z$ where **X** refers to all explanatory variables. Note that this is a more general model that introduces moderation, as in Equation 9 compared to Equation 1. Derivation for less general models yields the same conclusion.

For logit models:

$$P\left(Y=1|\boldsymbol{X}\right) = \frac{e^{\boldsymbol{\beta X}}}{1+e^{\boldsymbol{\beta X}}}$$

First order condition:

$$\frac{\delta P}{\delta X} = \frac{e^{\boldsymbol{\beta X}} * \left(\beta_1 + 2\beta_2 X + \beta_3 Z + 2\beta_4 XZ\right)}{\left(1+e^{\boldsymbol{\beta X}}\right)^2} = 0$$

As the denominator and $e^{\boldsymbol{\beta X}}$ are strictly positive, $X^* = \frac{-\beta_1-\beta_3 Z}{2\beta_2+2\beta_4 Z}$, identical to the case of linear models. Differentiating X^* with respect to Z to determine how X^* changes as Z increases, the same conclusions as those in linear models can be obtained (see Appendix 3).

APPENDIX 2: THE DISTINCTIVENESS OF THE TWO MODERATION TYPES FOR MULTIPLICATIVE COMBINATIONS

For turning point shift, suppose that A is a negative linear function of X, where Z shifts the latent function A upwards:

$$A = a_0 - a_1 X + Z \qquad \text{(A2.1)}$$

And that B is a positive linear function of X:

$$B = b_0 + b_1 X \qquad \text{(A2.2)}$$

Interacting A and B then yields the following observed relationship:

$$Y = A * B = a_0 b_0 + b_0 Z + \left(a_0 b_1 - a_1 b_0 + b_1 Z\right) X - a_1 b_1 X^2 \qquad \text{(A2.3)}$$

The turning point of this relationship is:

$$X^* = \frac{a_0 b_1 - a_1 b_0 + b_1 Z}{2 a_1 b_1}$$

which depends on Z. Because b_1 is positive, the turning point shifts rightwards as Z increases in this illustration. As there is no interaction between X^2 and Z in Equation A2.3, no flattening or steepening occurs.

To observe a flattening, suppose that the moderator no longer affects the A-function, but instead weakens the slope of the B-function, such that it becomes:

$$B = b_0 + \left(b_1 - Z\right) X$$

The new observed relationship becomes:

$$Y = A * B = a_0 b_0 + \left(a_0 b_1 - a_1 b_0 - a_0 Z\right) X - \left(a_1 b_1 - a_1 Z\right) X^2 \qquad \text{(A2.4)}$$

Given that a_1 is positive, a flattening occurs as Z increases. The turning point of Equation A2.4 is at:

$$X^* = \frac{a_0}{2a_1} + \frac{b_0}{2Z - 2b_1}$$

If $b_0 = 0$, then no turning point shift occurs as the turning point then does not depend on Z.

APPENDIX 3: DETERMINING THE DIRECTION OF TURNING POINT SHIFT AS Z CHANGES

Given the specification:

$$Y = \beta_0 + \beta_1 X + \beta_2 X^2 + \beta_3 XZ + \beta_4 X^2 Z + \beta_5 Z \tag{A3.1}$$

First order condition:

$$\frac{\delta\, Y}{\delta\, \mathrm{X}} = \beta_1 + 2\beta_2 X + \beta_3 Z + 2\beta_4 XZ = 0$$

Solving for X yields the turning point,

$$X^* = \frac{-\beta_1 - \beta_3 Z}{2\beta_2 + 2\beta_4 Z} \tag{A3.2}$$

Taking the derivative of Equation A3.2 with respect to Z determines how the turning point changes as Z changes:

$$\frac{\delta\, \mathrm{X}^*}{\delta\, \mathrm{Z}} = \frac{-\beta_3\left(2\beta_2 + 2\beta_4 Z\right) - 2\beta_4\left(-\beta_1 - \beta_3 Z\right)}{\left(2\beta_2 + 2\beta_4 Z\right)^2}$$

$$= \frac{\beta_1\beta_4 - \beta_2\beta_3}{2\left(\beta_2 + \beta_4 Z\right)^2} \tag{A3.3}$$

As the denominator is strictly larger than zero, the direction of shift depends only on the sign of the numerator. If it is positive (negative), the turning point X^* will move to the right (left) as Z increases.

APPENDIX 4: DETERMINING WHETHER A FLATTENING OR STEEPENING OCCURS

Given the specification:

$$Y = \beta_0 + \beta_1 X + \beta_2 X^2 + \beta_3 XZ + \beta_4 X^2 Z + \beta_5 Z \tag{A4.1}$$

When $Z = Z_1$, the turning point is at:

$$X_1^* = \frac{-\beta_1 - \beta_3 Z_1}{2\beta_2 + 2\beta_4 Z_1} \tag{A4.2}$$

When $Z = Z_2$, the turning point is at:

$$X_2^* = \frac{-\beta_1 - \beta_3 Z_2}{2\beta_2 + 2\beta_4 Z_2} \tag{A4.3}$$

where $Z_2 > Z_1$. Assume Equation A4.1 is an inverted U-shape and remains so within the relevant range of Z. Then going the same distance a ($a > 0$) to the left of both turning points, and designating S_1 the slope at $X_1^* - a$ and S_2 the slope at $X_2^* - a$:

$$S_1 = \beta_1 + 2\beta_2\left(X_1^* - a\right) + \beta_3 Z_1 + 2\beta_4\left(X_1^* - a\right) Z_1$$

$$S_2 = \beta_1 + 2\beta_2\left(X_2^* - a\right) + \beta_3 Z_2 + 2\beta_4\left(X_2^* - a\right) Z_2$$

If $S_2 > S_1$ the inverted U-shape is steepening, and if $S_2 < S_1$ the inverted U-shape is flattening. By symmetry, the same holds if we move a ($a > 0$) to the right of the turning points. Then:

$$\begin{aligned} S_2 - S_1 &= 2\beta_2\left(X_2^* - X_1^*\right) + \beta_3\left(Z_2 - Z_1\right) \\ &\quad + 2\beta_4\left[\left(X_2^* - a\right) Z_2 - \left(X_1^* - a\right) Z_1\right] \end{aligned} \tag{A4.4}$$

Substituting Equations A4.2 and A4.3 for X_1^* and X_2^*, respectively in Equation A4.4, and collecting terms the following is obtained:

$$S_2 - S_1 = -2\beta_4\left(Z_2 - Z_1\right) a \tag{A4.5}$$

Because $Z_2 > Z_1$ and $a > 0$, the inverted U-shape is steepening if $\beta_4 < 0$ and flattening if $\beta_4 > 0$. For a U-shape it is steepening if $\beta_4 > 0$ and flattening if $\beta_4 < 0$.

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