

## THE RISK-RETURN PARADOX FOR STRATEGIC MANAGEMENT: DISENTANGLING TRUE AND SPURIOUS EFFECTS

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*The concept of risk is central to strategy research and practice. Yet, the expected positive association between risk and return, familiar from financial markets, is elusive. Measuring risk as the variance of a series of accounting-based returns, Bowman obtained the puzzling result of a negative association between risk and mean return. This finding, known as the Bowman paradox, has spawned a remarkable number of publications, and various explanations have been suggested. The present study contributes to this literature by showing that skewness of individual firm' return distributions has a considerable spurious effect on the empirically estimated mean-variance relationship. I devise a method to disentangle true and spurious effects, illustrate it using simulations, and apply it to empirical data. It turns out that the size of the spurious effect is such that, on average, it explains the larger part of the observed negative relationship. My results might thus help to reconcile mean-variance approaches to risk-return analysis with other, ex-ante, approaches. In concluding, I show that the analysis of skewness is linked to all three streams of literature devoted to explaining the Bowman paradox. Copyright © 2008 John Wiley & Sons, Ltd.*

## INTRODUCTION

The concept of risk is central to strategic management. In particular, the relationship between the risk and return of firms is highly relevant both to practitioners and scholars. One important strand of literature measures risk and return as variance and mean, respectively, of a series of returns on equity, assets, or sales. Employing such a mean-variance approach, Bowman (1980) obtained the puzzling result of a negative relationship between risk and return. Since this finding is at odds with the usual

and plausible assumption of risk-averse actors, he termed it the 'risk-return paradox.'

Subsequent work by numerous authors confirmed his result, and various explanations for the risk-return paradox have been proposed. Following the categorization by Andersen, Denrell, and Bettis (2007), these explanations can roughly be categorized into those based on prospect theory<sup>1</sup> (Bowman, 1982; Fiegenbaum and Thomas, 1988, 1990; Fiegenbaum, 1990; Jegers, 1991; Johnson, 1992; Gooding, Goel, and Wiseman, 1996), strategic and organizational factors (Bowman, 1980; Bettis and Hall, 1982; Bettis and Mahajan, 1985; Jemison, 1987; Andersen *et al.*, 2007), and model misspecifications (Rueffli, 1990; Wiseman and Bromiley,

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<sup>1</sup> See Kahneman and Tversky (1979). Also Sinha (1994) discusses the usefulness of prospect theory in the present context, but takes a critical stance.

1991; Oviatt and Bauerschmidt, 1991; Henkel, 2000). For a complete overview, see the reviews by Ruefli, Collins, and Lacugna (1999), Bromiley, Miller, and Rau (2001), and Nickel and Rodriguez (2002).

The present work explores a possible misspecification that so far, apart from a study by Henkel (2000), has received little attention: the effect of distribution skewness on the observed mean-variance relationship (more precisely, on the cross-sectional correlation between these quantities). It is a standard assumption in mean-variance analysis that firms' return distributions are normal (e.g., Bromiley, 1991a; Ruefli and Wiggins, 1994) or at least symmetric. If this assumption is violated in such a way that the return distributions exhibit nonzero skewness, then empirically estimated means and variances are spuriously correlated. In particular if, as suggested by Bowman (1980: 27) and deduced in a model analysis by Andersen *et al.* (2007), some maximum return is feasible in an industry, then most variance is in fact downward-variance from this upper bound. As a result, the distribution of returns is left-skewed, that is, the distribution density is characterized by a long tail to the left with its mean below the median. Now, if a year's return happens to lie far out in the left tail of the distribution, then the estimate of the respective firm's average return is decreased while the estimate of its variance goes up. Hence, a negative relationship between the *empirical estimates* of mean and variance would be observed in a sample of firms even if all had identical, but left-skewed, return distributions. As I will discuss in the concluding section, this (*spurious*) negative mean-variance relationship is entirely different from the (*genuine*) one that Andersen *et al.* (2007) analyze, even though both are linked to distribution skewness.

By taking strategic, long-term decisions, management defines the conditions for the firm's performance in subsequent years. In mathematical language, management implicitly sets the parameters of the firm's return distribution, in particular its mean and variance. It is *these* quantities that risk-return analysis seeks to relate to each other, since risk is inherently an *ex ante* concept (e.g., Ruefli *et al.*, 1999). I refer to this relationship as the *true* mean-variance relationship. However, empirical studies have to rely on (*ex post*) *estimates* of these quantities—and this is what leads to spurious results. I refer to this latter relationship

between the estimates of mean and variance as the *empirical*, or *observed* mean-variance relationship.

Summarizing the work by Henkel (2000), the spurious correlation between the estimates of mean and variance due to skewness is, first, analytically calculated for a sample of firms with *identical* return distributions. Then, I show how the overall estimated mean-variance relationship in a sample of firms with *heterogeneous* individual return distributions is made up of the true, sought-for relationship between (*ex ante*) means and variances of the firms' return distributions, and each firm's individual spurious correlation due to skewness. Note that, even with strongly left-skewed distributions, a negative observed mean-variance relationship need not be entirely spurious; however, it is in any case downward biased.

Next, a method to disentangle spurious and real effects is developed and demonstrated using simulated data. Finally, the theoretical results are applied in an empirical analysis. It turns out that from a total of 27 industries, 20 exhibit a negative estimated mean-variance relationship, which is significant for 12 of them. Of those 12 industries, 11 show a negative average skewness of firms' return distribution. This finding already strongly suggests that, in explaining the risk-return paradox, spurious effects due to skewness matter. Considering their size, I find that they can in fact explain the larger part of the observed negative mean-variance relationship—in several industries, even all of it. These results are even more pronounced when the analysis is restricted to those firms whose average return lies below the respective industry median.

The study proceeds as follows. First, I develop the analysis method and illustrate it using simulated data. Then, I analyze empirical data from the Compustat database. The final section concludes with a summary and a discussion.

## THEORETICAL ANALYSIS

### Spurious mean-variance correlation for individual firms

One of the usual assumptions in mean-variance analysis is that firms' return distributions are stable over time. If they were not, one would encounter an identification problem (Bromiley, 1991a; Ruefli, 1991; Ruefli and Wiggins, 1994). It would not be clear if, for example, a low return in one year was an unlucky draw from the same distribution

that was relevant in the years before, or if it was an average draw from a distribution that altogether had shifted downwards. The assumption of stable return distributions is also made in my analysis. Even though it is likely not fully correct, it can be justified for at least two important types of temporal instability, namely, time trends and serial correlation. Wiseman and Bromiley (1991) corrected for the effect of potential time trends in a firm's returns on the measure of variance, and found a negative risk-return relationship to persist. As to serial correlation between a firm's returns in consecutive years, as long as it affects all firms in the same way it will only reduce variance of returns overall, but will not impact the mean-variance association.

The second simplifying assumption that is usually made is that returns are normally, or at least symmetrically, distributed. This assumption is relaxed in my analysis. In particular, distributions may be skewed.

Following Henkel (2000), let the return of firm  $i$  in period  $t$ ,  $t = 1 \dots T$ , be given by the random variable  $r_{it}$ . The  $r_{it}$  are assumed independent for all  $i$  and  $t$ . For a particular firm  $i$  and all time periods  $t$ , the  $r_{it}$  are modeled to be identically distributed with expected value  $\mu_i$ , variance  $\sigma_i^2$ , and third and fourth central moment  $\alpha_i^3$  and  $\kappa_i^4$ , respectively. The goal of my analysis is to determine the relationship between  $\mu_i$  and  $\sigma_i^2$  across the sample of firms—the true mean-variance relationship.<sup>2</sup>

From firm  $i$ 's returns  $r_{it}$  over the time period  $t = 1 \dots T$ , estimates for this firm's expected return  $\mu_i$  as well as for its variance of returns,  $\sigma_i^2$ , can be calculated. The random variables  $m_i$  and  $s_i^2$  describe the distribution of these estimates:

$$m_i := \frac{1}{T} \sum_{t=1}^T r_{it} \quad (1)$$

$$s_i^2 := \frac{1}{T-1} \sum_{t=1}^T (r_{it} - m_i)^2 \quad (2)$$

These are dependent random variables, the joint distribution of which is induced by the distribution

<sup>2</sup> Alternatively to using the variance of per-period returns as a measure of risk, one could employ their standard deviation as done, e.g., by Sinha (1994) and Gooding *et al.* (1996). Both approaches have been widely used in earlier studies (see the overview by Nickel and Rodriguez, 2002). By employing variances, my analysis remains consistent with Bowman's (1980) original work as well as many subsequent studies. See the concluding section for a more detailed discussion.

of  $r_{it}$ . A realization, or 'draw,' of these random variables yields the familiar sample mean and sample variance, where sampling is performed over the time periods  $t = 1 \dots T$  with fixed  $i$ .

*Proposition 1:* For the variances, covariance, and correlation of  $m_i$  and  $s_i^2$ , the following holds:<sup>3</sup>

$$\text{Var } [m_i] = \frac{\sigma_i^2}{T} \quad (3)$$

$$\text{Var } [s_i^2] = \frac{1}{T} \left( k_i^4 - \sigma_i^4 \frac{T-3}{T-1} \right) \quad (4)$$

$$\text{Cov } [m_i, s_i^2] = \frac{\alpha_i^3}{T} \quad (5)$$

$$\text{Corr } [m_i, s_i^2] = \frac{\alpha_i^3}{\sqrt{\sigma_i^2 \left( k_i^4 - \sigma_i^4 \frac{T-3}{T-1} \right)}} \quad (6)$$

*Proof.* Equation (3) is obvious. A proof of (4) can be found, for example, in Mood, Franklin, and Boes (1974: 229). A proof of (5) is provided by Henkel (2000) and is reproduced in the Appendix at A1. Equation (6) follows from (3), (4), and (5).

These equations can be interpreted as follows. First assume all firms' return distributions are identical and characterized by the central moments  $\sigma_i^2$ ,  $\alpha_i^3$  and  $\kappa_i^4$  (note that the assumption of identical return distributions for all firms is relaxed in the following section). Then the expected correlation between the estimated values for mean and variance across firms is given by (6). For symmetric distributions such as the normal distribution, this quantity is zero—sample mean and sample variance are independent. For a left-skewed distribution, however, (5) and (6) are negative, even though there is no correlation at all between the underlying *true* means and variances of the firms' return distributions; these are all identical, equal to  $\mu_i$  and  $\sigma_i^2$ , respectively. Hence, a risk-return 'paradox' emerges as a pure artifact. The central point here is that, while (1) and (2) provide unbiased estimators of  $\mu_i$  and  $\sigma_i^2$ , respectively, (5) and (6) are biased estimators of covariance and correlation between  $\mu_i$  and  $\sigma_i^2$ . While the possible existence of such a bias and the resulting artifact

<sup>3</sup> Note that, for one particular firm  $i$ , neither mean and variance of its return distribution ( $\mu_i, \sigma_i^2$ ) can be correlated to each other, nor the empirical estimates of these quantities. In both cases, merely two numbers are given. However,  $m_i$  and  $s_i^2$  are random variables, and between these covariance and correlation can be calculated even when looking at only one firm ( $i$ ).

due to skewness may be obvious, its quantification as described by Equations (5) and (6) in Proposition 1 is far from trivial.

Using German firm data from 1988 to 1997, Henkel (2000) goes on to show that return distributions are, on average, skewed. Assuming, as above, *identical* return distributions for all firms, with a skewness equal to the empirically determined *average* skewness in the sample, he shows that the resulting spurious mean-variance association given by (5) and (6) is of the same order of magnitude as the empirically observed association. However, while this result gives an indication that the risk-return 'paradox' *may be* spurious and caused by skewness, the assumption of identical return distributions across firms is obviously not correct. Thus, more sophisticated means of analysis are required that go beyond Henkel's (2000) study. These are developed in the following.

### Disentangling spurious and real effects in samples

In any empirical sample, mean and variance of firms' return distributions vary across firms, such that a nontrivial relationship between the two quantities exists. To identify this true mean-variance relationship is the purpose of risk-return analysis. However, when employing *estimates* of mean and variance, the sought-for true relationship between means and variances *across* firms is, in general,

intertwined with the spurious effects (due to skewness) from *individual* firms' return distributions discussed above. How these effects can be disentangled and unbiased estimates can be obtained is shown below.

Consider  $(\mu_i, \sigma_i^2)$  as a random two-vector obtained by drawing one firm out of the population. That is, if firm  $i$  is drawn, then the vector contains the true mean  $\mu_i$  and the true variance  $\sigma_i^2$  of firm  $i$ 's return distribution. The distribution of  $(\mu_i, \sigma_i^2)$  for a population of  $N = 4$  firms is illustrated by the black dots in Figure 1, each denoting a point  $(\mu_i, \sigma_i^2)$ . In order to set the stage for the mathematical argument developed below and in Appendix A2., let  $\phi$  denote the distribution density of  $(\mu, \sigma^2)$  in  $(\hat{\mu}, \hat{\sigma}^2)$ -space (variable names with hats are axis labels), and  $E_\phi[\cdot]$  the operator generating the expected value of a function in  $(\mu, \sigma^2)$ -space with respect to the density  $\phi$ .<sup>4</sup> Due to the finite number of firms in the population,  $\phi$  is an atomistic distribution, that is, given by isolated points.

If a particular firm  $i$  is given, then the  $T$  random variables  $r_{it}$  ( $i = 1 \dots T$ ) yield, by Equations (1) and (2), the random two-vector  $(m_i; s_i^2)$ . Let the joint distribution of  $(m_i; s_i^2)$  be described by the

<sup>4</sup> That is, for any function  $g(\mu_i, \sigma_i^2)$ ,  $E_\phi[g] \equiv \int g(\hat{\mu}, \hat{\sigma}^2) \phi(\hat{\mu}, \hat{\sigma}^2) d\hat{\mu} d\hat{\sigma}^2$ . Introducing  $\phi$  and  $E_\phi[\cdot]$  allows to treat (a) drawing one firm  $i$  out of the population and (b) drawing the return values  $r_i, \dots, r_{iT}$  for this firm as two consecutive, linked random processes. This interpretation, in turn, allows setting up the proof of Proposition 2 in the way it is shown in Appendix A2.

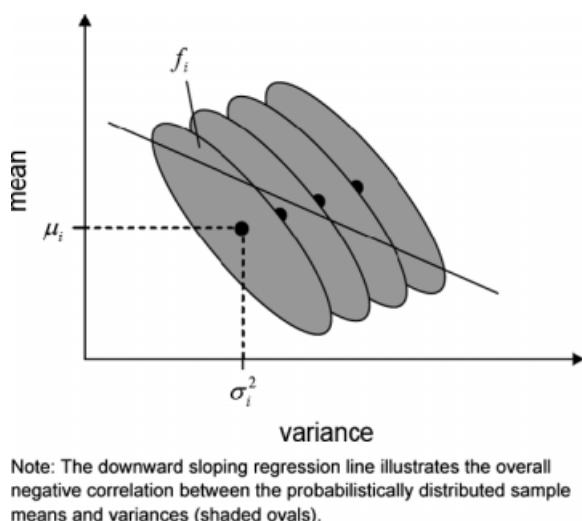


Figure 1. Illustration of  $(\mu_i, \sigma_i^2)$  (black dots) and probability distribution of the sample means and variances  $(m_i, s_i^2)$  (shaded ovals)

density function  $f_i$ , which is illustrated as the shaded oval area around the point  $(\mu_i, \sigma_i^2)$ .<sup>5</sup> Note that  $f_i$  is centered around  $(\mu_i, \sigma_i^2)$ , since this is the expected value of  $(m_i, s_i^2)$ . In particular,  $m_i$  and  $s_i^2$  are unbiased estimators of  $\mu_i$  and  $\sigma_i^2$ , respectively. In contrast, their *correlation* (cf. Footnote 3) is biased due to skewness, which is illustrated by the fact that the shaded ovals are downward sloping. That is, if a particular draw of  $r_{it}$ ,  $t = 1 \dots T$ , has yielded an estimate of  $\mu_i$  that is larger than the true value, then the estimate of  $\sigma_i^2$  is likely to be *smaller* than its true value. The same holds vice versa.

The two-stage process of randomly drawing one firm out of the population and then drawing the  $T$  return values  $r_{i1}, \dots, r_{iT}$  for this firm yields a random two-vector  $(m, s^2)$ . The distribution density  $f$  of this vector can be thought of as a superposition of the densities of the random vectors  $(m_i, s_i^2)$ , weighted with the probability  $1/N$  of drawing firm  $i$  in the first step of the process. Expressed in more mathematical terms, the distribution density  $f$  results from averaging the individual firms' distributions  $f_i$  over the population of all firms:

$$f(\hat{\mu}, \hat{\sigma}^2) \equiv \frac{1}{N} \left( \sum_{i=1}^N f_i(\hat{\mu}, \hat{\sigma}^2) \right).$$

Figure 1 illustrates this superposition. Due to negative skewness, which is assumed in the illustration, the ovals are downward sloping. The figure visualizes how the superposition of the individual firms' distributions of  $(m_i, s_i^2)$ , yielding the total shaded area, can lead to an overall *negative* relationship between mean and variance for  $(m, s^2)$ , even though a positive correlation exists for the true values  $(\mu_i, \sigma_i^2)$  (shown as black dots). The regression line in Figure 1 emphasizes this negative relationship. Put differently, overlaying the distribution  $\phi$  of  $(\mu, \sigma^2)$  (black dots) with each firm's distribution  $f_i$  of  $(m_i, s_i^2)$  leads, in this example, to a dominance of the spurious negative correlation due to skewness over the (true) positive

<sup>5</sup> In order to keep the illustration clear and transparent, the depiction of the density functions  $f_i$  is simplified. More correctly, the density plot should show several contour lines as in a topographic map (instead of just one for each  $i$ , as in Figure 1). For a given firm  $i$  in the illustrative example, these contour lines would be downward-sloping ovals centered around  $(\mu_i, \sigma_i^2)$ , some larger and some smaller in diameter than the contour line that is shown. The latter indicates one particular value of the density function.

correlation. The following proposition shows how these effects can be disentangled.

*Proposition 2.* *The sought-for true covariance and correlation between the two components of the random vector  $(\mu, \sigma^2)$ —that is, the true means and variances—can be obtained as follows:*

$$\text{Cov}[\mu, \sigma^2] = \frac{\text{Cov}[m, s^2] - \text{E}_\phi[\text{Cov}[m_i, s_i^2]]}{\text{E}_\phi[\text{Cov}[m_i, s_i^2]]} \quad (7)$$

$$\text{Corr}[\mu, \sigma^2] = \frac{\text{E}_\phi[\text{Cov}[m_i, s_i^2]]}{\sqrt{\text{Var}[m] - \text{E}_\phi[\text{Var}[m_i]]}} \quad (8)$$

$$\sqrt{\text{Var}[s^2] - \text{E}_\phi[\text{Var}[s_i^2]]}$$

*Proof.* See Appendix A2.

The first term on the right-hand side of Equation (7) is what is commonly measured in mean-variance analysis of returns. This measure is biased; the true covariance between means and variances across firms requires a correction. It is obtained by subtracting the average of the individual firms' spurious covariance (5)—the spurious contribution due to skewness—from the measured covariance. (Equivalently and in more mathematical terms,  $\text{E}_\phi[\text{Cov}[m_i, s_i^2]]$  stands for the expected value over  $\phi$  of the individual firms' covariance between the estimators of mean and variance). Similar equations hold for the 'true' variances of  $\mu$  and  $\sigma^2$  (see Appendix A2), which together with (7) lead to (8). When average skewness across the sample is negative, the corrected covariance is unambiguously larger than the uncorrected value. In particular, when the uncorrected covariance is negative—that is, a risk-return paradox seems to exist—the corrected value is either negative and smaller in absolute value than the uncorrected one, or positive. In the latter case, the apparent risk-return paradox is fully accounted for by the spurious effect of skewness, no matter if covariance or correlation is used as a measure. If the corrected covariance remains negative, then the corrected correlation depends on the interplay between the corrections in the nominator and the denominator of (8).

Equations (7) and (8) can readily be employed to obtain unbiased estimates of the desired quantities on the left-hand side. Estimates for  $\text{Var}[m]$ ,  $\text{Var}[s^2]$ , and  $\text{Cov}[m; s^2]$  are available; these are the commonly used quantities in mean-variance

analysis. Unbiased estimates for the expected values  $E_\phi[\dots]$  can be obtained by calculating the sample means of the quantities given in Equations (3), (4), and (5). In the following, these results will be employed in a simulation, then in an empirical analysis.

## Simulation

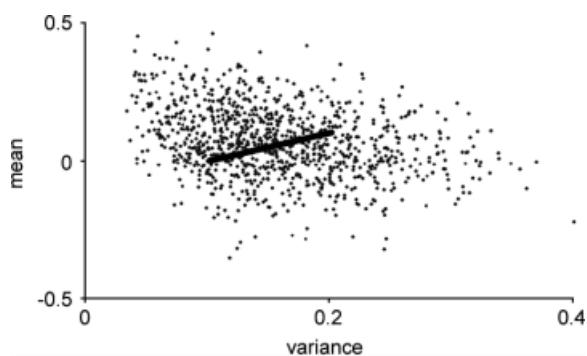
In order to illustrate how skewness can confound the measurement of the risk-return association and how its influence can be controlled for, a simulation is employed. Parameters are chosen such that the true association between mean and variance of return distributions across firms is positive, yet a negative correlation is obtained empirically due to skewness of firms' return distributions. I consider a sample of 1,000 firms. The true means  $\mu_i$  of the individual return distributions are equally distributed in  $[0, 0.1]$ . The true variances are given as  $\sigma_i^2 = \mu_i + 0.1$ , and are thus equally distributed in  $[0.1, 0.2]$ . The (true) values of  $\mu_i$  and  $\sigma_i^2$  (or, equivalently, the joint distribution  $\phi$  of the components of the random vector  $(\mu_i, \sigma_i^2)$  introduced earlier) are depicted as the bold line in Figure 2. The correlation between  $\mu_i$  and  $\sigma_i^2$  equals unity, the covariance equals 0.00833. These are the (*ex ante*) values that risk/return studies seek to identify, but which are typically confounded by the spurious effect of skewness.

To model skewness, I use a triangular distribution since this allows for explicit calculation of all required higher moments (any other left-skewed distribution would qualitatively yield the same result). For example, the single-period return values for the median firm in the simulation, which is characterized by  $\mu_i = 0.05$  and  $\sigma_i^2 = 0.15$ , are

distributed between -1.05 and 0.6, with the distribution density increasing from zero (at -1.05) linearly up to 1.212 (at 0.6). In time period  $t$ , firm  $i$ 's return obtains as a draw of the random variable  $r_{it}$ , which has mean  $\mu_i$  and variance  $\sigma_i^2$ . In order to distinguish random variables from actually observed (i.e., simulated) data, I denote the latter by capital letters. Hence, firm  $i$ 's observed return in period  $t$  is  $R_{it}$ . Figure 2 shows the result of the simulation, that is, the observed values  $(M_i, S^2_i)$ .

It is clearly visible from the scatter plot that the correlation between estimated values of mean and variance is negative. The statistical analysis confirms this observation, yielding a correlation of -0.292 and a covariance of -0.00251 (I report average values from 100 simulations). Hence, the spurious effect due to skewness has not only biased downwards covariance and correlation but has, in fact, reversed their signs: the data show an entirely spurious risk-return 'paradox.'

In order to estimate the spurious contribution of skewness using (5) and (7), the third central moment  $\alpha^3_i$  is estimated for each firm (see Appendix A3 for a description of this and further estimators). Its average over all firms, divided by the number of periods, yields an estimate of the spurious contribution to the measured covariance,  $E_\phi[\text{Cov}[m_i, s^2_i]]$  (see (7)). For the simulation shown in Figure 2, the spurious contribution is estimated as -0.00332. It is larger in absolute value than the observed covariance, -0.00251. Applying (7) yields a corrected value of 0.000815, which comes very close to the exact, theoretically calculated value of 0.000833. This example demonstrates that the method devised in the preceding section is indeed suited to disentangle true



Note: Bold line: 'true' mean-variance values  $(\mu_i, \sigma_i^2)$ .

Figure 2. Simulation of mean and variance of returns for a sample of 1,000 firms over  $T = 10$  periods

and spurious contributions to the mean-variance relationship.<sup>6</sup>

As a comparison of Equations (7) and (8) shows, correcting for the spurious effect is more complicated for the correlation between means and variances than for the covariance. In order to employ (8), the second, third, and fourth central moment of the return distribution were estimated for each firm (see Appendix A3 for the formulae employed), allowing in turn to determine estimates for  $\text{Var}[m_i]$  and  $\text{Var}[s_i^2]$  (3, 4). Averaging these over all firms yields estimates for  $E_\phi[\text{Var}[m_i]]$  and  $E_\phi[\text{Var}[s_i^2]]$ , which allow a corrected correlation value to be calculated according to (8).

Since the corrected covariance turned out positive in all 100 iterations, also the corrected correlation is positive in all cases. The question arises how to deal with iterations that (due to very small denominators in (8)) yielded a correlation greater than 1, or (due to negative arguments in one of the roots in the denominator) an undefined result. Given that the closest 'sensible' outcome for these results is the maximum possible correlation (i.e., 1), these outcomes were replaced by 1. Averaging over all 100 iterations then yields a corrected correlation of 0.797, with a standard deviation of 0.267.<sup>7</sup> Although the accuracy is, for reasons outlined above, clearly below that achieved for the covariance value, the corrected correlation comes much closer to the correct value of 1 than the uncorrected value of -0.292. In particular, the sign of the association is now correct. Due to the difficulties of estimating the denominator of (8) correctly, the focus in the following analysis will be put on the corrected value of covariance.

## EMPIRICAL ANALYSIS

### Data

For the empirical analysis, I employ U.S. firm data from Standard & Poor's Compustat Industrial

<sup>6</sup> In more detail, after 100 simulations I obtain for the uncorrected covariance an average value of -0.00251 and a standard deviation of 0.00024, for the spurious contribution an average value of -0.00332 with a standard deviation of 0.00015, and for the corrected covariance an average value of 0.000815 with a standard deviation of 0.00031.

<sup>7</sup> When instead, very conservatively, undefined outcomes and outcomes with results greater than 1 are discarded, an average corrected correlation of 0.577 obtains, with a standard deviation of 0.235.

Annual Database. The analysis is performed on the level of two-digit Standard Industrial Classification (SIC) codes as also done, for example, by Bowman (1980) and Fiegenbaum and Thomas (1986, 1988). In order to make the results comparable to those obtained by these and other authors (e.g., Oviatt and Bauerschmidt, 1991; Ruefli and Wiggins, 1994) I use data from the time period 1970 to 1979.<sup>8</sup>

In keeping with Bowman's (1980) original article and various later studies, I use return on equity (ROE) as a measure of return. Alternatively, return on assets (ROA) or some transformation of either one could be employed. However, existence of the risk-return paradox when using accounting data does not seem to hinge on the exact operationalization of risk (see, e.g., the overview by Nickel and Rodriguez, 2002).

ROE is calculated as income before extraordinary items available for common equity divided by total common equity. In order to restrict the influence of outliers, the one percent of individual ROE observations with extreme values is discarded. This is a somewhat sensitive step, since outliers have a strong influence on all moments of the distribution. Ideally, one would want to keep them in the analysis, but restrict their weight. A convenient way to do this is to employ rank correlation, but this would make it impossible to identify and correct for the effect of skewness. Hence, removing extreme values is the best compromise. Varying the percentage of extreme values that are discarded leaves the results qualitatively unchanged.<sup>9</sup> Since deleting an individual observation with an extreme ROE value from a firm's time series would strongly change this firm's observed

<sup>8</sup> Compared to other decades, this time period has been found to show more and stronger negative associations between mean and variance of firms' returns (Fiegenbaum and Thomas, 1986). My analysis is intentionally restricted to this decade, since the goal of this study is not to assess the mean-variance relationship in general, but rather to demonstrate how it is influenced by spurious effects and how one can, nonetheless, arrive at an unbiased estimate of it.

<sup>9</sup> Earlier authors have removed outliers in a similar manner in order to restrict their influence (e.g., Fiegenbaum, 1990; Miller and Chen, 2004; Andersen *et al.*, 2007). Note also that exactly how outliers are treated is not too critical for my analysis, for two reasons. First, removing firms with outlier return observations will typically reduce both the spurious effect and the observed mean-variance association, so the effects cancel at least partly. Second, the purpose of this study is to show that when risk-return analysis is performed using mean and variance, then one must take skewness into account and correct for it. This holds true irrespective of the exact number of outliers discarded.

distribution skewness, the respective firms are completely deleted in order to avoid bias. Generally, only those firms are kept in the sample for which data on ROE is available for each year in the respective period.

I analyze the full 10-year period 1970 to 1979 in order to obtain more precise estimates of the required higher moments. Note that using a longer time period than earlier authors (who mostly used five consecutive years) yields a *conservative* estimate of the spurious effect of skewness, which depends inversely on the number of periods (see (5)). I exclude firms under the one-digit SIC code 9, 'public administration,' because of their nonprofit nature, and keep only those industries for which there are at least 10 firms in the sample. This leaves 740 firms in 27 industries.<sup>10</sup>

### Corrected mean-variance association: results

Table 1 displays the results of the empirical analysis. The column 'measured covariance' confirms the familiar result that the majority of industries (20 out of 27) exhibit a negative estimated mean-variance relationship. The column 'spurious effect' displays the average skewness of firms' return distributions in the respective industry (divided by the number of periods)—and even a cursory look reveals that skewness matters. Relating the measured covariance to the spurious effect shows a strong correlation: in almost all cases (24 out of 27), the entries carry the same sign. Table 2 quantifies this point. Of those 12 industries where the covariance is significantly negative, 11 show a negative spurious contribution. On the other hand, in all of the five industries with a significantly positive covariance, the effect from skewness is positive. Already this finding strongly supports the proposition that skewness must be taken into account in analyzing the mean-variance relationship of returns.

The size of the artifact can further be quantified by comparing the size of measured covariance and

spurious effect, as done in the column 'ratio spurious/measured.' Averaging this ratio over those industries where the covariance is significantly different from zero yields 75 percent for the group of 11 industries with significant negative covariances; the median ratio is given by 69 percent. For four of these 11 industries, the ratio is even 99 percent or larger, such that the spurious effect completely accounts for the observed negative mean-variance association. These results shows that the larger part of the measured negative mean-variance relationship is in fact spurious.

Furthermore, also for industries with a *positive* observed mean-variance relationship the latter turns out to be spuriously influenced by skewness. For those five industries with a significantly positive relationship, I find that on average 38 percent of the covariance is due to the spurious effect. The last two columns of Table 1 juxtapose measured and corrected correlations, the latter estimated using (8). For those 20 industries where the measured correlation is negative, the corrected correlation is greater than the measured one in 11 cases (it either becomes positive or the absolute value of the negative correlation decreases), smaller in six cases, and not defined in three cases. Restricting the analysis to those 12 industries where the negative association is significant, one finds an increased corrected correlation in six cases, a decreased one in five cases, and an undefined one in one case. This result is less clear-cut than that obtained from the analysis of covariances, for two reasons. First, in cases where both the uncorrected mean-variance association and the spurious effect are negative (first block of lines in Table 1) but the latter is smaller in absolute value (ratio smaller than 100%), the correction terms in the denominator in (8) counteract the correction in the numerator by inflating the absolute value of the result. However, this inflation does not restore significance when the correction has reduced significance of the covariance (i.e., the numerator). Second, estimation errors in the denominator of (8) can leverage into very large errors of the whole expression.

By the sign of the measured mean-variance association and the sign of the spurious effect, the industries listed in Tables 1 and 2 can be classified into four groups. Doing so suggests a search for systematic differences between these groups. In particular, the one industry (rubber and plastics, SIC 30) with a significant negative risk-return

<sup>10</sup> The total number of firms in the initial data set was 1,858, over a time-period of 10 years. For 5,211 single-year observations, no ROE could be calculated because of missing data. From the remaining 13,369 observations, 134 were excluded because of extreme ROE values, the same number for the highest and lowest values. Next, all observations were deleted which belonged to incomplete time series, leaving 8,950 observations, or 895 firms, in the sample. Excluding industries with less than 10 firms, as well as SIC code 9, finally left 740 firms, as shown in Table 1.

Table 1. Empirical analysis of mean-variance relationship and spurious contributions due to skewness

| SIC   | industry <sup>a</sup>   | no. of firms | measured covar. | P     | spurious effect | corrected covar | ratio spurious/ measured | measured correl. | corrected correl. <sup>b</sup> |
|---|-------------------------|--------------|-----------------|-------|-----------------|-----------------|--------------------------|------------------|--------------------------------|
| Industries with a <i>negative</i> mean-variance relationship before correction  |                         |              |                 |       |                 |                 |                          |                  |                                |
| Industries with <i>negative</i> average skewness                                |                         |              |                 |       |                 |                 |                          |                  |                                |
| 36  | Electronics             | 61           | −0.0743         | 0.119 | −0.1578         | .00835          | 212%                     | −.20159          | .72083                         |
| 29  | Petroleum               | 15           | −0.01200        | 0.068 | −0.01908        | .00708          | 159%                     | −.48310          | 11.72696                       |
| 35  | Machinery, computers    | 62           | −0.00652        | 0.058 | −0.00953        | .00301          | 146%                     | −.24203          | .32700                         |
| 26  | Paper                   | 18           | −0.00485        | 0.455 | −0.00590        | .00104          | 121%                     | −.18795          | .06817                         |
| 50  | Wholesale, durables     | 27           | −0.03112        | 0.001 | −0.03433        | .00321          | 110%                     | −.61230          | .33876                         |
| 54  | Food stores             | 13           | −0.02378        | 0.036 | −0.02363        | −.00015         | 99%                      | −.58484          | −.01559                        |
| 65  | Real Estate             | 10           | −0.01148        | 0.273 | −0.01138        | −.00011         | 99%                      | −.38403          | −.01169                        |
| 27  | Printing, publishing    | 22           | −0.00011        | 0.174 | −0.00010        | −.00001         | 92%                      | −.30045          | −.11947                        |
| 13  | Oil, gas extraction     | 20           | −0.02640        | 0.076 | −0.02382        | −.00259         | 90%                      | −.40527          | −.08814                        |
| 37  | Transportation equipm.  | 42           | −0.00258        | 0.228 | −0.00225        | −.00034         | 87%                      | −.18995          | n.a.                           |
| 51  | Wholesale, nondurables  | 16           | −0.00263        | 0.044 | −0.00181        | −.00083         | 69%                      | −.50976          | −.34914                        |
| 48  | Communications          | 11           | −0.04696        | 0.001 | −0.03144        | −.01553         | 67%                      | −.84924          | −1.20724                       |
| 60  | Depository              | 29           | −0.00132        | 0.396 | −0.00083        | −.00049         | 63%                      | −.16385          | −.11265                        |
| 20  | Food                    | 38           | −0.02279        | 0.000 | −0.01420        | −.00859         | 62%                      | −.89423          | −2.41101                       |
| 59  | Misc. retail            | 12           | −0.08881        | 0.000 | −0.03486        | −.05395         | 39%                      | −.95290          | −.96114                        |
| 23  | Apparel                 | 13           | −0.00103        | 0.021 | −0.00040        | −.00063         | 39%                      | −.62836          | −.76915                        |
| 34  | Fabricated metal prod.  | 32           | −0.01685        | 0.000 | −0.00538        | −.01148         | 32%                      | −.72238          | −1.0049                        |
| Industries with <i>positive</i> average skewness                                |                         |              |                 |       |                 |                 |                          |                  |                                |
| 38  | Measuring, analyzing    | 27           | −0.00075        | 0.744 | .00315          | −.00390         | −418%                    | −.06603          | n.a.                           |
| 30  | Rubber, plastics        | 13           | −0.08259        | 0.026 | .02205          | −.10465         | −26%                     | −.61172          | n.a.                           |
| 25  | Furniture               | 12           | −0.00002        | 0.756 | .000002         | −.00002         | −12%                     | −.10038          | −.14443                        |
| Industries with a <i>positive</i> mean-variance relationship before correlation |                         |              |                 |       |                 |                 |                          |                  |                                |
| Industries with <i>positive</i> average skewness                                |                         |              |                 |       |                 |                 |                          |                  |                                |
| 58  | Eating, drinking places | 11           | .01227          | 0.546 | .05129          | −.03901         | 418%                     | .20463           | n.a.                           |
| 10  | Metal mining            | 10           | .37225          | 0.000 | .41397          | −.04171         | 111%                     | .93281           | n.a.                           |
| 73  | Business services       | 27           | .06963          | 0.225 | .05537          | .01427          | 80%                      | .24146           | n.a.                           |
| 28  | Chemicals               | 55           | .03248          | 0.000 | .01301          | .01947          | 40%                      | .70330           | .76029                         |
| 33  | Primary metal           | 25           | .30833          | 0.000 | .06305          | .24529          | 20%                      | .86287           | .87609                         |
| 53  | Gen. merchandise stores | 11           | .00013          | 0.065 | .00002          | .00011          | 13%                      | .57435           | .72770                         |
| 49  | Electric etc. services  | 108          | .00077          | 0.000 | .00006          | .00072          | 7%                       | .55425           | .63551                         |

<sup>a</sup> See the Appendix for a list of unabridged industry names.

<sup>b</sup> n.a.: result not defined, since one of the roots in equation (8) takes on a negative value.

Table 2. Counts of industries in Table 1 by sign of spurious effect and sign of measured covariance

| measured covariance |          |     |          |      |     |    |
|---------------------|----------|-----|----------|------|-----|----|
| negative            |          |     | positive |      |     |    |
|                     | 5%       | 10% | n.s.     | n.s. | 10% | 5% |
| spurious effect     | negative | 8   | 3        | 6    | —   | —  |
| positive            | 1        | —   | 2        | 2    | 1   | 4  |

relationship and yet a positive spurious effect might be a telling case. However, cursory inspection at least does not yield particular insights. A

more detailed analysis might reveal systematic differences between the groups, but is beyond the scope of this study.

## Results for below-average performers

One explanation for the risk-return paradox that has been advanced is based on prospect theory (Bowman, 1982; Fiegenbaum and Thomas, 1988, 1990; Fiegenbaum, 1990; Jegers, 1991; Johnson, 1992; Gooding *et al.*, 1996). It predicts a negative risk-return relationship for firms with a below-average performance, which indeed has been found empirically (Fiegenbaum and Thomas,

Table 3. Analysis of mean-variance relationship for below-median performing firms in each industry

| SIC   | industry <sup>a</sup>   | no. of firms | measured covar. | P     | spurious effect | corrected covar | ratio spurious/ measured | measured correl. | corrected correl. <sup>b</sup> |
|---|-------------------------|--------------|-----------------|-------|-----------------|-----------------|--------------------------|------------------|--------------------------------|
| Industries with a <i>negative</i> mean-variance relationship before correction  |                         |              |                 |       |                 |                 |                          |                  |                                |
| Industries with <i>negative</i> average skewness                                |                         |              |                 |       |                 |                 |                          |                  |                                |
| 27  | Printing, publishing    | 11           | -.00007         | 0.452 | -.00021         | .00014          | 308%                     | -.25312          | 13.11384                       |
| 29  | Petroleum               | 7            | -.01429         | 0.239 | -.04079         | .02649          | 285%                     | -.51255          | n.a.                           |
| 26  | Paper                   | 9            | -.00672         | 0.000 | -.01200         | .00527          | 178%                     | -.95616          | n.a.                           |
| 54  | Food stores             | 6            | -.03447         | 0.032 | -.05118         | .01671          | 148%                     | -.84878          | n.a.                           |
| 35  | Machinery, computers    | 31           | -.01523         | 0.000 | -.02040         | .00516          | 134%                     | -.61578          | n.a.                           |
| 13  | Oil, gas extraction     | 10           | -.04361         | 0.001 | -.04807         | .00445          | 110%                     | -.87780          | n.a.                           |
| 51  | Wholesale, nondurables  | 8            | -.00336         | 0.011 | -.00370         | .00034          | 110%                     | -.82759          | n.a.                           |
| 65  | Real Estate             | 5            | -.02137         | 0.064 | -.02358         | .00220          | 110%                     | -.85616          | n.a.                           |
| 37  | Transportation equipm.  | 21           | -.00536         | 0.009 | -.00566         | .00030          | 106%                     | -.55474          | n.a.                           |
| 48  | Communications          | 5            | -.06685         | 0.018 | -.06917         | .00231          | 103%                     | -.93774          | .49649                         |
| 36  | Electronics             | 30           | -.01421         | 0.000 | -.01315         | -.00105         | 93%                      | -.62305          | -.31353                        |
| 50  | Wholesale, durables     | 13           | -.05953         | 0.000 | -.04674         | -.01278         | 79%                      | -.97876          | -.124744                       |
| 73  | Business services       | 13           | -.14989         | 0.001 | -.10428         | -.04561         | 70%                      | -.82224          | n.a.                           |
| 20  | Food                    | 19           | -.04237         | 0.000 | -.02834         | -.01402         | 67%                      | -.97593          | -.275021                       |
| 60  | Depository institutions | 14           | -.00454         | 0.001 | -.00267         | -.00187         | 59%                      | -.79278          | -.81729                        |
| 23  | Apparel                 | 6            | -.00160         | 0.043 | -.00084         | -.00075         | 53%                      | -.82531          | -.105585                       |
| 59  | Misc. retail            | 6            | -.16333         | 0.000 | -.06977         | -.09356         | 43%                      | -.99003          | -.104052                       |
| 34  | Fabricated metal prod.  | 16           | -.02834         | 0.000 | -.01077         | -.01756         | 38%                      | -.92771          | n.a.                           |
| 25  | Furniture               | 6            | -.00005         | 0.338 | -.000002        | -.00005         | 5%                       | -.47731          | -.783511                       |
| Industries with <i>positive</i> average skewness                                |                         |              |                 |       |                 |                 |                          |                  |                                |
| 30  | Rubber, plastics        | 6            | -.13127         | 0.137 | .04949          | -.18076         | -37%                     | -.67930          | n.a.                           |
| 10  | Metal mining            | 5            | -.00033         | 0.114 | .00003          | -.00036         | -9%                      | -.78656          | -.152389                       |
| 28  | Chemicals               | 27           | -.00441         | 0.000 | .00019          | -.00460         | -3%                      | -.79214          | -.131417                       |
| Industries with a <i>positive</i> mean-variance relationship before correlation |                         |              |                 |       |                 |                 |                          |                  |                                |
| Industries with <i>positive</i> average skewness                                |                         |              |                 |       |                 |                 |                          |                  |                                |
| 58  | Eating, drinking places | 5            | .00667          | 0.801 | .06456          | -.05788         | 967%                     | .15645           | n.a.                           |
| 38  | Measuring, analyzing    | 13           | .00247          | 0.485 | .00665          | -.00418         | 269%                     | .21288           | n.a.                           |
| 53  | Gen. merchandise stores | 5            | .00001          | 0.640 | .00002          | -.00001         | 164%                     | .28638           | -.65619                        |
| 33  | Primary metal           | 12           | .00005          | 0.891 | .00008          | -.00003         | 148%                     | .04418           | n.a.                           |
| 49  | Electric etc. services  | 54           | .00004          | 0.023 | .00001          | .00003          | 23%                      | .30840           | .32053                         |

<sup>a</sup> See the Appendix for a list of unabridged industry names.

<sup>b</sup> n.a.: result not defined, since one of the roots in equation (8) takes on a negative value.

1988; Chang and Thomas, 1989; Fiegenbaum, 1990; Jegers, 1991; Gooding *et al.*, 1996). This and the results obtained above suggest the need to analyze the effect of distribution skewness separately for underperforming firms in each industry. The result is shown in Table 3. In line with earlier studies, a negative mean-variance relationship is now found for even more industries (22 out of 27), and is significant for 17 of them (see Table 4). Across these 17 industries, the contribution of the spurious effect to the measured covariance has a mean value of 88 percent, and a median of 93 percent. That is, return distributions in the 17 industries with significantly negative mean-variance

Table 4. Counts of industries in Table 3 by sign of spurious effect and sign of measured covariance

| measured covariance |                 |          |     |      |          |     |
|---------------------|-----------------|----------|-----|------|----------|-----|
|                     | spurious effect | negative |     |      | positive |     |
|                     |                 | 5%       | 10% | n.s. | n.s.     | 10% |
|                     | negative        | 15       | 1   | 3    | —        | —   |
|                     | positive        | 1        | —   | 2    | 4        | 1   |

Note: Only firms in lower half of each industry's performance ranking included.

association are, on average, so strongly negatively skewed that the resulting spurious contribution can largely explain the measured negative relationship between mean and variance. In eight of these industries, the spurious effect is in fact larger than the measured covariance, such that the corrected covariance turns out to be positive. For reasons discussed above, and since more than half of the values are undefined, the corrected correlation is not further interpreted.

This result suggests an alternative interpretation of the fact that the risk-return paradox is more marked for badly performing firms. In all but three industries, the spurious contribution is smaller—that is, in most cases, negative and larger in absolute value—in Table 3 than in Table 1. Hence, return distributions for poorly performing firms tend to be more negatively skewed than for the industry average. A possible interpretation would thus be that an industry's low performers are firms with a large negative skewness that had more bad years than others. Hence, also results concerning underperformers appear, at least partly, to be fallacious.

## SUMMARY AND DISCUSSION

Mean-variance analysis is a common approach in studying the relationship between risk and return. The present study has pointed out spurious effects in this analysis due to skewness of the individual firms' return distributions, which bias the observed relationship. It has been shown how these spurious effects can be corrected for to arrive at an unbiased estimate of the true relationship between means and variances of firms' return distributions. Using empirical data, I find that, on average, the spurious effects explain the larger part of the observed risk-return 'paradox.'

This study is closely related to the work by Andersen *et al.* (2007), since for both studies negative skewness of return distributions plays a pivotal role. However, the mechanisms analyzed in the two studies are entirely different. Andersen *et al.* (2007) argue that skewness results from firms' obtaining, randomly, a certain level of strategic fit (see below for more details), and that firms differ in their capabilities to obtain such fit. As a result, return distributions of firms with lower capabilities are further stretched to lower values, which implies

both a lower mean and a larger variance. This negative relationship between mean and variance is a *genuine* one, existing between characteristics of the return distributions (that is, *ex ante* quantities). In contrast, the *spurious* relationship that I analyze exists between *ex post* quantities, namely, the *empirical estimates* for mean and variance. In general, both effects will be intertwined.

A number of limitations of the present study need to be mentioned. First, the study was performed for a certain time period (1970–1979), one country (the United States), and ROE as the measure of return. Thus, while the aim of this work is not to provide a comprehensive revision of earlier studies, an extension and generalization would be of interest.

Second, the disentanglement of true and spurious effects devised here is analytically possible only when variance of returns is employed as the risk measure, while standard deviations might be preferable on the grounds of being homogeneous of degree one in returns. Still, using variances is not only a widely employed approach (see the overview by Nickel and Rodriguez, 2002), but also a valid one: when a monotonous relationship exists between mean and standard deviation, then the same is true for mean and variance. Hence, if a monotonous relationship exists, then it can be found using either variance or standard deviation of returns. Obviously, at most one of these relationships can be linear (though it is not clear which one). If the linear one is that between mean and standard deviation then using variance instead leads to linear regression models being misspecified. However, even though meaningful regression coefficients would be desirable, determining the *slope* of the relationship correctly is valuable in itself, and has been the subject of most of the literature on Bowman's paradox.

Third, Miller and coauthors have argued that managers are primarily focused on downside risk, such that to capture risk one should focus on the downside portion of the return distribution rather than on variance (Miller and Leiblein, 1996; Miller and Reuer, 1996). Without entering the discussion as to which approach is preferable, and acknowledging that downside risk measures may be more relevant than returns variance as proxies for risk perceptions, it should be noted that focusing on downside risk aggravates the spurious influence of skewness: downward variations (which tend to be large in cases of left-skewness) reduce average

returns and increase the risk measure, while (typically smaller) upward variations, which would mitigate the negative risk-return relationship if considered as contributing to risk, are excluded from the latter.

Finally, my study assumes that no longitudinal association between return and risk exists (which would arise, e.g., when the return in one year influences the risk level in the following year). If such longitudinal association is present, it appears plausible that left-skewness would aggravate the negative relationship between mean and variance that results when higher risk taking is a response to low performance. Disentangling the different effects will be more challenging in this case, yet, the spurious contribution of skewness should persist.

The analysis presented here is linked to all three research streams devoted to explaining the Bowman paradox, namely, those focusing on strategic and organizational factors, prospect theory, and model misspecifications. The link to strategic and organizational factors is likely the most obvious one since, given the strong effect of distribution skewness, the question arises why return distributions should be negatively skewed in the first place. Andersen *et al.* (2007) suggest an explanation based on the concept of strategic fit. They argue that a firm attains its individual performance maximum when its strategy and structure are optimally aligned with environmental conditions prevailing in the respective period. They assume the level of alignment as a random variable the density of which peaks at optimal alignment and decreases toward lower levels. Since performance (and thus return) corresponds to the level of strategic fit, a left-skewed return distribution obtains. As a second explanation, a firm's performance might follow positive swings in external conditions to a lesser degree than it follows negative swings. For example, capacity constraints might make it impossible to take full advantage of an upward shift in demand, while a downward shift has an undampened impact on profits. Finally, skewness may be the result of income smoothing. Managers might prefer to accumulate downward deviations from some target level of return, such that a series of relatively stable, high returns would be followed by a rather bad result.

The connection of the present study to the second research stream, referring to prospect theory, becomes apparent when translating the logic

applied here to the fourth moment of the return distribution. Assume firms' return distributions have a relatively large fourth moment (high kurtosis). That is, return distributions show flat but long tails to both sides. If a year's return value for a certain firm lies in one of these tails, then this firm's mean return is pushed toward the top or the bottom of the population, while its variance goes up. Across the sample of firms, this results in a U-shaped dependence of empirical variance on the mean—even if all firms had *identical* return distributions. Such U-shaped dependence has been observed in various empirical studies (Fiegenbaum and Thomas, 1988; Chang and Thomas, 1989; Fiegenbaum, 1990; Jegers, 1991; Gooding *et al.*, 1996), suggesting that this artifact may be of considerable empirical relevance. If, in addition, the return distribution is left-skewed, then the U-shape will exhibit a more pronounced left (falling) branch, which is precisely what the empirical studies cited above find. Fiegenbaum and Thomas (1988) interpret this asymmetry in the context of prospect theory. The effect of kurtosis discussed here suggests an artifact as an alternative, or at least complementing, interpretation.<sup>11</sup>

Finally, the present analysis also has an interesting link to the third research stream, focusing on model misspecifications. Oviatt and Bauerschmidt (1991) compare ordinary least squares (OLS) to three stage least squares (3SLS) estimates of the mean-variance relationship. While they find a negative relationship using OLS, the latter disappears when a 3SLS estimator is employed. The authors conclude that, in their model, both mean and variance of returns are influenced by industry conditions and business strategies, but do not affect each other. The results obtained in the present study allow to look at Oviatt and Bauerschmidt's (1991) findings from a new angle. OLS estimates are biased when there is a correlation between error terms and exogenous variables; 3SLS allows to remove this bias when appropriate instrumental variables are employed. In the case of mean and

<sup>11</sup> Denrell (2005) and Andersen *et al.* (2007), employing model analysis and simulations respectively, obtain a spurious U-shape by assuming that firms' risk propensities are *heterogeneous* in a particular way. Due to this heterogeneity, the U-shape is particularly pronounced in their cases. However, as mentioned above, even with identical return distributions across firms, and thus identical risk propensities, a U-shaped dependence of empirical variance on the mean can arise as a pure artifact.

variance of returns, such a correlation is caused by skewness (and, possibly, other factors). Indeed, Oviatt and Bauerschmidt (1991) mention, as an aside, a strong correlation between variance and skewness. Their observation suggests what has been shown rigorously here, namely, that spurious effects due to skewness constitute an important contribution to the observed mean-variance relationship.

In managers' strategic decision taking, risk is an *ex ante* concept. Accordingly, it is appropriate to consider *ex ante* measures of risk such as the variance of analysts' profit estimates (Bromiley, 1991b; Deephouse and Wiseman, 2000), the content of annual reports (Bowman, 1984), or strategic measures such as diversification, R&D intensity, and debt-to-equity ratio (Miller and Bromiley, 1990; Palmer and Wiseman, 1999) in studying the relationship between risk and return. Now, also the return distribution's variance, while being an unobserved, latent variable, is an *ex ante* correlate of risk. Like other *ex ante* risk measures, it must translate, by definition, into *ex post* variations of outcomes. This study has shown how the biased relation between *empirical estimates* of mean and variance, which are *ex post* quantities, can be corrected in such a way as to yield an unbiased estimate of the relation between the true means and variances of the underlying return distributions, which are *ex ante* quantities. Hence, while Ruefli *et al.* (1999) do have a certain point in criticizing mean-variance approaches as not being linked to managers' *ex ante* decision making, their criticism does not justify dismissing *ex post* measures of risk. Rather, one should strive to combine both measures (see, e.g., Miller and Bromiley, 1990; Palmer and Wiseman, 1999). Provided the *ex ante* estimates of risk are unbiased, it should be possible to establish consistency between the different approaches.

Such consistency could not be confirmed by Walls and Dyer (1996) in their study of the petroleum exploration industry. They found that, 'Ex ante risk propensity is not positively associated with the *ex post* risk measure, variance' (Walls and Dyer, 1996: 1018 [italics in original]). In light of their results, I believe the present study makes a very useful contribution by identifying a misspecification that possibly lies at the root of the above inconsistency. More generally, correcting for the influence of skewness might make *ex ante* and *ex*

*post* risk measures positively associated, thus reconciling *ex ante* and *ex post* approaches in strategic risk-return analysis.

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## APPENDIX

### A1. Proof of Equation (5)

Henkel (2000) provides the following proof, which is slightly modified here. The random variable  $r_{it}$  can be decomposed into its expected value,  $\mu_i$ , and a random variable  $\epsilon_{it}$  with mean zero:  $r_{it} \equiv \mu_i + \epsilon_{it}$ . For each  $\mu_i$ , the  $\epsilon_{it}$  are independently and identically distributed. For the random variables  $r_i$  and  $s_i^2$ , the sample mean and variance of the series of observations  $r_{i1}, \dots, r_{iT}$ , one then obtains:

$$\begin{aligned} r_i &= \frac{1}{T} \sum_t r_{it} \\ &= \mu_i + \frac{1}{T} \sum_t \epsilon_{it} \\ s_i^2 &= \frac{1}{T-1} \sum_t \left( r_{it} - \frac{1}{T} \sum_{t'} r_{it'} \right)^2 \\ &= \frac{1}{T-1} \sum_t \left( \epsilon_{it} - \frac{1}{T} \sum_{t'} \epsilon_{it'} \right)^2 \\ &= \frac{1}{T(T-1)} \left( T \sum_t \epsilon_{it}^2 - \left( \sum_t \epsilon_{it} \right)^2 \right) \end{aligned}$$

For the covariance of  $r_i$  and  $s_i^2$  the above yields (with  $E[\dots]$  denoting ‘expected value of’):

$$\begin{aligned} \text{Cov}[r_i, s_i^2] &= E[(r_i - E[r_i]) \cdot (s_i^2 - E[s_i^2])] \end{aligned} \quad (9)$$

$$= E \left[ \left( \frac{1}{T} \sum_t \epsilon_{it} \right) \cdot (s_i^2 - \sigma_i^2) \right] \quad (10)$$

$$\begin{aligned} &= \frac{1}{T^2(T-1)} E \left[ \left( \sum_t \epsilon_{it} \right) \cdot \left( T \sum_{t'} \epsilon_{it'}^2 - \left( \sum_{t'} \epsilon_{it'} \right)^2 - T(T-1)\sigma_i^2 \right) \right] \end{aligned} \quad (11)$$

$$\begin{aligned} &= \frac{1}{T^2(T-1)} E \left[ T \sum_{t,t'} \epsilon_{it} \epsilon_{it'}^2 - \left( \sum_t \epsilon_{it} \right)^3 - T(T-1)\sigma_i^2 \sum_t \epsilon_{it} \right] \end{aligned} \quad (12)$$

$$\begin{aligned} &= \frac{1}{T^2(T-1)} E \left[ T \sum_{t,t'} \epsilon_{it} \epsilon_{it'}^2 \right. \\ &\quad \left. - \sum_{t,t',t''} \epsilon_{it} \epsilon_{it'} \epsilon_{it''} \right] \end{aligned} \quad (13)$$

$$\begin{aligned} &= \frac{1}{T^2(T-1)} (T-1) E \left[ \sum_t \epsilon_{it}^3 \right] \\ &= \frac{\alpha_i^3}{T} \end{aligned} \quad (14)$$

For the step from (12) to (13), note that  $E[\epsilon_{it}] = 0$  for all  $i$  and  $t$ . Line (14) is identical to the preceding line (13) because (due to the fact that the  $\epsilon_{it}$  are independently distributed) the expected value is different from zero only for those summands in which all summation indices are identical (i.e.,  $t = t'$ ,  $t = t' = t''$ ). Finally, the last line obtains since, for each  $t$ ,  $E[\epsilon_{it}^3] \equiv E[(r_{it} - \mu_i)^3] \equiv \alpha_i^3$ . *Q.E.D.*

### A2. Proof of Proposition 2

Let in the following equations  $\bar{\mu}$  and  $\bar{\sigma}^2$  denote the expected values of the random variables  $\mu$  and  $\sigma^2$  (which describe randomly drawing one firm out of the population). That is,  $\bar{\mu}$  and  $\bar{\sigma}^2$  are the average values of  $\mu_i$  and  $\sigma_i^2$  across the population of firms:  $\bar{\mu} := \frac{1}{N} \left( \sum_{i=1}^N \mu_i \right)$  and  $\bar{\sigma}^2 := \frac{1}{N} \left( \sum_{i=1}^N \sigma_i^2 \right)$ . These are also the expected values of the random variables  $m$  and  $s^2$ , i.e.,  $\bar{\mu} = \bar{\mu}$  and  $\bar{\sigma}^2 = s^2$ , since the expected value of  $(\mu_i, s_i^2)$  is  $(\bar{\mu}, \bar{\sigma}^2)$  for all  $i$ . As a reminder, the random variables  $m$  and  $s^2$  are generated by the two-step process of first drawing one firm  $i$  out of the population, then drawing the  $T$  return values  $r_{i1}, \dots, r_{iT}$ , and finally applying Equations (1) and (2). The resulting random vector  $(m, s^2)$  has the joint distribution density  $f(\cdot, \cdot) \equiv \frac{1}{N} \sum_{i=1}^N f_i(\cdot, \cdot)$ . The variables  $\mu$  and  $\sigma^2$  are help variables over which the integrals run. Then  $\text{Cov}[m, s^2]$  can be decomposed as follows:

$$\begin{aligned} \text{Cov}[m, s^2] &= \int \int (\hat{\mu} - \bar{\mu}) \\ &\quad (\hat{\sigma}^2 - \bar{\sigma}^2) f(\hat{\mu}, \hat{\sigma}^2) d\hat{\mu} d\hat{\sigma}^2 \\ &= \frac{1}{N} \sum_{i=1}^N \int \int (\hat{\mu} - \bar{\mu}) \\ &\quad (\hat{\sigma}^2 - \bar{\sigma}^2) f(\hat{\mu}, \hat{\sigma}^2) d\hat{\mu} d\hat{\sigma}^2 \end{aligned} \quad (15)$$

$$\begin{aligned} & \left( \hat{\sigma}^2 - \bar{s}^2 \right) f_i(\hat{\mu}, \hat{\sigma}^2) d\hat{\mu} d\hat{\sigma}^2 \\ &= \frac{1}{N} \sum_{i=1}^N \int \int (\hat{\mu} - \mu_i + \mu_i - \bar{\mu}) \end{aligned} \quad (16)$$

$$\cdot \left( \hat{\sigma}^2 - \sigma_i^2 + \sigma_i^2 - \bar{\sigma}^2 \right) f_i(\hat{\mu}, \hat{\sigma}^2) d\hat{\mu} d\hat{\sigma}^2 \quad (17)$$

$$\begin{aligned} &= \frac{1}{N} \sum_{i=1}^N \left[ \int \int (\hat{\mu} - \mu_i)(\hat{\sigma}^2 - \sigma_i^2) \right. \\ &\quad f_i(\hat{\mu}, \hat{\sigma}^2) d\hat{\mu} d\hat{\sigma}^2 \\ &+ \left( \sigma_i^2 - \bar{\sigma}^2 \right) \int \int (\hat{\mu} - \mu_i) f_i(\hat{\mu}, \hat{\sigma}^2) d\hat{\mu} d\hat{\sigma}^2 \\ &+ (\mu_i - \bar{\mu}) \int \int (\hat{\sigma}^2 - \sigma_i^2) f_i(\hat{\mu}, \hat{\sigma}^2) d\hat{\mu} d\hat{\sigma}^2 \\ &+ (\mu_i - \bar{\mu}) \left( \sigma_i^2 - \bar{\sigma}^2 \right) \int \int \end{aligned} \quad (18)$$

$$f_i(\hat{\mu}, \hat{\sigma}^2) d\hat{\mu} d\hat{\sigma}^2 \quad (19)$$

$$= \mathbb{E}_\phi [\text{Cov} [m_i, s_i^2]] + \text{Cov} [\mu, \sigma^2] \quad (20)$$

The steps from (15) to (18) are obvious. The first line of (18) equals the first term in (19), which represents the average over the individual firms' skewness-induced covariance between mean and variance of its returns. The second and third line in (18) vanish, since the expected values of  $\hat{\mu}$  and  $\hat{\sigma}^2$  over the distribution  $f_i$  are  $\mu_i$  and  $\sigma_i^2$ , respectively. The fourth line in (18) is the sought-for true covariance  $\frac{1}{N} \left( \sum_{i=1}^N (\mu_i - \bar{\mu}) \left( \sigma_i^2 - \bar{\sigma}^2 \right) \right)$  variance of firms' returns. Note that, since this is a population quantity, not a sample quantity, the division is by the population size  $N$ , not  $N - 1$ . The last line is merely a reformulation. This completes the proof of equation (7) in Proposition 2.

To prove Equation (8), note that

$$\text{Var} [m] = \mathbb{E}_\phi [\text{Var}[m_i]] + \text{Var}[\mu] \text{ and} \quad (21)$$

$$\text{Var} [s^2] = \mathbb{E}_\phi [\text{Var}[s_i^2]] + \text{Var}[\sigma^2] \quad (22)$$

are proved along the same lines as equation (20). Taking (20) to (22) together yields (8).

*Q.E.D.*

### A3. Estimation of higher moments

Let  $W_i^k$  be defined as the average over all  $T$  periods of the  $k$ 'th power of the deviation of  $r_{it}$  from the mean ROE of firm  $i$ :

$$W_i^k = \frac{1}{T} \sum_{t=1}^T \left( r_{it} - \frac{1}{T} \sum_{\tau=1}^T r_{i\tau} \right)^k. \quad (23)$$

Then unbiased estimates for the third and fourth central moment of firm  $i$ 's return distribution can be expressed in terms of the  $W_i^k$  as follows (see Kenney and Keeping, 1951: 189 for these and the following equations):

$$\hat{\alpha}_i^3 = \frac{T^2}{(T-1)(T-2)} W_i^3 \quad (24)$$

$$\hat{\kappa}_i^4 = \frac{T(T^2-2T+3)W_i^4 - 3(2T-3)(W_i^2)^2}{(T-1)(T-2)(T-3)}. \quad (25)$$

The  $k$ -statistics  $\hat{\alpha}_i^3$  and  $\hat{\kappa}_i^4$  are defined as

$$k_i^2 = \frac{T}{T-1} W_i^2, \quad (26)$$

$$k_i^4 = \frac{T^2((T+1)W_i^4 - 3(T-1)(W_i^2)^2)}{(T-1)(T-2)(T-3)}. \quad (27)$$

Using these expressions, an unbiased estimator of the variance of  $s_i^2$ , which for known  $\sigma_i^2$  is given by (4), is

$$\widehat{\text{Var}[s_i^2]} = \frac{2T(k_i^2)^2 + (T-1)k_i^4}{T(T+1)}. \quad (28)$$

### A4. Industries in the sample, by two-digits SIC codes

- 10: Metal mining
- 13: Oil and gas extraction
- 20: Food and kindred products
- 23: Apparel and other finished products made from fabrics and similar materials
- 25: Furniture and fixtures
- 26: Paper and allied products
- 27: Printing, publishing, and allied industries
- 28: Chemicals and allied products
- 29: Petroleum refining and related industries
- 30: Rubber and miscellaneous plastics products
- 33: Primary metal industries

- 34: Fabricated metal products, except machinery and transportation equipment
- 35: Industrial and commercial machinery and computer equipment
- 36: Electronic and other electrical equipment and components, except computer Equipment
- 37: Transportation equipment
- 38: Measuring, analyzing, and controlling instruments; photographic, medical, and optical goods; watches and clocks
- 48: Communications
- 49: Electric, gas, and sanitary services
- 50: Wholesale trade—durable goods
- 51: Wholesale trade—nondurable goods
- 53: General merchandise stores
- 54: Food stores
- 58: Eating and drinking places
- 59: Miscellaneous retail
- 60: Depository institutions
- 65: Real estate
- 73: Business services