

2. Percentages and Ratios

Percentages

The term percentage means parts per 100 or "for every hundred". Thus, when we say a man made a profit of 20 percent we mean to say that he gained ₹ 20 for every hundred rupees he invested in the business, i.e., 20/100 rupees for each Rupee.

The abbreviation of percent is p.c. and it is generally denoted by %.

Important Results

1. A Percentage can be expressed as a Fraction.

10% can be expressed as 10/100 or 1/10.

To express a percentage as a fraction divide it by 100 $a\% = a/100$.

E1. Express the following as fraction

a. 25% b. $33\frac{1}{3}\%$

Sol. a. $25\% = \frac{25}{100}$ (since % means $\frac{1}{100}$) = $\frac{1}{4}$

b. $33\frac{1}{3}\% = \frac{100}{3}\% = \frac{100}{3 \times 100} = \frac{1}{3}$

2. To express a fraction as a percent multiply it by 100.

$= > \frac{a}{b} = [(\frac{a}{b}) \times 100]\%$

E2. Express 1/8 as a percentage.

Sol. $\frac{1}{8} = \frac{1}{8} \times \frac{100}{8}\% = \frac{25}{2}\% = 12\frac{1}{2}\%$

3. A Percentage can be expressed as a Decimal. To express percentage as a decimal we remove the symbol % and shift the decimal point by two places to the left. For example, 10% can be expressed as 0.1.

E3. Express $6\frac{1}{2}\%$ as a decimal.

Sol. $6\frac{1}{2}\% = \frac{13}{2}\% = 6.5\% = \frac{6.5}{100} = 0.065$

4. To express decimal as a percentage we shift the decimal point by two places to the right and write the number obtained with the symbol % or simply we multiply the decimal with 100.

E4. Express 0.7 as a percentage.

Sol. $0.7 = 0.7 \times 100\% = (7/10) \times 100 = 70\%$

5. If X is R% of a given number N, then $N = \frac{X \times 100}{R}$

E5. 25% of a number is 60. What is the number?

Sol. Let the number be X. According to the given condition

$$\frac{25}{100} \times X = 60 \Rightarrow X = \frac{60 \times 100}{25} = 240$$

6. If A's income is $r\%$ more than that of B, then B's income is $\frac{r}{(100+r)} \times 100$ less than that of A.

E6. The income of A is 50% more than that of B. Then B's income is less than A by what percent?

Sol. Let income of B be ₹ 100. Hence, income of A is ₹ 150. B's income is ₹ 50 less than that of A,

In percentage, $= \left(-\frac{50}{150}\right) \times 100 = -33.33\%$. (-ve sign shows that income of B is less than A)

Hence, B's income is 33.33% less than that of A.

7. If A's income is $r\%$ less than that of B, then B's income is $\frac{r}{(100-r)} \times 100$ more than that of A.

E7. If A's income is 20% less than that of B, then B's income is how much percent more than that of A?

Sol. Let income of B be ₹ 100. Hence, income of A is ₹ 80. B's income is ₹ 20 more than that of A. In percentage, $= \frac{20}{80} \times 100 = 25\%$

Hence, B's income is 25% more than that of A.

8. An increase of, say 25%, means that for each 100 units in the original value, there is an increase of 25 units, making the new value 125 units. Therefore,

$$\text{Increase \%} = \frac{\text{increase}}{\text{originalvalue}} \times 100$$

If X is increased by $r\%$ Then new value = $X + r\%$ of X

Also, New value = Original Value $\times (1 + \text{Increase})$.

The increase given is not to be taken in percentage, it should be taken as a fraction.

E8. The present population of a city is 14,00,000. The population of the city 3 years ago was 10,00,000. What is the percentage increase in the population of the city over the given period?

Sol. Applying the formula given above, we get

$$\text{Percentage Increase} = \frac{(1400000 - 1000000)}{1000000} \times 100 = \frac{300000}{1000000} \times 100 = 30\%$$

9. A decrease of, say 25%, means that for each 100 units in the original value, there is a decrease of 25 units, making the new value 75 units. Therefore,

$$\text{Decrease\%} = \frac{\text{Decrease}}{\text{originalValue}} \times 100$$

If X is decreased by r% Then new value = $X - r\% X$

Also, New value = Original Value $\times (1 - \text{Decrease})$

The decrease given is not to be taken in percentage; it should be taken as a fraction.

E9. The number of books published by a group dropped from 375000 units to 250000 units. What is the percentage decrease in the number of books published by the group?

Sol. Applying the formula given above, we get

$$\text{Percentage decrease} = \frac{(375000 - 250000)}{375000} \times 100 = \frac{125000}{375000} \times 100 = 33.3\%.$$

10. A change of, say + 25% (implies an increase) or -25% (implies a decrease), means that for each 100 units in the original value, there is a change of ± 25 units, making the new value 125 or 75 units respectively. Therefore, change% =

$$\frac{\text{change}}{\text{originalvalue}} \times 100$$

Hence the ratio of the new value to the original value = 125 : 100 or 75 : 100.

Also, change should be taken positive if it is an increase if it is a decrease

E10. A company manufacturing television sets observed that over the last three years there was an increase in the number of units sold in the local market. It was found that in the year 2000 the company was able to sell 90,000 units as against only 55,000 units sold by them in the year 1997. This increase could be mostly attributed to the decrease in the price per television set from ₹ 15,000 to ₹ 12,000 over the same period. What has been the percentage change in the price per television set and the number of units sold by the company ?

Sol. The percentage change in the PRICE of the television set and second, the percentage change in the NUMBER of units sold by the company have to be found

Answer I: percentage change in the price of the television set, Applying the formula, we get

$$\text{Percentage change} = \frac{12000 - 15000}{15000} \times 100 = \frac{-3000}{15000} \times 100 = -20\%$$

there is a decrease of 20% in the price of the television set.

Answer II: percentage change in the number of television sets sold, Applying the formula discussed above for percentage change we get

Percentage change = $\frac{9000-55000}{55000} \times 100 = \frac{35000}{55000} \times 100 = 64\%$ Hence, both the answers can be found.

11. Average percentage rate of change over a period = $\frac{\text{newvalue}-\text{oldvalue}}{\text{oldvalue}} \times \frac{100}{n} \%$ where n = no.of periods

Important: Remember that positive (+) percentage change implies a percentage increase, while a negative (-) percentage change implies a percentage decrease.

Average percentage increase or average annual growth rate over a given period

$$\text{A.P.I..} = \frac{\text{Finalvalue} - \text{Initialvalue}}{\text{Initialvalue}} \times \frac{100}{\text{Number of years}}$$

E11. The number of students selected from a particular institute at BANK tests has moved up to 1000 in the year 2000, from only 400 selections in the year 1996. What is the total percentage increase in the number of students selected from the institute? Also find the average annual growth rate for the institute.

Sol. Let's first find the total percentage increase in the number of selections for the institute.

Applying the percentage increase formula we get, percentage increase

$$= \frac{1000-400}{400} \times 100 = \frac{600}{400} \times 100 = 150\%$$

=> Average growth rate = $150/4 = 37.5\%$.

The average annual growth rate can be found out by standard formula also:

Average percentage Increase / Average annual growth rate over the given period

$$= \frac{1000-400}{400} \times \frac{100}{4} = \frac{600}{400} \times \frac{100}{4} = \frac{6 \times 100}{16} = 37\frac{1}{2}\%. \text{ Obviously, both answers are same.}$$

Crucial concepts in Percentages

Let's start with a number X (= 1X)

X increased by 10% would become $X + 0.1 X = 1.1 X$

X increased by 1% would become $X + 0.01 X = 1.01 X$

X increased by 0.1% would become $X + 0.001 X = 1.001 X$

X decreased by 10% would become $X - 0.1 X = 0.9 X$

X decreased by 1% would become $X - 0.01 X = 0.99 X$

X decreased by 0.1% would become $X - 0.001 X = 0.999 X$

X increased by 200% would become $X + 2X = 3X$

X decreased "by 300% would become $X - 3X = -2 X$

E12. Complete all entries in the following table.

Increased by Number	10%	25%	40%	100%	300%
50	55				
75		93.75			
150					
500				1000	
600					2400
1000				2000	
2000					
2500		3125			

Solution:

Increased by Number	10%	25%	40%	100%	300%
50	55	62.5	70	100	200
75	82.5	93.75	105	150	300
150	165	187.5	210	300	600
500	550	625	700	1000	2000
600	660	750	840	1200	2400
1000	1100	1250	1400	2000	4000
2000	2200	2500	2800	4000	8000
2500	2750	3125	3500	5000	10000

Some important formulae

- $\text{Average} = \frac{\text{Sum of the Observations}}{\text{Number of Observations}}$
- $\text{Capacity utilisation} = \frac{\text{Production}}{\text{Installed Capacity}}$
- $\text{Revenue} = \text{No. of items sold} \times \text{price per unit}$
- $\text{Turnover} = \text{Price} \times \text{quantity (volume)}$
- $\text{Deficit} = \text{Demand} - \text{Supply}$
- $\text{Income per share} = \frac{\text{Total Profit}}{\text{Number of Shares}}$
- $\text{Crop yield} = \frac{\text{Production of Crop}}{\text{Area under Cultivation}} \times \text{Area under Cultivation}$
- In case of pie-chart, $1\% = (18/5)^\circ$
- 1 million = 10 lakhs
- $\text{Average Speed} = \frac{\text{Distance Covered}}{\text{Time Taken}}$
- $\text{Total or gross profit} = \text{Sales} - \text{Expenditure} = (\text{S.P.} - \text{C.P.})$

- Net Profit = Revenue - Expenses
- Deficiency = Requirement - Availability
- Balance of trade or trade gap = Exports - Import
- Dividend per share = $\frac{\text{Total dividend}}{\text{Number of shares}}$
- Per capita income = $\frac{\text{Total income}}{\text{population}}$
- In case of pie-chart, $1^\circ = (5/18)\%$
- 1 billion = 100 corers = 1000 million

Important Abbreviations

- 1) GDP - Gross Domestic Product
- 2) GNP - Gross National Product
- 3) NP - Net Profit
- 4) CPI - Consumer Price Index
- 5) CI - Compound Interest
- 6) SI - Simple Interest
- 7) SP - Selling Price
- 8) CP - Cost Price
- 9) GPM - Gross Profit Margin
- 10) NPM - Net Profit Margin

Some more useful tipsto Save time!

- Learn tables up to 20 by heart.
- Approximate your calculations judiciously .
- Always look at the options. If they are sufficiently widely spaced, then you can make a quick choice.
- Always set an order of question sets that you have to attempt first, second, third etc.

Intelligent Strategy!

- If one question in a set (of 6 - 7 questions) is tough or involves very large calculations, leave it without a second thought. **Avoid blunders!**
- Learn reciprocals of numbers, equivalent percentages of fractions etc. to speed up your calculations.