Probability: Assignment 5

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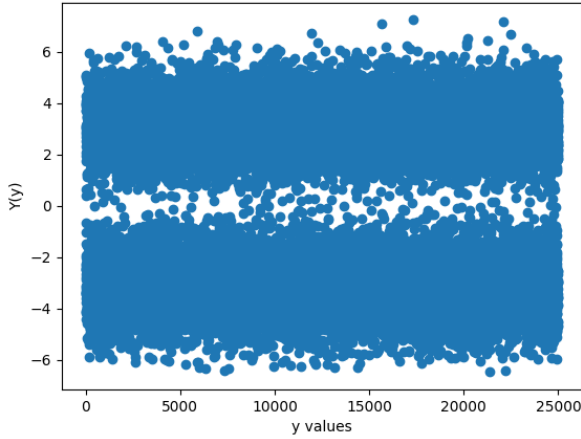
Fig. 1.3. The Plot of Y

1 MAXIMUM LIKELIHOOD

1.1 Generate equiprobable $X \in \{1, -1\}$.
Solution:
The samples are stored in 5_1.dat file.

Assignment 5/codes/equi.py

1.2 Generate
$$Y = AX + N, \qquad (1.2.1)$$

where $A = 5$ dB, and $N \sim \mathcal{N}(0, 1)$.
Solution: The samples are stored in 5_2.dat file.

Assignment 5/codes/equi.py

1.3 Plot Y using a scatter plot.
Solution: = The code for the plot in Figure 1.3 is below.

Assignment 5/codes/equi.py

1.4 Guess how to estimate X from Y.
Solution:
From the plot in Figure 1.3, we can see that

$$\hat{X} = \begin{cases} 1 & \text{for } Y < 0 \\ -1 & \text{for } Y > 0 \end{cases}$$

1.5 Find
$$P_{e|0} = \Pr\left(\hat{X} = -1 | X = 1\right) \qquad (1.5.1)$$
and
$$P_{e|1} = \Pr\left(\hat{X} = 1 | X = -1\right) \qquad (1.5.2)$$

Solution:
The code for finding the $P_{e|0}$ and $P_{e|1}$ is in

Assignment 5/codes/5_5.py

The values of $P_{e|0} = 0.00071$ and $P_{e|1} = 0.00081$.

1.6 Find P_e assuming that X has equiprobable symbols.
Solution:
Using law of total probability, we can find the value of P_e,. The value of $P(X = 1) = 0.5$ and $P(X = -1) = 0.5$ as X is equiprobable.

$$\begin{aligned}
P_e &= P(X = 1) \times P_{e|0} + P(X = -1) \times P_{e|1} \\
&= 0.5 \times 0.00071 + 0.5 \times 0.00081 \\
&= 0.000355 + 0.000405 \\
&= 0.00076 \\
&= 7.6 \times 10^{-4}
\end{aligned}$$

1.7 Verify by plotting the theoretical P_e with respect to A from 0 to 10 dB.
Solution:
The code for finding thre theoretical P_e wrt A from 0 to 10dB is in

Assignment 5/codes/5_6.py

Accoringly, the plot is in Figure 1.7.

1.8 Now, consider a threshold δ while estimating X from Y. Find the value of δ that maximizes the theoretical P_e.

1.9 Repeat the above exercise when
$$p_X(0) = p \qquad (1.9.1)$$
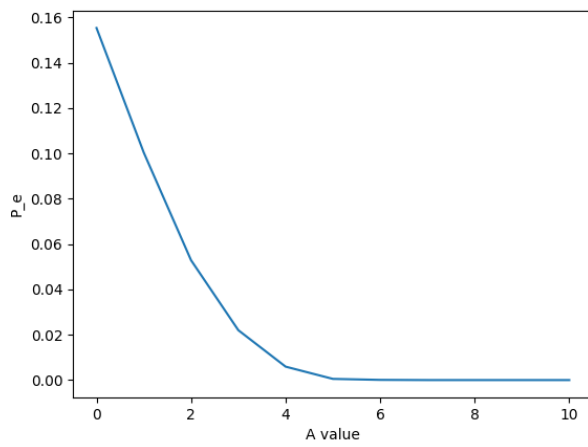
1.10 Repeat the above exercise using the MAP criterion.

Fig. 1.7. The Plot of P_e wrt A