Probability: Assignment 4

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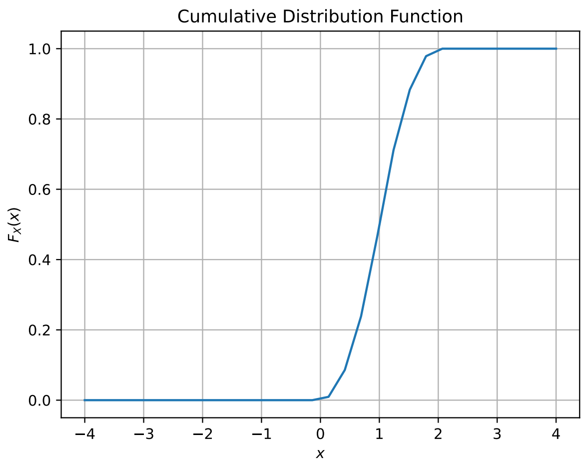

Fig. 1.2. The CDF of T

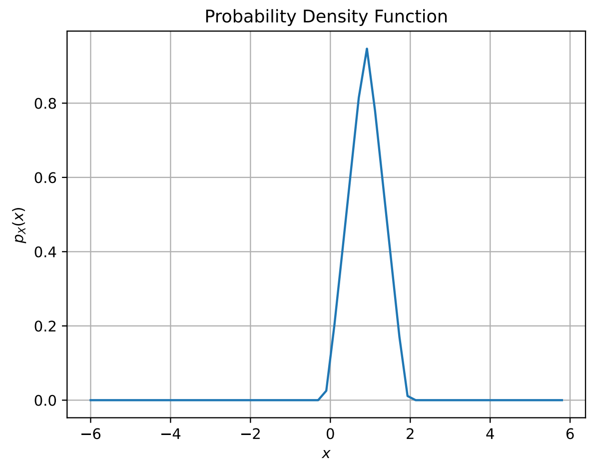

Fig. 1.3. The PDF of T

Contents

1 Triangular Distribution

1.1 Generate

$$T = U_1 + U_2 \tag{1.1.1}$$

Solution: The code of generating 10^6 random samples is in

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Assignment 4/codes/txrand.py
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1.2 Find the CDF of T.
Solution: The code for the CDF of T is in

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Assignment 4/codes/txrand.py
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1.3 Find the PDF of T.
Solution: The code for the PDF of T is in

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Assignment 4/codes/pdf_plot.py
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1.4 Find the theoretical expressions for the PDF and CDF of T.

Solution: We know that if X and Y are independent distributions, then we can use convolution formula for getting the PDF of F_{X+Y} as

$$f_{X+Y}(t) = \int_{\infty}^{\infty} f_X(x) f_Y(t-x) dx \tag{1.4.1}$$

And the CDF of F_{X+Y} is given as

$$F_{X+Y}(t) = \int_{-\infty}^{t} f_{X+Y}(x) dx \tag{1.4.2}$$

Accordingly, for our casee U_1 and U_2 are uniform i.i.d. random variables in [0, 1]. Then, $0 \le U_1 + U_2 \le 2$. Again we know that

$$f_{U_1}(x) = \begin{cases} 0 & \text{for x} < 0 \\ 1 & \text{for x} \ge 0 \end{cases}$$

$$F_{U_1}(x) = \Pr(U_1 \le x) = \begin{cases} 0 & \text{for x} < 0 \\ x & \text{for } x \in [0,1] \\ 1 & \text{for x} > 1 \end{cases}$$

and

$$f_{U_2}(x) = \begin{cases} 0 & \text{for x} < 0 \\ 1 & \text{for x} \geq 0 \end{cases}$$

$$F_{U_2}(x) = \Pr(U_2 \leq x) = \begin{cases} 0 & \text{for x} < 0 \\ x & \text{for } x \in [0,1] \\ 1 & \text{for x} > 1 \end{cases}$$

Now we deal with the interval from 0 to 2. It is useful to break this down into two cases (i) $0 \leq t < 1$ and (ii) $1 \leq t < 2$.

Case (i): $0 \leq t < 1$. The product $f_{U_1}(x)f_{U_2}(t-x)$ is 1 in someplaces and 0 elsewhere. So, for $f_{U_2}(t-x) = 1$ we need $t - x \geq 0$ and thus $x \leq t$ and therefore, we integrate from $x = 0$ and $x = t$.

$$\begin{aligned} f_T(t) &= \int_0^t 1.1dt = t \\ F_T(t) &= \int_0^t f_T(t)dt \\ &= \int_0^t tdt \\ &= \frac{t^2}{2} \end{aligned}$$

Case (ii): For $1 \leq t < 2$, Again for $f_{U_2}(t-x) = 1$ we need $t - x \leq 1$ and thus $x \leq t - 1$ and therefore, we integrate from $x = t-1$ and $x = 1$.

$$\begin{aligned} f_T(t) &= \int_{t-1}^1 1.1dx \\ &= [t]_{t-1}^1 = 2 - t \\ F_T(t) &= \int_0^t f_T(t)dt \\ &= \int_0^1 tdt + \int_1^t (2-t)dt \\ &= \left[\frac{t^2}{2}\right]_0^1 + \left[2t - \frac{t^2}{2}\right]_1^t \\ &= 2t - \frac{t^2}{2} - 1 \end{aligned}$$

So, the CDF for T is given by

$$F_T(t) = \Pr(T \leq t) = \begin{cases} 0 & x < 0 \\ \frac{t^2}{2} & 0 \leq t < 1 \\ 2t - \frac{t^2}{2} - 1 & 1 \leq t < 2 \\ 1 & x \geq 2 \end{cases} \tag{1.4.3}$$

and the PDF for T is given by

$$f_T(t) = \begin{cases} t & 0 \leq t < 1 \\ 2 - t & 1 \leq t < 2 \\ 0 & \text{Elsewhere} \end{cases} \tag{1.4.4}$$

1.5 Verify your results through a plot.
Solution: The Figures 1.2 and 1.3 suffice the purpose.