

Probability: Assignment 3

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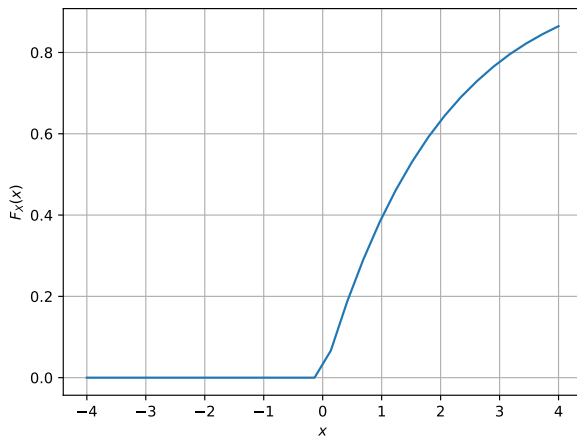


Fig. 1.1. The CDF of X

So, V is in $[0, \infty)$ and is an increasing function such that for $u \in U$ and $v \in V$ we have

$$v = -2\ln(1 - u)$$

$$\frac{-v}{2} = \ln(1 - u)$$

$$1 - u = e^{\frac{-v}{2}}$$

$$u = 1 - e^{\frac{-v}{2}}$$

Accordingly, $F_V(v) = F_V(1 - e^{\frac{-v}{2}})$

$$F_V(x) = \Pr(V \leq x) = \begin{cases} 0 & \text{for } x < 0 \\ 1 - e^{\frac{-x}{2}} & \text{for } x \geq 0 \end{cases}$$

1 FROM UNIFORM TO OTHER

1.1 Generate samples of

$$V = -2\ln(1 - U) \quad (1.1.1)$$

and plot its CDF.

Solution:

Assignment 3/codes/uxrand.py
Assignment 3/codes/coeffs.h

1.2 Find a theoretical expression for $F_V(x)$.

Solution: We know from 1.2 that

$$F_U(x) = \Pr(U \leq x) = \begin{cases} 0 & \text{for } x < 0 \\ x & \text{for } x \in [0, 1] \\ 1 & \text{for } x > 1 \end{cases}$$

$$0 \leq U \leq 1 \implies 1 - 0 \geq 1 - U \geq 1 - 1$$

$$\implies -\infty \leq \ln(1 - U) \leq 0$$

$$\implies \infty \geq -2\ln(1 - U) \geq 0$$

$$\implies 0 \leq -2\ln(1 - U) \leq \infty \implies 0 \leq V \leq \infty$$