# Modern Control Theory CH5120
# Project – 2

**Team Members**
**Sreedars S J ME19B174**
**Kavita Shekhavat AE20D401**
**Jeet Rajde CH17B067**
**Bharath Chandar N ME18B009**

## Model Predictive controller

### Introduction

MPC uses a model of the plant to make predictions about future plant outputs. It solves an optimization problem at each time step to find the optimal control action that drives the predicted plant output to the desired reference as close as possible.

Consider the system model

$$x(k+1) = Ax(k) + Bu(k) + Gdp(k)$$

$$y(k) = Cx(k) + Du(k) + Hdm(k)$$

where,
x(k) are the states
u(k) are manipulated inputs and,
y(k) are the measured outputs.
Vectors dp(k) and dm(k) are unmeasured disturbances to the state dynamics (process noise) and to the outputs (measurement noise), respectively.

### Cost function

The required input to the plant is calculated by minimizing a cost function defined along the prediction horizon, defined as a sum of quadratic future errors between the reference trajectory and predicted plant output and the predicted control effort.
The equation

$$J = (Rs - Y)T(Rs - Y) + \Delta UTR\Delta U$$

For MIMO system
Y=step prediction for output
Rs=Set-point trajectory into the future
∆U= future control moves
R⁻=Penalty on control moves

### MPC TUNING

The tuning parameters of the MPC controller are the cost function weight matrices uwt, R and Q, the control horizon Nu, the prediction horizon Np and the sampling time Ts for the discretization of the system.
R=The Measurement noise covariance matrix
Q=The process noise covariance matrix

$N_p$= The prediction horizon, determines the number of output predictions that are used in the optimization calculation.
$N_{c=}$ The control horizon determines the number of future control actions that are calculated in each optimization step.
$T_s$=Sampling time

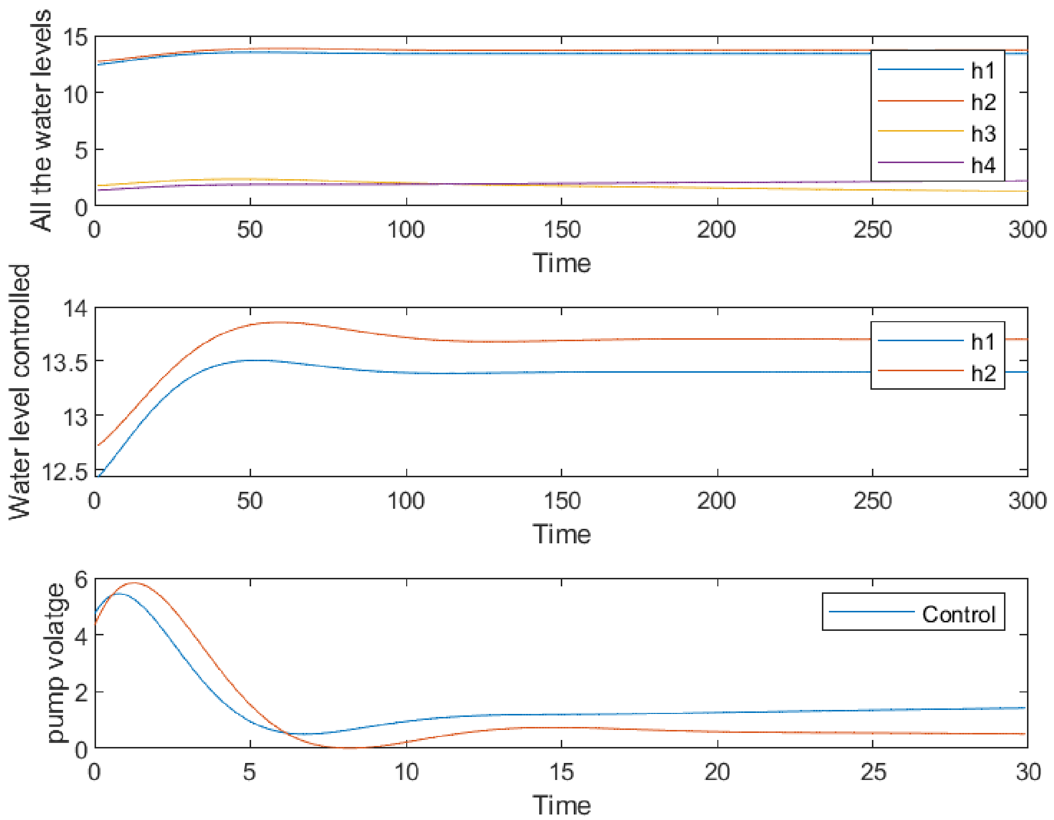
**The Four-Tank process**

This work presents an implementation of a stabilizing model predictive control. In this work, the quadruple-tank system has been considered. The coupled tank process is a two input two output process. The process inputs are v1 and v2 (input voltages to the pumps) and the outputs are h1 and h2 (voltages from level measurement devices).
In the 4 cases we are finding the set point of 2 tank height which are controlled and the other are measured.

**Results and Simulations**

**Note: (The first subplot in the figure shows all the tank heights together, the second subplot shows the controlled heights and third is the Pump Voltage.)**

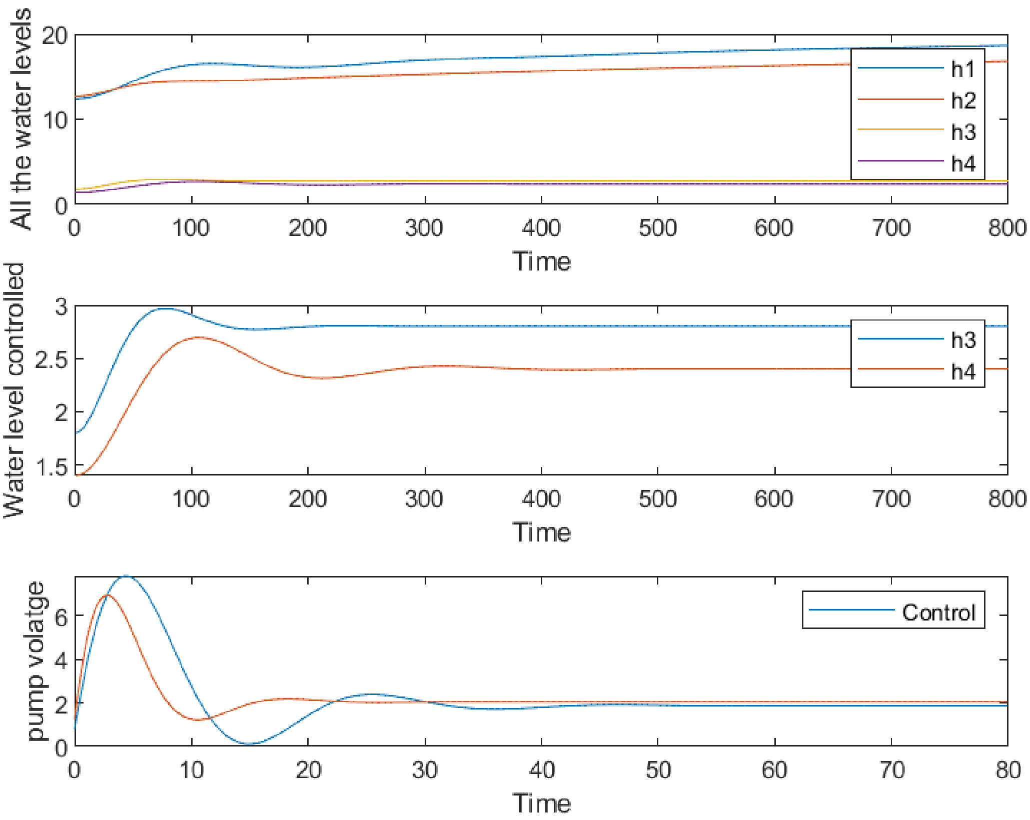
**Case 1: Constraint MPC to control h1, h2 when h3, h4 are measured; set-point for [h1 h2] is [13.4 13.7]**




i) The set point we set were [h1 h2] = [13.4 13.7] and by implementing the Controller after 300 iterations we were able to achieve the set points of [h1 h2] = [13.4 13.7006].

ii) So we can see very good control performance and the Np and Nc were tuned and found to be 30 and 15 respectively.

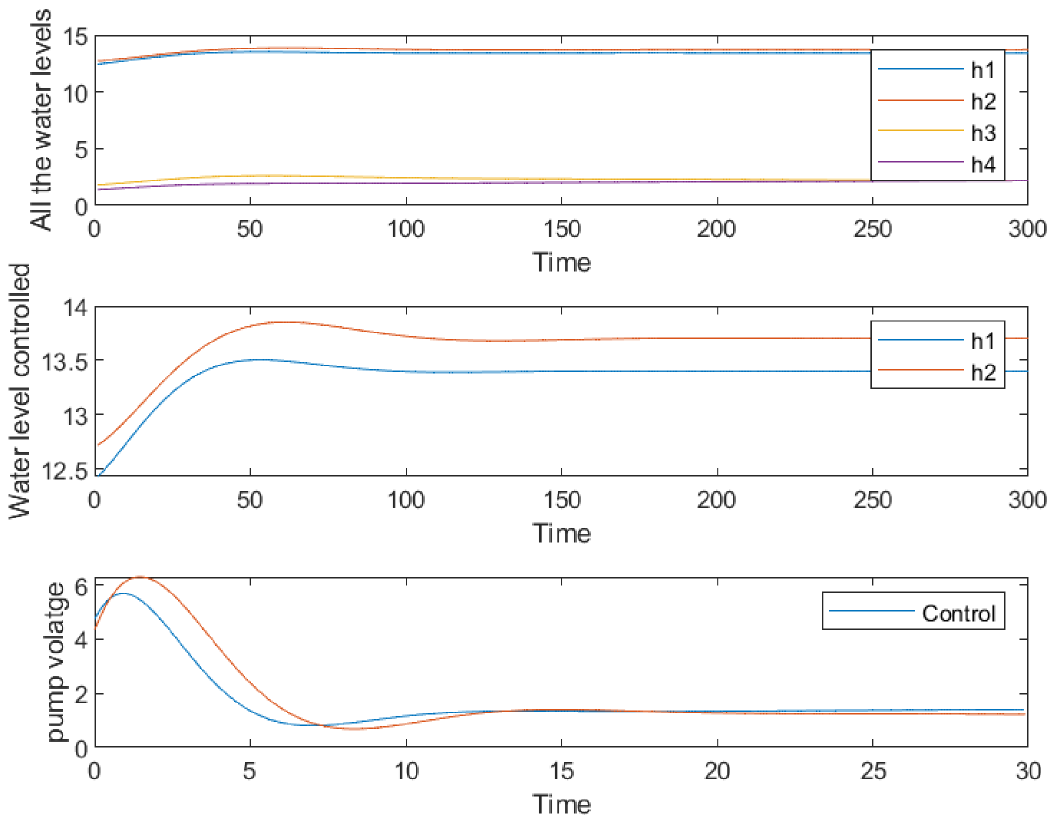
**Case 2: Constraint MPC to control h3, h4 when h1, h2 are measured; set-point for [h3 h4] is [2.8 2.4]**




i) The set point we set were [h1 h2]=[2.8 2.4] and by implementing the Controller after 800 iterations we were able to achieve the set points of [h1 h2]=[2.800012.4000].

ii) So we can see very good control performance and the Np and Nc were tuned and found to be 30 and 5 respectively.

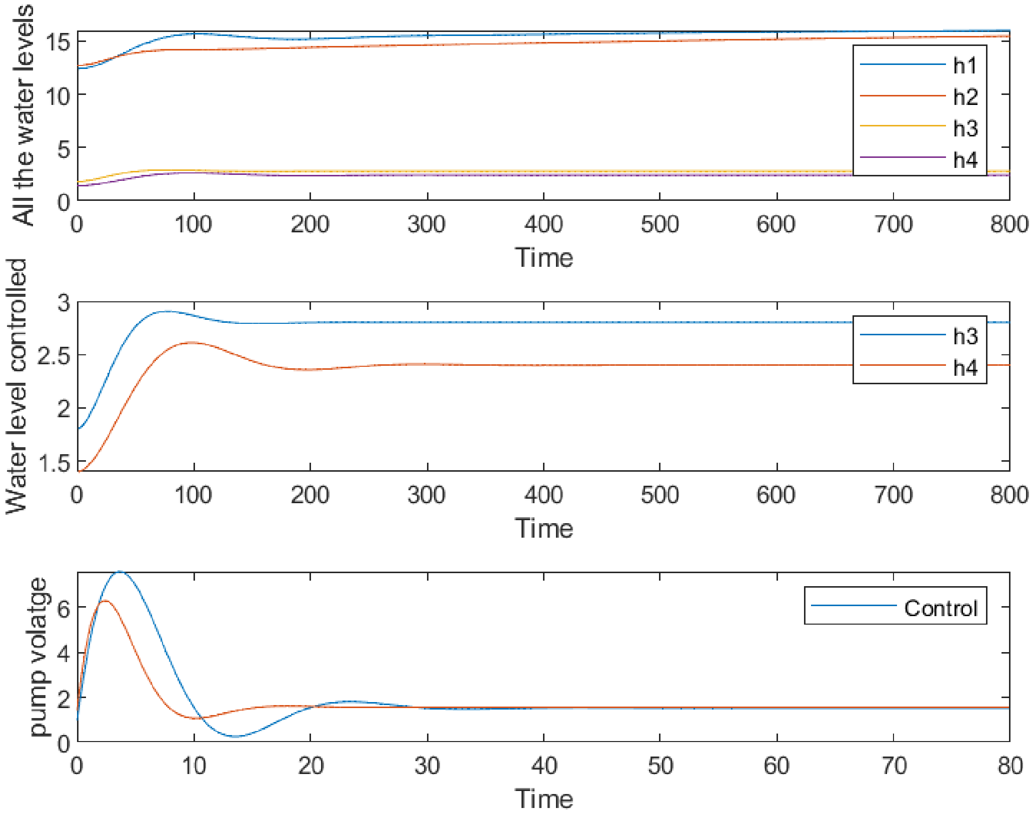
**Case 3: Constraint MPC to control h1, h2 when h1, h2 are measured; set-point for [h1 h2] is [13.4 13.7]**




i) The set point we set were [h1 h2]=[13.4 13.7] and by implementing the Controller after 300 iterations we were able to achieve the set points of [h1 h2]=[13.3996 13.7005].

ii) So we can see very good control performance and Np and Nc were tuned and found to be 30 and 15 respectively.

**Case 4: Constraint MPC to control h3, h4 when h3, h4 are measured; set-point for [h3 h4] is [2.8 2.4]**




i) The set point we set were [h3 h4]=[2.8 2.4] and by implementing the Controller after 800 iterations we were able to achieve the set points of [h1 h2]=[2.8000 2.4000].

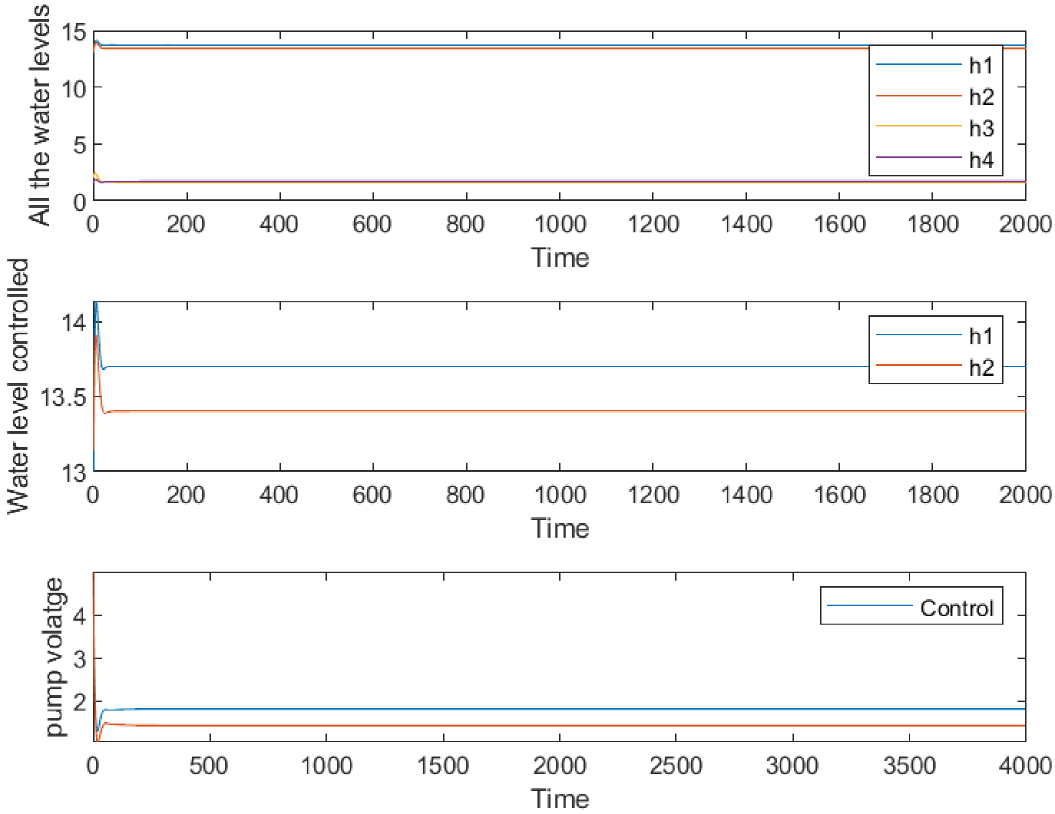
ii) So we can see very good control performance and the Np and Nc were tuned and found to be 35 and 8 respectively.

3) The Performance deteriotions between the cases can be due to both the Kalman Filter performance and MPC Performance but it is more significant due to **MPC Performance**

4) The few ways to improve the overall performance are by changing weight matrix, R and Q, the control horizon Nc, the prediction horizon Np and the sampling time Ts for the discretization of the system.
The prediction horizon Np determines the number of output predictions that are used in the optimization calculation.
The control horizon Nc determines the number of future control actions that are calculated in each optimization step.
Examples given below Note: (All the changes are made with respect to the first case)

**a)** The matrix Q penalizes the tracking errors and guides the servo performance of the control system. The matrix R is a move suppression factors that change the aggressiveness of the controller and assure a smooth control action.

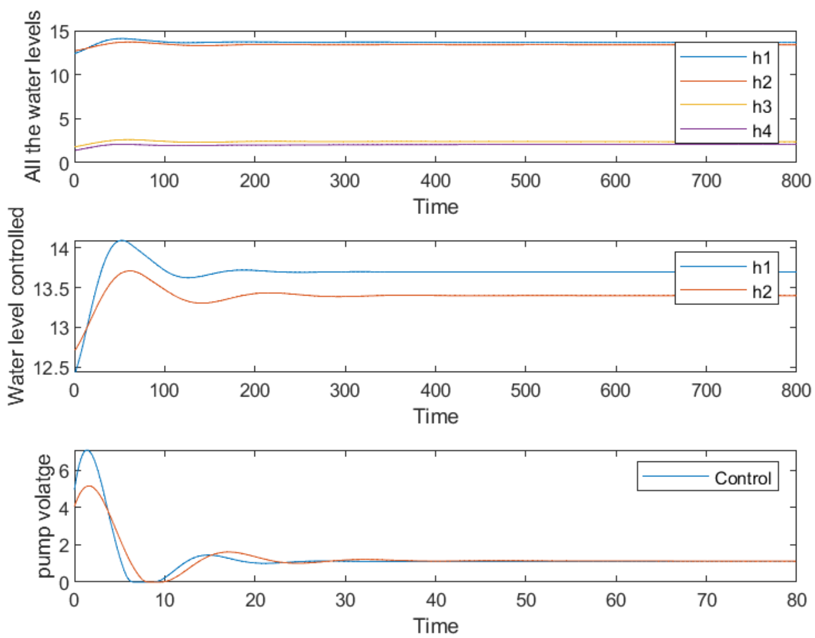
**b)** **On changing the sample time from 0.1 seconds to 2 seconds**



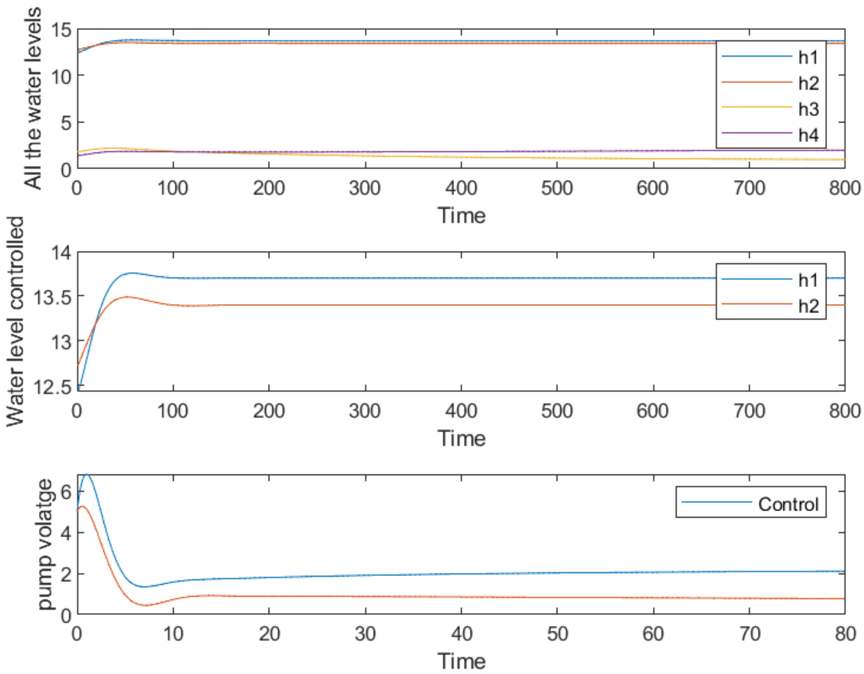

You can observe that Smaller sampling time Ts demand more aggressive control, while larger time constants result in less aggressive action.

**c)On changing Np and Nc**

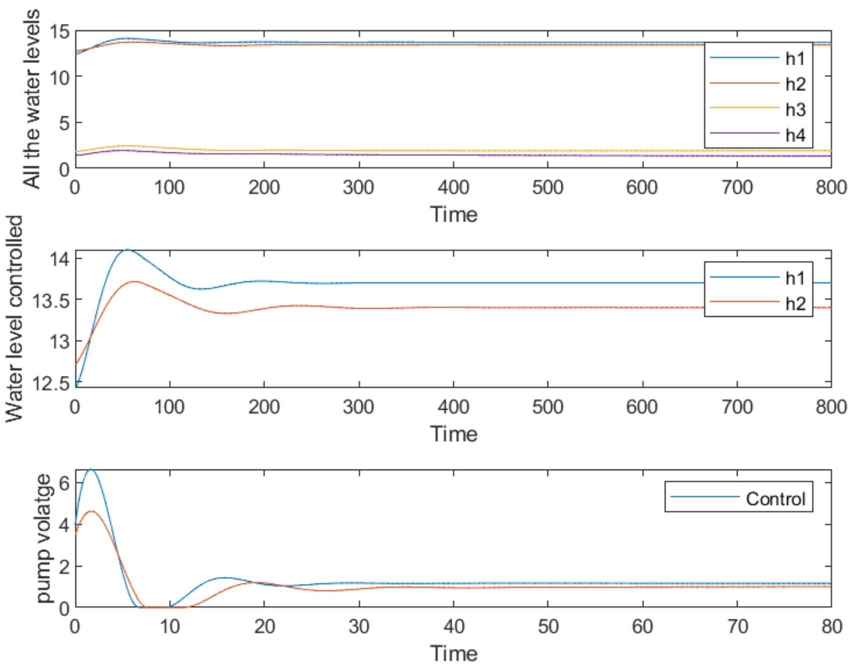
Initially the Np and Nc value were Np=20 and Nc=5 we get




On increasing Np and keeping Nc constant ie Np=30 and Nc=5 we get




On keeping Np constant and increasing Nc ie Np=20 and Nc=15 we get

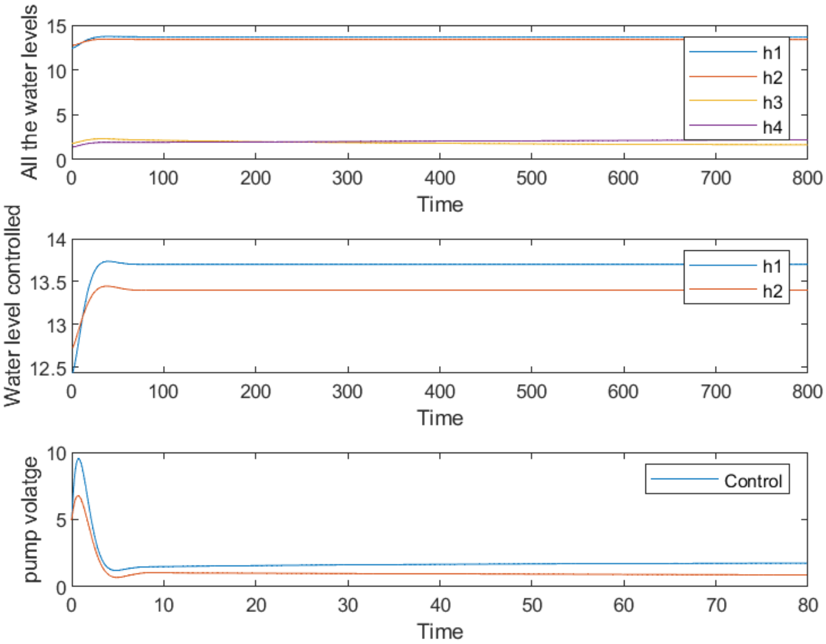



From these graphs we can infer that

i) A long prediction horizon (Np) leads to better performance and has a stabilizing effect, but it increases the computation burden.

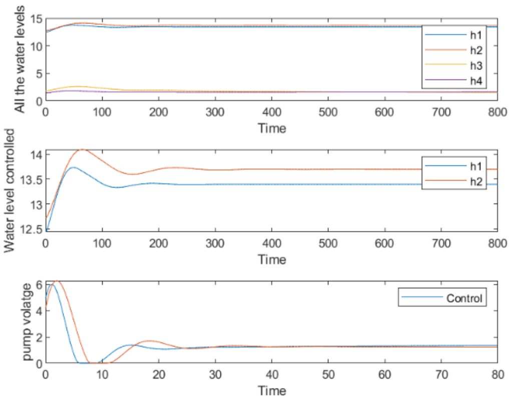
ii) A short control horizon (Nc) leads to a controller that is moderately insensitive to uncertainties and modelling errors, whereas a long control horizon results in unnecessary control action and long computation time.

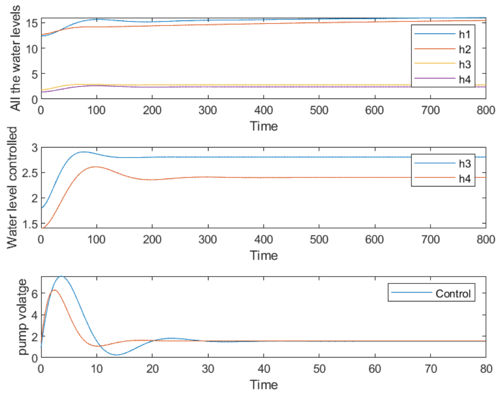
**d)On changing the weight matrix**
we change the weight matrix from [4 4] to [0.5 0.5] keeping the Np and Nc constant i.e. 20 and 5 resp.




We can see with fixed Np and Nc, Increasing the move weight slows down the system.

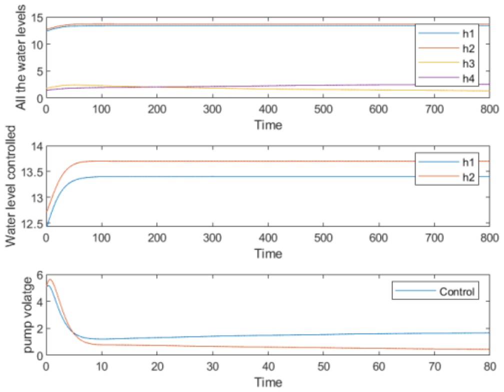
**B) Effect of changes of Control and Prediction Horizons in case 3 and 4.**
**Left Side= Case 3 and Right Side=Case 4**

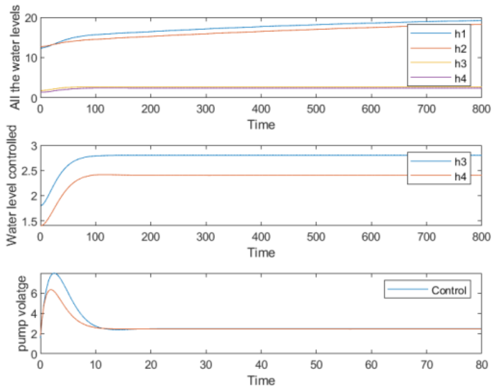


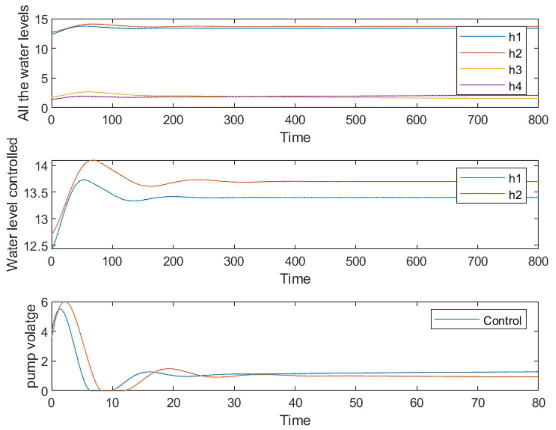

1)Initially Np=20 and Nc =5

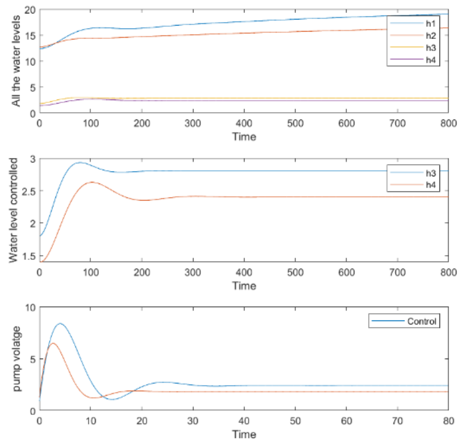
Initially Np=35 and Nc =8




2)Keeping Nc=5 constant increasing Np=40     Keeping Nc=8 constant increasing Np=55




3)Keeping Np=20 constant increasing Nc=15     Keeping Np=35 constant increasing Nc=30

From the above 3 cases we can infer that,

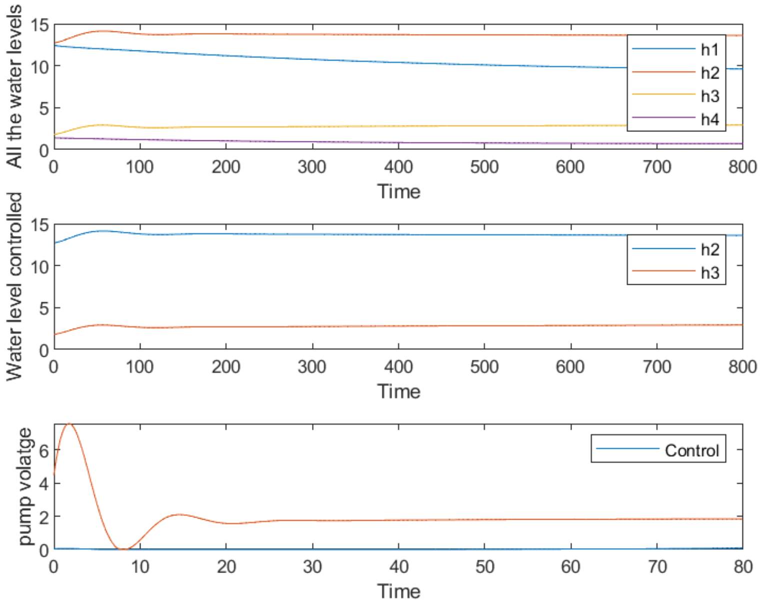
2)With fixed control horizon Nc, Increasing Np will slow down the system.
3)With fixed prediction Horizon Np, Increasing Nc speeds the system.

**C) Case 5: Constraint MPC to control h2 and h3 and h1,h4 are measured with set point for [h2 h3]=[13.7 2.8].**

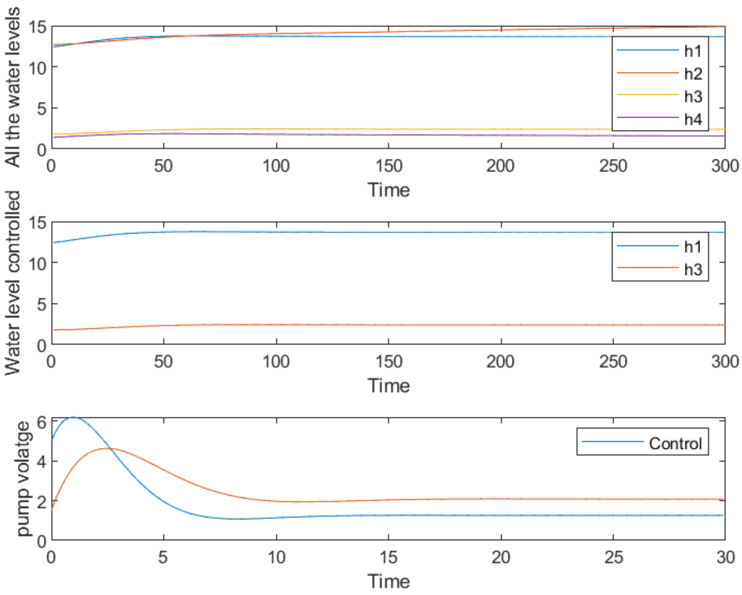
i) By implementing the Controller after 800 iterations the values it reached were [h2 h3]=[14.4414 1.8051] and Np=20 and Nc =5.

ii) We can infer that there is a significant variation in the settling values with h2 having an error of 5.42% and h3 with 35.5%.So this case is Uncontrollable.

iii) The possible reason behind the unreachability to set point is probably the tanks h1 and h4 are filled by one valve and the tanks h2 and h3 by other and both are not connected with each other. So since here we are asked to control h2, h3 when measuring h1 and h4, the MPC was not able to track the set point.




**Case 6: Constraint MPC to control h1 and h3 and h1, h4 are measured with set point for [h2 h4]=[13.7 2.4].**




i) The set point we set were [h2 h4]=[13.7 2.4] and by implementing the Controller after 300 iterations we were able to achieve the set points of [h2 h4]=[13.7 2.4].

ii) So we can see very good control performance and the Np and Nc were tuned and found to be 45 and 20 respectively.

## Conclusion

In all the cases except 5 we were able to achieve the set point tracking, but in case 5 where h2 and h3 were controlled and h1,h4 were measured the MPC was not able to achieve the set point.