

**Q1.** Suppose a child psychologist claims that the average time working mothers spend talking to their children is at least 11 minutes per day. You conduct a random sample of 1000 working mothers and find they spend an average of 11.5 minutes per day talking with their children. Assume prior research suggests the population standard deviation is 2.3 minutes. Conduct a test with a level of significance of  $\alpha = 0.05$ .

Given,

$$\begin{aligned}\bar{X} &= 11.5 \quad (\text{sample mean}) \\ \mu &= 11 \quad (\text{population mean under the null hypothesis}) \\ \sigma &= 2.3 \quad (\text{population standard deviation}) \\ n &= 1000 \quad (\text{sample size}) \\ \alpha &= 0.05 \quad (\text{level of significance})\end{aligned}$$

Null Hypothesis  $H_0: \mu \geq 11$

Alternative Hypothesis  $H_1: \mu < 11$

We will use the Z-test for the hypothesis test since we know population standard deviation.

$$\begin{aligned}Z &= \frac{\bar{X} - \mu}{\sigma / \sqrt{n}} \\ &= \frac{11.5 - 11}{2.3 / \sqrt{1000}} = \frac{0.5}{2.3 / \sqrt{1000}} \\ &= \frac{0.5}{0.0728} \approx \underline{\underline{6.87}}\end{aligned}$$

The critical value of  $z$  from the table is  $-1.645$ .

Here

$$6.87 > -1.645$$

⇒ We accept the null hypothesis.

⇒ The average time working mothers spend talking to their children is at least 11 minutes per day.

**Q2.** A coffee shop claims that their average wait time for customers is less than 5 minutes. To test this claim, a sample of 40 customers is taken, and their wait times are recorded. The sample mean wait time is found to be 4.6 minutes with a standard deviation of 0.8 minutes. Perform a hypothesis test at a significance level of 0.05 and determine whether there is enough evidence to support the coffee shop's claim.



Given,

$$\bar{X} = 4.6$$

$$\mu = 5$$

$$\sigma = 0.8$$

$$n = 40$$

$$\alpha = 0.05$$

Null Hypothesis  $H_0: \mu \geq 5$

Alternative Hypothesis  $H_1: \mu < 5$

We will use the Z-test for the hypothesis test since we know the sample standard deviation.

$$Z = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{4.6 - 5}{\frac{0.8}{\sqrt{40}}}$$

$$= \frac{-0.4}{\frac{0.8}{\sqrt{40}}}$$

$$= \frac{-0.4}{0.1265} \approx \underline{\underline{-3.16}}$$



The critical value of  $z$  from the table is  $-1.645$

Here,

$$-3.16 < -1.645$$

$\Rightarrow$  We reject the null hypothesis

$\Rightarrow$  Average wait time for customers is less than 5 minutes. Therefore, coffee shop's claim is valid.