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Faculty of Engineering, Architecture and Science

Department of Electrical and Computer Engineering

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ASSIGNMENT NO.	**3**

Report Title	Fourier Series Analysis Using MATLAB

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Group No.	N/A
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PART A

A1. (Moiz)

A.1 $x_1(t) = \cos\frac{3\pi}{10}t + \frac{1}{2}\cos\frac{\pi}{10}t$. derive expression for the exponential fourier series D_n

$$= \frac{1}{2}e^{j\frac{3\pi}{10}t} + \frac{1}{2}e^{-j\frac{\pi}{10}t} + \frac{1}{2}\left(\frac{1}{2}e^{j\frac{\pi}{10}t} + \frac{1}{2}e^{-j\frac{\pi}{10}t}\right)$$

$$= \frac{1}{2}e^{j\frac{3\pi}{10}t} + \frac{1}{2}e^{-j\frac{3\pi}{10}t} + \frac{1}{4}e^{j\frac{\pi}{10}t} + \frac{1}{4}e^{-j\frac{\pi}{10}t}$$

Fundamental frequency:

$$\frac{3\pi}{10}\cdot\frac{10}{\pi} = 3 \qquad T_0 = \frac{2\pi}{\left(\frac{\pi}{10}\right)} = 2\pi\cdot\frac{10}{\pi} = 20$$

$$\frac{GCE}{LCM} = \frac{\pi}{10} = \frac{\pi}{10}$$

$$j\frac{\pi}{10}t = j\frac{3\pi}{10}t \qquad j\frac{\pi}{10}t = j\frac{\pi}{10}t$$

$$n = 3, -3 \qquad n = 1, -1$$

$$D_3 = \frac{1}{2} \qquad D_{-3} = \frac{1}{2} \qquad D_1 = \frac{1}{4} \qquad D_{-1} = \frac{1}{4}$$

$$D_n = \frac{1}{20}\int_{-10}^{10}\left[\frac{1}{2}e^{j\frac{3\pi}{10}t} + \frac{1}{2}e^{-j\frac{3\pi}{10}t} + \frac{1}{4}e^{j\frac{\pi}{10}t} + \frac{1}{4}e^{-j\frac{\pi}{10}t}\right]e^{-j\frac{\pi}{10}t}$$

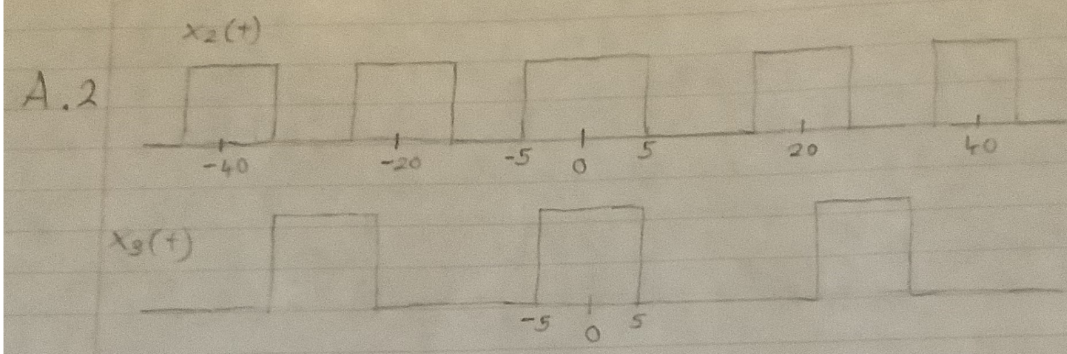
$$D_n = \frac{1}{20}\left[\frac{e^{j(3-n)\pi} - e^{-j(3-n)\pi}}{2j(3-n)\frac{\pi}{10}} + \frac{e^{j(3+n)\pi} - e^{-j(3+n)\pi}}{2j(3+n)\frac{\pi}{10}} + \frac{e^{j(1+n)\pi} - e^{-j(1+n)\pi}}{4j(1+n)\frac{\pi}{10}} + \frac{e^{j(1-n)\pi} - e^{-j(1-n)\pi}}{4j(1-n)\frac{\pi}{10}}\right.$$

$$D_n = \frac{1}{2}\left[\mathrm{sinc}\left[(3-n)\pi\right] + \mathrm{sinc}\left[(3+n)\pi\right] + \frac{1}{2}\mathrm{sinc}\left[(1+n)\pi\right] + \frac{1}{2}\mathrm{sinc}\left[(1-n)\pi\right]\right.$$

Figure 1: The expression for the Exponential Fourier Series coefficients of x1(t)

A2. (Moiz)

A.2



$x_2(t)$: $T_0 = 20$ $\quad \omega_0 = \frac{2\pi}{20} = \frac{\pi}{10}$

$$D_n = \frac{1}{20}\left[\int_{-5}^{5} (1) e^{-jn\frac{\pi}{10}t}\, dt\right] = \frac{1}{20}\left[\frac{1}{-jn\frac{\pi}{10}} e^{-jn\frac{\pi}{10}t}\right]_{-5}^{5}$$

$$D_n = \frac{1}{20}\left[-\frac{10}{jn\pi} e^{-jn\frac{\pi}{2}} + \frac{10}{jn\pi} e^{jn\frac{\pi}{2}}\right] = \frac{1}{n\pi}\sin\left(\frac{\pi n}{2}\right)$$

$x_3(t)$: $T_0 = 40$ $\quad \omega_0 = \frac{2\pi}{40} = \frac{\pi}{20}$ $\quad \Big| \quad D_n = \frac{1}{40}\int_{-5}^{5} (1) e^{-jn\frac{\pi}{20}t}\, dt$

$$D_n = \frac{1}{40}\left[\frac{1}{-jn\frac{\pi}{10}} e^{-jn\frac{\pi}{10}t}\right]_{-5}^{5} \qquad D_n = \frac{1}{40}\left[-\frac{20}{jn\pi} e^{-\frac{jn\pi}{4}} + \frac{20}{jn\pi} e^{\frac{jn\pi}{4}}\right]$$

$$\therefore D_n = \frac{1}{n\pi}\sin\left(\frac{n\pi}{4}\right)$$

Figure 2: The expression for the Exponential Fourier Series coefficients of x2(t) and x3(t)

A3. (Moiz)

```
A3.m

function [D]=Dn(d,n)
D1 = [0.5,0,-0.5*1i,0,0.5*1i,0,0.5];
D2 = (1/(n.*pi)*sin((n*pi)/2));
D3 = (1/(n.*pi)*sin((n*pi)/4));
if (d==1)
    D=D1;end
if (d==2)
    D=D2;end
if (d==3)
    D=D3;end
end
```

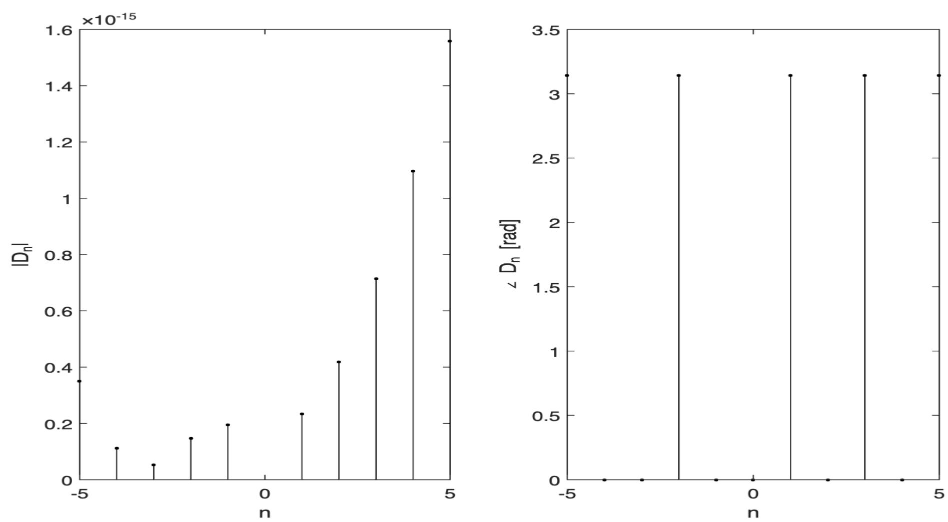
Figure 3: Code for A3

A4. (Sreeja)

a) -5 <= n <= 5;

```
clf;
n = (-5:5);
D_n = 1./2.*((1./(pi.*n)).*sin((3-n).*pi)) + (1./pi.*n).*sin((3+n).*pi) + (1./(2.*n.*pi).*sin((1+n).*pi)) + (1./(2.*n.*pi).*sin((1-n).*pi));
subplot(1,2,1); stem(n,abs(D_n),'.k');
xlabel('n'); ylabel('|D_n|');
subplot(1,2,2); stem(n,angle(D_n),'.k');
xlabel('n'); ylabel('\angle D_n [rad]');
```

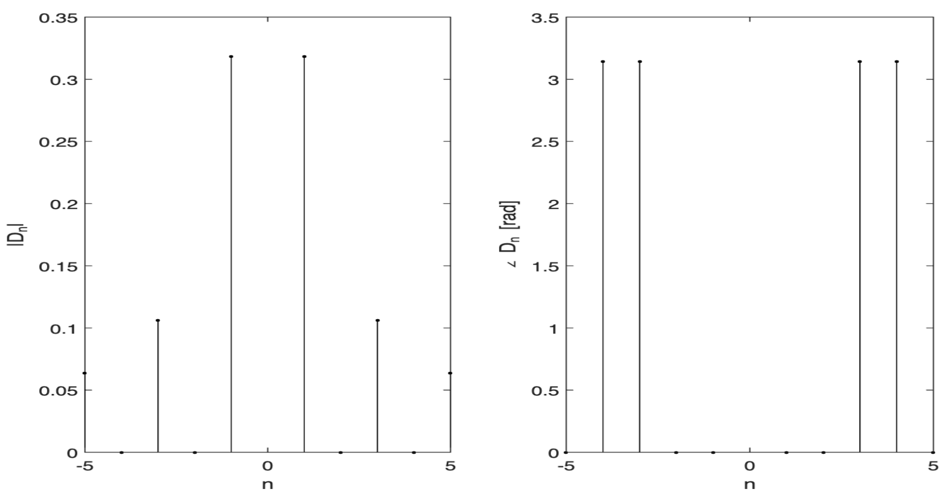
Figure 4: Code for x1(t) for the interval [-5:5]



Graph 1: Graph for Figure 4.

```
clf;
n = (-5:5);
D_n = (1./(n.*pi).*sin((n.*pi)./2));
subplot(1,2,1); stem(n,abs(D_n),'.k');
xlabel('n'); ylabel('|D_n|');
subplot(1,2,2); stem(n,angle(D_n),'.k');
xlabel('n'); ylabel('\angle D_n [rad]');
```

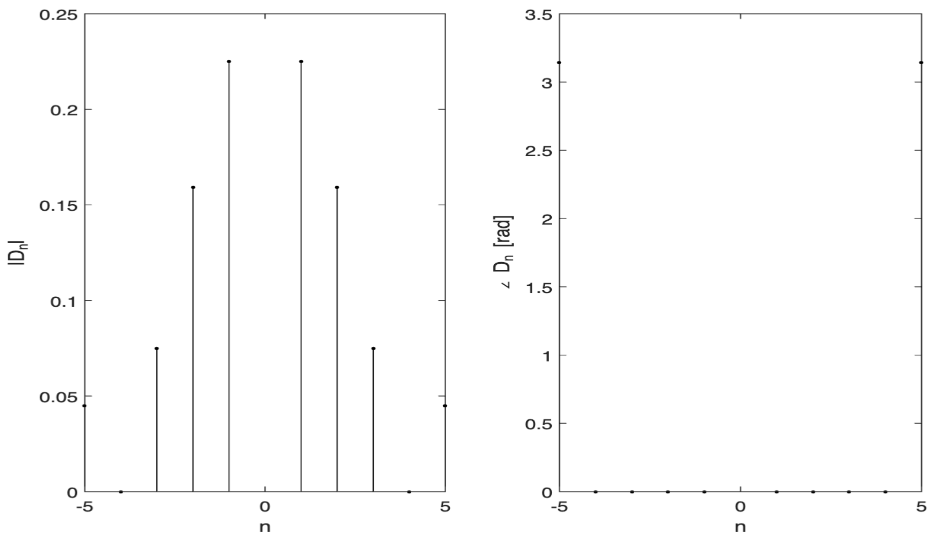
Figure 5: Code for x2(t) for the interval [-5:5]



Graph 2: Graph for Figure 5.

```
clf;
n = (-5:5);
D_n = (1./(n.*pi).*sin((n.*pi)./4));
subplot(1,2,1); stem(n,abs(D_n),'.k');
xlabel('n'); ylabel('|D_n|');
subplot(1,2,2); stem(n,angle(D_n),'.k');
xlabel('n'); ylabel('\angle D_n [rad]');
```

Figure 6: Code for x3(t) for the interval [-5:5]

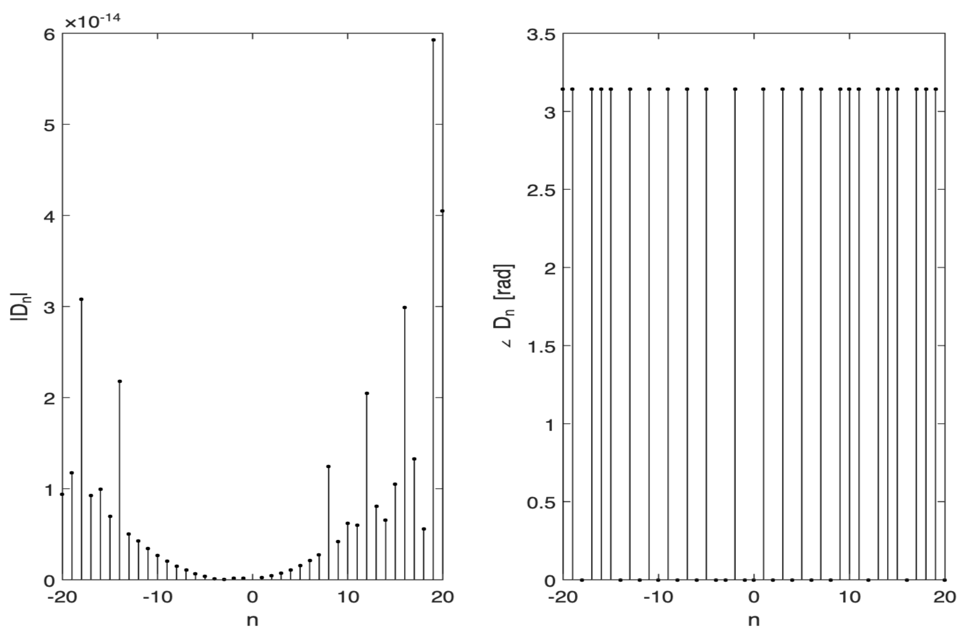


Graph 3: Graph for Figure 6.

b) -20 <= n <= 20;

```
clf;
n = (-20:20);
D_n = 1./2.*((1./(pi.*n)).*sin((3-n).*pi)) + (1./pi.*n).*sin((3+n).*pi) + (1./(2.*n.*pi).*sin((1+n).*pi)) + (1./(2.*n.*pi).*sin((1-n).*pi));
subplot(1,2,1); stem(n,abs(D_n),'.k');
xlabel('n'); ylabel('|D_n|');
subplot(1,2,2); stem(n,angle(D_n),'.k');
xlabel('n'); ylabel('\angle D_n [rad]');
```

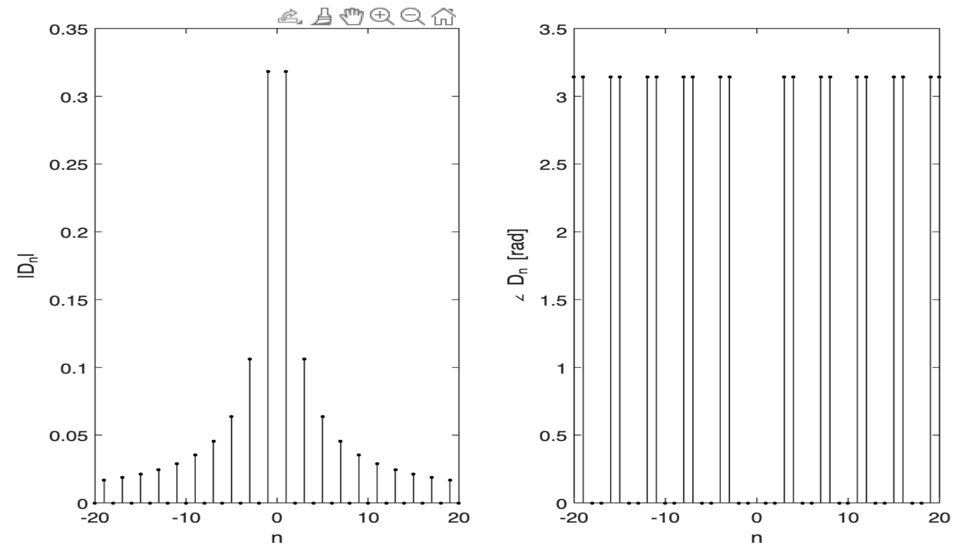
Figure 7: Code for x1(t) for the interval [-20:20]



Graph 4: Graph for Figure 7.

```
clf;
n = (-20:20);
D_n = (1./(n.*pi).*sin((n.*pi)./2));
subplot(1,2,1); stem(n,abs(D_n),'.k');
xlabel('n'); ylabel('|D_n|');
subplot(1,2,2); stem(n,angle(D_n),'.k');
xlabel('n'); ylabel('\angle D_n [rad]');
```

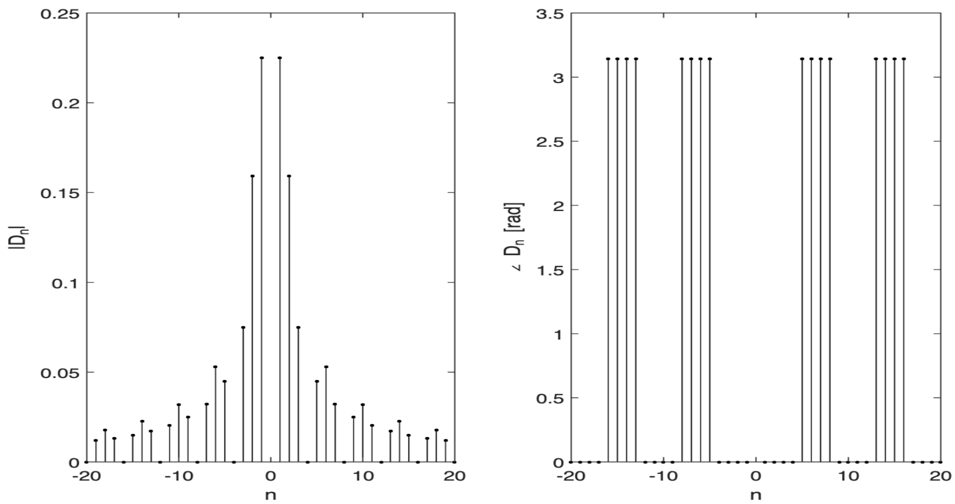
Figure 8: Code for x2(t) for the interval [-20:20]



Graph 5: Graph for Figure 8.

```
clf;
n = (-20:20);
D_n = (1./(n.*pi).*sin((n.*pi)./4));
subplot(1,2,1); stem(n,abs(D_n),'.k');
xlabel('n'); ylabel('|D_n|');
subplot(1,2,2); stem(n,angle(D_n),'.k');
xlabel('n'); ylabel('\angle D_n [rad]');
```

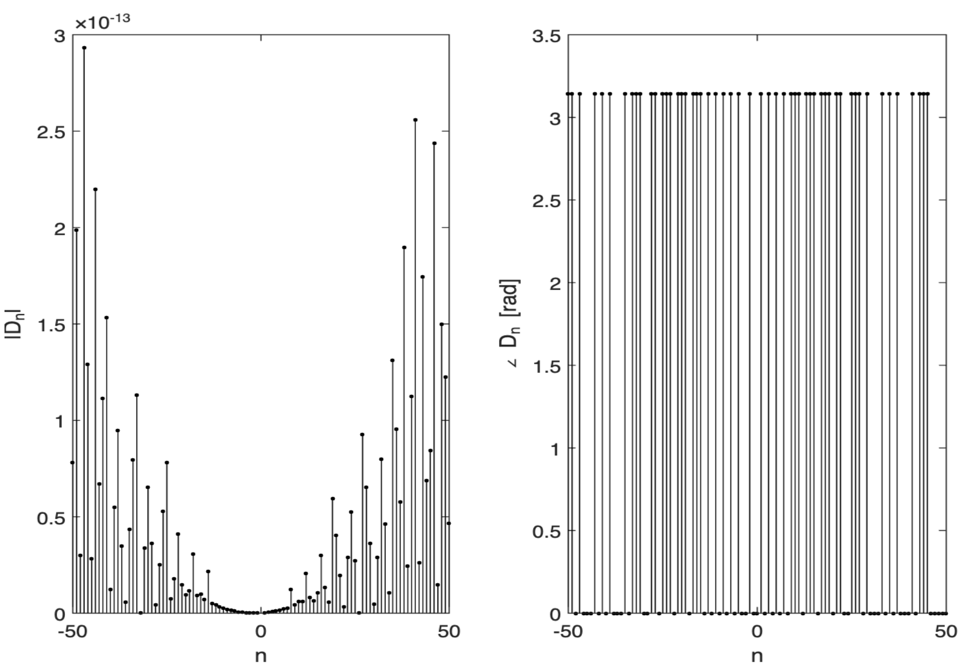
Figure 9: Code for x3(t) for the interval [-20:20]



Graph 6: Graph for Figure 9.

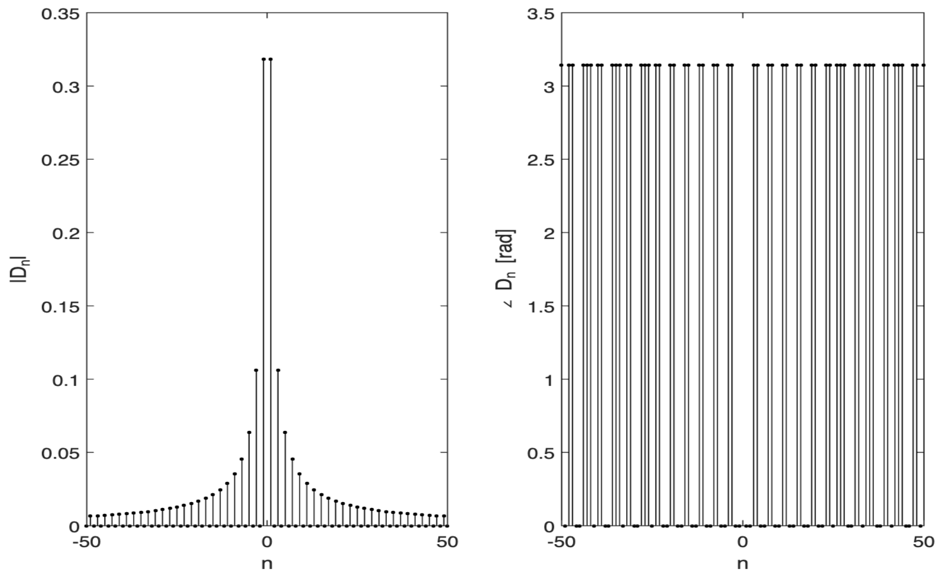
c) -50 <= n <= 50;

The code is the same but the interval will be like n = (-50:50);



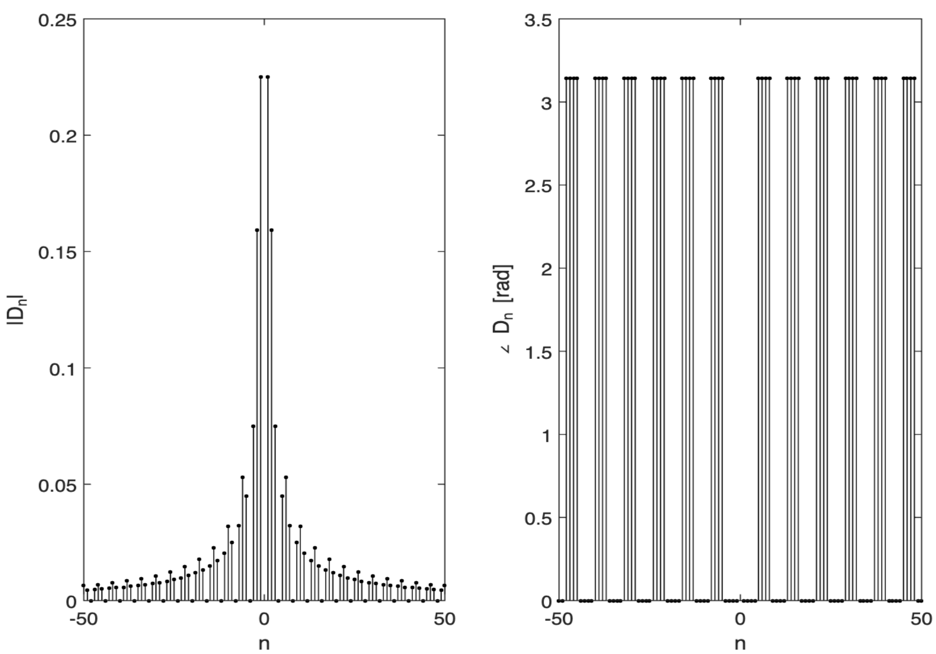
Graph 7: Graph for the function x1(t) where n = [-50,50].

The code is the same but the interval will be like n = (-50:50);



Graph 8: Graph for the function x2(t) where n = [-50,50].

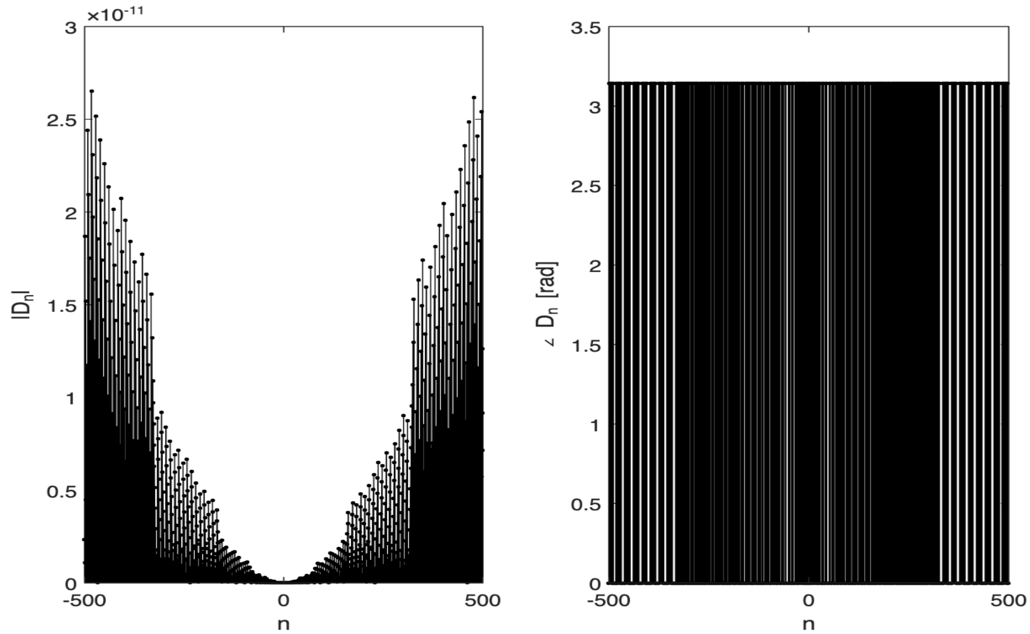
The code is the same but the interval will be like n = (-50:50);



Graph 9: Graph for the function x3(t) where n = [-50,50].

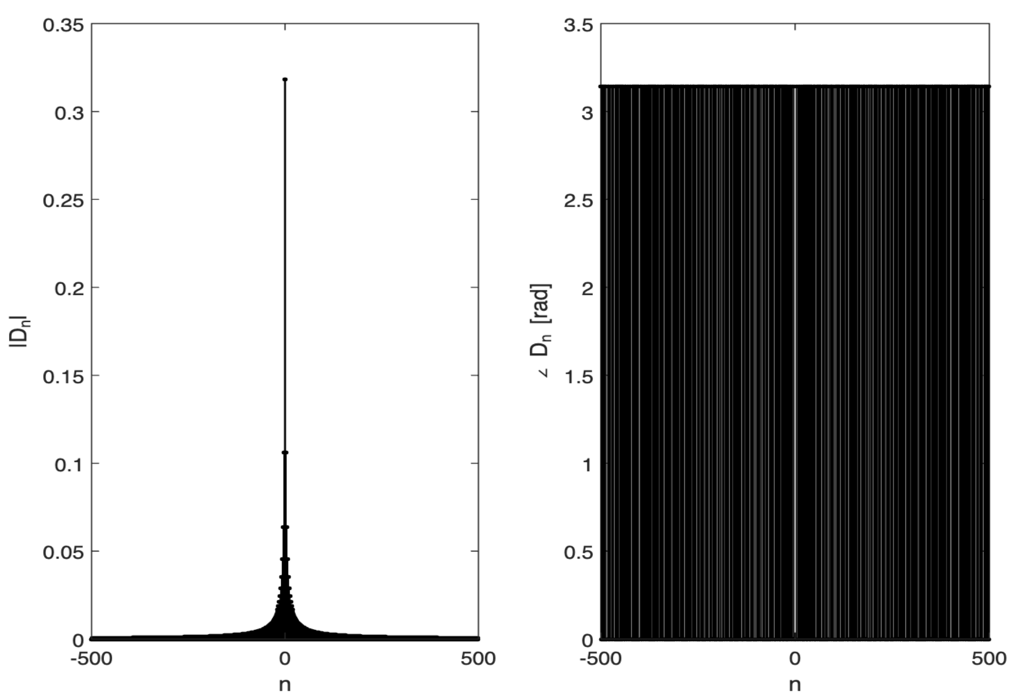
d) -500 <= n <= 500;

The code is the same but the interval will be like n = (-500:500);



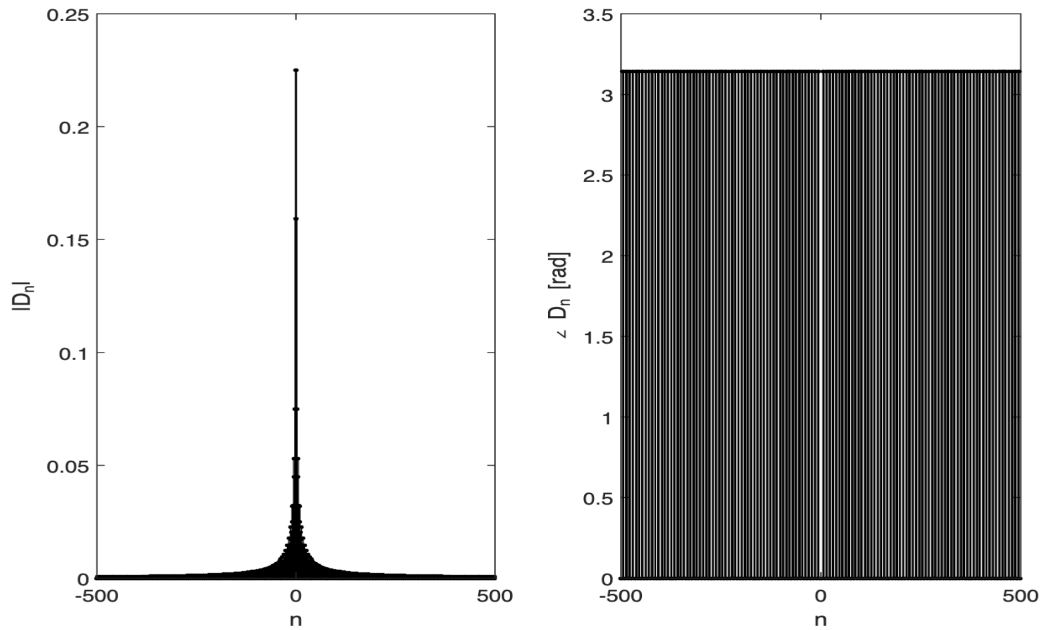
Graph 10: Graph for the function x1(t) where n = [-500,500].

The code is the same but the interval will be like n = (-500:500);



Graph 11: Graph for the function x2(t) where n = [-500,500].

The code is the same but the interval will be like n = (-500:500);



Graph 12: Graph for the function x3(t) where n = [-500,500].

A5. (Sreeja)

Fourier series is an alternate way of representing periodic signals. Specifically, the Fourier series allows a periodic signal to be expressed as a linear combination of harmonically related complex exponential functions. Let x(t) be a periodic signal with period T_o and the corresponding fundamental frequency $\omega_o = 2\boldsymbol{\pi}/T_o$.

Using the Fourier series expansion, x(t) can be expressed by the synthesis equation:

$$x(t) \ = \ \sum_{n=-\infty}^{+\infty} D_n e^{jn\omega ot}, \quad \text{[Equation 1]}$$

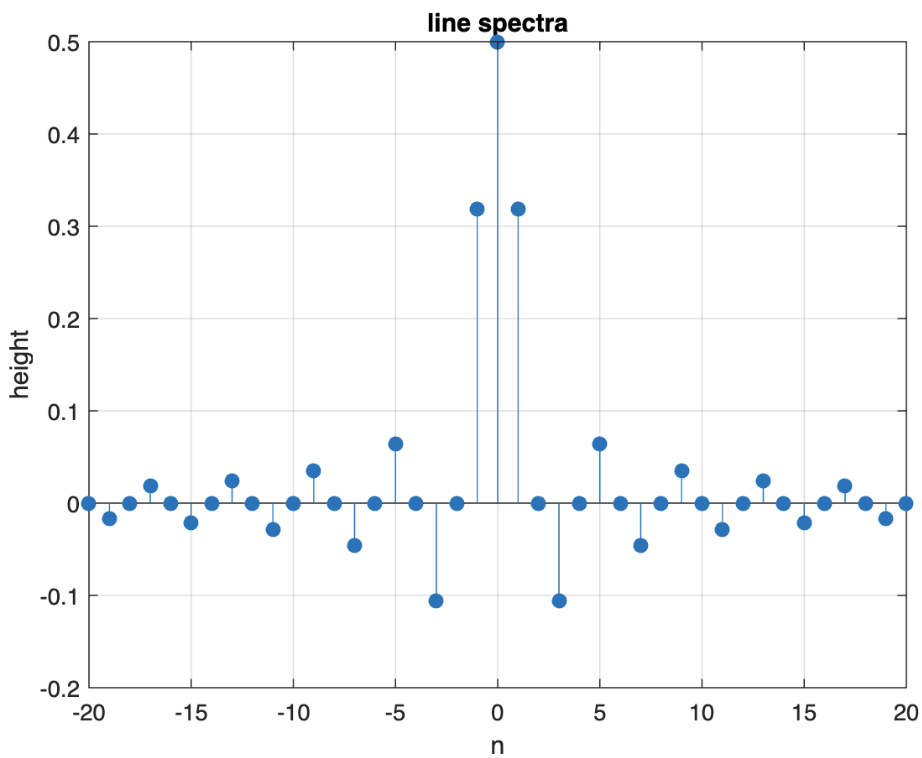
Where $\{D_n, n = 0, \pm1, \pm2, \pm3, ...\}$ is the set of Fourier coefficients that define the periodic signal x(t). We can calculate the Fourier coefficients using the analysis equation:

$$D_n \ = \ \frac{1}{T_o} x(t)\, e^{-jn\omega ot} dt, \qquad n \ = \ 0, \pm1,\ \pm2,\ \pm3, ... \qquad \text{[Equation 2]}$$

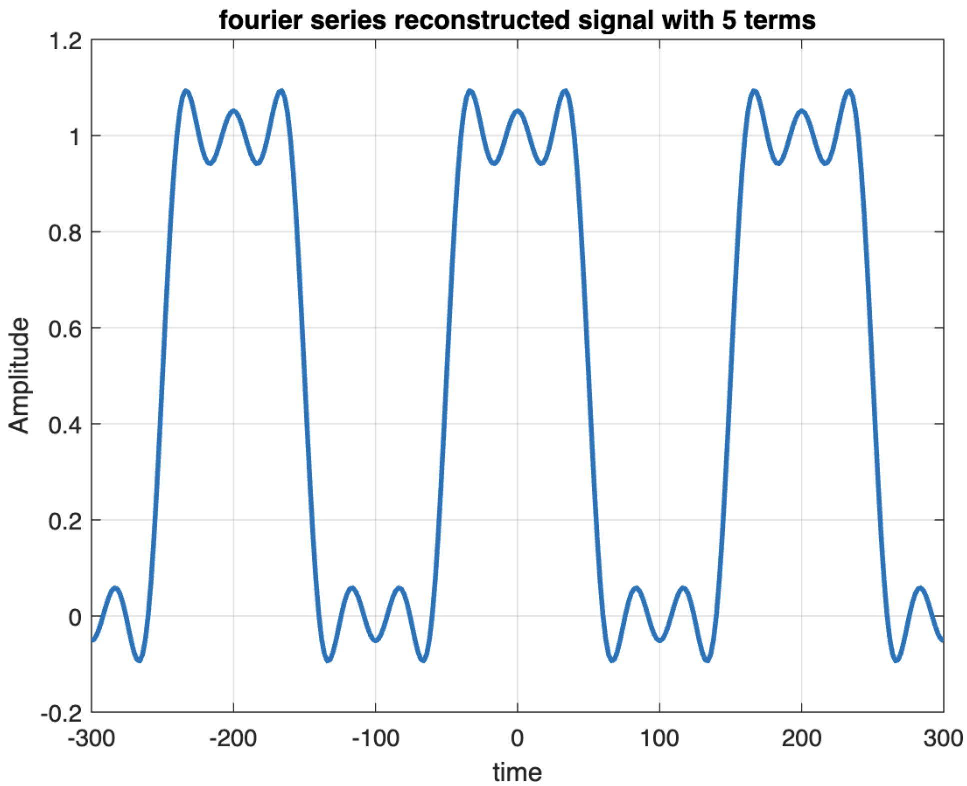
Where the integral can be evaluated over any interval of T_o duration.

```
% define the signal parameters
T = 200;
w0 = 2*pi/T;
A = 1;
tau = T/4;
% define the time vector
t = -300:T/100:300; % total 4 periods of the signal
% define the maximum number of coefficients
nmax = 20;
n = -nmax:nmax;
% define the fourier series coefficients
```

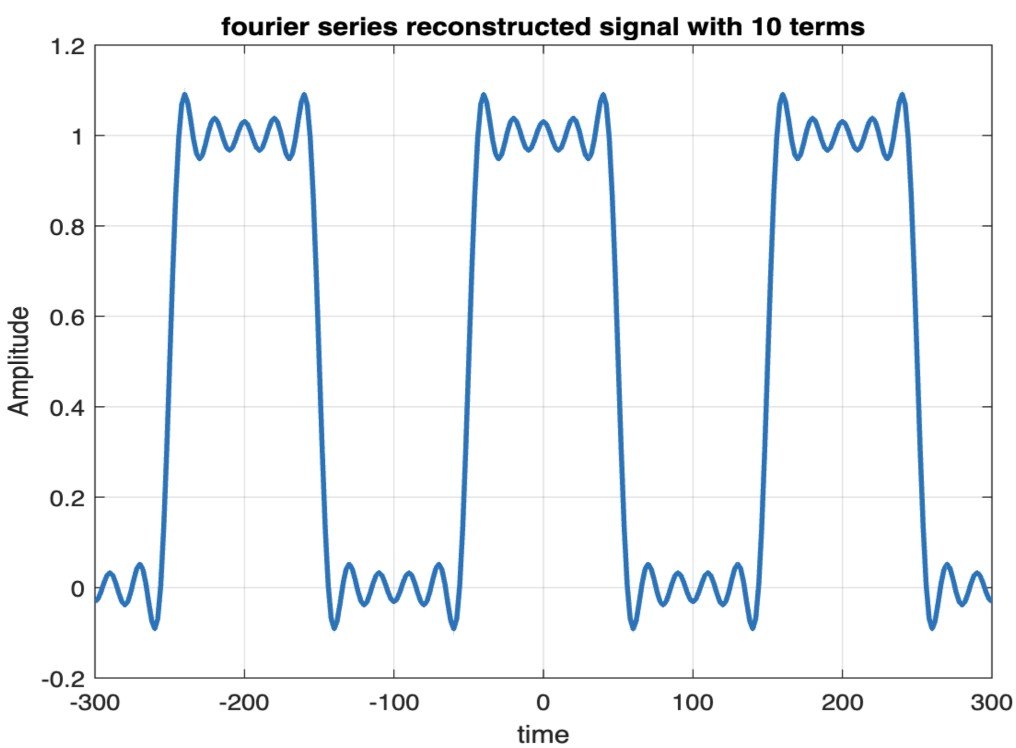
```
D = A./(n*pi).*sin(2*n*pi*tau/T);
D(nmax+1) = 2*A*tau/T; % this corresponds to n = 0
% plot the fourier series
figure;
stem(n,D,'fill');xlabel('n');ylabel('height');title('line spectra');
grid on;
% define the number of terms
N = [5 10 20];
% Now fourier series reconstruction
for k = 1:length(N)
n = -N(k):N(k);
for i = 1:length(t)
terms_range = D((nmax+1-N(k)):(nmax+1+N(k)));
x(i) = sum(terms_range.*exp(1j*n*w0*t(i)));
end
figure;
plot(t,x,'linewidth',2);    % Plot results
title(['fourier series reconstructed signal with ',num2str(N(k)),' terms',]);
xlabel('time');ylabel('Amplitude');grid on;%ylim([-1 2])
end
```



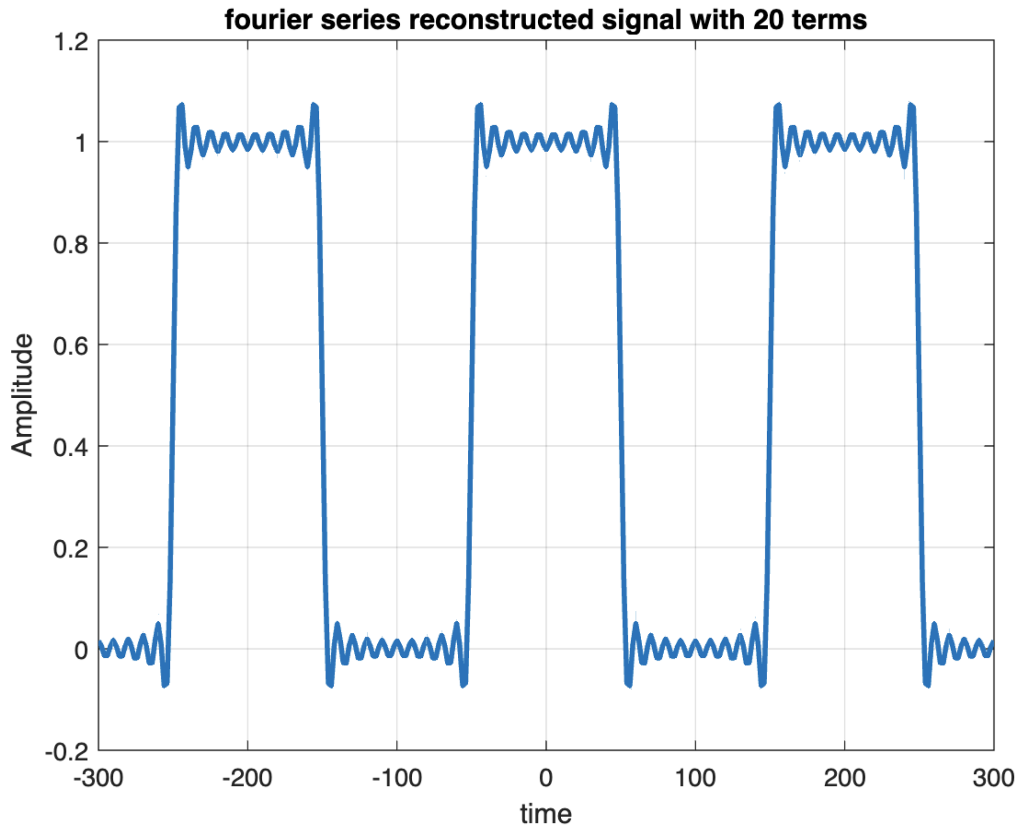
Graph 13: Graph for the line spectra.



Graph 14: Graph for the fourier series reconstructed signal with 5 terms



Graph 15: Graph for the fourier series reconstructed signal with 10 terms



Graph 16: Graph for the fourier series reconstructed signal with 20 terms

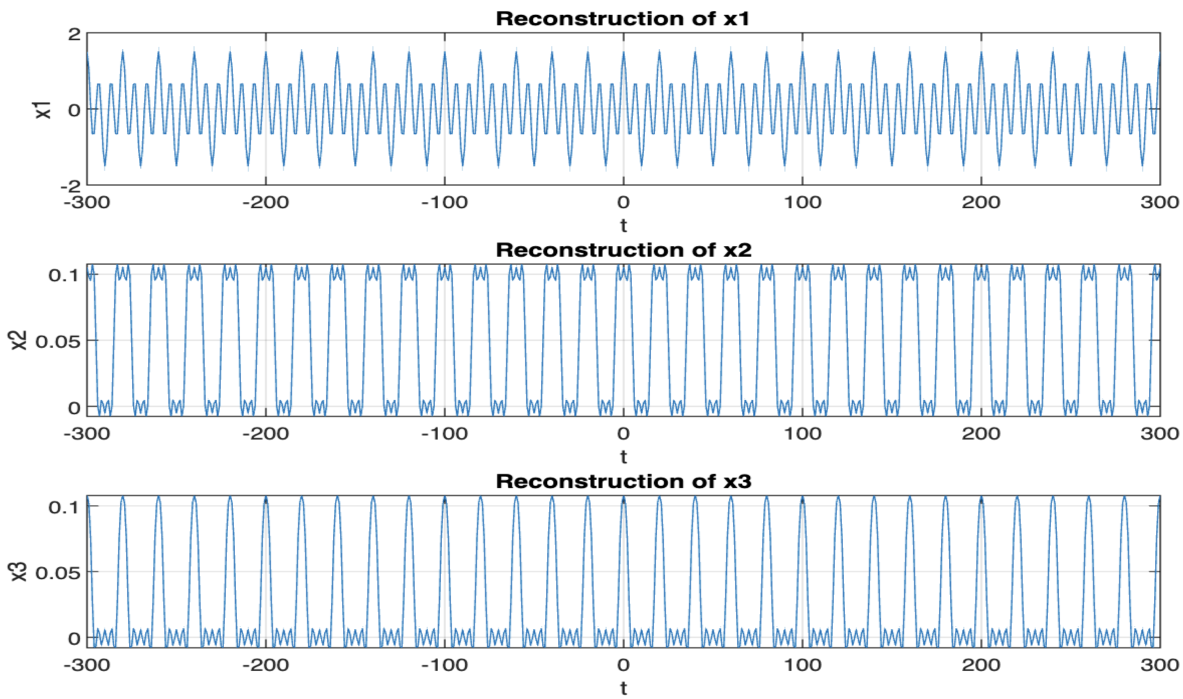
A6. (Sreeja)

Time domain signals for the following:

1. **-5 <= n <= 5**

```
n = -5:5; %larger n values allow for better representation of signal
Dn1 = 0.5*(abs(n)==3) + 0.25*(abs(n)==1);
t = (-300:300);
w = pi/10;
xo = zeros(size(t));
for i = 1:length(n) %looping through n values in the summation
   temp = Dn1(i)*exp(1j*n(i)*w*t);
   xo = xo +temp;
end
figure();
subplot(3,1,1);
plot(t,xo); title('Reconstruction of x1');
xlabel('t');ylabel('x1'); grid;
Dn2 = 0.05*sinc(n/2);
w2 = pi/10;
x2 = zeros(size(t));
for i = 1:length(n)%looping through n values in the summation
   temp = Dn2(i)*exp(1j*n(i)*w*t);
   x2 = x2 +temp;
end
subplot(3,1,2);
plot(t,x2);title('Reconstruction of x2');
xlabel('t');ylabel('x2'); grid;
```

```
Dn3 = 0.025*sinc(n/4);
w3 = pi/20;
x3 = zeros(size(t));
for i = 1:length(n)%looping through n values in the summation
   temp = Dn3(i)*exp(1j*n(i)*w*t);
   x3 = x3 +temp;
end
subplot(3,1,3);
plot(t,x3); title('Reconstruction of x3');
xlabel('t');ylabel('x3'); grid;
```

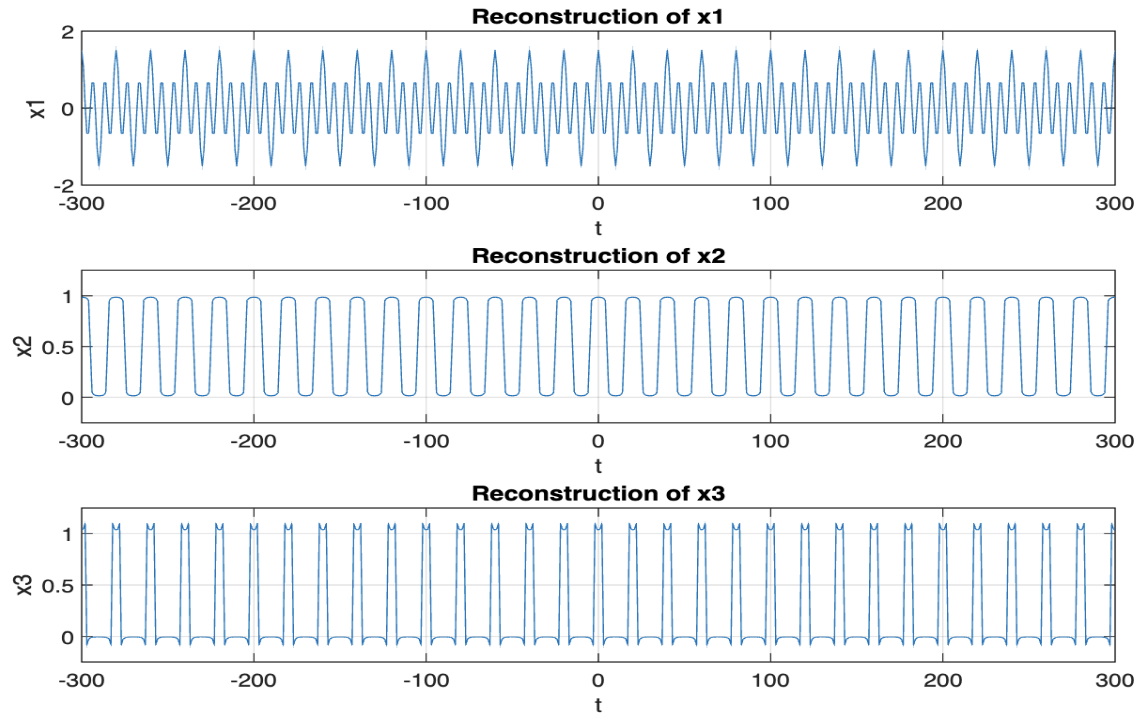


Graph 16: Graph for the reconstruction for x1(t), x2(t), x3(t) where n[-5,5].

2. -20 <= n <= 20

```
n = -20:20;%larger n values allow for better representation of signal
Dn1 = 0.5*(abs(n)==3) + 0.25*(abs(n)==1);
t = (-300:300);
w = pi/10;
xo = zeros(size(t));
for i = 1:length(n)%looping through n values in the summation
   temp = Dn1(i)*exp(1j*n(i)*w*t);
   xo = xo +temp;%these two lines execute the sum of the Dn*exp values
end
figure();
subplot(3,1,1);
plot(t,xo); title('Reconstruction of x1');
xlabel('t');ylabel('x1'); grid;
Dn2 = 0.5*sinc(n/2);
w2 = pi/10;
x2 = zeros(size(t));
for i = 1:length(n)%looping through n values in the summation
   temp = Dn2(i)*exp(1j*n(i)*w*t);
   x2 = x2 +temp; %these two lines execute the sum of the Dn*exp values
end
subplot(3,1,2);
plot(t,x2);title('Reconstruction of x2');axis([-300 300 -0.25 1.25]);
xlabel('t');ylabel('x2'); grid;
```

```
Dn3 = 0.25*sinc(n/4);
w3 = pi/20;
x3 = zeros(size(t));
for i = 1:length(n)%looping through n values in the summation
   temp = Dn3(i)*exp(1j*n(i)*w*t);
   x3 = x3 +temp;%these two lines execute the sum of the Dn*exp values
end
subplot(3,1,3);
plot(t,x3); title('Reconstruction of x3');axis([-300 300 -0.25 1.25]);
xlabel('t');ylabel('x3'); grid;
```

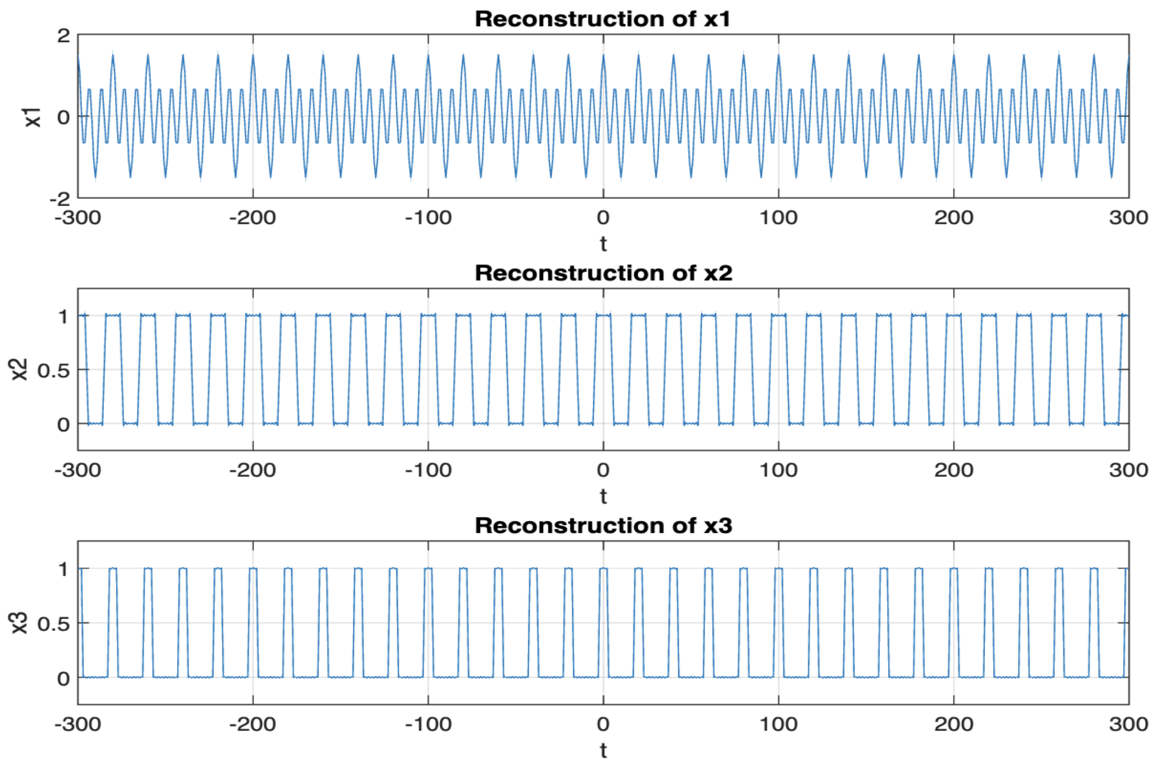


Graph 17: Graph for the reconstruction for x1(t), x2(t), x3(t) where n[-20,20].

3. -50 <= n <= 50

```
n = -50:50;%larger n values allow for better representation of signal
Dn1 = 0.5*(abs(n)==3) + 0.25*(abs(n)==1);
t = (-300:300);
w = pi/10;
xo = zeros(size(t));
for i = 1:length(n)%looping through n values in the summation
   temp = Dn1(i)*exp(1j*n(i)*w*t);
   xo = xo +temp;%these two lines execute the sum of the Dn*exp values
end
figure();
subplot(3,1,1);
plot(t,xo); title('Reconstruction of x1');
xlabel('t');ylabel('x1'); grid;
Dn2 = 0.5*sinc(n/2);
w2 = pi/10;
x2 = zeros(size(t));
for i = 1:length(n)%looping through n values in the summation
   temp = Dn2(i)*exp(1j*n(i)*w*t);
   x2 = x2 +temp;%these two lines execute the sum of the Dn*exp values
end
subplot(3,1,2);
```

```
plot(t,x2);title('Reconstruction of x2');axis([-300 300 -0.25 1.25]);
xlabel('t');ylabel('x2'); grid;
Dn3 = 0.25*sinc(n/4);
w3 = pi/20;
x3 = zeros(size(t));
for i = 1:length(n)%looping through n values in the summation
   temp = Dn3(i)*exp(1j*n(i)*w*t);
   x3 = x3 +temp;%these two lines execute the sum of the Dn*exp values
end
subplot(3,1,3);
plot(t,x3); title('Reconstruction of x3');axis([-300 300 -0.25 1.25]);
xlabel('t');ylabel('x3'); grid;
```

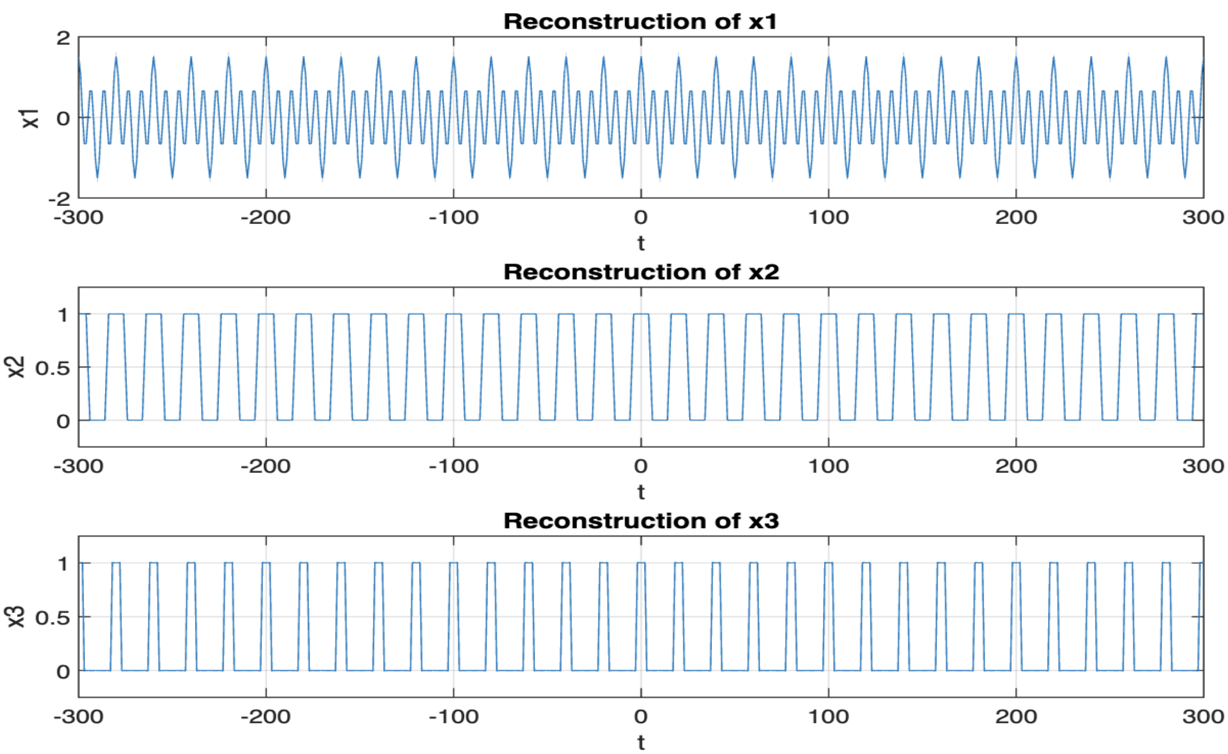


Graph 18: Graph for the reconstruction for x1(t), x2(t), x3(t) where n[-50,50].

4. -500 <= n <= 500

```
n = -500:500;
Dn1 = 0.5*(abs(n)==3) + 0.25*(abs(n)==1);
t = (-300:1:300);
w = pi/10;
xo = zeros(size(t));
for i = 1:length(n)%looping through n values in the summation
   temp = Dn1(i)*exp(1j*n(i)*w*t);
   xo = xo +temp;%these two lines execute the sum of the Dn*exp values
end
figure();
subplot(3,1,1);
plot(t,xo); title('Reconstruction of x1');
xlabel('t');ylabel('x1'); grid;
Dn2 = 0.5*sinc(n/2);
w2 = pi/10;
x2 = zeros(size(t));
for i = 1:length(n)%looping through n values in the summation
   temp = Dn2(i)*exp(1j*n(i)*w*t);
```

```
    x2 = x2 +temp;%these two lines execute the sum of the Dn*exp values
end
subplot(3,1,2);
plot(t,x2);title('Reconstruction of x2');axis([-300 300 -0.25 1.25]);
xlabel('t');ylabel('x2'); grid;
Dn3 = 0.25*sinc(n/4);
w3 = pi/20;
x3 = zeros(size(t));
for i = 1:length(n)%looping through n values in the summation
    temp = Dn3(i)*exp(1j*n(i)*w*t);
    x3 = x3 +temp;%these two lines execute the sum of the Dn*exp values
end
subplot(3,1,3);
plot(t,x3); title('Reconstruction of x3');axis([-300 300 -0.25 1.25]);
xlabel('t');ylabel('x3'); grid;
```



Graph 19: Graph for the reconstruction for x1(t), x2(t), x3(t) where n[-500,500].

PART B GRAPHS

B.1 (Sreeja)

$x_1(t) = cos(\frac{3\pi}{10})t \;+\; \frac{1}{2}cos(\frac{\pi}{10})t,$

$\omega_{o1} = \frac{3\pi}{10}, \; \omega_{o2} = \frac{\pi}{10}$

$\omega_o = \frac{G.C.F\ of\ numerator}{L.C.M\ of\ denominator} = \frac{\pi}{10} = 0.314$ rad/s.

For $x_2(t) \rightarrow T_o = 20s$

$\omega_o = \frac{\pi}{10} = 0.314$ rad/s.

For $x_3(t) \rightarrow T_o = 40s$
$\omega_o = \frac{\pi}{20} = 0.157$ rad/s.

B.2 (Moiz)
The main difference between the Fourier coefficients of x1(t) and x2(t) is that one is made up of sinc functions, while the other is made up of sin functions. In addition, x1(t) has four unique Fourier series coefficients, but x2(t) for Dn has infinite Fourier coefficients.

B.3 (Moiz)
For its Fourier coefficients, signal x3(t) has a lower fundamental frequency value than signalx2(t).

B.4 (Moiz)
Signal x4(t), generated from x2(t), has Do= 0.5.

B.5 (Moiz)
Because x1(t) has a finite number of Dn values, increasing the Fourier coefficients has no effect. Increasing the value of Dn, on the other hand, improves the accuracy of x2(t) and x3(t).

B.6 (Moiz)
Because x1(t) contains a limited number of Dn values, complete reconstruction would only require four Fourier series coefficients in this example. For x2(t) and x3(t), however, an unlimited number of Dn would be required for flawless reconstruction.

B.7 (Moiz)
A periodic signal is not viable since it has an endless number of Dn values. If it is finite, such as x1(t), the values of Dn can be saved. However, for signals with a high number of finite Dn values, this is not advised since it wastes space.