RYERSON UNIVERSITY

Faculty of Engineering, Architecture and Science

Department of Electrical and Computer Engineering

Course Number	**ELE 532**
Course Title	**Signals and Systems I**
Semester/Year	**Fall 2021**
Instructor	**Dr. Javad Alirezaie**

ASSIGNMENT NO.	**2**

Report Title	System Properties and Convolution

Section No.	02
Group No.	N/A
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Name	Student No.	Signature
Naga Sreeja Kurra	xxxx20707	K.N.S
Moiz Kharodawala	xxxx68133	M.K.

PART A (SREEJA)

A1.

Use MATLAB command **poly**.

```
clc;
close all;


R = [1e4, 1e4, 1e4];
C = [1e-9, 1e-6];

A0 = 1;
A1 = (1/R(1)+1/R(2)+1/R(3))/C(2);
A2 = 1/(R(1)*R(2)*C(1)*C(2));

A = [A0 A1 A2];

lambda = roots(A);
p = poly(A);
```

Figure 1: Code for using command poly.

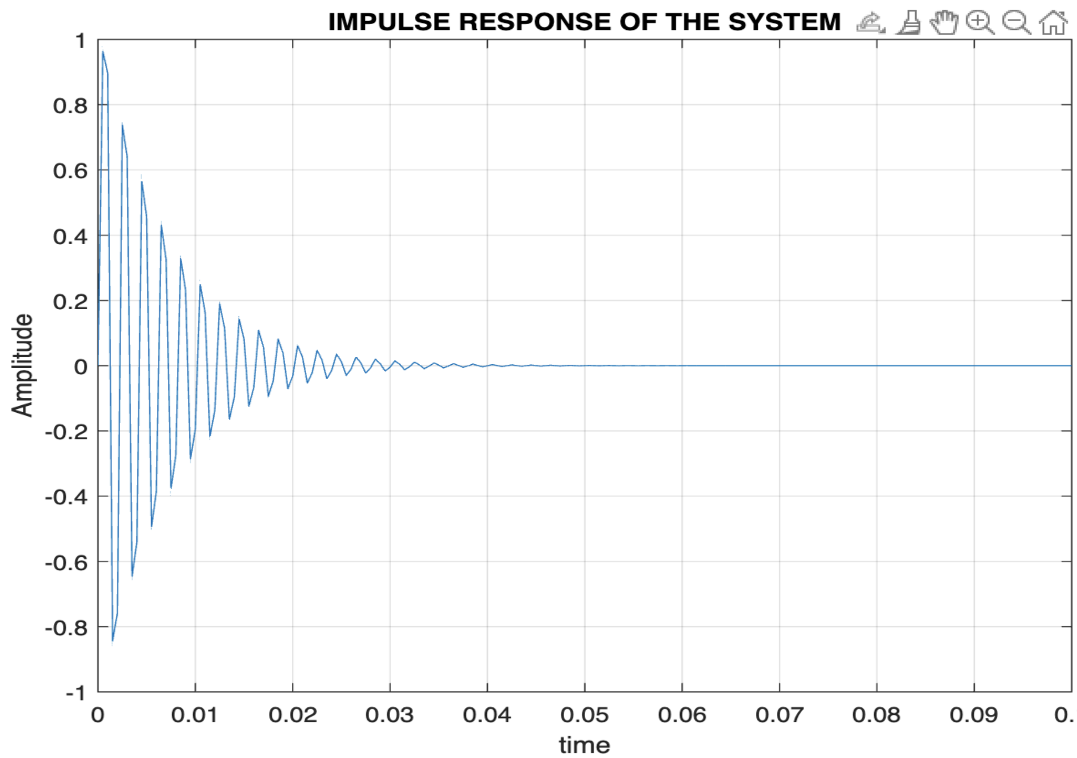
```
>> p

p =

   1.0e+09 *

    0.0000   -0.0100    3.0100   -3.0000

>> A

A =

   1.0e+07 *

    0.0000    0.0000    1.0000
```

Figure 2: Output for p,A

A2.

```
clc;
close all;


R = [1e4, 1e4, 1e4];
C = [1e-9, 1e-6];

A0 = 1;
A1 = (1/R(1)+1/R(2)+1/R(3))/C(2);
A2 = 1/(R(1)*R(2)*C(1)*C(2));

A = [A0 A1 A2];

lambda = roots(A);
H = tf(A2,A);

t = 0:0.0005:0.1;
u = (t==0);
y = lsim (H,u,t);
figure;plot(t,y);grid;xlabel('time');ylabel('Amplitude');
title('IMPULSE RESPONSE OF THE SYSTEM');
```

Figure 3: Code for t[0:0.0005:0.1]



Graph 1: Impulse Response of the System

A3.

CH2MP1.m

```
% CH2MP1.m : Chapter 2, MATLAB Program 1
% Script M-file determines characteristic roots of op-amp circuit.
% Set component values:
R = [1e4, 1e4, 1e4];
C = [1e-6, 1e-6];
% Determine coefficients for characteristic equation:
A = [1, (1/R(1)+1/R(2)+1/R(3))/C(2), 1/(R(1)*R(2)*C(1)*C(2))];
% Determine characteristic roots:
lambda = roots(A);
```

Figure 4: Code from the pg 214 with C = [$1e^{-6}$, $1e^{-6}$]

CH2MP1.m

```
% CH2MP1.m : Chapter 2, MATLAB Program 1
% Script M-file determines characteristic roots of op-amp circuit.
% Set component values:
R = [1e4, 1e4, 1e4];
C = [1e-9, 1e-6];
% Determine coefficients for characteristic equation:
A = [1, (1/R(1)+1/R(2)+1/R(3))/C(2), 1/(R(1)*R(2)*C(1)*C(2))];
% Determine characteristic roots:
lambda = roots(A);
```

Figure 5: Code from the pg 214 with C = [$1e^{-9}$, $1e^{-6}$]

```
>> CH2MP1
>>
>> lambda

lambda =

  -261.8034
   -38.1966

>> CH2MP1
>> lambda

lambda =

   1.0e+03 *

  -0.1500 + 3.1587i
  -0.1500 - 3.1587i
```

Figure 6: Output for C = [$1e^{-9}$, $1e^{-6}$] and C = [$1e^{-9}$, $1e^{-6}$]

A script file is created by placing these commands in a text file, which in this case is named CH2MP1.m. After execution, all the resulting variables are available in the workspace. Script files permit simple or incremental changes, thereby saving significant effort. So the above two different outputs are for the values of e.

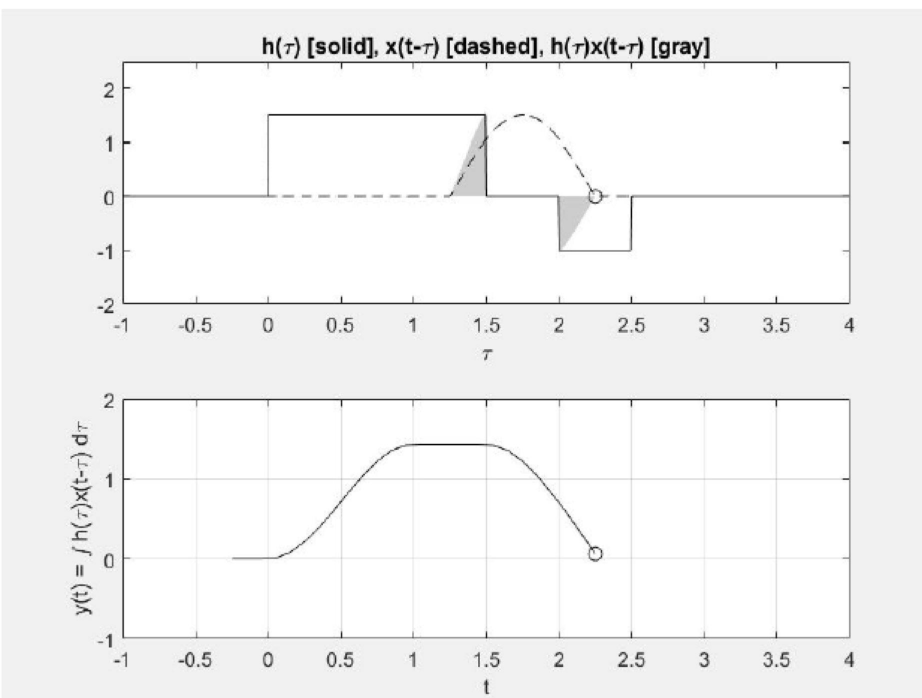
The imaginary portion of dictates ƛ an oscillation rate of 3158.7 rad/s or about 505Hz. The real portion dictates the rate of decay. The time expected to reduce the amplitude to 25% is approximately t = ln 0.25/ Re(ƛ) ~ 0.01 second.

PART B GRAPHS (MOIZ)

B.1

```
u = @(t) 1.0*(t>=0);
h = @(t) 1.5*(u(t)-u(t-1.5))-u(t-2)+u(t-2.5);
x = @(t) 1.5*sin(pi*t).*(u(t)-u(t-1));
dtau = 0.005;
tau = -1:dtau:10.5;ti = 0;
tvec = -.25:.1:3.75; y = NaN*zeros(1,length(tvec));
for t = tvec
    ti = ti+1;
    xh = x(t-tau).*h(tau);
    lxh = length(xh);
    y(ti) = sum(xh.*dtau);
    subplot(2,1,1),plot(tau,h(tau),"k-",tau,x(t-tau),"k--",t,0,"ok");
    axis([tau(1) tau(end) -2.0 2.5]);
    patch([tau(1:end-1);tau(1:end-1);tau(2:end);tau(2:end)],...
    [zeros(1,lxh-1);xh(1:end-1);xh(2:end);zeros(1,lxh-1)],...
    [.8 .8 .8],"edgecolor","none");
    xlabel("\tau"); title("h(\tau) [solid], x(t-\tau) [dashed], h(\tau)x(t-\tau) [gray]");
    c = get(gca,'children'); set(gca,'children',[c(2);c(3);c(4);c(1)]);
    subplot(2,1,2),plot(tvec,y,"k",tvec(ti),y(ti),"ok");
    xlabel("t"); ylabel("y(t) = \int h(\tau)x(t-\tau) d\tau");
    axis([tau(1) tau(end) -1.0 2.0]); grid;
    drawnow;
end
```

Figure 7: Code for B.1 (from textbook)

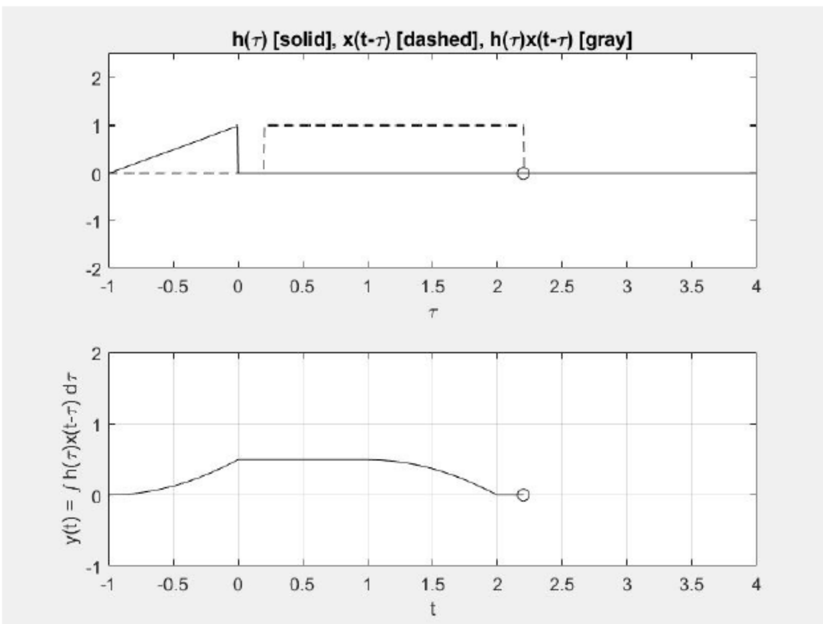


Graph 2: Graph output for Figure 7

B.2

```
u = @(t) 1.0*(t>=0);
x = @(t) u(t)-u(t-2);
h = @(t) (t+1).*(u(t+1)-u(t));
dtau = 0.005; tau = -1:dtau:4;
ti = 0; tvec = -.25:.1:3.75;
y = NaN*zeros(1,length(tvec));
```

Figure 8: Code for B2 (for loop is same as B1)

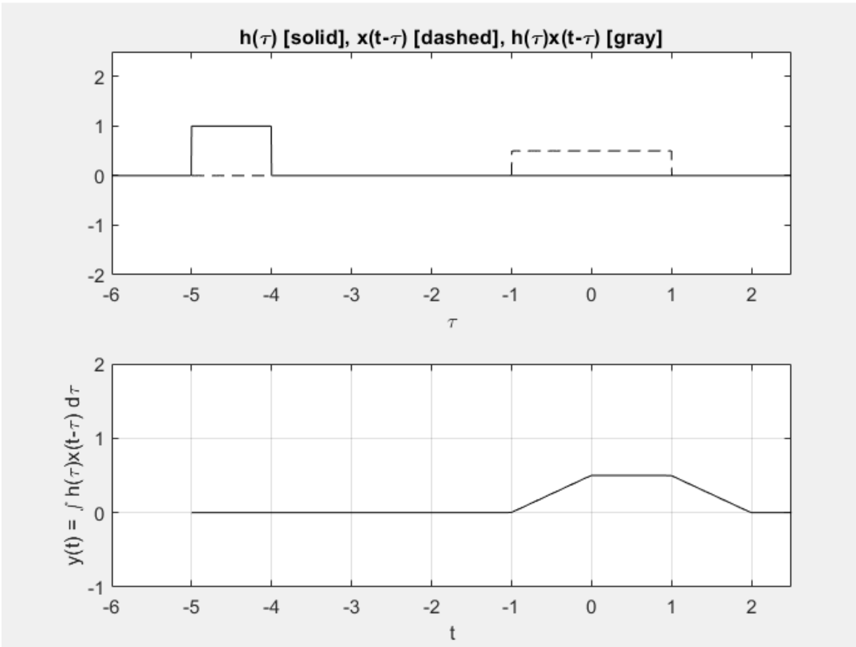


Graph 3: Graph output for Figure 8

B.3

```
u = @(t) 1.0*(t>=0);
A = 0.5; B=1;
x = @(t) A*(u(t-4)-u(t-6));
h = @(t) B*(u(t+5)-u(t+4));
dtau = 0.005; tau = -6:dtau:2.5;
ti = 0; tvec = -5:.1:5;
y = NaN*zeros(1,length(tvec));
```

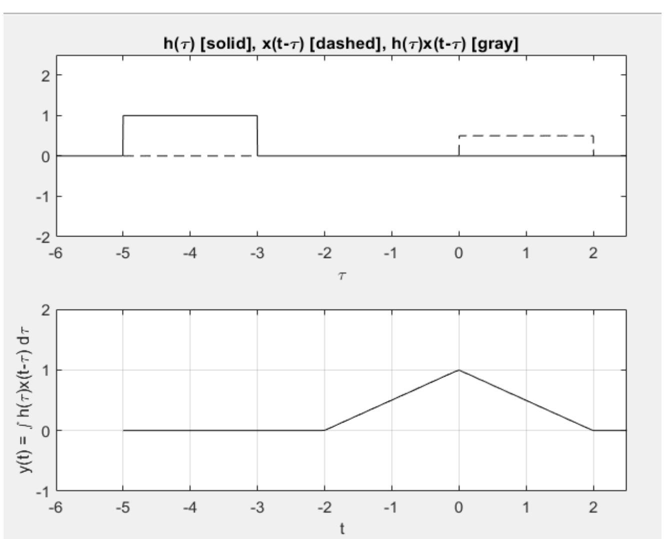
Figure 9: Code for B3a (Assuming A = 0.5 and B = 1)(for loop is same as B1)



Graph 4: Graph output for Figure 9

```
u = @(t) 1.0*(t>=0);
A = 0.5; B=1;
x = @(t) A*(u(t-3)-u(t-5));
h = @(t) B*(u(t+5)-u(t+3));
dtau = 0.005; tau = -6:dtau:2.5;
ti = 0; tvec = -5:.1:5;
y = NaN*zeros(1,length(tvec));
```

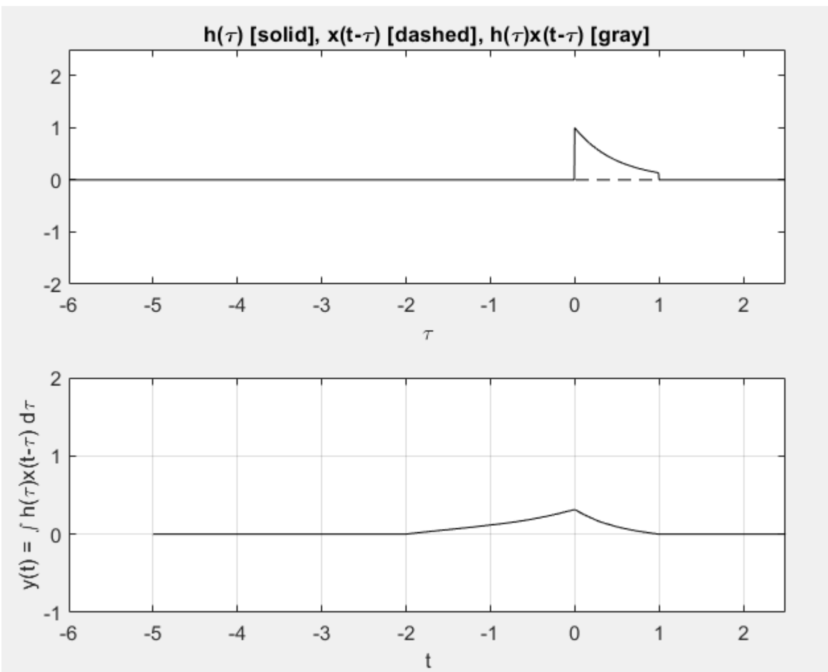
Figure 10: Code for B3b (Assuming A = 0.5 and B = 1)(for loop is same as B1)



Graph 5: Graph output for Figure 10

```
u = @(t) 1.0*(t>=0);
x = @(t) exp(t).*(u(t+2)-u(t));
h = @(t) exp(-2*t).*(u(t)-u(t-1));
dtau = 0.005; tau = -6:dtau:2.5;
ti = 0; tvec = -5:.1:5;
y = NaN*zeros(1,length(tvec));
```

Figure 11: Code for B3h (Assuming A = 0.5 and B = 1)(for loop is same as B1)



Graph 6: Graph output for Figure 11

PART C (SREEJA)

C1.

Consider LTI systems for the following:

$$\mathbf{h_1(t) = e^{t/5}u(t) = S_1}$$

$$\mathbf{h_2(t) = 4e^{-t/5}\ u(t) = S_2}$$

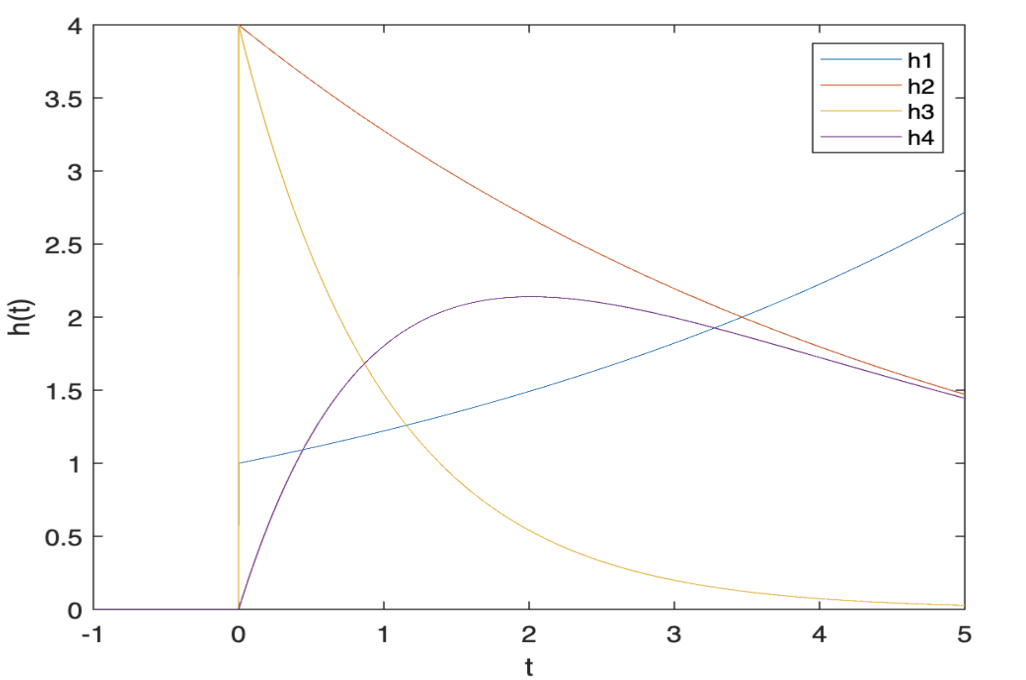
$$\mathbf{h_3(t) = 4e^{-t}\ u(t) = S_3}$$

$$\mathbf{h_4(t) = 4\ (e^{-t/5} - e^{-t})\ u(t) = S_4}$$

Response function t = [-1:0.001:5]

```
t = [-1:0.001:5];

% Create function
u = @(t) 1.0.* (t>=0);
h1 = @(t) exp(t/5).*u(t);
h2 = @(t) 4*exp(-t/5).*u(t);
h3 = @(t) 4*exp(-t).*u(t);
h4 = @(t) 4*(exp(-t/5) - exp(-t)).*u(t);

plot(t,h1(t));
xlabel("t");
ylabel("h(t)");
hold on;

plot(t,h2(t));
plot(t,h3(t));
plot(t,h4(t));

legend("h1", "h2", "h3", "h4");
%Plot all the graphs
hold off;
```

Figure 12: MATLAB Code for the above LTI systems



Graph 8: Graph for the above MATLAB code.

C2.

$$h_1(t) = e^{t/5}u(t) = S_1$$
$$h_2(t) = 4e^{-t/5}\,u(t) = S_2$$
$$h_3(t) = 4e^{-t}\,u(t) = S_3$$
$$h_4(t) = 4\,(e^{-t/5} - e^{-t})\,u(t) = S_4$$

Characteristic eigenvalues for the above S_1-S_4 functions:

$\underline{S_1}$

$\lambda_1 = 1/5$

$\underline{S_2}$

$\lambda_1 = -1/5$

$\underline{S_3}$

$\lambda_1 = -1$

$\underline{S_4}$

$\lambda_1 = -1/5$

$\lambda_2 = -1$

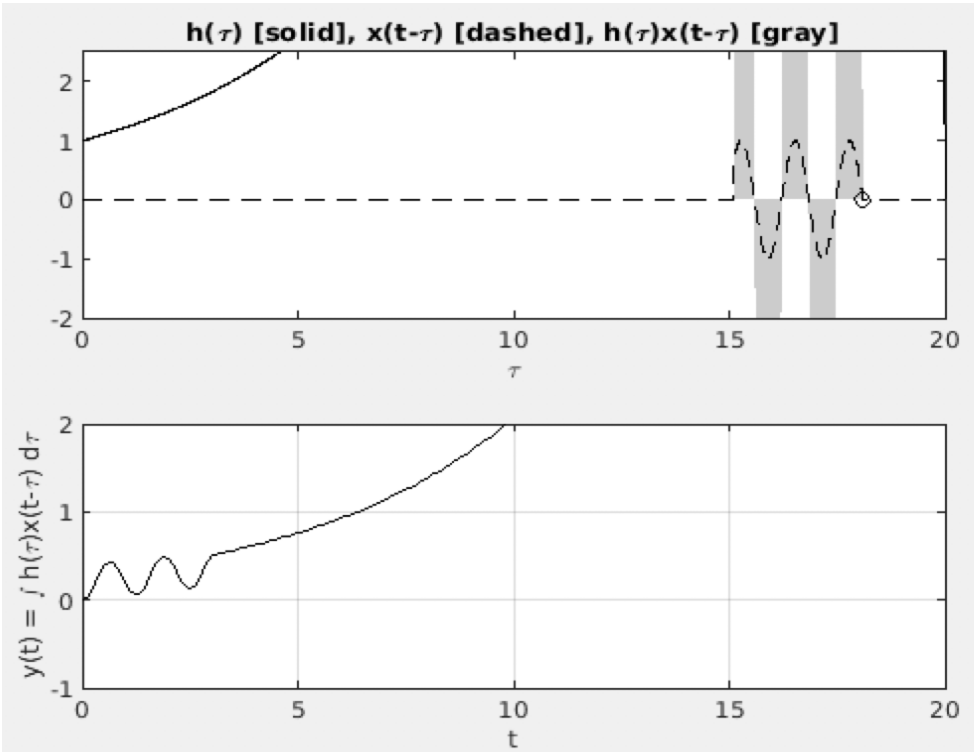
C3.

Convolution for the above LTI systems.

1. **$h_1(t) = e^{t/5}u(t)$**

```
% First create the u(t) function
u = @(t) 1.0.* (t>=0);

% Create the x(t) function.
x = @(t) sin(5*t).*(u(t) - u(t - 3));

% Now truncate each of the impluse response funtions.
h = @(t) exp(t/5).*(u(t)-u(t-20));
% Modified CH2MP4 from B.1
dtau = 0.005;
tau = 0:dtau:20; ti = 0;
tvec = 0:.1:20; y = NaN*zeros(1,length(tvec));
% Pre-allocate memory
for t = tvec
    ti = ti+1; % Time index
    xh = x(t-tau).*h(tau);
    lxh = length(xh);
    y(ti) = sum(xh.*dtau);
    % Trapezoidal approximation of convolution integral
    subplot(2,1,1),plot(tau,h(tau),"k-",tau,x(t-tau),"k--",t,0,"ok");
    axis([tau(1) tau(end) -2.0 2.5]);
    patch([tau(1:end-1);tau(1:end-1);tau(2:end);tau(2:end)],...
        [zeros(1,lxh-1);xh(1:end-1);xh(2:end);zeros(1,lxh-1)],...
        [.8 .8 .8],"edgecolor","none");
    xlabel("\tau"); title("h(\tau) [solid], x(t-\tau) [dashed], h(\tau)x(t-\tau) [gray]");
    c = get(gca,'children'); set(gca,'children',[c(2);c(3);c(4);c(1)]);
    subplot(2,1,2),plot(tvec,y,"k",tvec(ti),y(ti),"ok");
    xlabel("t"); ylabel("y(t) = \int h(\tau)x(t-\tau) d\tau");
    axis([tau(1) tau(end) -1.0 2.0]); grid;
    drawnow;
end
```

Figure 13: MATLAB Code for the convolution for H1.

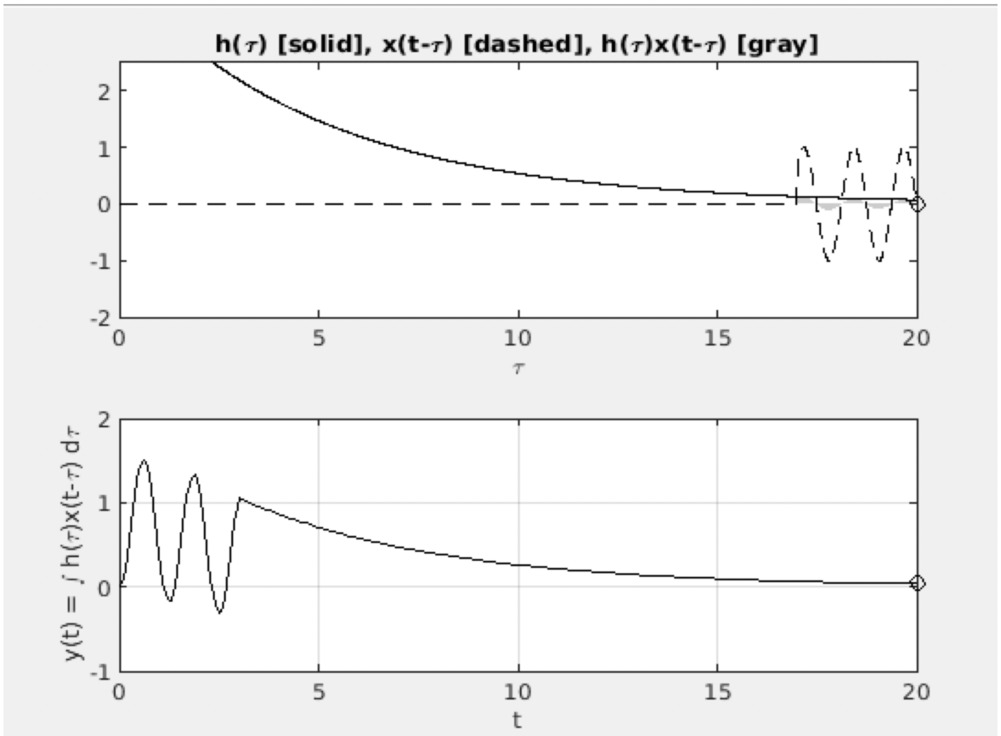


Graph 9: Graph for the above MATLAB code.

2. $h_2(t) = 4e^{-t/5}\,u(t)$

```
% First create the u(t) function
u = @(t) 1.0.* (t>=0);

% Create the x(t) function.
x = @(t) sin(5*t).*(u(t) - u(t - 3));

% Now truncate each of the impluse response funtions.
h = @(t) 4*exp(-t/5).*(u(t)-u(t-20));
% Modified CH2MP4 from B.1
dtau = 0.005;
tau = 0:dtau:20; ti = 0;
tvec = 0:.1:20; y = NaN*zeros(1,length(tvec));
% Pre-allocate memory
for t = tvec
    ti = ti+1; % Time index
    xh = x(t-tau).*h(tau);
    lxh = length(xh);
    y(ti) = sum(xh.*dtau);
    % Trapezoidal approximation of convolution integral
    subplot(2,1,1),plot(tau,h(tau),"k-",tau,x(t-tau),"k--",t,0,"ok");
    axis([tau(1) tau(end) -2.0 2.5]);
    patch([tau(1:end-1);tau(1:end-1);tau(2:end);tau(2:end)],...
        [zeros(1,lxh-1);xh(1:end-1);xh(2:end);zeros(1,lxh-1)],...
        [.8 .8 .8],"edgecolor","none");
    xlabel("\tau"); title("h(\tau) [solid], x(t-\tau) [dashed], h(\tau)x(t-\tau) [gray]");
    c = get(gca,'children'); set(gca,'children',[c(2);c(3);c(4);c(1)]);
    subplot(2,1,2),plot(tvec,y,"k",tvec(ti),y(ti),"ok");
    xlabel("t"); ylabel("y(t) = \int h(\tau)x(t-\tau) d\tau");
    axis([tau(1) tau(end) -1.0 2.0]); grid;
    drawnow;
end
```

Figure 14: MATLAB Code for the convolution for H2.

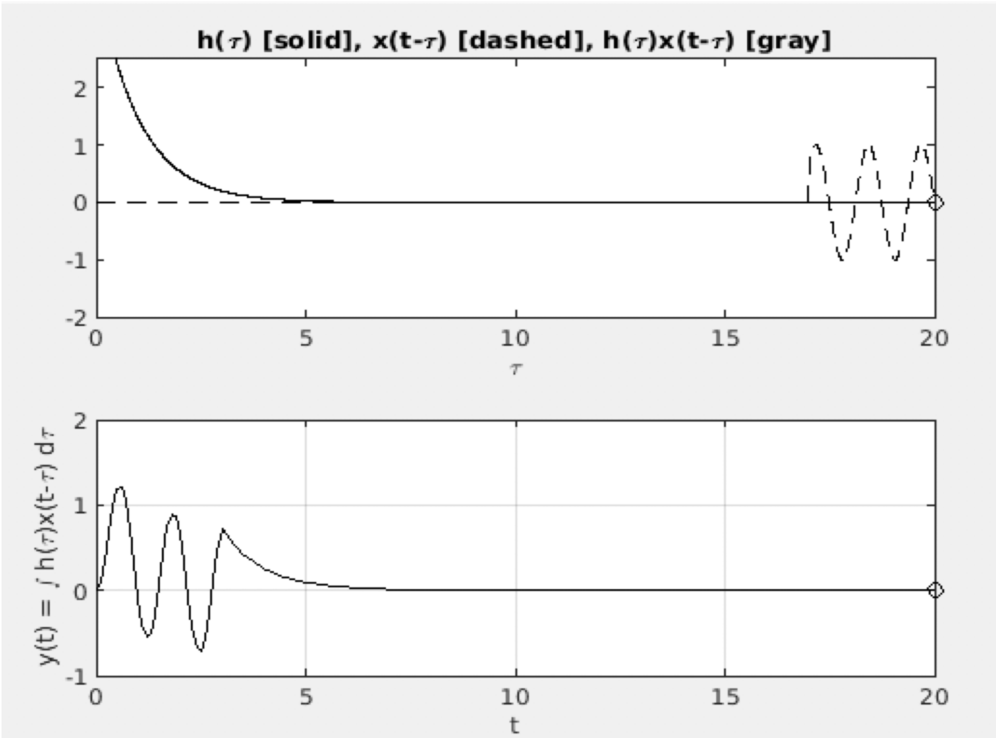


Graph 10: Graph for the above MATLAB code.

3. $h_3(t) = 4e^{-t}\,u(t)$

```
% First create the u(t) function
u = @(t) 1.0.* (t>=0);

% Create the x(t) function.
x = @(t) sin(5*t).*(u(t) - u(t - 3));

% Now truncate each of the impluse response funtions.
h = @(t) 4*exp(-t).*(u(t)-u(t-20));
% Modified CH2MP4 from B.1
dtau = 0.005;
tau = 0:dtau:20; ti = 0;
tvec = 0:.1:20; y = NaN*zeros(1,length(tvec));
% Pre-allocate memory
for t = tvec
    ti = ti+1; % Time index
    xh = x(t-tau).*h(tau);
    lxh = length(xh);
    y(ti) = sum(xh.*dtau);
    % Trapezoidal approximation of convolution integral
    subplot(2,1,1),plot(tau,h(tau),"k-",tau,x(t-tau),"k--",t,0,"ok");
    axis([tau(1) tau(end) -2.0 2.5]);
    patch([tau(1:end-1);tau(1:end-1);tau(2:end);tau(2:end)],...
        [zeros(1,lxh-1);xh(1:end-1);xh(2:end);zeros(1,lxh-1)],...
        [.8 .8 .8],"edgecolor","none");
    xlabel("\tau"); title("h(\tau) [solid], x(t-\tau) [dashed], h(\tau)x(t-\tau) [gray]");
    c = get(gca,'children'); set(gca,'children',[c(2);c(3);c(4);c(1)]);
    subplot(2,1,2),plot(tvec,y,"k",tvec(ti),y(ti),"ok");
    xlabel("t"); ylabel("y(t) = \int h(\tau)x(t-\tau) d\tau");
    axis([tau(1) tau(end) -1.0 2.0]); grid;
    drawnow;
end
```

Figure 15: MATLAB Code for the convolution for H3.

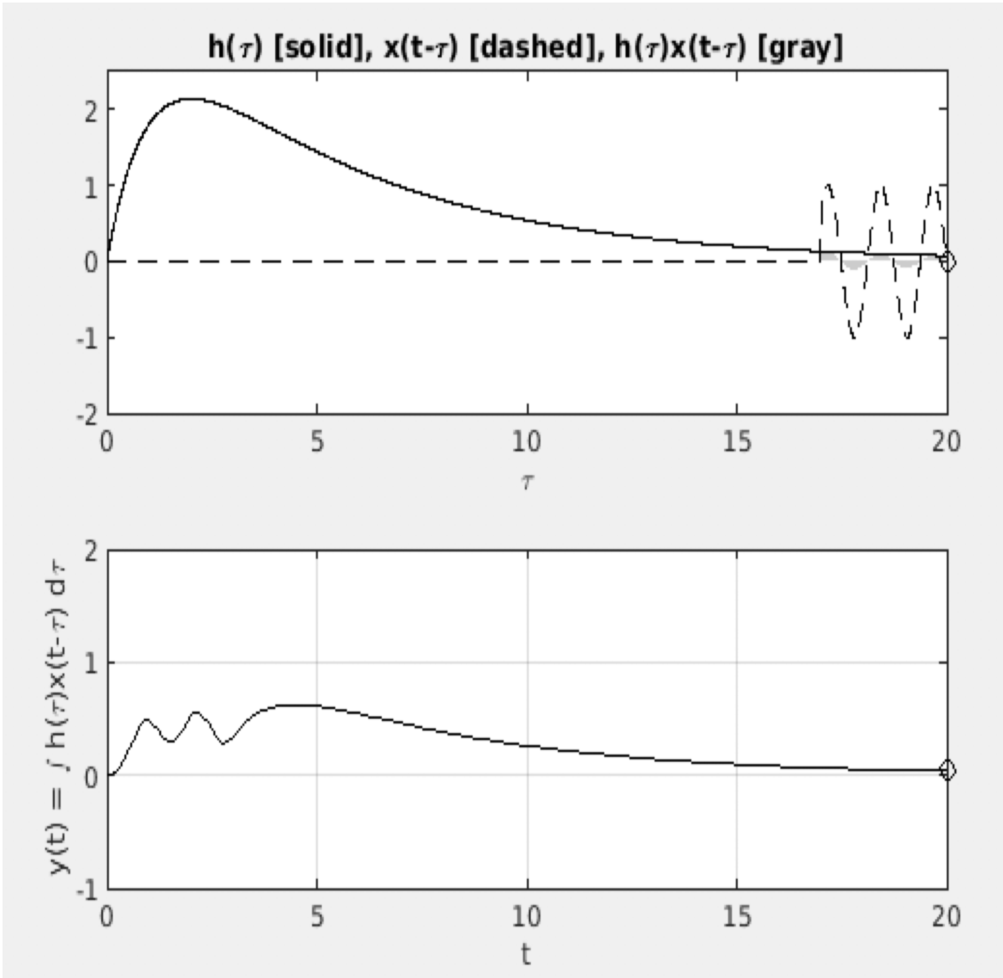


Graph 11: Graph for the above MATLAB code.

4. $h_4(t) = 4\,(e^{-t/5} - e^{-t})\,u(t)$

```
% First create the u(t) function
u = @(t) 1.0.* (t>=0);

% Create the x(t) function.
x = @(t) sin(5*t).*(u(t) - u(t - 3));

% Now truncate each of the impluse response funtions.
h = @(t) 4*(exp(-t/5)-exp(-t)).*(u(t)-u(t-20)); |
% Modified CH2MP4 from B.1
dtau = 0.005;
tau = 0:dtau:20; ti = 0;
tvec = 0:.1:20; y = NaN*zeros(1,length(tvec));
% Pre-allocate memory
for t = tvec
    ti = ti+1; % Time index
    xh = x(t-tau).*h(tau);
    lxh = length(xh);
    y(ti) = sum(xh.*dtau);
    % Trapezoidal approximation of convolution integral
    subplot(2,1,1),plot(tau,h(tau),"k-",tau,x(t-tau),"k--",t,0,"ok");
    axis([tau(1) tau(end) -2.0 2.5]);
    patch([tau(1:end-1);tau(1:end-1);tau(2:end);tau(2:end)],...
        [zeros(1,lxh-1);xh(1:end-1);xh(2:end);zeros(1,lxh-1)],...
        [.8 .8 .8],"edgecolor","none");
    xlabel("\tau"); title("h(\tau) [solid], x(t-\tau) [dashed], h(\tau)x(t-\tau) [gray]");
    c = get(gca,'children'); set(gca,'children',[c(2);c(3);c(4);c(1)]);
    subplot(2,1,2),plot(tvec,y,"k",tvec(ti),y(ti),"ok");
    xlabel("t"); ylabel("y(t) = \int h(\tau)x(t-\tau) d\tau");
    axis([tau(1) tau(end) -1.0 2.0]); grid;
    drawnow;
end
```

Figure 16: MATLAB Code for the convolution for H4.

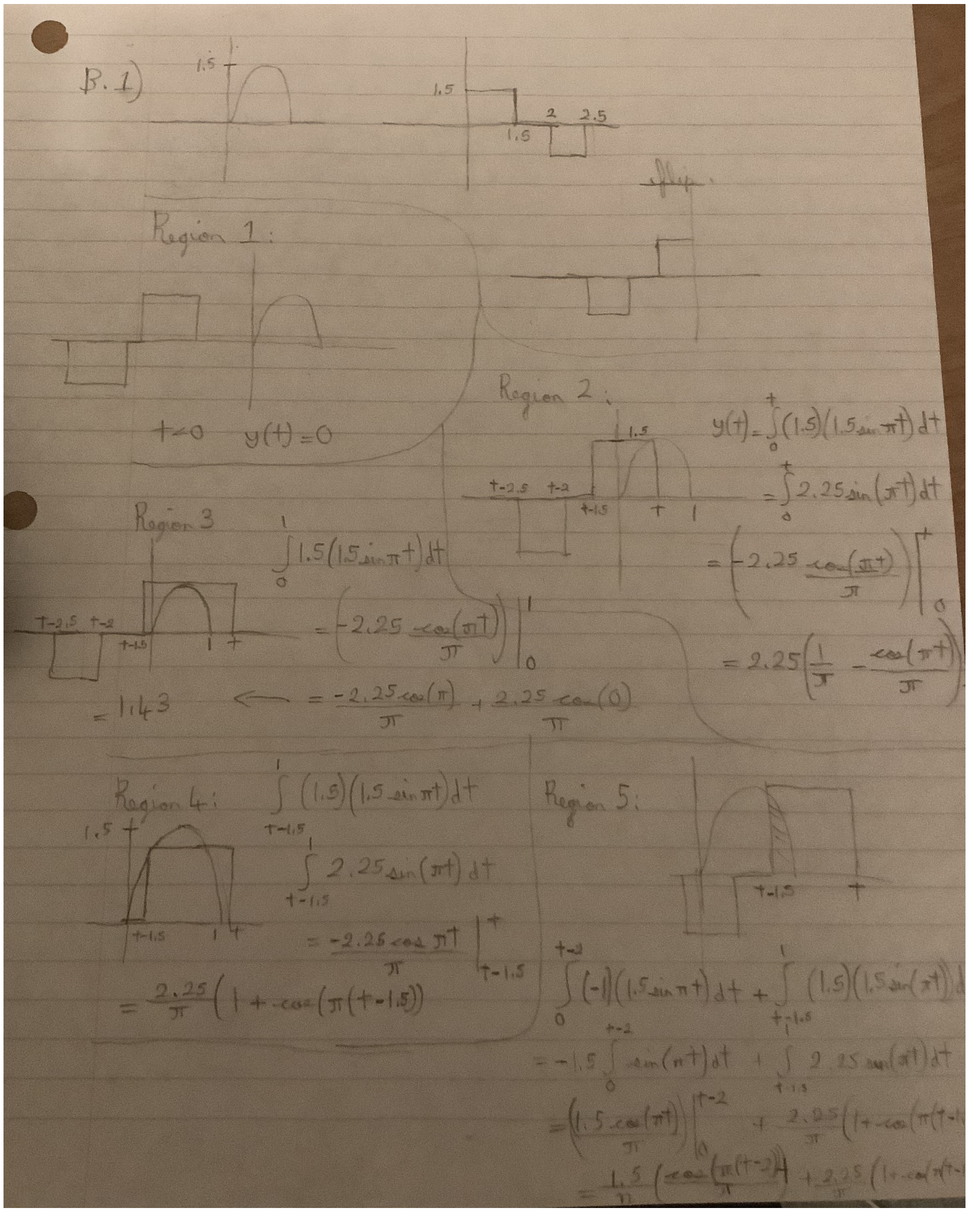


Graph 12: Graph for the above MATLAB code.

I observed that all the above convolutions are a part of the sin(t) function and it is similar to it. It is also observed that the duration of the signal resulting from the convolution of two signals is that the duration of the convolution is equal to the sum of the duration of the functions. There is a similar relationship between S2, S3 and S4. S2 and S3 have almost the same convolution but whereas S4 is similar to the S2 and S3.

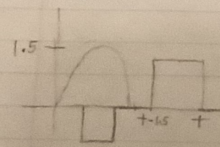
PART D (MOIZ)

D.1



Region 6:

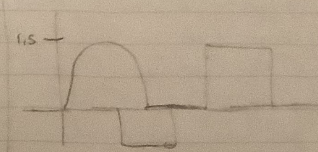
$$\int_{t-2.5}^{t-2} 1.5\sin(\pi t)\,dt$$



$$= \left(\frac{1.5\cos(\pi t)}{\pi}\right)\Bigg|_{t-2.5}^{t-2}$$

$$= \frac{1.5}{\pi}\left(\cos(\pi(t-2)) - \cos(\pi(t-2.5))\right)$$

Region 7:

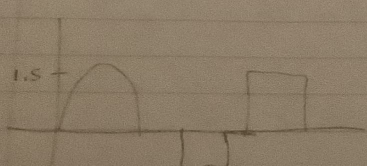


$$\int_{t-2.5}^{1} (-1)\,1.5\sin(\pi(t))\,dt$$

$$= \left(\frac{1.5\cos(\pi(t))}{\pi}\right)\Bigg|_{t-2.5}^{1}$$

$$= \frac{1.5}{\pi}\left(-1 - \cos(\pi(t-2.5))\right)$$

Region 8.



no overlap $y(t)=0$

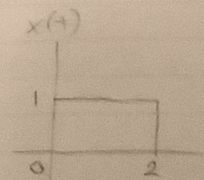
Total:

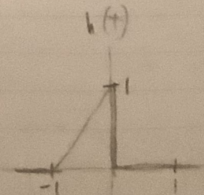
$$y(t) = 0 + 2.25\left(\frac{1}{\pi} - \frac{\cos(\pi t)}{\pi}\right) + 1.43 + \frac{2.25}{\pi}\left(1 + \cos(\pi(t-1.5)\right) + \frac{1.5}{\pi}\left(\cos(\pi(t-2)-1\right)$$

$$+ \frac{2.25}{\pi}\left(1 + \cos(\pi(t-1.5)\right) + \frac{1.5}{\pi}\left(\cos\pi(t-2) - \cos(\pi(t-2.5))\right) + \frac{1.5}{\pi}\left(-1 - \cos(\pi(t-2.5\right)$$

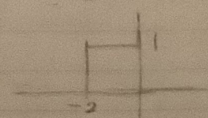
$$+0$$

B.2)





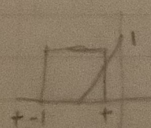
↳ flip



Region 1:

no overlap $y(t) = 0$

Region 2:

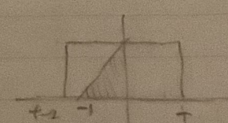


$$\int_{-1}^{t} (t+1)\,dt$$

$$\left(\frac{t^2}{2}+t\right)\Bigg|_{-1}^{t} = t\left(\frac{t}{2}+1\right)\Bigg|_{-1}^{t}$$

$$= \left(\frac{t^2}{2}+t\right) - \left(\frac{1}{2}-1\right) = \frac{t^2}{2}+t+\frac{1}{2}$$

Region 3:

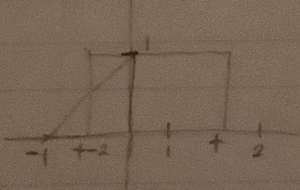


$$\int_{-1}^{0} (t+1)\,dt$$

$$\left(\frac{t^2}{2}+t\right)\Bigg|_{-1}^{0}$$

$$= -\frac{1}{2}$$

Region 4:

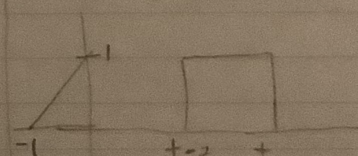


$$\int_{t-2}^{0} (1)(t+1)$$

$$= \left(\frac{t^2}{2}+t\right)\Bigg|_{t-2}^{0}$$

$$= -\left[\frac{(t-2)^2}{2}+(t-2)\right]$$

Region 5

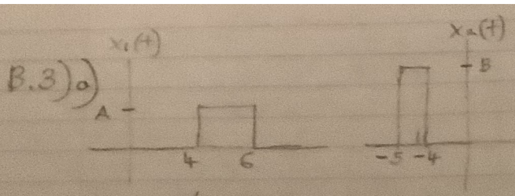


no overlap

Total:

$$y(t) = 0 + \frac{t^2}{2} + t + \frac{1}{2} + \frac{1}{2} - \left[\frac{(t-2)^2}{2} + (t-2)\right] + 0$$

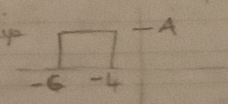
B.3) a)



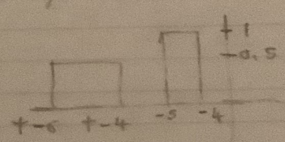
$x_1(t) = \frac{1}{2}(t-4) - u(t-6)$

$x_2(t) = 1\,[u(t+5) - u(t+4)]$

↳ flip

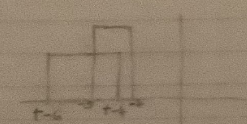


Region 1:



no overlap, $y(t) = 0$

Region 2:

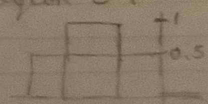


$$\int_{-5}^{t-4} 0.5\,dt = (0.5t)\Big|_{-5}^{t-4}$$

$$= \frac{1}{2}(t-4) - \frac{1}{2}(-5)$$

$$= \frac{1}{2}t + \frac{1}{2}$$

Region 3:



$$\int_{-5}^{-4} 0.5\,dt$$

$$= (0.5t)\Big|_{-5}^{-4}$$

$$= 0.5(-4) - 0.5(-5) = 0.5$$

Region 4:

$$\int_{t-6}^{-4} 0.5\,dt$$

$$= (0.5t)\Big|_{t-6}^{-4}$$

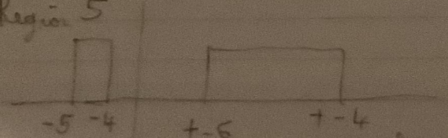
$$= 0.5(-4) - 0.5(t-6)$$

$$= -2 - 0.5t + 3 = -\frac{1}{2}t + 1$$

Total:

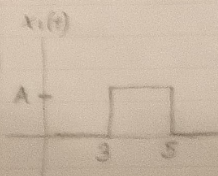
$$0 + \left(\frac{1}{2}t + \frac{1}{2}\right) + \frac{1}{2} + \left(-\frac{1}{2} + 1\right) + 0$$

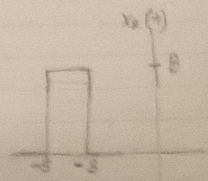
Region 5



no overlap $y(t) = 0$

B.3) b)

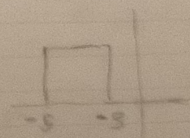




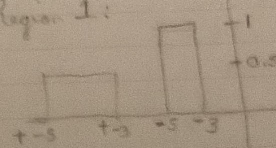
$x_1(t) = \frac{1}{2}(u(t-3) - u(t-5))$

$x_2(t) = 1(u(t+5) - u(t+3))$

flip

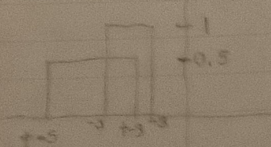


Region 1:



no overlap, $y(t) = 0$

Region 2:



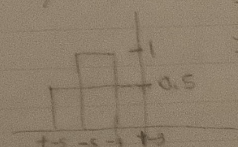
$\int_{-5}^{t-3} 0.5\,dt = (0.5t)\Big|_{-5}^{t-3}$

$= \frac{1}{2}(t-3) - \frac{1}{2}(-5)$

$= \frac{1}{2}t - \frac{3}{2} + \frac{5}{2}$

$= \frac{1}{2}t + 1$

Region 3:



$\int_{-5}^{-3} 0.5\,dt$

$= (0.5t)\Big|_{-5}^{-3}$

$= \frac{1}{2}(-3) - \frac{1}{2}(-5) = 1$

Region 4:

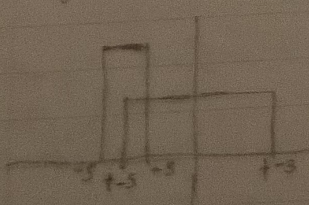
$\int_{t-5}^{-3} 0.5\,dt$

$= (0.5t)\Big|_{t-5}^{-3}$

$= \frac{1}{2}(-3) - \frac{1}{2}(t-5)$

$= -\frac{3}{2} - \frac{1}{2}t + \frac{5}{2}$

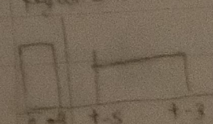
$= -\frac{1}{2}t + 1$



total:

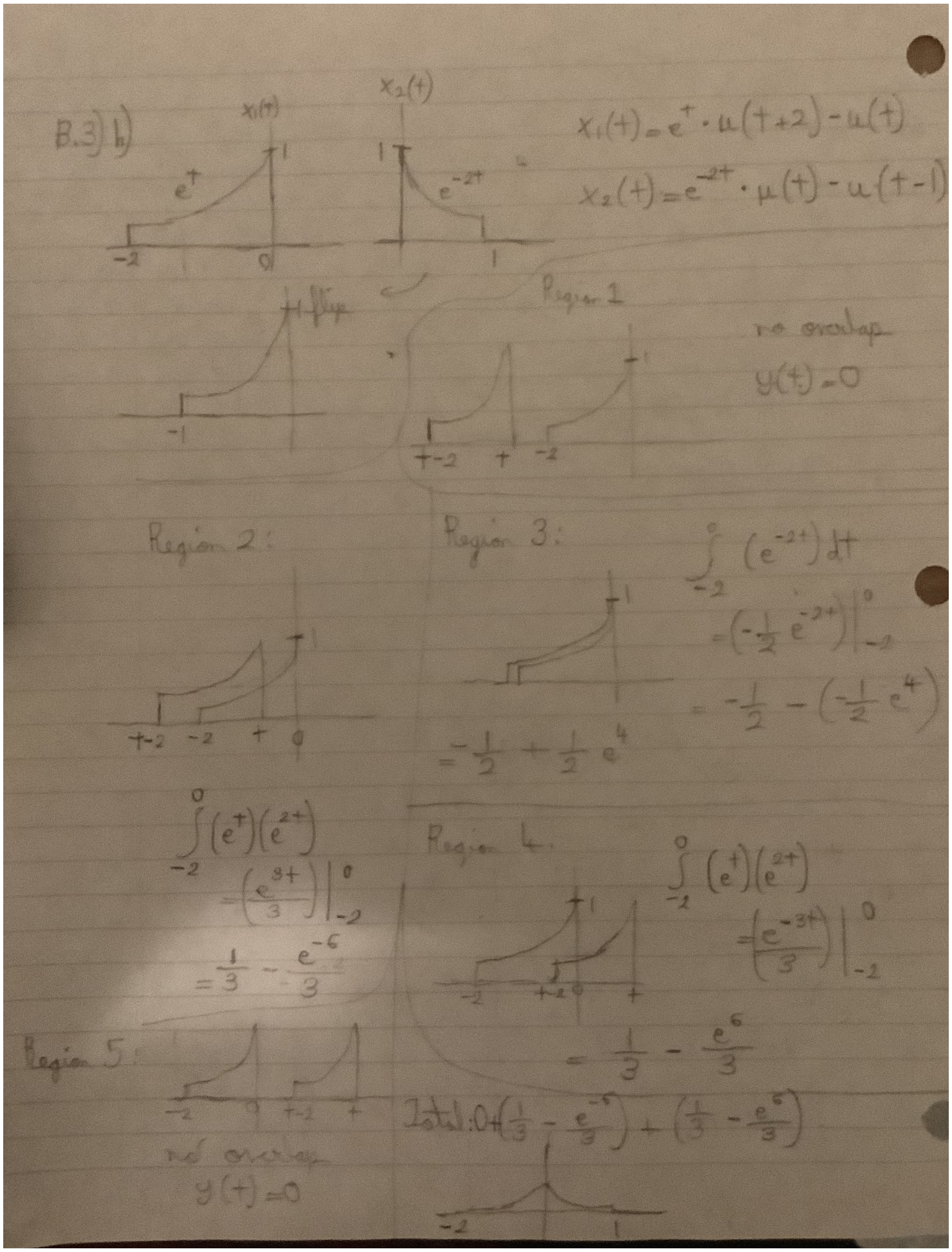
$0 + (\frac{1}{2}t + 1) + 1 + (-\frac{1}{2}t + 1) + 0$

Region 5:



no overlap

$y(t) = 0$



The calculations from the above pictures are similar to the experimental results.

D.2

The width/duration of the signal coming from the convolution of two signals may be determined by observing that the convolution's duration is equal to the sum of the functions' durations.