7.1 Translation

- Three dimensional transformation matrix for translation with homogeneous coordinates is as given below. It specifies three coordinates with their own translation factor.

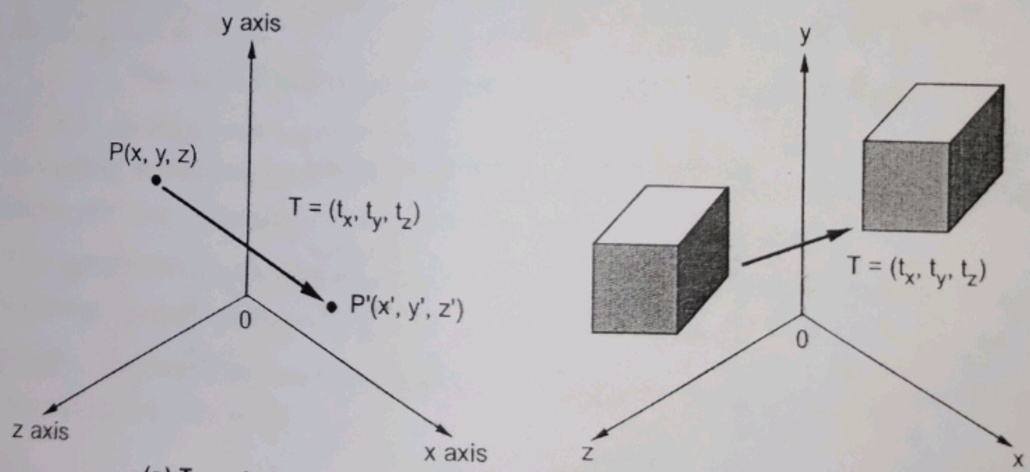
$$T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ t_x & t_y & t_z & 1 \end{bmatrix}$$

$$\therefore \quad P' = P \cdot T$$

$$\therefore [x' \ y' \ z' \ 1] = [x \ y \ z \ 1] \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ t_x & t_y & t_z & 1 \end{bmatrix}$$

$$= [x + t_x \quad y + t_y \quad z + t_z \quad 1] \qquad \ldots (7.1.1)$$

- Like two dimensional transformations, an object is translated in three dimensions by transforming each vertex of the object.



(a) Translating point
(b) Translating object

Fig. 7.1.1 3D translation

Review Question

1. *Obtain the 3 D transformation matrix for translation.*

7.2 Scaling

- Three dimensional transformation matrix for scaling with homogeneous co-ordinates is as given below.
- It specifies three co-ordinates with their own scaling factor.

$$S = \begin{bmatrix} S_x & 0 & 0 & 0 \\ 0 & S_y & 0 & 0 \\ 0 & 0 & S_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

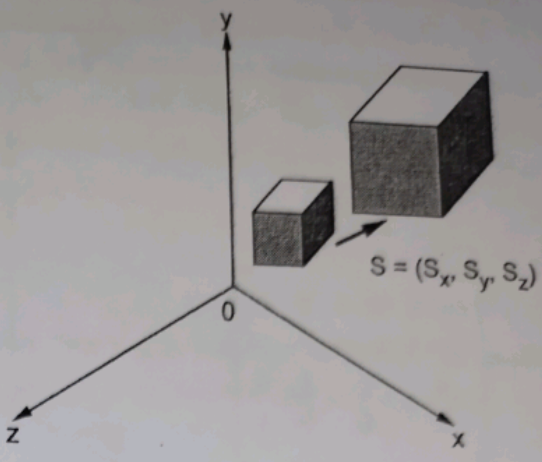


Fig. 7.2.1 3D scaling

$$\therefore \qquad P' = P \cdot S$$

$$\therefore [x' \ \ y' \ \ z' \ \ 1] = [x \ \ y \ \ z \ \ 1] \begin{bmatrix} S_x & 0 & 0 & 0 \\ 0 & S_y & 0 & 0 \\ 0 & 0 & S_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= [x \cdot S_x \quad y \cdot S_y \quad z \cdot S_z \quad 1] \qquad \text{... (7.2.1)}$$

- A scaling of an object with respect to a selected fixed position can be represented with the following transformation sequence.
  1. Translate the fixed point to the origin.
  2. Scale the object.
  3. Translate the fixed point back to its original position.

Review Question

1. *Obtain the 3D transformation matrix for scaling.*

7.3 Rotation

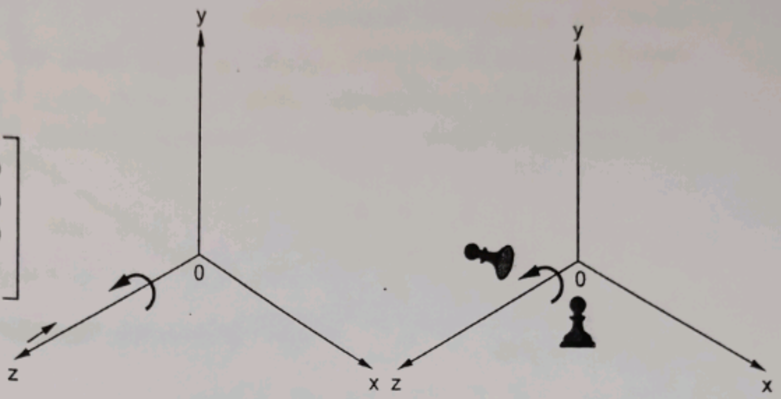
- Unlike two dimensional rotation, where all transformations are carried out in the xy plane, a three-dimensional rotation can be specified around any line in space. Therefore, for three dimensional rotation we have to specify an axis of rotation about which the object is to be rotated along with the angle of rotation.
- The easiest rotation axes to handle are those that are parallel to the co-ordinate axes. It is possible to combine the co-ordinate axis rotations to specify any general rotation.

Co-ordinate axes rotations

- Three dimensional transformation matrix for each co-ordinate axes rotations with homogeneous co-ordinate are as given below.

$$R_z = \begin{bmatrix} \cos\theta & \sin\theta & 0 & 0 \\ -\sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(a)



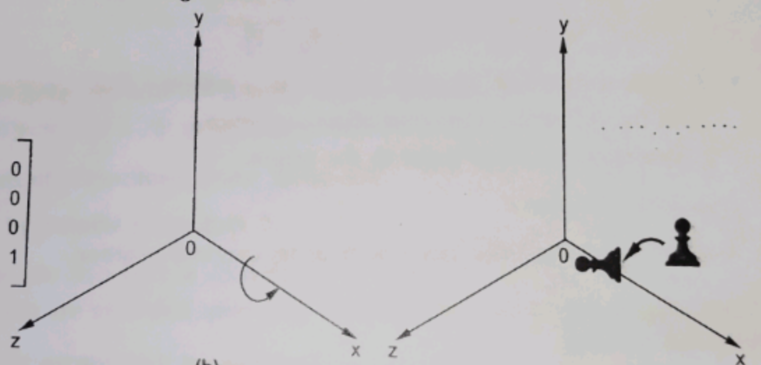
(b) (c)

Fig. 7.3.1 Rotation about z axis

- The positive value of angle θ indicates counterclockwise rotation. For clockwise rotation value of angle θ is negative.

$$R_x = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & \sin\theta & 0 \\ 0 & -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(a)

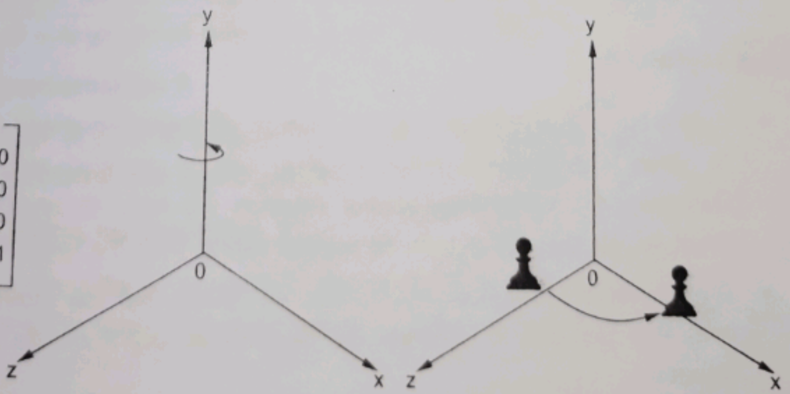


(b) (c)

Fig. 7.3.2 Rotation about x axis

$$R_y = \begin{bmatrix} \cos\theta & 0 & -\sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ \sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(a)



(b) (c)

Fig. 7.3.3 Rotation about y axis

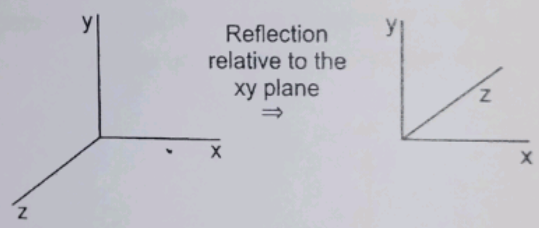
Review Questions

1. *Obtain the 3D transformation matrix for rotation about z-axis.*
2. *Obtain the 3D transformation matrix for rotation.*
3. *Explain the rotation about all co-ordinate axis.*

7.4 Reflection

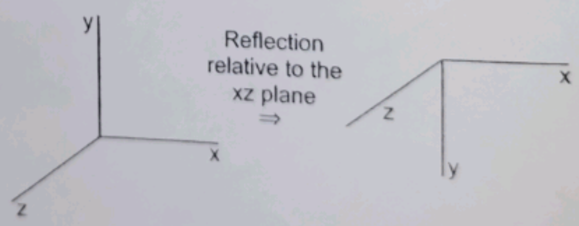
- A three-dimensional reflection can be performed relative to a selected reflection axis or with respect to a selected reflection plane.
- The three-dimensional reflection matrices are set up similarly to those for two dimensions.
- Reflections relative to a given axis equivalent to 180° rotations about that axis.
- Reflections with respect to a plane are equivalent to 180° rotations in four dimensional space.
- The reflections relative to xy, yz and zx planes are as shown in Fig. 7.4.1.

$$RF_z = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



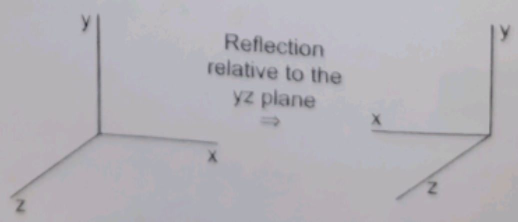
(a) Reflection relative to xy plane

$$RF_y = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



(b) Reflection relative to xz plane

$$RF_x = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



]5.

(c) Reflection relative to yz plane

Fig. 7.4.1