# Tutorial 1

## 4.1 Truth tables and semantics

**Exercise 4.1.** *Convert any formula into an equivalent formula which uses only symbols* $\wedge$ *and* $\neg$.

**Exercise 4.2.** *Write a formula which satisfies the following condition*

$$\begin{array}{r} p_1p_2\dots p_n \\ + \\ q_1q_2\dots q_n \\ = \\ r_1r_2\dots r_n \end{array}$$

*View* $p_1,\dots,p_n,q_1,\dots,q_n,r_1,\dots,r_n$ *as propositions.*

**Exercise 4.3.** *Use propositional logic to answer the following question:*

*You are trapped in a room. There are two doors. Either the doors lead to an exit or to a lion (note that both leading to an exit or to a lion are also possible). In Door 1, it is written "This door is exit and other door leads to lion". In Door 2, it is written "One of the rooms lead to exit, the other to a lion". You are told that only one of the written statements are true and the other false. Which door would you choose?*

**Exercise 4.4** (Pigeon hole principle)**.** *Consider* $n+1$ *pigeons and* $n$ *holes. Let proposition* $p_{i,j}$ *denote the fact that the* $i^{th}$ *pigeon is in the* $j^{th}$ *hole. Write a propositional logic formula to encode the pigeon hole principle.*

**Exercise 4.5.** *Show that the following statments are equivalent.*

1. $\alpha_1,\alpha_2,\dots,\alpha_n \models \beta$
2. $T \models \Big(\alpha_1 \Rightarrow (\alpha_2 \Rightarrow \dots(\alpha_n \Rightarrow \beta))\Big)$
3. $T \models (\alpha_1 \wedge \alpha_2 \wedge \dots \alpha_n) \Rightarrow \beta$

## 4.2 Natural deduction

**Exercise 4.6.** *Derive natural deduction proofs for the following*

1. $T \vdash \alpha \vee \neg\alpha$.

2. $\neg(\alpha \wedge \beta) \dashv\vdash \neg\alpha \vee \neg\beta$.

3. $T \vdash \alpha \Rightarrow (\beta \Rightarrow \alpha)$

4. $\alpha \vee \beta, \alpha \vee \neg\beta \vdash \alpha$

5. $\neg\alpha \vdash \alpha \Rightarrow \beta$

6. $\neg\alpha \vdash \neg(\alpha \wedge \beta)$

**Exercise 4.7.** *Show that the following are equivalent*

1. $\alpha_1, \alpha_2, \ldots, \alpha_n \vdash \beta$

2. $T \vdash \Big(\alpha_1 \Rightarrow \big(\alpha_2 \Rightarrow (\ldots(\alpha_n \Rightarrow \beta))\big)\Big)$

3. $T \vdash (\alpha_1 \wedge \alpha_2 \wedge \ldots \alpha_n) \Rightarrow \beta$

**Exercise 4.8.** *Answer the following questions*

1. *Let $\Psi$ be a formula over only the proposition $p$. Assume that $p \vdash \psi$ and $\neg p \vdash \psi$. Show that, $T \vdash \psi$.*

2. *This is an extension of the previous one. Let $\psi$ be a formula over proposition $P$. Let $\Gamma$ be the set of all formulas over $P$. If $\alpha \vdash \psi$ for all $\alpha \in \Gamma$, then $T \vdash \psi$.*


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