



Quiz 2 (CS 315)

Advanced Algorithms (CS 315)

Questions: 2, Marks: 25

**Instructions:** Only YOUR notebook allowed (No electronic devices, textbooks, printed/photocopies).

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## Answer all questions

1. Roll a single fair dice (a dice is a cube with 6 faces, each face having a number from 1 to 6) and let  $X$  be the outcome. Answer the following question with explanations.
  1. (2 marks) What is  $E[X]$ ?
  2. (2 marks) What is  $Var[X]$ ?
  3. (2 marks) What is the  $Prob[X \geq 6]$ ?
  4. (2 marks) Use Markov's inequality to calculate  $Prob[X \geq 6]$ .
  5. (2 marks) Use Chebyshev's inequality to calculate  $Prob[X \geq 6]$ .
2. (15 marks) Consider the following problem: Let  $U$  be a universe of  $n$  elements and  $S_1, S_2, \dots, S_m$  be subsets of  $U$ . We color every element in  $U$  with red color or green color with equal probability (that is, for an element  $v \in U$ , the  $Prob[v \text{ is colored red}] = Prob[v \text{ is colored green}] = \frac{1}{2}$ ). We measure the *Badness* of a set  $S_i$ , for an  $i \leq m$  as follows

$$Badness(S_i) = \left| \frac{cardinality(S_i)}{2} - \text{number of elements in } S_i \text{ colored red} \right|$$

Show, with clear explanation, that

$$Prob [\text{there exists an } i \text{ such that } Badness(S_i) \geq \sqrt{3n \log m}] \leq \frac{2}{m}$$