An example of layered min-sum decoder

$$H = \left[\begin{array}{cccccccc} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ \hline 0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 1 & 1 & 0 \end{array}\right] \begin{array}{l} \left.\vphantom{\begin{array}{c}1\\1\end{array}}\right\} \text{layer 1} \\ \left.\vphantom{\begin{array}{c}1\\1\end{array}}\right\} \text{layer 2} \end{array}$$

$$\bar{r} = \begin{bmatrix} 0.8 & 0.2 & -1.3 & 0.4 & -0.6 & -0.1 & 0.9 & -0.7 \end{bmatrix}$$

④ Iteration = 1, Layer 1

Initialization similar to earlier example but only on layer 1

$$L = \begin{bmatrix} 0.8 & 0.2 & -1.3 & 0.4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -0.6 & -0.1 & 0.9 & -0.7 \\ & & & & & & & \end{bmatrix}$$

min sum operation on each row: (row 1 and row 2)

$$L = \begin{bmatrix} -0.2 & -0.4 & 0.2 & -0.2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.1 & 0.6 & -0.1 & 0.1 \\ & & & & & & & \end{bmatrix}$$

$\bar{r}$ is updated after each layer, using only the column elements of the completed layer.

$$\text{updated } \bar{r} = \begin{bmatrix} 0.6 & -0.2 & -1.1 & 0.2 & -0.5 & 0.5 & 0.8 & -0.6 \end{bmatrix}$$

(B) iteration = 1 , layer 2

layer 2 is initialized with updated $\bar{\gamma}$.

updated $\bar{\gamma}$ (from layer 1) = $[0.6 \;\; -0.2 \;\; -1.1 \;\; 0.2 \;\; -0.5 \;\; 0.5 \;\; 0.8 \;\; -0.6]$

$$L = \begin{bmatrix} & & & & & & & \\ & & & & & & & \\ 0 & -0.2 & 0 & 0.2 & -0.5 & 0 & 0 & -0.6 \\ 0.6 & 0 & -1.1 & 0 & 0 & 0.5 & 0.8 & 0 \end{bmatrix}$$

minsum operation on each row : (row 3 and 4)

$$L = \begin{bmatrix} & & & & & & & \\ & & & & & & & \\ 0 & 0.2 & 0 & -0.2 & 0.2 & 0 & 0 & 0.2 \\ -0.5 & 0 & 0.5 & 0 & 0 & 0.6 & 0.5 & 0 \end{bmatrix}$$

$\bar{\gamma}$ is updated :

$$\bar{\gamma} = [0.1 \;\; 0 \;\; -0.6 \;\; 0 \;\; -0.3 \;\; -0.1 \;\; 0.3 \;\; -0.4]$$

(C) iteration = 2 , layer 1 :

from iteration 2 onwards, there is an additional step of adjusting $\bar{\gamma}$ before placing them on storage matrix L.

new $\bar{\gamma}$ = $\bar{\gamma}$ − layer 1 ;
(after last iteration)

Layer 1 after last iteration :

$$= \begin{bmatrix} -0.2 & -0.4 & 0.2 & -0.2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.1 & 0.6 & -0.1 & 0.1 \end{bmatrix} \quad \text{— (1)}$$

$\bar{r}$ after last iteration :

$$\bar{r} = \begin{bmatrix} 0.1 & 0 & -0.6 & 0 & -0.3 & -0.1 & 0.3 & -0.4 \end{bmatrix} \quad \text{— (2)}$$

$$\text{updated } \bar{r} = \begin{bmatrix} 0.3 & 0.4 & -0.8 & 0.2 & -0.4 & -0.7 & 0.4 & -0.5 \end{bmatrix} \rightarrow (2)-(1)$$

This $\bar{r}$ is used to initialise layer 1 in iteration 2 .

incoming $\bar{r}$ ~~(at (1))~~ already had information from layer 1 in last iteration , that needs to be corrected .

$$L = \begin{bmatrix} 0.3 & 0.4 & -0.8 & 0.2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -0.4 & -0.7 & 0.4 & -0.5 \\ & & & & & & & \end{bmatrix}$$

minimum operation on layer 1 :

$$L = \begin{bmatrix} -0.2 & -0.2 & 0.2 & -0.3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.4 & 0.4 & -0.4 & 0.4 \\ & & & & & & & \end{bmatrix}$$

$$\text{updated } \bar{r} = \begin{bmatrix} 0.1 & 0.2 & -0.6 & -0.1 & 0 & -0.3 & 0 & -0.1 \end{bmatrix}$$

(D) iteration 2, layer 2

incoming $\bar{r} = \begin{bmatrix} 0.1 & 0.2 & -0.6 & -0.1 & 0 & -0.3 & 0 & -0.1 \end{bmatrix}$

layer 2
(after last iteration)
$= \begin{bmatrix} 0 & 0.2 & 0 & -0.2 & 0.2 & 0 & 0 & 0.2 \\ -0.5 & 0 & 0.5 & 0 & 0 & -0.6 & -0.5 & 0 \end{bmatrix}$

Corrected $\hat{r} = \begin{bmatrix} 0.6 & 0 & -1.1 & 0.1 & -0.2 & 0.3 & 0.5 & -0.3 \end{bmatrix}$

$$L = \begin{bmatrix} 0 & 0 & 0 & 0.1 & -0.2 & 0 & 0 & 0.3 \\ 0.6 & 0 & -1.1 & 0 & 0 & 0.3 & 0.5 & 0 \end{bmatrix}$$

minsum operation operation on each row:

$$L = \begin{bmatrix} 0 & 0 & 0 & 0.2 & -0.1 & 0 & 0 & -0.1 \\ -0.3 & 0 & 0.3 & 0 & 0 & -0.5 & -0.3 & 0 \end{bmatrix}$$

updated $\bar{r} = \begin{bmatrix} 0.3 & 0 & -0.8 & 0.3 & -0.3 & -0.2 & 0.2 & -0.4 \end{bmatrix}$