TOPUP, A GLICK AND A YUP

$$S(t) \propto \int_{x}^{\infty} \int_{y}^{\infty} \rho(x, y) e^{iy[\Delta B(x,y) + G_{f}(x,y,t) + G_{p}(x,y,t)]} dxdy$$

$$\underbrace{\mathbf{s}}_{m \times 1} = \underbrace{\mathbf{A}}_{m \times n_{x}n_{y}} \underbrace{\mathbf{\rho}}_{n_{x}n_{y} \times 1}$$

$$\mathbf{A} = \begin{bmatrix} e^{iy[\Delta B_0(x_1,y_1)t_1 + G_f(x_1,y_1,t_1) + G_p(x_1,y_1,t_1)]} & e^{iy[\Delta B_0(x_2,y_1)t_1 + G_f(x_2,y_1,t_1) + G_p(x_2,y_1,t_1)]} & \cdots & e^{iy[\Delta B_0(x_n,y_n,t_1) + G_f(x_n,y_n,t_1) + G_p(x_n,y_n,t_1)]} \\ e^{iy[\Delta B_0(x_1,y_1)t_2 + G_f(x_1,y_1,t_2) + G_p(x_1,y_1,t_2)]} & e^{iy[\Delta B_0(x_2,y_1)t_2 + G_f(x_2,y_1,t_2) + G_p(x_2,y_1,t_2)]} & \cdots & e^{iy[\Delta B_0(x_n,y_n,t_2) + G_f(x_n,y_n,t_2) + G_p(x_n,y_n,t_2)]} \\ \vdots & \vdots & \ddots & \vdots \\ e^{iy[\Delta B_0(x_1,y_1)t_m + G_f(x_1,y_1,t_m) + G_p(x_1,y_1,t_m)]} & e^{iy[\Delta B_0(x_2,y_1)t_m + G_f(x_2,y_1,t_m) + G_p(x_2,y_1,t_m)]} & \cdots & e^{iy[\Delta B_0(x_n,y_n,t_1) + G_f(x_n,y_n,t_1) + G_p(x_n,y_n,t_1) + G_p(x_n,y_n,t_1)]} \end{bmatrix}$$

$$\underbrace{\mathbf{f}}_{n^2 \times 1} = \underbrace{\mathbf{F}}^H \quad \underline{\mathbf{A}} \quad \underline{\boldsymbol{\rho}}_{} = \underbrace{\mathbf{K}}_{n^2 \times n^2} \quad \underline{\boldsymbol{\rho}}_{}^2 .$$



SIMPLIFICATIONS

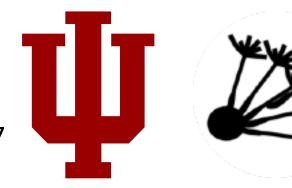
- t_i used to multiply B0 by in A increase only in discrete steps for each phase-encode step
- Ignore any susceptibility in frequency-encode direction

$$\mathbf{K} = \begin{bmatrix} \frac{\mathbf{K}_{1}}{n \times n} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \frac{\mathbf{K}_{2}}{n \times n} & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \frac{\mathbf{K}_{n}}{n \times n} \end{bmatrix} \quad \hat{\boldsymbol{\rho}}_{i} = \mathbf{K}_{i}^{+} \mathbf{f}_{i}$$

$$\hat{\boldsymbol{\rho}}_i = \mathbf{K}_i^+ \mathbf{f}_i$$

$$\mathbf{F}_{jk} = e^{-2\pi\sqrt{-1}} \frac{\left(j - \frac{n}{2} - 1\right)\left(k - \frac{n}{2} - 1\right)}{n},$$
 $j, k = 1, 2, \dots, n$

$$[\mathbf{A}_i]_{jk} = e^{-2\pi\sqrt{-1}\left(\frac{\left(j-\frac{n}{2}-1\right)\left(k-\frac{n}{2}-1\right)}{n}+\frac{j}{n}\Delta B_0(x_i,y_k)\right)}$$



$$\boldsymbol{A}^+ = (\boldsymbol{A}^T \boldsymbol{A})^{-1} \boldsymbol{A}^T$$
 Kadah and Hu, 1997