TOPUP, A CLICK AND A YUP

$$S(t) \propto \int_x \int_y \rho(x,\, y) e^{i\gamma[\Delta B(x,y)+G_f(x,y,t)+G_p(x,y,t)]} dx dy$$

$$\underbrace{\mathbf{s}}_{m\times 1} = \underbrace{\mathbf{A}}_{m\times n_x n_y} \underbrace{\boldsymbol{\rho}}_{n_x n_y \times 1}$$

$$\mathbf{A} = \begin{bmatrix} e^{i\gamma[\Delta B_0(x_1,y_1)t_1+G_f(x_1,y_1,t_1)+G_p(x_1,y_1,t_1)]} & e^{i\gamma[\Delta B_0(x_2,y_1)t_1+G_f(x_2,y_1,t_1)+G_p(x_2,y_1,t_1)]} & \cdots & e^{i\gamma[\Delta B_0(x_{n_x},y_{n_y})t_1+G_f(x_{n_x},y_{n_y},t_1)+G_p(x_{n_x},y_{n_y},t_1)]} \\ e^{i\gamma[\Delta B_0(x_1,y_1)t_2+G_f(x_1,y_1,t_2)+G_p(x_1,y_1,t_2)]} & e^{i\gamma[\Delta B_0(x_2,y_1)t_2+G_f(x_2,y_1,t_2)+G_p(x_2,y_1,t_2)]} & \cdots & e^{i\gamma[\Delta B_0(x_{n_x},y_{n_y})t_2+G_f(x_{n_x},y_{n_y},t_2)+G_p(x_{n_x},y_{n_y},t_2)]} \\ \vdots & \vdots & \ddots & \vdots \\ e^{i\gamma[\Delta B_0(x_1,y_1)t_m+G_f(x_1,y_1,t_m)+G_p(x_1,y_1,t_m)]} & e^{i\gamma[\Delta B_0(x_2,y_1)t_m+G_f(x_2,y_1,t_m)+G_p(x_2,y_1,t_m)]} & \cdots & e^{i\gamma[\Delta B_0(x_{n_x},y_{n_y})t_m+G_f(x_{n_x},y_{n_y},t_m)+G_p(x_{n_x},y_{n_y},t_m)]} \end{bmatrix}$$

$$\underbrace{\mathbf{f}}_{n^2\times 1} = \underbrace{\mathbf{F}^H}_{n^2\times n^2} \underbrace{\mathbf{A}}_{n^2\times n^2} \underbrace{\boldsymbol{\rho}}_{n^2\times 1} = \underbrace{\mathbf{K}}_{n^2\times n^2} \underbrace{\boldsymbol{\rho}}_{n^2\times 1} .$$

SIMPLIFICATIONS

- t_i used to multiply B0 by in A increase only in discrete steps for each phase-encode step
- Ignore any susceptibility in frequency-encode direction

$$\mathbf{K} = \begin{bmatrix} \underbrace{\mathbf{K}_1}_{n\times n} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \underbrace{\mathbf{K}_2}_{n\times n} & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \underbrace{\mathbf{K}_n}_{n\times n} \end{bmatrix}$$

$$\hat{\boldsymbol{\rho}}_i = \mathbf{K}_i{}^+\mathbf{f}_i$$

$$\mathbf{F}_{jk} = e^{-2\pi\sqrt{-1}\frac{\left(j-\frac{n}{2}-1\right)\left(k-\frac{n}{2}-1\right)}{n}}, \quad j,k = 1,2,\ldots,n$$

$$[\mathbf{A}_i]_{jk} = e^{-2\pi\sqrt{-1}\left(\frac{\left(j-\frac{n}{2}-1\right)\left(k-\frac{n}{2}-1\right)}{n}+\frac{j}{n}\Delta B_0(x_i,y_k)\right)}$$

$$A^+ = (A^T A)^{-1} A^T$$

Kadah and Hu, 1997