X Ray

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Objective

This lab uses Bragg's diffraction equation to study data taken from diffraction of X-rays on NaCl (Sodium Chloride) and Al (Aluminum).

Theory and Setup

Figure 1: A very expensive thing

Creating the x-rays

The tube that creates the x-rays consists of a cathode, an anode, and a Molybdenum target in between. When the machine is turned on, the cathode is heated and emits electrons which strike the Molybdenum and create x-rays. These x-rays are then passed through a Collimator to parallelize the beam ensuring that the diffraction pattern we observe is due to Bragg Diffraction alone and not from slit-diffraction or interference between non-parallel x-rays. Once the x-rays pass the collimator, they are incident on the crystal sample and are recorded by the Geiger tube.
The x-ray beam is generated by a Molybdenum target that is bombarded with an electron beam. Electromagnetic radiation in such cases occurs when an electron is excited to a higher energy state and then returns to it's ground state releasing the excess energy in the form of photons. The energy of the released photon corresponds to the energy difference between the two energy states. The peaks in the x-ray spectrum for various materials are named after the shell that they return to (K, L, M etc), and the number of energy levels it drops (K_α corresponds to the drop from L to K, while K_β corresponds to the drop from M to K).
As the high-energy electrons from the electron beam pass through the Molybdenum target, they experience a decrease in kinetic energy due to interactions between the Molybdenum atoms. This lost energy also manifests as x-rays radiated from the target. In both these cases the energy of the x-rays created has a maximum possible energy given by:

$$E = eV_{acc} = h\frac{c}{\lambda}$$

where e is the charge of an electron, V_{acc} is the voltage applied to create the electron beam, and λ is the wavelength of the released x-ray.

Bragg Diffraction

The Geiger tube and sample rotate with respect to the incident beam such that the angle between the Geiger tube and the sample is equal to the angle of incidence of the beam on the sample (which is the same as the expected angle of emergence).
When the x-rays are incident on the crystal sample, the electron cloud surrounding the atom redistributes. This redistribution of the electrons creates more x-rays that have the same frequency (and therefore energy) as the incident rays. These re-emitted x-rays are blurred slightly due to changes the direction of propagation of the re-emitted x-rays. This blurring allows the x-rays to interact with each other and produce a diffraction pattern that is referred to as Bragg Diffraction. Typically, diffraction refers to a phenomenon that occurs when a wave encounters an obstacle and changes it's path. In this respect, Bragg Diffraction is significantly different from regular diffraction because the change in direction of the wave arises from the absorption and re-emission of electro-magnetic radiation as opposed to a collision with an obstacle.
Since Bragg diffraction produces the same pattern as regular diffraction, the same equations can be used to describe their relationship between wavelength and diffraction angle and we can tell that constructive interference occurs when at:

$$m\lambda = 2d\sin\theta$$

Where m is an integer that represents divisibility of the right-side term by the wavelength. d is the spacing between the atoms in a sample.

Moseley's equation gives us a relationship between the atomic number and the frequencies of the K_α and K_β lines:

$$\frac{f}{c} = \frac{1}{\lambda} = R(Z-1)^2\left(1 - \frac{1}{n^2}\right)$$

Where R refers to the Rydberg constant ($R = 1.097 \times 10^7 \mathrm{m}^{-1}$), Z is the atomic number, and n corresponds to the energy state from which the electron returns to ground state i.e. $n = 2$ corresponds to the K_α line and $n = 3$ corresponds to the K_β line.

Procedure

1. The NaCl crystal was placed in the sample tray and the accelerating voltage was set for the desired values ($V = 20, 25, 30, 35\mathrm{kV}$)
2. The current was set to 1 mA
3. The scanning angles were set to vary from 2 to 30 degrees in steps of 0.1 degree each second.
4. The process was repeated for an aluminum crystal.

Data Analysis

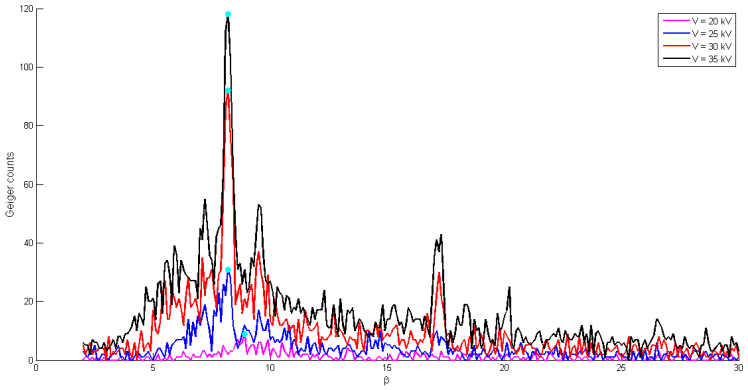

Figure 2: Aluminum peaks with different voltages in different colors

For Aluminum, we observed the following values at the minima:

V (in kV)	$\frac{\lambda}{2}$ **(in nm)**	θ_{min}**(in °)**	**d (in nm)**
20	$3.10 \times 10^{-}11$	6.3 ± 0.4	0.283 ± 0.018
25	$2.48 \times 10^{-}11$	5.0 ± 0.4	0.285 ± 0.023
30	$2.07 \times 10^{-}11$	4.2 ± 0.4	0.283 ± 0.027
35	$1.77 \times 10^{-}11$	3.8 ± 0.4	0.268 ± 0.028

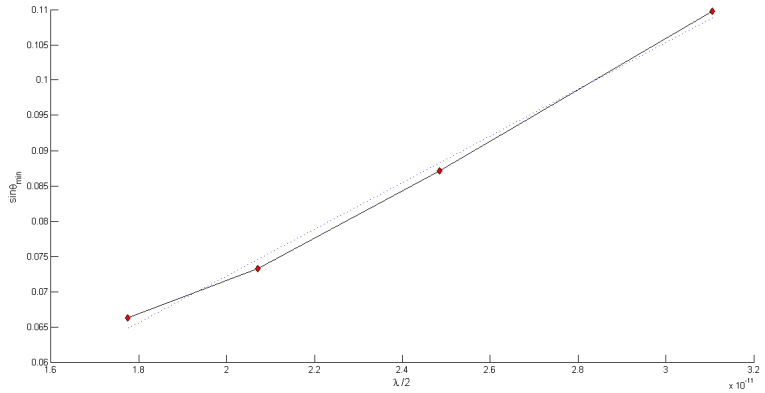

Figure 3: Plot of $\sin\theta_{min}$ against $\frac{hc}{2eV_{acc}}$ giving a slope of 0.302nm

This gives us an average particle separation of $d_{avg} = 0.280 \pm 0.008$nm

$$\lambda_\alpha = d\sin\theta_\alpha$$
$$\lambda_\beta = d\sin\theta_\beta$$

Peak	$\theta_{m=1}$**(in °)**	$\lambda_{theoretical}$ **(in nm)**	λ **(in nm)**
K_α	8.3 ± 0.2	0.072	0.080 ± 0.002
K_β	7.2 ± 0.2	0.061	0.070 ± 0.001

Here $\lambda_{theoretical}$ was found using Moseley's equation and pugging in numbers for the Molybdenum $K_\alpha and K_\beta$ peaks, while λ was found by using Bragg's diffraction law for the first minimum (m=1). For successive values of m, one would expect to find θ_m at arcsin $\left(\frac{m\lambda}{d}\right)$

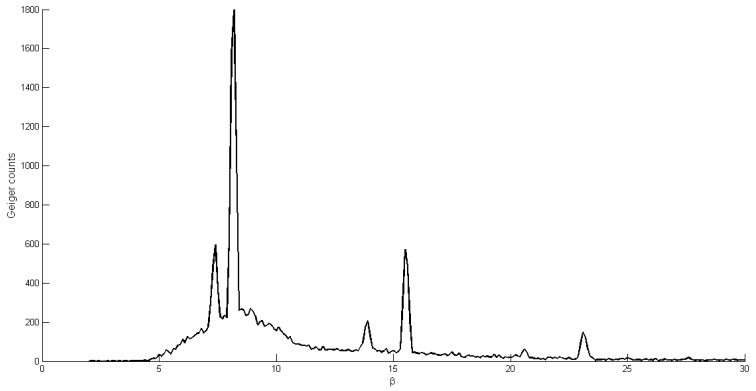

Figure 4: Counts vs Diffraction angle for NaCl sample with a step of 0.1° along θ per step and a step length of 1s and an accelerating voltage of 35kV

For the salt sample, most of our data was lost due to a technical issue, but for the data set we had (V=35 kV, and $\frac{\lambda}{2} = \frac{hc}{2eV_{acc}} = 1.77 \times 10^{-11}$m), the first minimum for Geiger counts was found at: $\theta = 4.8 \pm 0.2$. Putting this into:

$$d = \frac{\lambda}{2}\frac{1}{\sin\theta} = \frac{1.77 \times 10^{-11}}{\sin(4.8)} \approx 2.12 \times 10^{-10}\text{m} = 0.212 \pm 0.009\text{nm}$$

Peak	$\theta_{m=1}$**(in °)**	$\lambda_{theoretical}$ **(in nm)**	λ **(in nm)**
K_α	8.2 ± 0.2	0.072	0.060 ± 0.001
K_β	7.4 ± 0.2	0.061	0.054 ± 0.001

Error Analysis

The uncertainty in atomic distance was found using error propagation:

$$\delta d = \delta\theta\frac{\partial d}{\partial\theta} = \delta\theta\left(\frac{hc}{2eV_{acc}}\right)\frac{1}{\sin^2\theta}\cos\theta$$

and

$$\delta\lambda = \sqrt{(\delta d)^2\left(\sin\theta\right)^2 + (\delta\theta)^2\left(d\cos\theta\right)^2} = \sqrt{(\delta d)^2\sin^2\theta + (\delta\theta)^2 d^2\cos^2\theta}$$

The uncertainty in θ was in large part due to the uncertainty in alignment. Due to background radiation, the peaks in radiation counts are far more easily distinguishable and therefore have less uncertainty compared to the radiation minima. The large number of counts with the Salt sample suggests that the sample holder was far better aligned for this part of the experiment than for the Aluminum part. This made it easier to find the points of maximum and the first minimum, reducing the uncertainty in the measurements of the θ.

In the part of the experiment using the Aluminum sample, we plotted the $\sin\theta_{min}$ against the $\lambda/2$ and found the line of best fit. This line had a slope (which would give a value of d) of 0.302nm, putting the expected value of 0.286nm within a reasonable range of uncertainty.