KeyCorp
Model Risk Governance

Development, Documentation and Validation

Standards and Guidelines

December 2013

Revised 2013

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12/27/2012	Added clarification about data quality review	G. Rondon
12/30/2013	2.3 All sections updated	G. Rondon

Table of Contents

I. Executive Summary

The KeyBank code for Model Development, Documentation and Validation sets the minimum standards, recommendations and guidelines for model building, documentation and validation. The incorporation of the guidelines establishes certain specific requirements on business units developing models impacting the bank's decision making.

The main principal governing the application of the guidelines is reduction of model risk by providing best practice methods and processes to ensure the ongoing accuracy, suitability and reliability of KeyCorp's models.

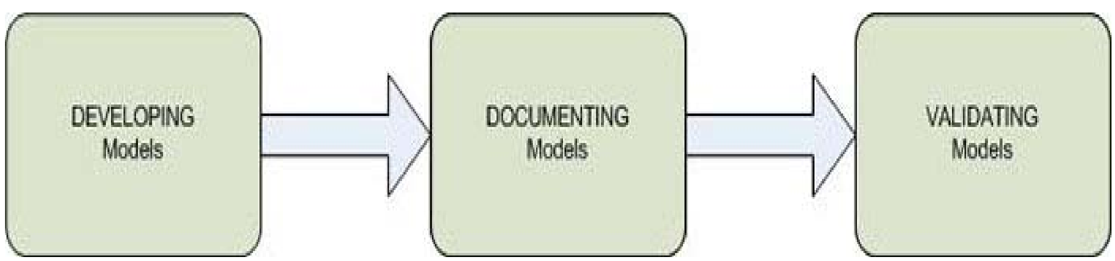
KeyCorp has established a Model Risk Committee (MRC) to ensure KeyCorp employs models that are suited for their intended purpose, properly implemented, and operating as intended. The MRC establishes standards, responsibility, and corporate governance over the approval, management, and validation of KeyCorp proprietary or vendor provided models.

The MRC membership is comprised of one representative from each of the following groups:

Group Head of Quantitative Risk Analysis – Chair

Commercial Credit Risk Management
Consumer/Retail Credit Risk Management
Operational Risk Management
Asset Recovery Group (ALLL)
Model Risk Control
Credit Portfolio Management
Asst/Liability Management
Capital Stress Testing
Internal Audit

For the purposes of this document, the standards and guidelines pertain to:



Model Risk Control continuously conducts training to line of business (LOB) for model development, documentation, and validation. This is consistent with Risk Management's 2nd line of Defense activities, which includes providing independent centralized oversight over all risk categories by aggregating, analyzing, and reporting risk information as part of Key's overall Enterprise Risk Management (ERM) framework.

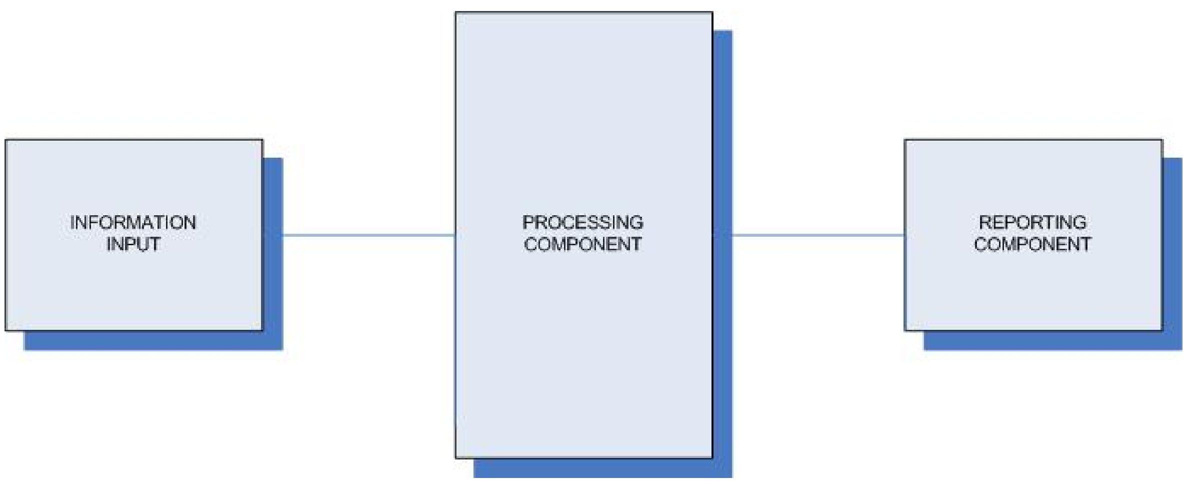
II. Definition of a Model

Models are simplified representations of real world relationships among observed characteristics, values, and events.

A model refers to a quantitative method, system, or approach that applies statistical, economic, financial, or mathematical theories, techniques, and assumptions to process input data into quantitative estimates. Thus, a model may use uncertain parameters and/or produce uncertain results.

A system that is capable of producing a value without the need of assumptions is not a model. Assumptions in this context refer to uncertain inputs or presupposed relationships. Thus, applications used to aggregate or reformat data so it can be used for other purposes are not models. Also, an application that performs calculations based on common algebraic formulas without any mathematical sophistication using actual customer information is not a model.

A model generally consists of three primary components:



Information input component, which delivers parameters and data to the model.

Processing component, which contains the theoretical model, then transforms inputs into estimates via the computer instructions (code).

Reporting component, which translates mathematical estimates into useful business information.

The definition of *model* also covers quantitative approaches whose inputs are partially or wholly qualitative or based on expert judgment, provided that the output is quantitative in nature.

III. Model Risk

<u>Model Risk</u> – Model Risk is the potential for adverse consequences such as financial loss, missed opportunities for generating earnings, reputational damage, or poor business and strategic decision-making caused by a model failure in one or more of the following model risk categories:

<u>Misspecification</u> – The data, inputs, and assumptions that are used by a model are improperly sourced, defined too broadly, or improperly calibrated.

> ***Example*:** A model works well at predicting default rates in normal economic condition, but does bad in extreme conditions.

<u>Misapplication</u> – A model is not appropriate for use by a particular segment or line of business. Possible causes of misapplication include a lack of understanding of a model's intended operating purpose, confidence limits, and level of accuracy.

> ***Example*:** The outputs from a vendor model are used in ways that do not conform to the model's intended purpose. For example, using RiskCalc for calculating the Probability of Default for a contractor using percentage of completion accounting method would result in misapplication of the model. RiskCalc hasn't been validated on financial statements that use these accounting methods.

<u>Incorrect Implementation</u> – A new model is implemented or updates/modifications are made to an existing model without adequate controls, model documentation, testing, and training to ensure proper integration into the broader model landscape.

> ***Example*:** A new risk rating model was built based on an old risk policy. The risk policy is changed during the model implementation period. However, the model is continuously implemented without considering the policy change.

Additionally, user-developed applications, such as spreadsheets or ad hoc database applications used to generate quantitative estimates are particularly prone to model risk.

The following is intended as a rough guide to the common types of model risks and how to overcome them.

PRIMARY RISK TYPE	MITIGATION GUIDELINES
Inapplicability Models are abstractions of reality. Models include assumptions and simplifications and necessarily exclude aspects of the real world that could be important components of the situation being modeled. Some aspects of the world are not subject to mathematical modeling. In some cases there is no model. *Example*: In order to model the price of a mortgage security, investors need to model the default risk of mortgagors. However, in 2010, the default risk of mortgagors was influenced by U.S. government policy. As U.S. government policy is not subject to the laws of mathematics, a model of mortgage securities could be inapplicable.	Provide a discussion of the applicability of the model to the situation being modeled. In particular, discuss the simplifying assumptions and the abstractions chosen and discuss why it was safe to ignore other aspects of the situation being modeled.
Incorrect solution A technical mistake in the derivation of a model could lead to an incorrect solution to a model. *Example*: A sign could be flipped or an error could be made copying a formula from a journal article. Or the incorrect model was used and does not accurately describe the reality being modeled.	Don't ignore small discrepancies in the output from the expected output. This could be a sign of technical errors in the model derivation.
Inappropriate use Using a model in a situation where the assumptions of the model are violated is an inappropriate use of the model. *Example*: In a linear regression model, there is an explicit assumption of homoscedasticity or constant variance. Consequently, it would be inappropriate to use the model to regress data with heteroscedasticity. There is a risk related to an inaccurate parameter estimation of an otherwise correct model.	Provide thorough documentation on correct uses of the model. Be explicit about situations where it would be inappropriate to use the model.

PRIMARY RISK TYPE	MITIGATION GUIDELINES
Bad approximations Many mathematical problems require numerical approximations to their solutions. Numerical methods are prone to inaccuracies due to round-off, truncation and discretization (transforming continuous problems to discrete problems), stability and convergence issues. The risk appears when numerical methods are used to solve a model.	Don't ignore small discrepancies in the output from the expected output. Choose algorithms that are known for stability. Investigate formula for subtraction of two nearly equal quantities from each other, or division of two quantities that are nearly zero. *Example*: Given the polynomial $p(x) = ax^2 + bx + c$ and seeking solutions of $p(x) = 0$, it is well-known that if a and c are both small then computing the quantity can lead to round-off error.
Software bugs Software bugs are flaws in computer programs that cause programs to produce incorrect results or behave in unintended ways. Software bugs are common and are a major source of model risk.	What procedures were followed to reduce the likelihood of significant software bugs? Are tests conducted regularly during the development/maintenance process? Do these tests test the model against inputs for which there is a known solution? *Example*: Note that a "line-by-line proofreading of the code" is not an effective way to reduce the likelihood of significant software bugs. Validate your code using different method or software
Bad data All models require input data. If the input data contains errors, the model will produce erroneous results. Additionally, the data could be unstable, so that data used to "fit" or "train" a model could be inappropriate as history might not be a good predictor of the future.	There should be rigorous assessment of data quality and relevance, and appropriate documentation. *Example*: Seasonal data requires adjustment for seasonality before or during analysis; not adjusting the data may lead to inaccurate estimates that do not capture the effects of seasonality.

IV. Roles and Responsibilities

At the forefront of best practices in risk policy governance is a sharp focus on three effective lines of defense.

1st Line of Defense

Line of Business (LOB)

Model Owner:

The role of model owner involves ultimate accountability to:

- Take ownership of model risks and assume ultimate accountability for model use and performance
- Take ownership of vendor models and assume full responsibility for model use and performance
- Manage within tolerances
- Identify which models to use to manage the business
- Manage the ultimate use of model results
- Ensure appropriate due diligence regarding suitability and accuracy of the model
- Coordinate with risk management and/or model user in the selection and approval of models
- Engage Corporate Sourcing to facilitate the purchase or lease of vendor models
- Ensure appropriate security and change control procedures are in place for all models
- Inform Model Risk Control of any updated model information that will entail an update in the model inventory. Updates that should be reported are:
    - Model changes
    - Model validation results
    - New model use

Model Developer:

The role of model developer involves ultimate accountability to:

- Develop models as required by model owners
- Test the components of a model and its overall functioning to determine whether the model is performing as intended. Testing should include:
    - Checking the model's accuracy
    - Demonstrating that the model is robust and stable
    - Assessing potential limitations and evaluating the model’s behavior over a range of input values
    - Assessing the impact of assumptions and identifying situations where the model performs poorly or becomes unreliable
- Coordinate with the LOB and Model Risk Control in the validation of models during development and subsequent stages of a model’s life cycle

• Ensure integrity of the model logic, including the algorithm and the data
• Create (or obtain in the case of vendor models) and maintain model documentation

The Line of Business (Model User):
The role of The Line of Business (model user) involves ultimate accountability to:
• Use the model strictly for intended business purpose
• Monitor and check integrity and accuracy of the model output
• Escalate actual or potential model weaknesses or failure to Model Risk Control, the independent Model Validation Group, and senior management
• Coordinate with the model developer and/or Model Risk Control in the validation of models during development and subsequent stages of a model's life cycle
• Inform Model Risk Control of any changes (including additions and deletions) that need to be made to the model inventory

2nd Line of Defense

Model Risk ControlModel Risk COntrol is responsible to:

• Develop Key's model risk control standards, which includes ways to control model risks
• Develop Key's overall view/position on model risk, which includes measurement methodologies
• Perform independent validations for models not developed in the Quantitative Risk Analysis group. The Strategic Analytics group will be the primary group responsible for performing the independent validations of models developed by Quantitative Risk Analysis
• Provide ongoing advisory services or feedback to model developers when warranted and requested
• Maintain ownership of the corporate model inventory
• Establish an independent risk-based annual model validation plan
• Draft (or coordinate drafting of) any proposed revisions to the Model Risk Policy
• Develop (if needed), maintain and implement processes to enable standard reporting and resolution of compliance issues, as outlined in Model Policy Section 3.3
• Develop (if needed), manage and implement the Certification process as outlined in Model Policy Section 3.3
• Provide guidance on interpretation issues as outlined in Model Policy Section 3.3
• Coordinate and execute reviews of the Model Risk Policy in compliance with Model Policy Section 3.4

3rd Line of Defense

Risk Review

Risk Review Group is responsible for:

- Providing an independent perspective on Key's model processes and risks by assessing the effectiveness of and adherence to Model Risk policies, practices and controls by model users, developers, owners (collectively the first line of defense) and model control units (second line of defense) across the enterprise
- Verifying model procedures and governance practices adhere to the Interagency guidelines (OCC 2000-16, FRB SR Letter 09-01 and OCC 2011-12 and others)
- Assessing Model Risk activities compliance with Key's Enterprise Risk Management Program requirements and expectations
- Issuing reports to the business unit, senior management, Risk Management, the MRC, and the Audit Committee on the results of its validation activities
- Making recommendations to enhance Model Risk practices
- Following up recommendations, including any made by regulators or external auditors
- Providing ongoing advisory services or feedback to Risk Management and/or Model Developers when warranted and requested

In accordance with its corporate charter, the Risk Review Group has complete and unrestricted access to all models, technical and operating documentation, computer code, developers, and other information necessary in the discharge of its duties.

At KeyCorp, the model owner, model developer, model user, and Risk Management share validation responsibilities.

V. Model Documentation

Model documentation consists of the technical and operating documents that detail and explain a model's technical specifications, assumptions, theory, code, inputs/outputs, weakness/strength, and operating procedures.

NOTE: ***The documentation must be sufficiently detailed to mitigate key-person risk, thus enabling a knowledgeable reader to replicate and operate the model to a level of precision consistent with its use, impact, and complexity within a reasonable period of time.***

Documentation provides for continuity of operations, makes compliance with policy transparent, and helps track recommendations, responses, and exceptions. Developers, users, control and compliance units, and supervisors are all served by effective documentation.

Model developers should have responsibility during model development for thorough documentation, which should be kept up-to-date as the model and application environment changes. For cases in which models from a vendor or other third party are used, the bank should ensure that appropriate documentation of the third-party approach is available so that the model can be appropriately validated.[1]

Documentation also includes non-technical reports of model results for management analysis and decision making. Finally, the policies and procedures, analysis, conclusions, and other work papers from the business unit and independent validations are integral components of documentation.

Model Documentation for all models should contain a discussion of *each* *model's*:

- Primary types of model risk
- Steps taken during model development and implementation to mitigate these primary types of model risk
- Key assumptions made in the structure and development of the model
- Assessment of the risk of violating key assumptions

[1] Taken from the OCC Bulletin 2011-12 Supervisory Guidance On Model Risk Management

V.1. Model Documentation Components

Effective model documentation should be organized into the following sections:

I. Executive Summary

II. Evaluation of Conceptual Soundness

- **i. Statement of Purpose**
- **ii. Methodology and Data**
- **iii. Critical Risks and Model Assumptions**

IV. Ongoing Monitoring

V. Outcome Analysis

- **i. Backtesting**
- **ii. Sensitivity Testing**

VI. Appendix

- **A. Contact and Identifying Information**
- **B. Model Validation**
  - **i. Validating computer code**
  - **ii. Validating model output and performance**
- **C. Model Output**

V.2. Model Documentation Guidelines

The check list below is provided as general guidance for the type of information that should be given in each section of the documentation. Model documentation contents will vary depending on factors such as type of model methodology in use or whether the model is third-party provided.

The **Executive Summary** section of the model documentation should provide a *summarized* version of the other sections of the model. It should state the purpose and use of the model.

Through the Executive Summary, management should be made aware of conceptual approaches and limitations, including key assumptions.

Evaluation of Conceptual Soundness involves assessing the quality of the model design and construction. It entails review of documentation and empirical evidence supporting the methods used and variables selected for the model. Documentation and testing should convey an understanding of model limitations and assumptions. This section of model documentation should include statement of purpose and use of model, methodology, and developmental data.

Methodology

- ☐ Reference any well-accepted theories on which the model is based.
- ☐ Show the comparison with alternative theories and approaches for a sound modeling process
- ☐ Indicate axioms and deductions. **Known results that can be verified do not need to be included, but should be referenced.**
- ☐ Explain the use of judgmental or qualitative adjustments as part of model development
- ☐ Evaluation of the various components of the model and its overall functioning to determine whether the model is performing as intended, including checking the model's accuracy, demonstrating that the model is robust and stable, assessing potential limitations, and evaluating the model's behavior over a range of input values

Data

- ☐ Data sources and Range of time covered by the data
- ☐ Rigorous assessment of data quality and relevance/suitability for the model, and consistency with the theory behind the approach and with the chosen methodology. **The discussion of data quality includes instances where internal data can't be used. In these cases, the documentation should include a discussion of the issues identified.**
- ☐ Data proxies used, how they were carefully identified and justified
- ☐ Adjustments made to the data and information – whether from an internal or external source - elaborate on the potential limitations

- ☐ If development data was limited, show the behavior of the model with stressed data inputs

The **<u>Critical Risks and Assumptions</u>** section of the model documentation should include an *<u>extensive discussion</u>* of:

- ☐ Where and in what circumstances it is appropriate to use the model
- ☐ Key assumptions made in the model structure and development
- ☐ The assessment of risks in relation to use of the model when the key assumptions are violated or absent. Explaining the impact of assumptions and identifying situations where the model performs poorly or becomes unreliable
- ☐ Results from sensitivity / what-if analysis using reasonable alternatives to major assumptions. This helps to communicate the robustness or fragility of model outputs.
- ☐ Rationale for use of vendor-provided assumptions or the decision to override those assumptions
- ☐ Rationale for use of market-generated assumptions or the decision to override those assumptions
- ☐ Assumptions based on the output of a separate model
- ☐ Assumptions about diversification or dependencies across risks and a rationale
- ☐ Assumptions characteristic of pricing models

The **<u>Ongoing Monitoring</u>** section is essential to evaluate whether changes in products, exposures, activities, clients, or market conditions necessitate adjustment, redevelopment, or replacement of the model and to verify that any extension of the model beyond its original scope is valid.

The **<u>Outcome Analysis</u>** Section of the model compares the model outputs to corresponding actual outcomes.

- ☐ Conduct Backtesting
- ☐ Conduct Stresstesting

In the **<u>Appendix</u>** section of the model documentation:

<u>Contact and Identifying Information</u>

- ☐ Indicate model owner name
- ☐ Indicate model developer name
- ☐ Indicate model user community/names
- ☐ Indicate date model began usage (implementation)

<u>Model Validation</u>

- ☐ Provide the results from independently reproducing key portions of the code
- ☐ Provide results from testing the model output with a known set of data
- ☐ Provide results from monitoring key performance statistics over time
- ☐ Provide the model triggers that may signal model breakdown

Model Output

- ☐ Provide examples of model output
- ☐ Provide outputs of estimates or forecasts including confidence bands around the estimate/forecast

Example of Model Assumptions and Discussion Guidelines

- With regard to vendor provided models, provide a rationale for use of vendor-provided assumptions or the decision to override those assumptions.

  Example: The bank chooses to derive its own assumptions by studying its own customer base regarding mortgage prepayments instead of using market-implied numbers available from various vendors about national or regional populations.

- Assumptions may be the output of a separate model. The inputs, processing and outputs of these models should be validated using sound validation principles.

  Examples: Prepayment functions for loan-valuation models,
  Core-deposit decay functions for cost-of-funds models.

- Some models contain assumptions about management actions or responses to a particular modeled situation. These need to be detailed.

  Example: Capital model assumptions regarding the sources or uses of additional capital.

- Assumptions about diversification or dependencies across risks should be acknowledged and a rationale supplied.

  Examples:
  - Infinite granularity in asymptotic single risk factor (ASRF) capital model.
  - Correlation between credit and market risk in an economic capital

- Assumptions characteristic of pricing models:
  - General equilibrium pricing models imply assumption of agreement on specification and parameter values. These assumptions may not be realistic.

    Example: The Black-Scholes model assumes risk-free interest rate, no arbitrage opportunities, and no transaction fees.
  - Such models are very effective when adjusted to reflect real world market conditions.

VI. Model Development

The modeling exercise is often a multidisciplinary activity drawing on economics, finance, statistics, mathematics, and other fields. Models are employed in real-world markets and events and therefore should be tailored for specific applications and informed by business uses.

Guidelines

Model development may be seen as spanning these four distinct milestones:

Business Proposition
Define business purpose, goals, and role of model – both the immediate, identified need and potential extensions of usage; and business-oriented measures of success.

> *Example:* Identify goals: define the problem, find out the intended usage of the model, the end users/audience. (The purpose/usage of the model often influences the choice of modeling tool/method.)

Development - Process: The process-perspective of model development from modeling problem definition, iterative building/refinement of analysis, through documentation

Development - Product: The immediate model and broader solution, where product is viewed as features/benefits to end users

Deployment & Application: Internal selling of model, deployment, and ongoing use, monitoring/tracking, and quality control of the model and broader business process.

The following provides documentation examples of the: a) Development - Process and b) Development – Product.

Development - Process:

Example: To build a behavior score model, a dependent variable of interest is established. Then a process of exploratory data analysis, isolation of potential explanatory variables, and data cleansing is undertaken. Candidate model formulations are developed, and sensitivity to the data sample is tested.

Elements of the development process, typically covered in model documentation, include:

- Definition of inputs and outputs of the model, and overall process
- Business assumptions agreed upon by stakeholders
- Data descriptive analysis, data cleansing, transformations
- Determination of data: input, output variables, timeframes, products, geographies, etc. (e.g. establish model development sample which reflects population on which model will be used; justify excluded sub-populations/observations; ensure surrogates for target variable are not included as explanatory variables)
- Summary of model framework, assumptions

Qualitative elements of good process development include:

- The process is orderly, tractable, and repeatable (e.g. on new data)
- The building process is independent of any given data or timeframe
- Stability is established (e.g. oversampling or bootstrapping used for sparse data scenarios)
- Assumptions (necessary or practical) are tested or challenged with alternatives (e.g. sensitivity analysis)
- Development of a challenger model that covers the critical analysis of objective, model limitations and assumptions, and produce appropriate changes.

Development - Product:

Example: A loss forecasting model formulation is finalized. Components of this product are the dependent and independent terms, the formulation, and the output capabilities. Then, in a broader sense the model is used in a production setting so the product is not only the core model but the overall solution: code to execute the modeling, diagnostic reports generated with the modeling, automated data loading, scoring, decisioning, regular updates of data (e.g. economic data), and live or periodic process reporting derived via the model.

Qualitative elements of an effective model or analytic solution include:

- Model is as parsimonious as possible
- Model is independent of data, timeframe, etc. (e.g. sample data used in model building is representative of data during application).
- Model performance is stable. Measures and metrics of usage (fit, scoring, forecasting, prediction, etc.) are attuned to original business proposition
- Model is sufficiently documented

VI.1. Model Development Checklist

☐ Make sure that the components work as intended, are appropriate for the intended business purpose, and are conceptually sound and mathematically and statistically correct.

☐ Comparing alternative theories and approaches is a fundamental component of a sound modeling process.

☐ Perform a rigorous assessment of data quality and relevance. Demonstrate that such data and information are suitable for the model and that they are consistent with the theory behind the approach and with the chosen methodology.

☐ Carefully identify data proxies, justify and document them.

☐ Testing the various components of a model and its overall functioning to evaluate to determine whether the model is performing as intended.

☐ Check the model's accuracy, demonstrating that the model is robust and stable, assessing potential limitations, and evaluating the model's behavior over a range of input values.

☐ Assess the impact of assumptions and identify situations where the model performs poorly or becomes unreliable.

☐ Test the model by applying actual circumstances under a variety of market conditions, including scenarios that are outside the range of ordinary expectations, and should encompass the variety of products or applications for which the model is intended.

VII. Model Validation

Model validation is a set of processes and activities intended to verify that models are performing as expected, in line with their design objectives and business uses. Effective validation helps ensure that models are sound. It also identifies potential limitations and assumptions, and assesses their possible impact.

All model components, including input, processing, and reporting, should be subject to validation; this applies equally to models developed in-house and to those purchased from or developed by vendors or consultants. The rigor and sophistication of validation should be commensurate with the organization's overall use of models, the complexity and materiality of its models, and the size and complexity of the bank's operations.

Effective model validation helps reduce model risk by identifying model errors, corrective actions, and appropriate use. It also provides an assessment of the reliability of a given model, based on its underlying assumptions, theory, and methods. In this way, it provides information about the source and extent of model risk. Validation can also reveal deterioration in model performance over time and set thresholds for acceptable levels of error, through analysis of the distribution of outcomes around expected or predicted values. If outcomes fall consistently outside this acceptable range, then the models should be redeveloped.[2]

Key Elements of Comprehensive Model Validation

An effective model validation framework should include three core elements:

- Evaluation of conceptual soundness, including developmental evidence

- Ongoing monitoring, including process verification and benchmarking

- Outcome analysis, including back-testing and stress-testing

[2] Taken from the OCC Bulletin 2011-12 Supervisory Guidance On Model Risk Management

1. Evaluation of Conceptual Soundness

Validation should ensure that judgment exercised in model design and construction is well informed, carefully considered, and consistent with published research and with sound industry practice. Developmental evidence should be reviewed before a model goes into use and also as part of the ongoing validation process, in particular whenever there is a material change in the model.

A sound development process will produce documented evidence in support of all model choices, including the overall theoretical construction, key assumptions, data, and specific mathematical calculations. As part of model validation, those model aspects should be subjected to critical analysis by both evaluating the quality and extent of developmental evidence and conducting additional analysis and testing as necessary. Comparison to alternative theories and approaches should be included.[3]

2. Ongoing Monitoring

The second core element of the validation process is ongoing monitoring. Such monitoring confirms that the model is appropriately implemented and is being used and is performing as intended.

Ongoing monitoring is essential to evaluate whether changes in products, exposures, activities, clients, or market conditions necessitate adjustment, redevelopment, or replacement of the model and to verify that any extension of the model beyond its original scope is valid. The organization should design a program of ongoing testing and evaluation of model performance along with procedures for responding to any problems that appear. This program should include process verification and benchmarking.[4]

3. Outcomes Analysis

The third core element of the validation process is outcomes analysis, a comparison of model outputs to corresponding actual outcomes. The precise nature of the comparison depends on the objectives of a model, and might include an assessment of the accuracy of estimates or forecasts, an evaluation of rank-ordering ability, or other appropriate tests. In all cases, such comparisons help to evaluate model performance, by establishing expected ranges for those actual outcomes in relation to the intended objectives and assessing the reasons for observed variation between the two.[5]

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[3] Taken from the OCC Bulletin 2011-12 Supervisory Guidance On Model Risk Management
[4] Taken from the OCC Bulletin 2011-12 Supervisory Guidance On Model Risk Management
[5] Taken from the OCC Bulletin 2011-12 Supervisory Guidance On Model Risk Management

VII.1. Model Validation Best Practices Checklist

It is expected that all of these validation practices will take place within the line of business responsible for model development.

A. Pre-production Model Validation Guidelines

- ☐ Validate by independent review (within the line of business) the modeling technique and methodology chosen as applicable for the intended purpose.
- ☐ Independently review the model for logical and conceptual soundness.
- ☐ Validate the reasonableness of the assumptions underlying the model.
- ☐ Validate model development data focusing on data quality and coverage.
- ☐ Ensure the model is adequately documented.
- ☐ Ensure the model is correctly installed for production use.
- ☐ Ensure new models are added to the corporate model catalog.

B. Ongoing Model Validation Guidelines

- ☐ Ongoing model validation is to be performed by a staff (within the line of business) organizationally independent of model development personnel to the extent practical.
- ☐ After a model has been in use for a period of time, model results are to be compared to actual outcomes at least once a year.
- ☐ Industry standard validation metrics should be computed regularly and compared to expected results. The tests should be understandable to Risk Management reviewers.
- ☐ Validation results should be regularly reviewed by management at a seniority level commensurate with the risk associated with the model.
- ☐ Models that show weakening predictive results should be reviewed and considered for re-development.

C. Model Output Guidelines

Model output should provide:

- ☐ Output/results of model calculations.
- ☐ Indication of the model’s power / reliability.
- ☐ Indication of confidence in the model’s output.
- ☐ Context or assumptions in which results should be understood.

1. **Results of model calculations**

- ☐ Output may include checking the statistics to indicate whether the model is working correctly.

  Example: Average backtest forecast error was less than 10%.

- ☐ Reports may provide additional detail to suggest the source or reason for a given model outcome.

  Example: Changes to stressed losses were primarily due to changes in projected unemployment rates.

- ☐ Reports can contain a breakdown by modeled or non-modeled segmentation.

  Examples:
  Loss forecasts broken down by geographic region (geographic region is not a part of the model specification).

2. **Model power / reliability**

- ☐ Output may include statistics on model power.

  Example: R^2 for a regression model.

- ☐ Some model outputs are conditional on forecasts or projections external to the model. Conditional outputs can be compared to actual results conditional on actual external factors.

  Example: Interest income conditional on a forecast of libor rates compared to actual interest income conditional on actual libor rates.

- ☐ Output may include statistics on past or current model performance relative to a management objective.

  Example: Percentage by which model forecast losses deviated from actual losses in recent quarters.

3. **Confidence in model output**

- ☐ Statistics to indicate the robustness or reliability of the output should be included.

  Example: Estimates of loan recovery are based on a sample of only 5 loans.

- ☐ Where possible, confidence in an estimate can be expressed in terms of confidence bands or intervals.

  Example: Sensitivity test indicates model performs optimally within 2 standard deviations of the error term.

4. Context and assumptions

- ☐ To ensure proper understanding of output, it should be reported in the context of key assumptions.

- ☐ Best practice reporting should contain sensitivity/what-if analysis using reasonable alternatives to major assumptions. This helps to communicate the robustness or fragility of model outputs.

VIII. Second Line of Defense Validation

Once KeyCorp's model inventory has been centralized within the Model Risk Control group, a model validation by the 2^{nd} Line of Defense will transpire. The second line test the effectiveness of the1st line of defense validations by reviewing the validation work performed on the models.

All models in the Bank's Model Inventory will be subject to a second line of defense validation. The models will be prioritized based on a pre-defined set of criteria and the most critical models will be validated first.

Validation is typically completed before a model is put into use and also on an ongoing basis to ensure the model continues to perform as intended. Refer to the Independent Model Validation Standard guide for more details.

The second line of defense validation will focus on:

- The review of the **reasonableness of the conceptual approach** and quantification techniques of the model itself.

- The **verification of model process** including: data inputs, the workings of the model itself, and model output reporting.

- **The review of outcomes to assess the predictive accuracy of the model.**

Checklist for a second line of defense validation

- ☐ Detailed background of proposed use of model
- ☐ Explain rational behind model assumptions, choice of approach, and examples of known models built on such rational.
- ☐ Demonstrate the appropriateness of the logic, quantification techniques, and conceptual soundness of the model
- ☐ Conduct statistical testing to check validity of model assumptions. For example:
    - Homoscedasticity test for a linear regression model
- ☐ Test the model's sensitivity to key assumptions, parameters and data inputs used.
- ☐ Perform a sanity check of model process. For example:
    - evaluation of controls
    - accuracy of program coding
- ☐ Perform outcome analysis. For example:
    - Back-testing and how it affects model
    - Benchmarking of model output with predicted results from other models or sources
- ☐ Robust documentation that discusses choice of approach, code implementation, related known models or equations, relative effect of statistical results on the model. For example:
    - Time series model was used due to autocorrelation of data
    - How a test result of 0.87 p-value affects the strength of the model
    - Detailed comments on codes used
- ☐ Ensure the documentation is up to date, comprehensive, detailed, and meets KeyBank's minimum standards
- ☐ Detailed steps planned to ensure ongoing monitoring of model usage

Appendix

Model Development, Documentation, Validation Example

MODEL NAME: Linear Regression Model

MODEL OWNER: ________________________

MODEL DEVELOPER(S): _________________

MODEL USERS: ________________________

MODEL IMPLEMENTATION DATE: ________

MODEL REVISION DATE: _______________

MODEL VALIDATOR: ___________________

VALIDATION DATE: ____________________

Table of Contents

1. Executive Summary

In the following document, we developed a model to estimate how changes in gross domestic product (GDP) affects the unemployment rate. We started by using linear regression to model the relationship and later used time series analysis with input series.

Results of regression analysis may be misleading without further probing of data, which could divulge relationships that a casual analysis could overlook. In the initial analysis using linear regression, after data transformation, the regression appeared very linear on graph (figure 2). However after further probing of data and linear regression, assumptions were violated.

Time series models are ideal for equally time spaced data with autocorrelation. Times Series analysis with input series (ARIMAX model) was used to model the relationship between unemployment and GDP. The data was transformed (differenced) using growth rates to stabilize the variance before conducting testing. Two viable ARIMAX models were attained after time series testing. Both models did fairly well with backtesting, and very well with sensitivity testing.

2. Scope

Linear regression models require that Gauss-Markov assumptions are not violated. In the event of violations, other models should be considered. Due to the presence of serial autocorrelation in our data, we used time series models with input series to model the relationship between unemployment and GDP.

Time series models with input series require the dependent variable and the input (independent variable) to be jointly stationary or cointegrated. Our variables are cointegrated, and the soundness of the ARIMAX model was tested by backtesting and sensitivity testing.

3. Evaluation of Conceptual Soundness

3.1 Statement of Purpose

The purpose of this paper is to show the effect of GDP on the unemployment rate, and forecast the unemployment rate. We also show other steps to take if the assumptions of linear regression are violated. We outline steps for conducting linear regression, and checking for its critical assumptions: best linear unbiased estimates (BLUE) and Gauss-Markhov assumptions.

3.2 Methodology and Data

Seasonally adjusted quarterly data of unemployment and GDP from 1949 to 2012 were collected from the U.S. Department of Commerce and U.S. Department of Labor respectively. A statistical data analysis test was conducted, and the outliers were deliberately included in the data to capture volatility in macroeconomic variables. Unemployment data was regressed on GDP to observe the graph and test for linear relationship (linearity). The graph was not as linear (see figure 1) with R^2 value of 0.0756, so a data transformation was necessary. To achieve data transformation, data for quarterly changes from previous years was uploaded, and the quarterly change in unemployment from previous years was regressed against quarterly change in real GDP from previous years (see figure 2).

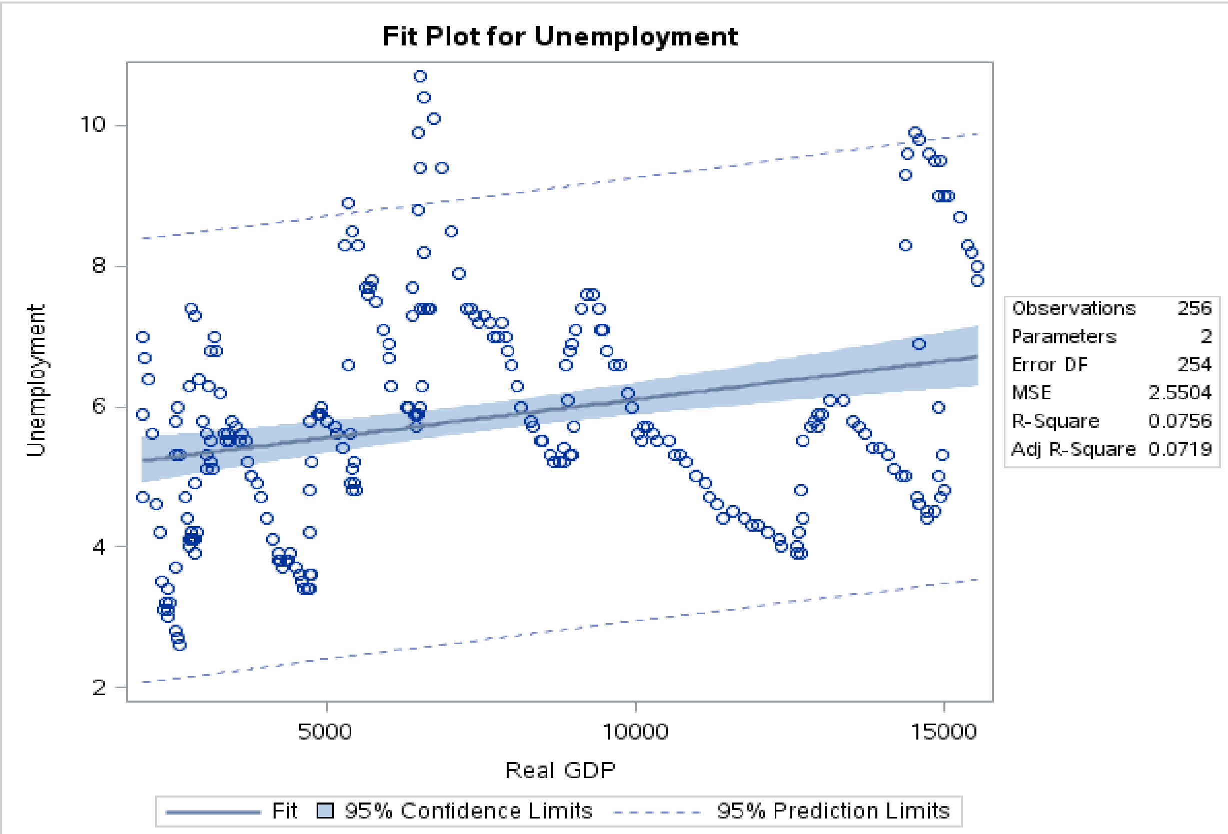

Figure 1

3.3 Critical Model Assumptions

In order to determine our independent and dependent variable, we observe how macroeconomic variables affect each other. We assumed everybody wants to work, and the rate of not obtaining a job depends on how the economy is performing or GDP. Unemployment was regressed on GDP: unemployment was the dependent variable while GDP was the independent variable.

The assumptions necessary for a linear regression model are best linear unbiased estimates (BLUE) and Gauss-Markov assumptions, which are stated below:

1. Linearity: The regression model is linear in parameters. $Y_i = \beta_1 + \beta_2 X_i + U_i$
2. Independence of errors: The regressor X_i and error term U_i are independent: $Cov(X_i, U_i) = 0$
3. No autocorrelation between error terms: $Cov(U_i, U_j | X_i, X_j) = 0$
4. Homoscedasticity: The variance of the error term U_i is constant: $Var(U_i) = \sigma^2$
5. Normality of the error distribution: $U_i \sim N(0, \sigma^2)$

4. Model Testing and Validity of Assumptions.

4.1 Testing for Linearity

We plotted historical quarterly data of unemployment against quarterly data of Real GDP, and we had a regression that was not as linear with R square of 0.0756 as seen above in figure 1. The low R-Square value suggests poor fit of data, so we transformed our data linearly for better fitting of data.

Linear transformation involves using deterministic mathematical function to change measurement scale, while preserving the linear relationship or correlation within the variables. The new function is usually invertible and continuous, and the purpose of the transformation is to achieve linearity. We achieved linearity by using data of changes in the quarterly values from the previous years, which is shown in Figure 2 below.

The output below suggests some autocorrelation might exist in the residuals as we observe some concentration around (5, 0). This concentration indicates some form of pattern or trend may exist in the residuals.

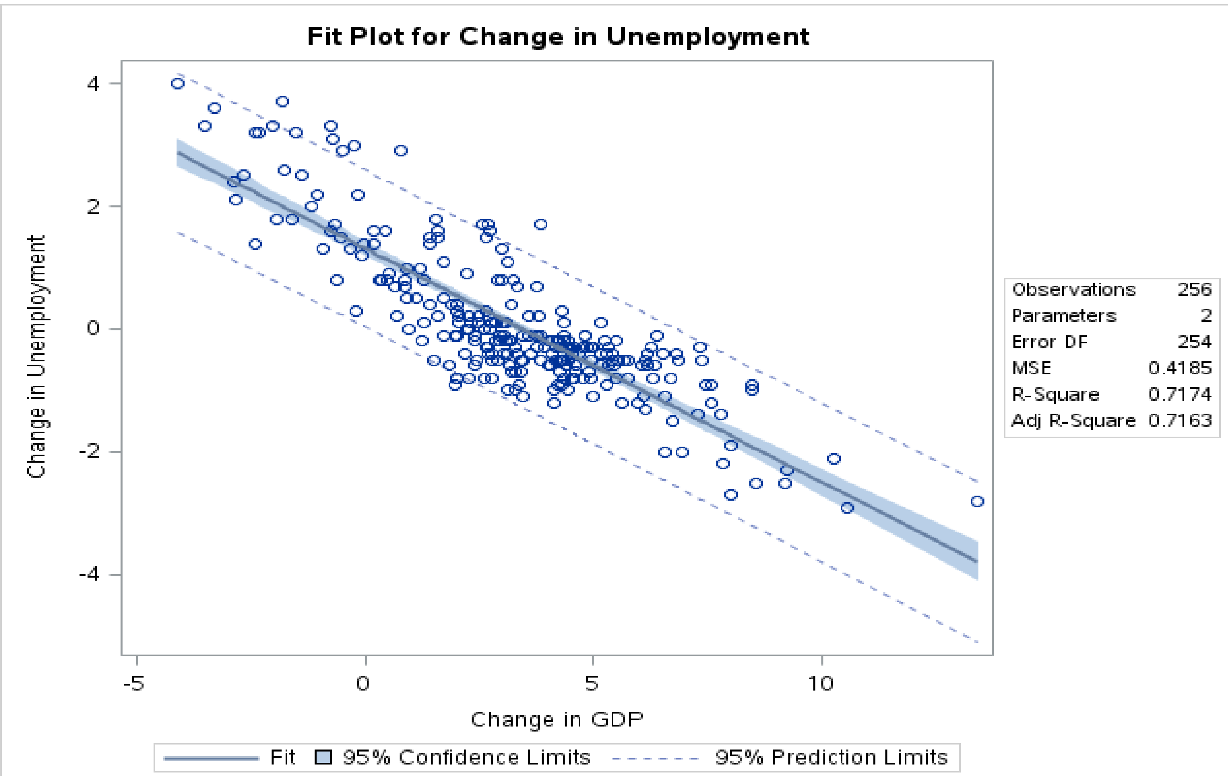

Figure 2

4.2 Testing for Independence

A first and second moment specification test the null hypothesis of constant variance and independence of error terms, and a higher than chosen significance level indicates that the chi-square test fails to reject the null hypothesis. Prob > Chisq of 0.0032 < 0.05 indicates the error terms are not independently and identically distributed. Thus, it supports presence of autocorrelation in residuals as p-value of 0.0032 is less than 0.05 (95% confidence interval). Therefore, the assumption for independence of regressor and error term is violated.

Test of First and Second Moment Specification		
DF	**Chi-Square**	**Pr > ChiSq**
2	**11.48**	**0.0032**

Figure 3

4.3 Testing for Autocorrelation

The Durbin-Watson statistic tests for first order correlation in a large data sample. The test statistic *d* value lies between 0 and 4. A *d* value of 2 indicates no autocorrelation and *d* value much less than 2 indicates serial autocorrelation. Our d value was 0.417, which is much less than 2 and suggests presence of autocorrelation.

Durbin-Watson D	**0.417**
Number of Observations	**256**
1st Order Autocorrelation	**0.781**

Figure 4

4.4 Testing for Homoscedasticity

Homoscedasticity is the presence of constant variance, and when the variance is non-constant the term heteroscedasticity is used. There are numerous ways to check for homoscedasticity. The first method we used is called Time Honored Method of Inspection (THMI), which involves looking for patterns in a plot of the residuals of the regression. From figure 5 below, we observe some levels of concentration or trend around (5, 0) which implies that the variance of the error term is non-constant.

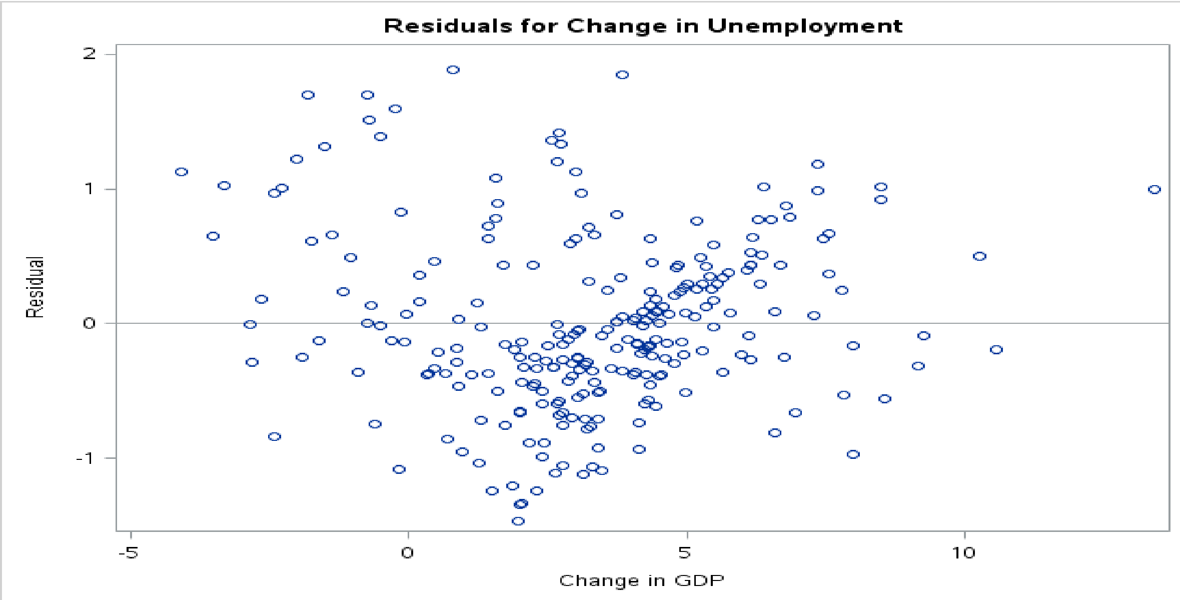

Figure 5

The more formal test for homoscedasticity is White's General test (White 1980) and the Breusch-Pagan test (Breusch and Pagan 1979). Based on the SAS output below, both the White test (Prob>Chisq, 0.0002) and the Breusch-Pagan test (Prob>Chisq, 0.0004) reject the null hypothesis of no heteroscedasticity at 95% confidence interval. Therefore, the assumption for constant variance is violated.

Heteroscedasticity Test

Equation	**Test**	**Statistic**	**DF**	**Pr > ChiSq**	**Variables**
Y	White's Test	17.53	2	0.0002	Cross of all vars
	Breusch-Pagan	12.77	1	0.0004	1, X, x2

Figure 6

4.5 Testing for Normality

There are numerous tests that check the error terms for a normal $N(0, \sigma^2)$ distribution. The normal probability plot (Q-Q plot), which plots the distribution of the residuals against a normal distribution can be used to check for normality and goodness of fit of data. According to figure 7 below, the residuals look linear and falls moderately in-line with the normal distribution. The plot of Distribution of Residual for change in unemployment (figure 8) also shows a moderately bell curve fit. The results of the Q-Q plots are not strong enough to confirm normality, so we conducted further testing.

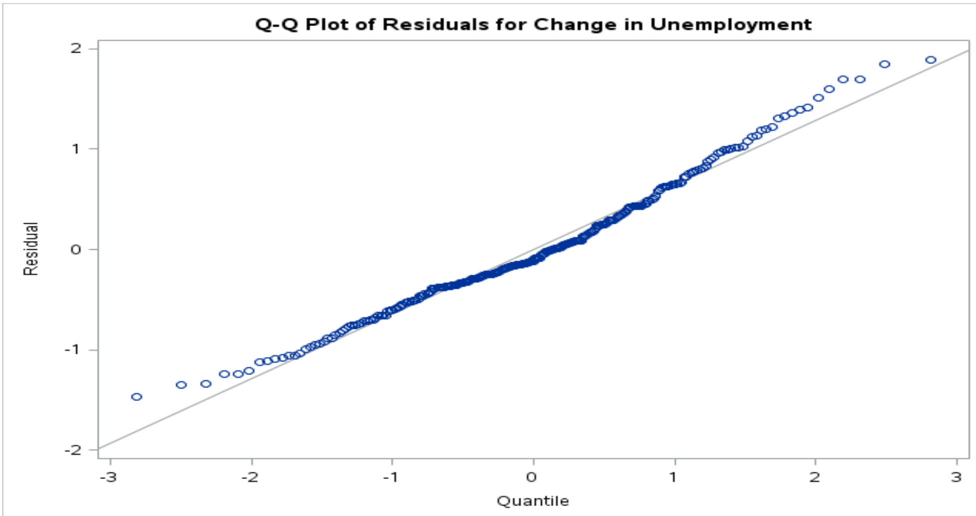

Figure 7

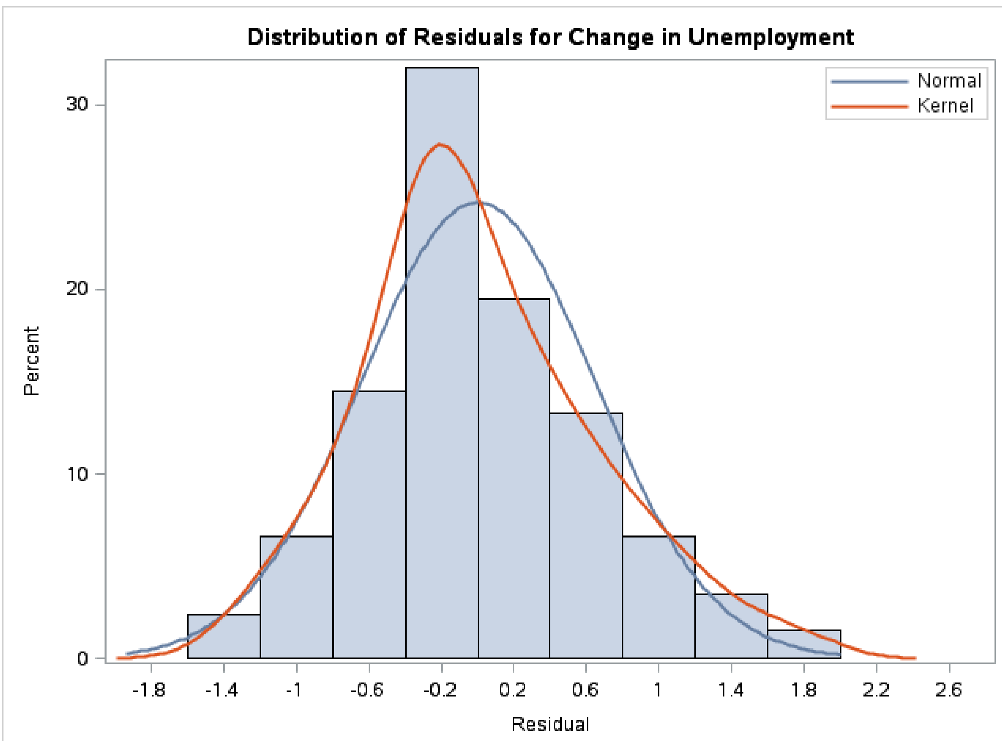

Figure 8

Test results of the Shapiro-Wilk test, Kolmogorov-Smirnov, Cramer-von Mises, and Anderson-Darling test are shown below. Of the four test results, the Shapiro-Wilk test has the best statistical power for any given significance, so the Shapiro-Wilk test is given the most consideration among the test results. All test results show p-value much lesser than 0.05, and the Shapiro-Wilk test shows an even lesser result of 0.0022. The result confirms that the assumption of normality is violated.

Tests for Normality				
Test	Statistic		p Value	
Shapiro-Wilk	W	0.981698	Pr < W	0.0022
Kolmogorov-Smirnov	D	0.07619	Pr > D	<0.0100
Cramer-von Mises	W-Sq	0.290622	Pr > W-Sq	<0.0050
Anderson-Darling	A-Sq	1.530618	Pr > A-Sq	<0.0050

Figure 9

4.6 Assumption Testing Conclusion

The tests of assumptions showed that 4 of 5 assumptions were violated. The major causes of the violations were due to non-constant variance in the error term and presence of serial autocorrelation. Presence of serial autocorrelation is a common trend in time series data, which implies the data is time dependent. Therefore, time series analysis would be better suited to analyze the data.

5. Time Series Analysis

A time series is a sequence of data points measured at successive equally spaced time intervals like the closing values of S&P index. Time series data is usually associated with autocorrelations and volatility clustering, that is, wide changes tend to be followed by wide changes, and small changes followed by small changes alike.

Time series analysis captures the lingering effects of earlier values and shocks (random process), and are analyzed using the ARIMA (p,d,q) model. The auto-regressive element, p, represents the lingering effects of preceding values, the integrated element, d, represents trends in the data that is removed by differencing, and the moving average element, q, represents the lingering effects of preceding random shocks.

In our case, we would be using the ARIMA model with the input series since GDP is our input or independent variable. This family of models is also known as the transfer function model, regression model with ARMA error, Box-Tiao model, and ARIMAX models. Pankratz (1991) refers to these models as dynamic regression.

5.1. Critical Assumptions

1. Observations are equally spaced in time
2. Data is stationary or jointly stationary; mean and variance of the process are constant over time
3. Error term is white noise after estimating parameters p,d, and q.

5.2. Model Testing and Validity of Assumptions

Logarithm transformation of data or variables is a common practice in time series analysis. The purpose of the logarithm transformation is to stabilize or correct heteroscedasticity in the variance of the variables. While logarithm transformation is a common practice, it could alter the estimates of parameters by estimating the log of the parameters. In our analysis, we used growth rate to transform our data and stabilize the variance.

We used data of the seasonally adjusted quarterly unemployment rate (end of period) and seasonally adjusted quarterly GDP values. The growth rates were computed by $(Y_1(t+1) - Y_1 t)/ Y_1 t$. By taking growth rates, differencing (d=1) is implied in our model. Since our data was already seasonally adjusted by U.S. Department of Commerce and U.S. Department of Labor, we ensured it was by observing the graph of the data. No further readjustment of data was required.

5.2.1. Testing for Stationarity

A stationary time series is one whose statistical properties such as mean and variance are constant over time. Since we already differenced our data, our data is expected to be stationary. To test for stationarity, we first observe the autocorrelation function plot (ACF), which shows how values of the series are correlated with past values of the series, and also how ACF value decays. A slow decay indicates nonstationarity, while a fast decay indicates stationarity. In ARIMAX analysis, we look at the cross correlation function plot (CCF). According to Figure 10, the decay is fast which implies stationarity.

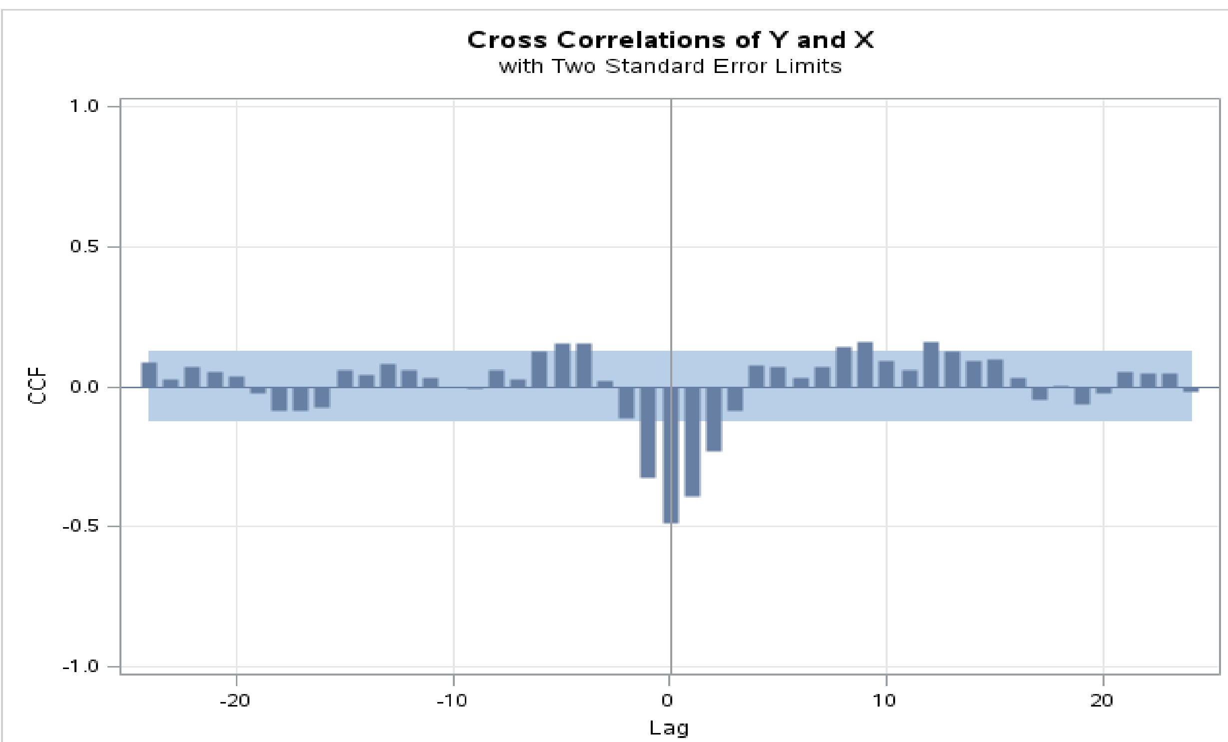

Figure 10

5.2.2. Cointegration Test

A more formal and accurate testing for stationarity can be conducted using the Augmented Dickey Fuller test. In the ARIMAX time series with input series, a cointegration test is conducted. Cointegration means that the variables are jointly stationary and share a stochastic trend. Engle and Granger (1987) suggested a two-step process to test for cointegration:

- Run OLS regression and get Residuals
- Run dickey fuller test on residuals

When the residuals are stationary, we can conclude that a relationship exists between the variables, and one of the variables granger causes the other in at least one direction. Based on the Augmented Dickey Fuller test on the residual below of (Prob<Tau 0.001), we reject the null hypothesis at the 95% confidence interval that unit root exists. Therefore, the residuals are stationary, GDP and Unemployment are jointly stationary, and one variable granger causes the other.

Seasonal Augmented Dickey-Fuller Unit Root Tests

Type	Lags	Rho	Pr < Rho	Tau	Pr < Tau
Zero Mean	**0**	-255.055	0.0001	-10.92	<.0001
	1	-251.442	0.0001	-10.81	<.0001
	2	-253.375	0.0001	-10.85	<.0001
	3	-253.105	0.0001	-10.88	<.0001
Single Mean	**0**	-254.933	0.0001	-10.86	<.0001
	1	-251.012	0.0001	-10.74	<.0001
	2	-252.863	0.0001	-10.78	<.0001
	3	-252.681	0.0001	-10.82	<.0001

Figure 11

5.2.3. Identification

We return to figure 10 to observe the CCF correlogram of GDP and unemployment to identify spikes in the lags. The spikes in the lags can be used to estimate values of p and q in the ARIMA model. Two major spikes are identified in the first two lags of CCF, and since our data has already been differenced by growth rate transformation, our ARIMAX model is likely to be of order (0,1,2) or (2,1,0). We would use diagnostic checking to check for the right model.

5.2.4. Estimation and Diagnostic Checking

We used maximum likelihood method to estimate the parameters for unemployment ARIMAX model (0,1,2) and (2,1,0) with one lag of GDP (X_{t-1}). The estimates and equations for Model 1 and Model 2 are shown below:

Model 1, ARIMAX (0,1,2): $(1-B)Y_t = u + (1-\theta_1 B - \theta_2 B^2)e_t + (\omega_0 - \omega_1 B)X_t$

Where B is the back shift operator, Y_t is the growth rate of GDP at time t, X_t is the growth rate in unemployment at time t, and e_t is the white noise. The estimates of the parameters $u, \theta_1, \theta_2, \omega_0, \omega_1$ are shown below.

Maximum Likelihood Estimation

Parameter	Estimate	Standard Error	t Value	Approx Pr > |t|	Lag	Variable	Shift
MU	0.05484	0.0087764	6.25	<.0001	0	Y	0
MA1,1	-0.08758	0.10143	-0.86	0.3879	1	Y	0
MA1,2	-0.02224	0.10168	-0.22	0.8268	2	Y	0
NUM1	-0.03995	0.0062016	-6.44	<.0001	0	X	0
NUM1,1	0.02502	0.0062344	4.01	<.0001	1	X	0

Figure 12

Model 2, ARIMAX (2,1,0): $(1-B)Y_t = u + \frac{1}{(1-\phi_1 B - \phi_2 B^2)}e_t + (\omega_0 - \omega_1 B)X_t$

Where B is the back shift operator, Y_t is the growth rate of GDP at time t, X_t is the growth rate in unemployment at time t, and e_t is the white noise. The estimates of the parameters $u, \phi_1, \phi_2, \omega_0, \omega_1$ are shown below.

Maximum Likelihood Estimation

Parameter	Estimate	Standard Error	t Value	Approx Pr > |t|	Lag	Variable	Shift
MU	0.05415	0.0090519	5.98	<.0001	0	Y	0
AR1,1	0.09624	0.10132	0.95	0.3422	1	Y	0
AR1,2	0.03981	0.10153	0.39	0.6950	2	Y	0
NUM1	-0.03977	0.0062215	-6.39	<.0001	0	X	0

Maximum Likelihood Estimation							
Parameter	**Estimate**	**Standard Error**	**t Value**	**Approx Pr > |t|**	**Lag**	**Variable**	**Shift**
NUM1,1	0.02458	0.0062664	3.92	<.0001	1	X	0

Figure 13

5.2.5. Comparison of Model 1 and Model 2

The process of selecting the best model among competing models involves observing Akaike information criterion (AIC) values and residuals of the model. AIC is a measure of the relative quality of a statistical model, for a given set of data, and the model with the lowest positive or highest negative value is considered.

According to Figure 14 below, Model 2 has AIC value of -503.761, which is slightly better than the AIC value of Model 1 of -503.595. Comparably, the difference is not significant to conclude that Model 2 is better.

Model 1	
Constant Estimate	0.054841
Variance Estimate	0.00772
Std Error Estimate	0.087862
AIC	-503.595
SBC	-485.968
Number of Residuals	251

Model 2	
Constant Estimate	0.046784
Variance Estimate	0.007714
Std Error Estimate	0.087832
AIC	-503.761
SBC	-486.134
Number of Residuals	251

Figure 14

We also observed our residual for presence of white noise, which indicates that useful information is not present in the shock term. We want our residuals to be white noise. According to Figure 15 and 16 below, the residuals are white noise for both Model 1 and Model 2.

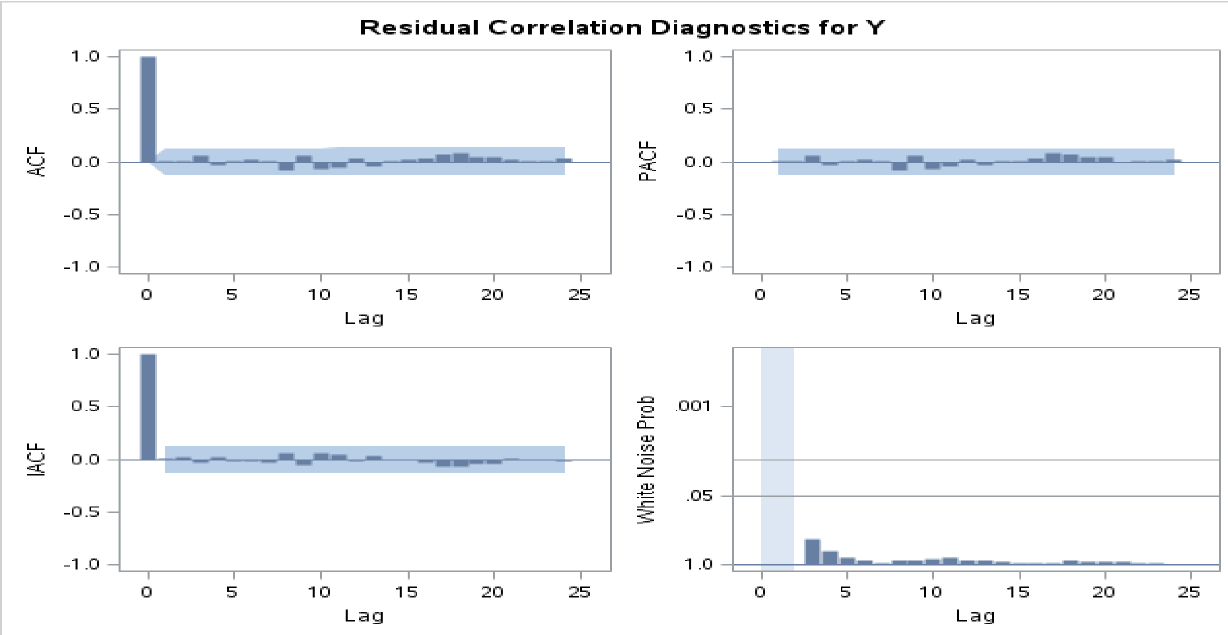

Figure 15

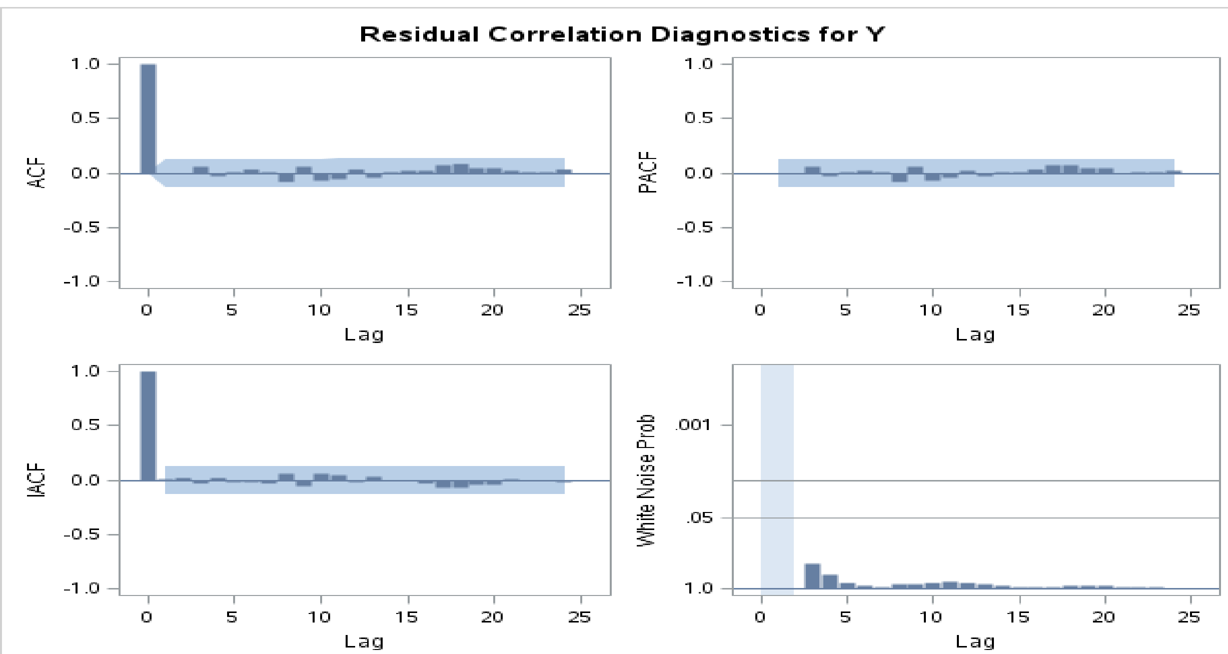

Figure 16

6. Backtesting and Validity of Model Results

We validated our model performance by backtesting to compare the predicted model results with actual for the two viable models; Model 1 and Model 2. Model 2 has average forecast error of 79.33%, which is less than the average forecast error of Model 1 79.50%. This difference is not significant to conclude that Model 2 is better than Model 1.

				Model 1		Model 2	
Obs	Actual	Forecast	Std Error	BackTest Error	ABS (BackTest Error)	BackTest Error	ABS(BackTest Error)
223	-0.0656	-0.0313	0.0879	52.29%	52.29%	55.03%	55.03%
224	0.0175	0.0026	0.0882	85.71%	85.71%	83.43%	83.43%
225	-0.0345	0.0093	0.0882	126.96%	126.96%	126.67%	126.67%
226	-0.0357	0.0006	0.0882	101.96%	101.96%	101.40%	101.40%
227	0	-0.0008	0.0882	0.08%	0.08%	0.10%	0.10%
228	-0.037	-0.0099	0.0882	73.24%	73.24%	72.97%	72.97%
229	-0.0385	0.0057	0.0882	114.55%	114.55%	114.29%	114.29%
230	0	0.0086	0.0882	-0.86%	0.86%	-0.83%	0.83%
231	-0.02	0.0123	0.0882	161.50%	161.50%	160.50%	160.50%
232	-0.0408	-0.0075	0.0882	81.62%	81.62%	81.13%	81.13%
233	-0.0213	0.0121	0.0882	156.34%	156.34%	155.87%	155.87%
234	-0.0217	0.0436	0.0882	300.92%	300.92%	298.62%	298.62%
235	-0.0222	0.0215	0.0882	196.85%	196.85%	194.59%	194.59%
236	0	0.0327	0.0882	-3.27%	3.27%	-3.23%	3.23%
237	0.0455	0.0224	0.0882	50.55%	50.55%	51.65%	51.65%
238	0.0217	0.0087	0.0882	60.37%	60.37%	61.29%	61.29%
239	0.0638	0.0234	0.0882	63.32%	63.32%	63.95%	63.95%
240	0.02	0.0725	0.0882	-262.50%	262.50%	-259.50%	259.50%
241	0.098	0.0518	0.0882	47.14%	47.14%	48.06%	48.06%
242	0.0893	0.0622	0.0882	30.35%	30.35%	30.91%	30.91%
243	0.1967	0.1532	0.0882	22.11%	22.11%	22.78%	22.78%
244	0.1918	0.1642	0.0882	14.39%	14.39%	15.38%	15.38%
245	0.092	0.0938	0.0882	-2.07%	2.07%	-0.65%	0.65%
246	0.0316	0.0448	0.0882	-41.77%	41.77%	-39.56%	39.56%
247	0.0102	0.0087	0.0882	14.71%	14.71%	18.63%	18.63%
248	0	0.0151	0.0882	-1.51%	1.51%	-1.49%	1.49%
				Average BackTest Error	**79.50%**		**79.33%**

7. Sensitivity Analysis

Sensitivity analysis can be conducted by changing input variables or parameters. We conducted sensitivity analysis by checking sensitivity to parameters. We calculated sensitivity by adding and subtracting one standard deviation in each parameter.

7.1 Sensitivity Analysis of Model 1

Model 1 is of the form: $(1-B)Y_t = u + (1-\theta_1 B - \theta_2 B^2)a_t + (\omega_0 - \omega_1 B)X_t$

The estimates of the parameters $u, \theta_1, \theta_2, \omega_0, \omega_1$ and their corresponding standard errors are listed in the following table.

Parameter	Estimate	Standard Error
u	0.05484	0.0087764
θ_1	-0.08758	0.1014300
θ_2	-0.02224	0.1016800
ω_0	-0.03995	0.0062016
ω_1	0.02502	0.0062344

Model 1 has five estimates, and each estimate can be of three forms namely: original estimate, estimate plus its standard error, and estimate minus its standard error. Therefore, there are $3^5 - 1 = 242$ possible combinations.

It is not necessary to go over all 242 possible combinations. We tested 8 of them, and their results are listed in the table below. We took the percentage difference of the forecasted standard error from the previous standard error in the original model. Based on the result below, the highest absolute number is 4.65%, which is under10% and indicates that the model is stable.

					Standard Error	Sensitivity	
u	θ_1	θ_2	ω_0	ω_1	Forcast	Previous	difference
0.05484	-0.08758	-0.02224	-0.03995	0.02502	0.0879	0.0882	0.34%
0.05484	-0.08758	0.07944	-0.03995	0.018786	0.0891	0.0882	-1.02%
0.05484	-0.08758	0.07944	-0.03995	0.031254	0.0883	0.0882	-0.11%
0.05484	0.01385	0.07944	-0.03375	0.018786	0.0902	0.0882	-2.27%
0.063616	0.01385	-0.12392	-0.03375	0.031254	0.0895	0.0882	-1.47%
0.046064	-0.18901	-0.12392	-0.04615	0.031254	0.0923	0.0882	-4.65%
0.063616	-0.18901	-0.02224	-0.03375	0.02502	0.0905	0.0882	-2.61%
0.046064	-0.08758	-0.12392	-0.04615	0.018786	0.0910	0.0882	-3.17%

7.2 Sensitivity Analysis of Model 2

Model 2 is of the form: $(1-B)Y_t = u + \frac{1}{(1-\phi_1 B - \phi_2 B^2)} a_t + (\omega_0 - \omega_1 B)X_t$

The estimates of the parameters $u, \phi_1, \phi_2, \omega_0, \omega_1$ and their corresponding standard errors are listed in the following table.

Parameter	Estimate	Standard Error
u	0.05415	0.0090519
ϕ_1	0.09624	0.1013200
ϕ_2	0.03981	0.1015300
ω_0	-0.03977	0.0062215
ω_1	0.02458	0.0062664

Model 2 has five estimates, and each estimate can be of three forms namely: original estimate, estimate plus its standard error, and estimate minus its standard error. Therefore, there are $3^5 - 1 = 242$ possible combinations.

We tested 8 of them, and their results are listed in the table below. We took the percentage difference of the forecasted standard error from the previous standard error in the original model. Based on the result below, the highest absolute number is 4.30%, which is under 10% and indicates that the model is stable. Both Model 1 and 2 are very suitable for modeling growth in the unemployment rate. Results might be improved if an extra variable is added to the input, such as education rate.

					Standard Error Sensitivity		
u	ϕ_1	ϕ_2	ω_0	ω_1	**Forcast**	**Previous**	**difference**
0.05415	0.09624	0.03981	-0.03977	0.02458	0.0883	0.0883	0.00%
0.05415	0.09624	0.14134	-0.03977	0.018314	0.0897	0.0883	-1.59%
0.05415	0.09624	0.14134	-0.03977	0.030846	0.0899	0.0883	-1.81%
0.05415	0.19756	0.14134	-0.03355	0.018314	0.0921	0.0883	-4.30%
0.063202	0.19756	-0.06172	-0.03355	0.030846	0.0905	0.0883	-2.49%
0.045098	-0.00508	-0.06172	-0.04599	0.030846	0.0913	0.0883	-3.40%
0.063202	-0.00508	0.03981	-0.03355	0.02458	0.0894	0.0883	-1.25%
0.045098	0.09624	-0.06172	-0.04599	0.018314	0.0894	0.0883	-1.25%

8. Ongoing Monitoring

The ongoing monitoring of model usage and performance is vital to ascertain that the model is performing as expected and appropriately implemented. We plan to revisit and update the model quarterly as macro-economic data changes, and to ensure the model has not been implemented wrongly.

In general, ongoing monitoring is necessary for updates in changes in products, exposures, activities, clients, extension of model, or market conditions. Consequently, the model might be adjusted, redeveloped, or replaced if necessary.

9. References

[1] Breusch, T. and Pagan, A. (1979), ``A Simple Test for Heteroscedasticity and Random Coefficient Variation," Econometrica, 47, 1287-1294.

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[3] SAS/ETS 12.1 User’s Guide, SAS Documentation, SAS Institute Inc., 2012.