Assignment - II

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1. List and Explain the various phases of OR problems
→ The various phases of OR problem Solving are:

1. Operations Research follows a Systematic Scientific approach to solve real-world decision-making problems.

2. The process consists of Several phases that Ensure Clarity accuracy and practical implementation.

problem Definition and Formulation.
1. the first phase identifies and understands the real problem in the System.
- objectives such as minimizing Cost or maximizing profit and Clearly Stated.
- Constraints related to resources, time and capacity are recorded.
- Decision variables that influence the outcome are identified.
- A Clear problem Statement guides all further Steps.

2. Model Building.
→ the real world problem is translated into a mathematical or Logical model.
- the model Expresses relationships among variables. Constraints & objectives
- the model Simplifies reality while capturing essential Characteristics.

3. Model Solution
→ Suitable Mathematical. methods are applied to obtain

the optimal Solution
- Examples Include Simplex method, Dynamic programming or Branch and Bound.

4. Validation and Testing
→ the model is tested against real or historical data
- Validation ensures the model behaves like the actual System
- Sensitivity analysis checks how changes in input affect output
- If outcomes are unrealistic, assumptions are revised.

5. Implementation
- The optimal Solution is introduced into the real operating environment.
- implementation involves training Staff and modifying procedures
- practical adoption determines the real success of the OR study

6. Maintenance and Control
- The System's performance is monitored Continuously after implementation.
- Changing Conditions may require updates to the model or Solution.
- feedback mechanisms help detect deviations
- Continuous review ensures long-term effectiveness
- OR becomes an ongoing improvement cycle rather than a one-time activity

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2. What are the limitations and Charcteristics of OR?

Charcteristics of OR:
- Quantitative approach to decision making.
- Interdisciplinary in nature.
- use Mathematical models and Scientific Methods.
- Helps in optimization of resources
- Suitable for large complex problems.
- Requires Computer programming for implementation

Limitations of OR:
- High implementation cost.
- Time-consuming process
- Doesn't consider non-quantifiable factors
- Solutions may not be practical for all situations.
- Requires skilled professionals.
- Assumptions may not always reflect reality.
- Dependence on Accurate Data.
- Complexity of models.
- High Cost and Time Requirements.
- Implementation Difficulties.

3. Define operation research and discuss its scope.

→ Operations Research is Scientific approach to decision making that employs Mathematical and Statistical models to find optimal or near-optimal solutions to complex organizational problems

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Scopes

1. Production and Manufacturing
→ Helps in production planning, Scheduling, inventory Control facility, layout and machine collection

2. Finance and Budgeting
- used in portofolio Selection, capital budgeting, cash flow analysis, and risk minimization

3. Marketing and Sales
- Supports market research, demand forecasting, advertise-ment planning and price optimization.

4. Human Resource Management
- Helps in manpower Planning, job assignment, Shift Scheduling and workload balancing.

5. Transportation and Logistics.
- Include vehicle routing, transportation cost minimi-zation, distribution planning and Supply Chain optimization

6. Military and Defense
- Historically the root of OR, used for Strategy planning, resource deployment and Logistic management

7. Healthcare and Hospital management
Supports ambulance routing, Staff Scheduling, patient flow optimization and inventory control of medicines.

4. Solve the following Lpp by graphical method

$Z_{max} = 5x_1 + 4x_2$

Subject to Constraint:

$x_1 + x_2 \geq 2$

$x_1 \leq 8$

$x_1 \leq 9$

$x_1, x_2 \geq 0$

1. plot the Constraints:
   - line 1 : $x_1 + x_2 = 2$
   - line 2 : $x_1 = 8$ (vertical line)
   - line 3 : $x_2 = 9$ (horizonal line)

2. Identify feasible region: The intersection of all Constraint regions Satisfied inequalities.

3. Corner points of feasible Region:
   - point A : $(2,0) \rightarrow Z = 5(2) + 4(0) = 10$.
   - point B : $(8,0) \rightarrow Z = 5(8) + 4(0) = 40$
   - point C = $(8,1) \rightarrow Z = 5(8) + 4(1) = 44$
   - point D : $(8,9) \rightarrow Z = 5(8) + 4(9) = 76$ ← optimal.
   - point E : $(0,9) \rightarrow Z = 5(0) + 4(9) = 36$

optimal Solution $x_1 = 8$, $x_2 = 9$ Maximum $Z = 76$

6. Solve the following Lpp by graphical method

Minimize $Z = 2x_1 + 1.7x_2$

Subject to Constraints:

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$0.15X_1 + 0.10X_2 \geq 1.0$

$0.7X_1 + 1.70X_2 \geq 7.5$

$1.30X_1 + 1.10X_2 \geq 10$

$X_1, X_2 \geq 0$

→ 1. plot Constraint lines and identify feasible region

2. Find corner points:

- Intersection of Constraints 1 and 2: Solve Simultaneously
  - $0.15X_1 + 0.10X_2 = 1.0$
  - $0.7X_1 + 1.70X_2 = 7.5$
  - Solution: $X_1 \approx 3.33$, $X_2 \approx 3.33$

- Intersection of Constraints 2 and 3:
  - $X_1 \approx 6.67, X_2 = 0.71$

- Intersection of Constraint 1 & 3
  - $X_1 \approx 6.67, X_2 \approx 0.71$

3. Evaluate at Corner points:
   - At $(3.33, 3.33)$: $Z = 2(3.33) + 1.7(3.33) \approx 12.32$
   - At $(6.67, 0.71)$: $Z = 2(6.67) + 1.7(0.71) \approx 14.32$

Optimal Solution: Minimum $Z \approx 12.32$ at $X_1 \approx 3.33, X_2 \approx 3.33$

7. A farmer has 200 acres of acres of land in which he can grow tomato, potato and radish what all he can grow can be sold in market he can get Rs 8 per kg of tomato, Rs 10 per kg of potato and Rs 12 per kg of radish, the average the average yield per acre is 2000 kg of tomato 2500 kg of potato and

1500 kg of radish. Fertilizer required for tomato potato and radish per acre is 100kg 125kg and 150 kg respectively the cost of fertilizer is rs 10 per kg. labour required for sowing cultivating and harvesting per acre 25, 30 and 35 man days for tomato, potato and radish. a total of 600 man-days are available at Rs100 per man-day formulate this as this as an LP model to maximize the profit

Decision variables:

- x_1 = acres allocated to tomato
- x_2 = acres allocated to potato
- x_3 = acres allocated to radish

profit calculation per acre

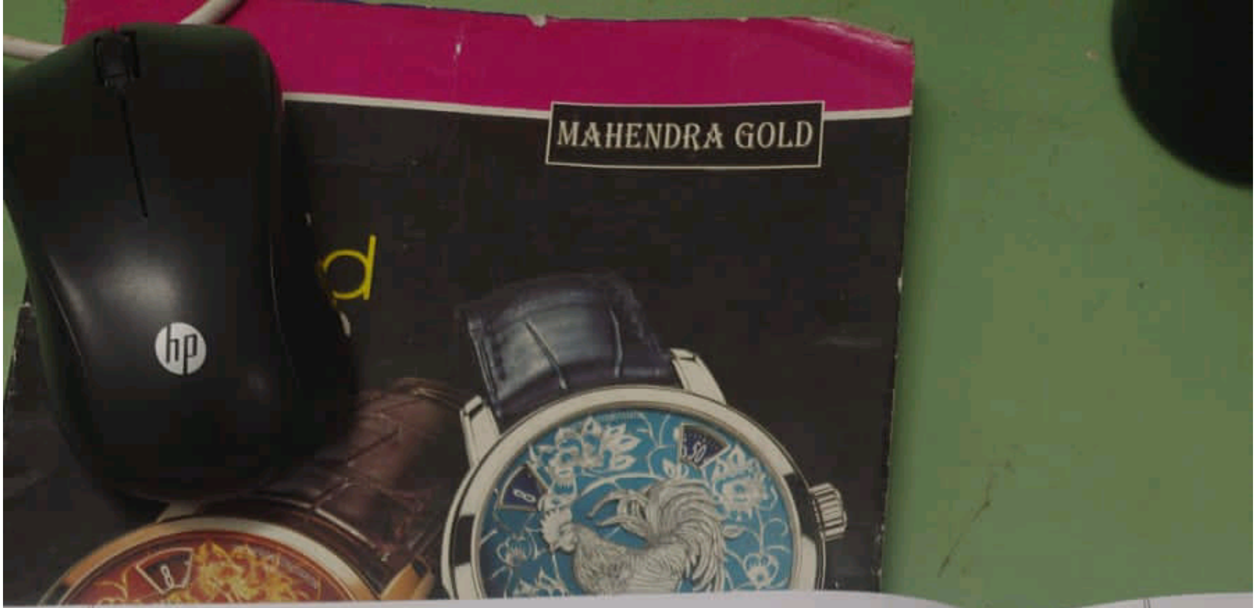
- Tomato = Revenue = 8X2000 = RS. 16,000
  Cost = Fertilizer (100X10) + Labour (25X100) = Rs 3,500
  profit = RS 12,500

- potato: Revenue = 10X 2500 = RS 25,000
  Cost = Fertilizer (125 X 10) + Labour (30X100) = RS 4,250
  profit = RS 20,750

- Radish: Revenue = 12 X 1500 = RS 18,000
  cost = Fertilizer (150X10) + Labour (35X100) = Rs 5,500
  profit = RS 12,500

LP Model:

Maximize : $Z = 12{,}500x_1 + 20{,}750x_2 + 12{,}500x_3$



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Subject to:

- $x_1+x_2+x_3 \le 200$ (Land Constraint)
- $25x_1+30x_2+35x_3 \le 600$ (Labour Constraint)
- $x_1, x_2, x_3 \ge 0$ (Non-negativity)

8) A Sales man has to visit five cities 1,2,3,4 and 5. The distance between any two cities are given in the following table. The Sales man starts from A and has to come back to A after visiting all other cities in a cycle which route has to select so that total distance travelled by him is minimum.

Distance Matrix

City	1	2	3	4	5
1	-	2	5	7	1
2	6	-	3	8	2
3	8	7	-	4	7
4	12	4	6	-	5
5	1	3	2	8	-

Solution by Nearest Neighbour Heuristic Starting from city 1

1. Start at 1
2. Nearest to 1: City 5 (distance 1)
3. Nearest unvisited to 5: City 2 (distance 3)
4. Nearest unvisited to 2: City 3 (distance 3)
5. Nearest unvisited to 3: City 4 (distance 4)

6. Return to 1 : Distance 12

Route: 1 → 5 → 2 → 3 → 4 → 1
total Distance: 1+3+3+4+12 = 23km.

10. A Sales man has to visit five cities 1, 2, 3 and 4 and he does not want to visit same city twice before completing tour of all cities and wishes to come back to city from where he has started cost of going city to another in Rs is given in following matrix find least cost row-

	1	2	3	4
1	-	3	8	5
2	4	-	14	3
3	4	5	-	2
4	7	8	13	-

Solution using Nearest Neighbour

Starting from City 1.

1. From 1 → 2 (cost 3)
2. From 2 → 4 (cost 3)
3. From 4 → 3 (cost 13)
4. From 3 → 1 (cost 4)

Route: 1 → 2 → 4 → 3 → 1.
Minimum Cost = 3 + 3 + 13 + 4 = 23 Rs.

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4. A project Consist of the following jobs and their duration.

Activity	precedence	Duration (in days)
A	—	8
B	—	10
C	—	8
D	A	10
E	A	16
F	B, D	17
G	C	18
H	G	14
I	G, F	9

a) Draw a network diagram
b) Identify the Critical path
c) Find the project duration.
d) Find Earliest and latest Start and finish time for each activity

a) Start → A → D → F → I → End

A → E

Start → B → F

Start → C → G → I → End

G → H → End

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b) Identify the Critical path.

Calculate Earliest Start and Finish time for forward pass

Activity	Duration	EST	EFT = EST + Duration
A	8	0	8
B	10	0	10
C	8	8 (after A)	8
D	10	8 (after A)	18
E	16	max (EFT B, D) = max (10, 18) = 18	24
F	17	8 (after C)	35
G	18	8 (after C)	26
H	14	0	22
I	9	max (EFT, G, F) = max (26, 35) = 35	44

time of activities = 44 days

Critical path activities have zero slack (difference between LST and EST zero), identified by longest path.

- A (0-8) → D (8-18) → F (18-35) → I (35-44)

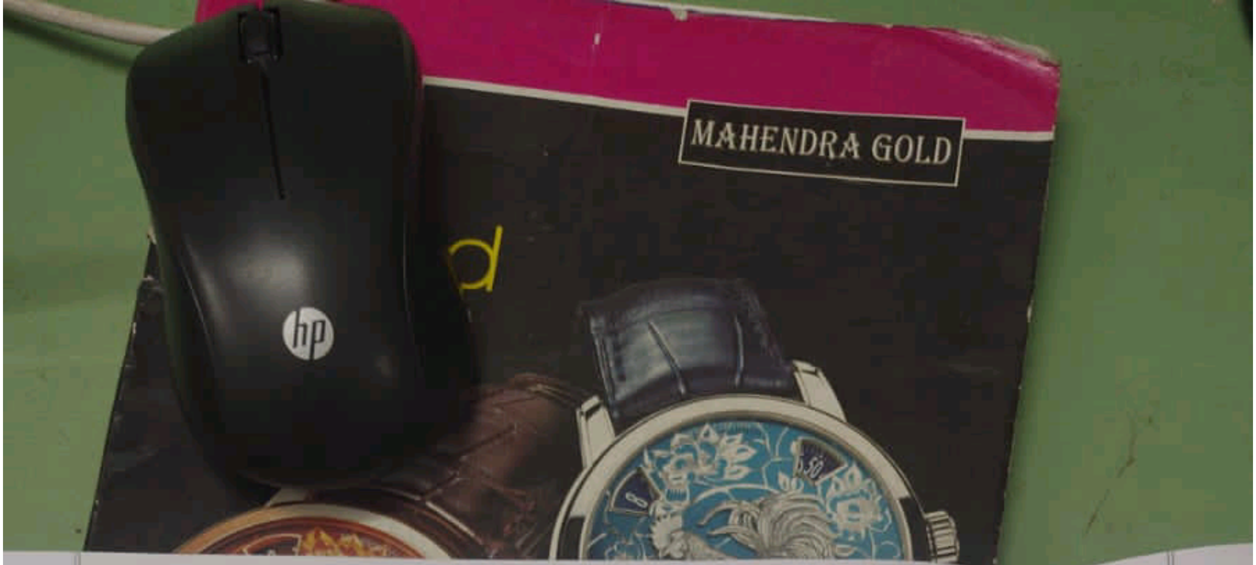
Critical path is: A → D → F → I

c) Project Duration

→ the project duration is 44 days as per critical path length.

d) Earliest and Latest Start and Finish Times

→ Calculate latest finish time (backward pass) starting from project completion (44 days):



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Activity	Duration	LFT	LST=LFT-Duration
I	9	44	35
F	17	35	18
G	18	35 (since I = 35 start)	17
H	14	22 (must finish before end)	8
E	16	24	8
D	10	18	8
A	8	min	0
B	10	18 (LST F)	8
C	8	min(LST G, LST H)=	0

Table of EST, EFT, LST, LFT:

Activity	Duration	EST	EFT	LST	LFT	LST-EST
A	8	0	8	0	8	0
B	10	0	10	8	18	8
C	8	0	8	0	8	0
D	10	8	18	8	18	0
E	16	8	24	8	24	0
F	17	18	35	18	35	0
G	18	8	26	17	35	9
H	14	8	22	8	22	0
I	9	35	44	35	44	0

Activities with Zero Slack form the critical path
$A \rightarrow D \rightarrow F \rightarrow I$

2. Define the following (i) processing time (ii) Idle time of Machine (iii) Total elapsed time (iv) No passing rule.

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- processing Time: The actual time required for a Machine to process job. It does not include waiting time or idle time.

- Idle Time of Machine: The time during which a machine remains idle (unutilized) because there is no job available for processing or the job is still being processed by another machine.

- Total Elapsed Time: The time from the start of the first job on the last machine. It represents the total duration to complete all jobs.

- No passive Rule: A rule in job sequencing where the relative order of jobs cannot be changed once they enter the production system. jobs must be processed the same sequence on all machines.

13. There are five jobs, each of which is to be processed through three Machines A, B and C in order ABC. The processing time in hours are:

Job	A	B	C
1	3	4	7
2	8	5	9
3	7	1	5
4	5	2	6
5	4	3	10

Determine optimum sequence for the five jobs and the minimum elapsed time. Also find the idle time for three machines.

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Job5: waits 3 hours at Machine B.

14) Find the sequence that minimize the total elapsed time required to complete the following jobs. processing time in hours.

Job	M1	M2
1	4	6
2	8	3
3	3	7
4	6	2
5	7	8
6	5	4

Using Johnson's Rule for 2 Machine.

1. Find minimum processing time : Job4 on $M_2 = 2$
2. Since minimum is on M2 (last machine), place Job4 at end
3. Continue: Job1 on $M_2 = 6$ (place after 4)
4. Job 6 on $M_2 = 4$ (place after 1)
5. Job 5 on $M_2 = 8$ (place after 6)
6. Job 3 on $M_1 = 3$, place at beginning
7. Job 2 next.

Optimal Sequence: 3 → 2 → 6 → 5 → 1 → 4.

Schedule:

Job	M1 Start	M1 End	M2 Start	M2 End
3	0	3	3	10

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e) Maximum principle: Strategy adopted by player A (maximizer) to maximize the minimum payoff player A choose the row with highest minimum value guaranting the best worst-case outcome

f) MiniMax principle: Strategy adopted by player B (minimizer) to minimize the maximum loss. player B chooses the column with the lowest maximum value, limiting potential losses.

16. List the difference between PERT and CPM

ASPECT	PERT	CPM
Full form.	program Evaluation and Review Techniques	Critical path method.
Nature	probablistic (Involves uncert-ainity)	Deterministic (precise activities.
time Estimate	Three estimates: optimistic, pessimistic, Most likely.	Assumes fixed activity durations
uncertainity	Handles uncertainity in project duration.	Assumes fixed activity durations.
Activity duration.	uses Expected time formula. (O+4M+P)/6	uses actual or estimated time

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project type	Research and development innovative projects.	Routine, well-defined projects.
cost Analysis	limited cost consideration	Include cost-time trade-off
History	Originated in 1958 (defence project)	Developed in 1956 (industry)
Accuracy	Higher due to three estimates.	May be less accurate for uncertain activities.

17) Define a) Network b) Activity c) Event d) Slack of an Event. e) float of an activity

a) Network : A graphical representation of a project showing the sequence of activities and their dependencies. It displays the logical flow using nodes and arrows. It displays the logical flow of the project

b) Activity : A task or job that takes time and resources to complete. It's represented by an arrow in the network diagram. ex: planning, Design, Construction.

c) Event : A specific point in time marking the completion of one or more activities and the beginning of one or more new activities. It's represented by a node/circle in the network diagram.

d) Slack of an Event : the difference between the latest time an

Finding optimal Strategies:

1. player A's Analysis (Maximizer):
- if A plays H: worst outcome $= -1/2$.
- if A plays T: worst outcome $= -1/2$
- Maximin value $= -1/2$

2. player B's Analysis (minimizer):
- if B plays H: Best outcome for $B = 1/2$.
- if B plays T: Best outcome for $B = 1/2$
- miniMax value $= 1/2$

Mixed Strategy Solution.
Let player A play H with probability p and T with probability(1-p)
Let player B play H with probability q and T with probability(1-q)

Expected payoff $= pq(1) + p(1-q)(-1/2) + (1-p)q(-1/2) + (1-p)(1-q)(0)$

for player B's indifference:

- payoff if B plays $H = p(1) + (1-p)(-1/2) = 1.5p - 0.5$
- payoff if B plays $T = p(-1/2) + (1-p)(0) = -0.5p$

Setting Equal: $1.5p - 0.5 = -0.5p$.
$2p = 0.5$, $p = 1/4$

For player A's indifference
- payoff if A plays $H = q(1) + (1-q)(-1/2) = 1.5q - 0.5$
- payoff if A plays $T = q(-1/2) + (1-q)(0) = -0.5q$

Setting Equal $q = 1/4$

optimal strategies

- player A: play H with probability 1/4 and T with probability 3/4
- player B: play H with probability 1/4 and T with probability 3/4

value of Game: $v = 1/4 \times 1/4 \times 1 + 1/4 \times 3/4 \times (-1/2) + (3/4) + 1/4 \times (-1/2) + 3/4 \times 3/4 \times 0$

$V = 1/16 - 3/32 - 3/32 = 1/16 - 6/32 = -1/16$

19) Solve the game by graphical method

	B1	B2	B3	B4	B5
A1	-3	5	0	-1	8
A2	8	-4	-1	6	-5

1. Check for saddle point: None Exists
2. Eliminate dominated strategies:
   - B4 dominates A1 (compare columns)
   - Strategy B5 for player B should be Evaluated.
3. For 2xn game, graph each column as a line
4. Find upper Envelope for player A and lower Envelope for player B

this requires plotting lines and finding the intersection point representing optimal mixed Strategy.

→ player A should use Mixed Strategy Combining A_1 and A_2

Q10. Solve the game by graphical method.

	1	2	3	4	5
1	-3	0	6	-1	7
2	-1	5	-2	2	1

optimal mixed strategies and value
- player A's optimal strategy.
- play row 1 with probability $p = 1/9$
- play row 2 with probability $1-p = \frac{8}{9}$

- player B's optimal strategy (only column 4 and 5 used):
- column 4 with probability $\frac{2}{3}$
- column 5 with probability $\frac{1}{3}$

- value of the game:

$v = \frac{5}{3}$ (in favour of A).

Q11 Solve the game graphically whose pay off matrix for the player A is given in the table

	I	II
I	2	4
II	2	3
III	3	2
IV	-2	6

Step 1: Check for dominance
Row II (2,3) is dominated by row I (2,4), so eliminate row II

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Step 2: Set up B's problem.

Let B play Column I with probability q and column II with probability $1-q$. B wants to minimize the maximum expected loss (A's gain):

- Row I: $E_I(q) = 2q + 4(1-q) = 4 - 2q$
- Row III: $E_{III}(q) = 3q + 2(1-q) = 2 + q$
- Row IV $= E_{IV}(q) = -2q + 6(1-q) = 6 - 8q$

Step 3: Find the minimax point.

plot these three lines and find the upper envelope is minimized. The critical intersection is between rows I and III:

$$4 - 2q = 2 + q \Rightarrow q = \frac{2}{3}$$

$$\text{At } q = \frac{2}{3}; E_I = E_{III} = \frac{8}{3} \approx 2.67.$$

Step 4: Find A's optimal strategy

- Only rows I and III are active. Let A play row I with probability p and row III with probability $1-p$. Make B indifferent between columns:

$$2p + 3(1-p) = 4p + 2(1-p) \Rightarrow p = \frac{1}{3}$$

Solution:

- Value of the game $v = \frac{8}{3} \approx 2.67.$

22. Use the graphical method to minimize needed to process the following jobs on the machine shown below i.e for each machine

Find the job which should be done first. Also Calculate the total time needed to complete both the jobs.

Job I	Sequence of Machine:	A	B	C	D	E
	time	2	3	4	6	2
Job 2	Sequence of Machine:	C	A	D	E	B
	Time	4	5	3	2	6

Soln

Machine	Job1 Start	Job1 End	Job2 Start	Job2 End
A	0	2	6	11
B	2	5	17	23
C	5	9	0	4
D	9	15	11	14
E	15	17	14	16

Order to minimize elapsed time:

- on Machine A: Job1 first (2 hours), then Job2 (5 hours)
- on machine B: Job1 first (3 hrs), then Job2 (6 hours)
- on machine C: Job2 first (4 hrs), then Job1 (4 hours)
- on Machine D: Job1 first (6 hrs), then Job2 (3 hours)
- on machine E: Job1 first (2 hrs), then Job3 (2 hours)

Total Elapsed Time: 23 hours.

234 Find the total elapsed time using the CDS heuristic for the

following flow shop problem.

Job	M/C (Hrs)	M/C 2 (Hrs)	M/C (3 Hrs)	M/C 4 (Hrs)
1	8	6	14	16
2	6	14	4	10
3	2	4	8	14
4	6	8	6	4

CDS Algorithm.

Iteration 1 (M/C 1 and M/C 2 Combined):
- Combined times: Job1(14), Job2(20), Job3(6), Job4(14)
- Sequence: 3 → 1 → 4 → 2

Iteration 2 (M/C 1, 2 and M/C 3, 4 Combined):
- Refine based on additional constraints
- May yield: 3 → 4 → 1 → 2 or similar.

Calculate elapsed time for best sequence using sequence 3 → 4 → 1 → 2:

Job	M1 Start	M1 End	M2 Start	M2 End	M3 Start	M3 End	M4 Start	M4 End
3	0	2	2	6	6	14	14	28
4	2	8	8	16	16	22	22	26
1	8	16	16	22	22	36	36	52
2	16	22	22	36	36	40	52	62

Minimum Elapsed Time: 62 hours.

Q 4) A project schedule has the following characteristics:

Activity	1-2	1-3	2-4	3-4	3-5	4-9	5-6	5-7	6-8	7-8	8-10	9-10
Time	4	1	1	1	6	5	4	8	1	2	5	[illegible]

Forward pass (EST/EFT):

- Event 1: EST = 0
- Event 2: EST = 0+4 = 4
- Event 3: EST = 0+1 = 1
- Event 4: EST = Max (4+1, 1+1) = 5
- Event 5: EST = 1+6 = 7
- Event 6: EST = 7+4 = 11
- Event 7: EST = 7+8 = 15
- Event 8: EST = max (11+1, 15+2) = 17
- Event 9: EST = 5+5 = 10
- Event 10 EST = max (17+5, 10+7) = 22.

Backward pass (LST/LFT):

- Event 10: LST = 22
- Event 9: LST = 22-5 = 17
- Event 8: LST = 22-7 = 15
- Event 7: LST = 17-2 = 15
- Event 6: LST = 17-1 = 16
- Event 5: LST = min(15-8, 16-4) = 7
- Event 4: min(15-5, 17-6) = 1 (wait, recalculate: LST of 4 = min(... so check contr... = 10 if going to 9; also 1+1 = 2 but EST of 4 is 5,
- Event 3: LST = min(7-6, 5-1) = 1
- Event 2: LST = 5-1 = 4
- Event 1: LST = 0.

Event Slack = LST − EST

Event	EST	LST	Slack
1	0	0	0
2	4	4	0
3	1	1	0
4	5	5	0
5	7	7	0
6	11	16	5
7	15	15	0
8	17	17	0
9	10	15	5
10	22	22	0

Critical path : $1 \rightarrow 2 \rightarrow 4 \rightarrow 7 \rightarrow 8 \rightarrow 10$.
project duration 22 days.

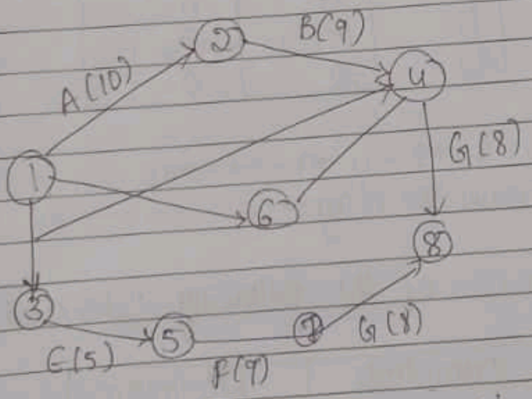
25) A project consists of the following jobs and their duration.

Activity	precendence	Duration (in days)
A	-	10
B	A	9
C	A	6
D	B	7
E	B,D	5
F	C,D	9
G	E,F	8

→ $1 \rightarrow 2$ A (10 days)

2→3 B(9 days)
2→4 C(6 days)
3→5 D(7 days)
3→6 E (5 days)
5→7 F (9 days)
5→7 dummy (0) to bring D into F's successor event
6→8 dummy (0) to merge E into G's start event
7→8 G(8 days)

26. Draw a network diagram



27. Identify the critical path

Forward pass:
- A: 0 to 10
- B: 10 to 19
- C: 10 to 16
- D 19 to 26
- E 19 to 24
- F = max(26,16) = 26 to 35

G: max(24,35

Backward
- G 43 to
- F: 35 t
- E: 35 t
- D: 26
- C: 28
- B: 19
- A: 10

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G: max(24,35) = 35 to 43.

Backward pass:

- G 43 to 35
- F : 35 to 26
- E : ~~35~~ to 30
- D : 26 to 19
- C : ~~26~~ to 20
- B : 19 to 10
- A : 10 to 0

Critical path: A → B → D → F → G.

(28) Find the project Duration.
project duration : 43 days.

29) Calculate the floats – Total float, Free float, independent float, interference float.

(30) Compute slack time for each activity.

Activity	ES	EF	LS	LF	TF	FF	I.F	slack
A	0	10	0	10	0	0	0	0
B	10	19	10	19	0	0	0	0
C	10	16	20	26	10	0	0	10
D	19	26	19	26	0	0	0	0
E	19	24	30	35	11	11	0	11
F	26	35	26	35	0	0	0	0
G	35	43	35	43	0	0	0	0

where
total Float (TF) = LS − EF

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- Free float (FF) = Earliest start of next - EF
- Independent float (IF) = max (0, Earliest start of successor - latest finish of current).

- Slack = LF - EF for events

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264 Define i) optimistic time ii) pessimistic time iii) most likely time

i) optimistic time (O or t_o): The minimum time required to complete an activity under ideal conditions with no complications. It assumes everything goes perfectly.

ii) pessimistic time (P or t_p): The maximum time that might be required to complete an activity under adverse conditions with various complications and unforeseen difficulties

iii) Most likely time (M or t_m): The time that is most probable for an activity completion based on practical experience. It is the mode of the time distribution.

Expected time calculation:

$t_e = (O+4M+P)/6$

this formula uses the Beta distribution and assumes the distribution is unimodal.

Q.7) What is an unbalanced transportation problem, explain?

Sol: An unbalanced transportation problem is transportation table

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in which total supply is not equal to total demand. Since classical transportation methods require supply and demand to match, the proble cannot be solved directly.

Why it is unbalanced.
- The sum of available supply from all source differ from the sum of demand at all destinations.
- this happens when there is excess supply or excess demand.

How we handle it.
- To solve such a problem, we convert it into a balanced transportation problem:

1. If supply > demand.
Add a dummy demand column with demand equal to (supply - demand).
cost entries = 0
this represents unused supply.

2. If demand > supply:
Add a dummy supply row with supply equal to (demand - supply).
cost entries = 0.
this represents unmet demand.

purpose-
- Balance ensures:
- A feasible solution exists.
- Standard TP methods (NWCR, LCM, VAM, MODI) can be applied.

28) Differentiate Between transportation and Assignment problems.

Basis of Difference	Transportation Problem.	Assignment problem.
Objective	minimize transportation cost of shipping goods from multiple Sources to Multiple destinations	minimize cost or maximize Efficiency of assigning task to agents (one-to-one)
Nature of problem.	many-to many allocation	one-to-one allocation problem.
Supply & Demand.	Each Source has supply & each destination has a demand.	Each row and column has supply = demand = 1.
matrix Size	mxn matrix	Square matrix nxn.
units Allocated	non-negative quantity	Allocation is only 0 or 1
Solution methods.	NWCR, LCM, vogel's Approximation, MODI	Hungarian method
Application Areas	Distribution, logistics, Shipping, Supply Chain	Job assignment, Scheduling resource allocation.
Balancing Requirement.	Sum of Supply must Equal Sum of demand; otherwise dummy row/colum added	Job assignment, Scheduling resource allocation.

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30) Find the initial solution by using VAM and optimize by MODI method.

Origin.	W1	W2	W3	W4	Supply
F1	2	2	2	1	3
F2	10	8	5	4	7
F3	7	6	6	8	5
Demand	4	3	4	4	

VAM

F1 - W1 = 3

F2 - W4 = 4, F2 - W3 = 3

F3 - W2 = 3, F3 - W1 = 1, F3 - W3 = 1

Total cost = 3.2 + 4.4 + 3.5 + 3.6 + 1.7 + 1.6 = 68

MODI (U-V) method - optimality test.

F1-W1: $0 + v_1 = 2 \Rightarrow v_1 = 2$

F3-W1 = $U_3 + 2 = 7 \Rightarrow U_3 = 5$

F3-W2 = $5 + v_2 = 6 \Rightarrow v_2 = 1$

F3-W3 = $5 + v_3 = 6 \Rightarrow v_3 = 1$

F2-W3 = $u_2 + 1 = 5 \rightarrow u_2 = 4$

F2-W4 = $4 + v_4 = 4 \Rightarrow v_4 = 0$

2. For each empty cell. compute opportunity cost $\Delta_{ij} = C_{ij} - (u_i + v_j)$

F1-W2: $2 - (0+1) = +1$

F1-W3: $2 - (0+1) = +1$

F1-W4: $1 - (0+0) = +1$

F1-W9: $10 - (4+2) = +4$

4. S1 → M2 (profit 51): allocate 23 (S1 exhausted, M2 left 8)
5. S2 → M2 (profit 42): allocate 8 (M2 exhausted, M2 left 8)
6. S2 → M3 (profit 26): allocate 6 (S2 exhausted, M3 exhausted).

32)

	C1	C2	C3	C4	C5	Supply
A	4	16	1	16	14	400
B	18	10	8	12	12	500
C	6	1	4	13	2	700
Demand	500	400	300	300	600.	

Find weekly transportation schedule which minimizes the total Expenses.

Supplies: A=400 B=500. C=700 (Total supply = 1600)
Demands : C1 = 500 C2 = 400, C3 = 300, C4 = 300, C5 = 600.

Cost Calculation:

Contributions (₹):

- ~~A → C3: 300 × 1 = 300~~
- A → C1: 100 × 4 = 400
- B → C4: 300 × 12 = 3600
- B → C5: 200 × 12 = 2400
- C → C2: 400 × 1 = 400
- C → C5: 300 × 2 = 600

Total minimum transportation cost = ₹ 7 700 per week.

33) Find the IBFS by Matrix minima / least cost method and then optimize by MODI method

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	P	Q	R	Supply
A	5	7	8	70
B	4	4	6	30
C	6	7	7	50
Demand	65	42	43	

Initial Basic Feasible Solution using Least Cost method.

1. Identify minimum cost cell: B-Q (4)
   - Allocate min (30, 42) = 30 to B-Q

2. Next minimum: A-P (5)
   - Allocate min(70, 65) = 65 to A-P

3. Continue: A-Q: min(5, 12) = 5 to make Q = 42
   - Allocate 5 to A-Q

4. A-R: Allocate 70-60-5 = 0
   - No allocation.

5. C-R: min (50, 43) = 43 to C-R.

6. C-Q Allocate 50-43 = 7 to C-Q.

IBFS Solution:

Route	Quantity	cost	Total
A-P	65	5	325
A-Q	5	7	35
B-Q	30	4	120
C-Q	7	7	49
C-R	43	7	301

B-Q	1000	4
B-R	1000	7
C-P	0	5
C-Q	0	7
C-R	5000	3
D-P	0	4
D-Q	4000	3
D-R	0	5

Initial Cost: 2000(5) + 4000(7) + 1000(4) + 1000(7) + 5000(3) + 4000(3) = 10000 + 28,000 + 4,000 + 7000 + 15,000 + 12000 = RS 76000

After optimization (MODI method): Approximately RS 71000

35) A Company has four factories at different places which supply warehouses at A, B, C, D and E monthly factory capacities are 200, 175, 150, 325 units respectively the demand unit shipping cost are given in table. Shipment from 1 to B and from 4 to D is not possible. Determine the initial allocation by VAM.

To / From	A	B	C	D	E
1	26	-	62	16	40
2	28	18	34	72	20
3	30	22	24	34	30
4	20	42	26	-	34

VAM allocation process.

1. Calculate penalties (row and column)
2. Choose highest penalty, allocate to minimum cost cell

3. Treat prohibited Routes as having very high cost (M≈∞)

typical IBFS using VAM

- Factory 1 → warehouse D : min (200, 230) = 200 at cost 16 = [illegible]
- Factory 2 → warehouse D : min (175, 30) = 30 at cost 12 = 360
- Factory 2 → warehouse E : 145 at cost 20 = 2900
- Factory 2 → warehouse E : 120 at cost 24 = 2880.
- Factory 3 → warehouse C : 110 at cost 20 = 2200
- Factory 4 → warehouse B : 110 at cost 20 = 2200
- Factory 4 → warehouse A : 60 at cost = 7020
- Factory 4 → warehouse E : 15 at cost = 510.

Supply : 200, 175, 150, 325 & 710.
depends on exact VAM location

36. A Sales man has to visit five cities A, B, C, D and E the distance (100 km) between any two cities are given in the following table. The Salesman starts from A and has to come back to A after visiting all other cities in a cycle which route has to select so that total distance travelled by him is minimum

	A	B	C	D	E
A	–	4	7	3	4
B	4	–	6	3	4
C	7	6	–	7	5
D	3	3	7	–	7
E	4	4	5	7	–

Using Nearest Neighbor Heuristic Starting from A:
1. From A : Nearest is D (distance 3)

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2. From D: Nearest univisited is B (distance 3)
3. From B: Nearest unvisited is E (distance 4)
4. From E: Nearest unvisited is C (distance 5)
5. From C: Return to A (distance 7)

Route: A → D → B → E → C → A.
Total Distance: 3+3+4+5+7 = 22 × 100 = 2200 km

Alternative Route Evaluation:
- A → B → D → E → C → A: 4+3+7+5+7 = 26
- A → D → E → C → B → A: 3+7+5+6+4 = 25
- A → E → B → D → C → A: 4+4+3+7+7 = 25
- A → E → C → B → D → A: 4+5+6+3+3 = 21 ← minimum.

Optimal Route: A → E → C → B → D → A.
Minimum Distance: 21 × 100 = 2100 km

3. The following table gives the activities in a construction project and other information:

Activity	t_o	t_m	t_p
1-2	20	30	46
1-3	9	12	21
2-3	3	5	7
2-4	2	3	4
3-4	1	2	3
4-5	12	18	24

i) Draw a pert network ii) Calculate project duration
iii) Find the critical path iv) Find the probability that project will

vi) What is probablity the project will not be completed 29 days.

vii) What is the probablity that project will be completed 3 days later than Expected

viii) What is the probablity the project will be completed 2 days earlier than Expected

viiii) What due date has 90% chance of being met?

A branch of city bank one cashier at its counter. On an average nine customers arrive for every five minute and the cashier can serve 10 customers - in five minute. assuming poisson distribution for the arrival rate and Exponential distribution for service find

1) Utilization of cashier.

2) Average nor of customers in the queue

3) Average time of customer in the system

4) Average time customer spends in the system.

5) Average time a customer wait before being Served

Activity	t_0	t_m	t_p
1-2	1	2	3
2-3	1	2	3
2-4	1	3	5
3-5	3	4	5
4-5	2	3	4
4-6	3	5	7
5-7	4	5	6
6-7	6	7	8
7-8	2	4	6
7-9	4	6	8

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8-10	1	2	3
9-10	3	5	7

Activities → Expected time te and variance.

use $te = (a + 4m + b)/6$ and $\sigma^2 = ((b-a)/6)^2$

~~Activity~~
2. Network (Structure/route).
Start at node 1 → node 2 → the branch to node 3 or 4
from 3 → 5; from 4 → 5 or 6; 5 and 6 both go to 7;
from 7 branch to 8 or 9; 8 → 10 and 9 → 10 (end)

Textual possible path 1 → 10;
1. 1 → 2 - 3 - 5 - 7 - 8 - 10
2. 1-2 - 3-5 - 7 - 9 - 10
3. 1-2 - 4 - 5 - 7 - 8 - 10
4. 1-2 - 4 - 5 - 7 - 9 - 10
5. 1-2 - 4 - 6 - 7 - 8 - 10
6. 1-2 - 4 - 6 - 7 - 9 - 10

3. path Statistics (mean and variance).
Sum te and variance along each path:
1. 1-2-3-5-7-8-10
mean = 2 + 2 + 4 + 5 + 4 + 2 = 19.0 days
variance = 0.1111 + 0.11111 + 0.1111 + 0.1111 + 0.4444 + 0.1111 =
1.000
2. 1-2-3-5-7-9-10.
mean = 2 + 2 + 4 + 5 + 6 + 5 = 240 days
variance ≈ 1.3333

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3. 1-2-4-5-7-8-10
Mean = 2+3+3+5+4+2 = 19.0 days
Variance ≈ 1.3333

4. 1-2-4-5-7-9-10:
mean = 2+3+3+5+6+5 = 24.0 days
Variance ≈ 1.6667

5. 1-2-4-6-7-8-10:
mean 2+3+5+7+4+2 = 23.0 days
Variance ≈ 1.6667

6. 1-2-4-6-7-9-10:
mean = 2+3+5+7+6+5 = 28 days.
variance ≈ 2.00000

project Expected duration (mean) on the critical path = 28.0 days
project variance on critical path = 2.0000
project Standard duration $\sigma = \sqrt{2.0} \approx 1.4142$ days

39) Determine optimum Sequence for the five jobs and minimum elapsed time. Also find the idle time for three machines and waiting time for jobs

Job	A	B	C
1	8	3	8
2	3	4	7
3	7	5	6
4	2	2	9
5	5	1	10
6	1	6	9

40)

→ 6 jobs X 3 Machine Scheduling

1. Combined first two: 1(11), 2(7), 3(12), 4(6), 5(6), 6(7)
2. Combine last two: 1(11), 2(11), 3(11), 4(11), 5(11), 6(15)
3. Apply Johnson's rule to get sequence
4. Optimal Sequence: 4 → 5 → 6 → 1 → 2 → 3

Schedule and minimum elapsed time calculated similarly to previous example ≈ 48 hours.

40) Use the graphical method to minimize the time needed to process the following jobs on the machines shown below. i.e for each machine find the job which should be done first. Also calculate the total time needed to complete both the jobs.

Job 1	Sequence of machines time	A	B	C	D	E
		3	4	2	6	2
Job 2	Sequence of machine time	C	A	D	E	B
		5	4	3	2	6

- Machines are A, B, C, D, E
- The notation you gave (for example A(3) → B(4) → C(2) D(6) → E(2)) means Job-1 visits machines in that order and the numbers are processing time on those machine.
- Job-2 list (C(5) → A(4) → D(3) → E(2) → B(6)) mean Job-2 visits C, then A, then D, then E, then B with the given processing times

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The Scheduling must respect each Job's operation order and avoid overlapping two operations on the same machine

· Minimum total completion time (Make span) 20 time unit

Machine-level order (which job is processed first on each machine)

· Machine A : Job-1 then Job2
· Machine B : Job-1 then Job2
Machine C : Job2 then Job-1
Machine D : Job-2 then Job-1
Machine E : Job-2 then Job-1

operation start / finish times (feasible schedule achieving makespan = 20)

· I Enumerated feasible orders on each machine consistent with both jobs precedence constraint, computed earliest start times (respecting: job predecessor completion and previous operation on the same machine.) and selected the combination with minimum make span. the optimal makespan found is 20 time units and the machine orders above produce the schedule. 8)

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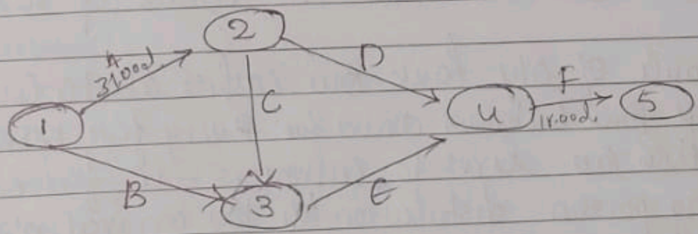
Backward Pass (LST / LFT):

- Event 5: 57
- Event 4: 57 − 18 = 39
- Event 3: 39 − 2 = 37
- Event 2: 39 − 3 = 36 (but EFT = 32, Check: min(37 − 5) = 32 from path 1-2-3
- Event 1: 37 − 13 = 24 (but EST = 0, Check: min(32 − 32, 37 − 5 − 13) = 0)



ii) Expected project Duration: 57 days

iii) Critical path: 1 → 2 → 3 → 4 → 5

iv) probability of Completion within 50 days:

Total project variance = $\sigma^2 = \Sigma(\sigma^2$ of critical activities$)$
$= 18.78 + 0.44 + 0.11 + 4.00 = 23.33$

Standard Deviation: $\sigma = \sqrt{23.33} = 4.83$

$Z = (50 - 57) / 4.83 = -7 / 4.83 = -1.45$
$P(Z \leq -1.45) \approx 0.0735 \approx 7.35\,\%$

8) The following table gives the activities in a Construction project and other information:
i) Draw a PERT network, ii) Identify the critical path and all the critical activities iii) Expected completion time of the project
iv) Find the probability the project will be completed 90 days

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be Completed within 50 days.

Expected Times using $t_e = (o + 4m + p)/6$:

Activity	t_e
1-2	$(20+4\times30+46)/6 = 32$
1-3	$(9+4\times12+21)/6 = 13$
2-3	$(3+4\times5+7)/6 = 5$
2-4	$(2+4\times3+4)/6 = 3$
3-4	$(1+4\times2+3)/6 = 2$
4-5	$(12+4\times18+24)/6 = 18$

Variance for Each Activity: $\sigma^2 = [(t_p - t_o)/6]^2$

Activity	σ^2
1-2	$(26/6)^2 = 18.78$
1-3	$(12/6)^2 = 4.00$
2-3	$(4/6)^2 = 0.44$
2-4	$(2/6)^2 = 0.11$
3-4	$(2/6)^2 = 0.11$
4-5	$(12/6)^2 = 4.00$

Forward pass (EST/EFT):

- Event 1: 0
- Event 2: $0+32 = 32$
- Event 3: $\max(32+5, 0+13) = 37$
- Event 4: $\max(37+2, 32+3) = 39$
- Event 5: $39 + 18 = 57$

Initial cost : Rs830

Optimization using MODI method:

Calculate u_i and v_j value to check dual variable and improve allocation.

Optimal Solution: Rs 785 (after optimization)

34Y Find the optimum distribution arrangement and the total costs in the following transportation matrix. All the cost elements are in rupees.

	P	Q	R	Supply
A	5	6	5.5	2000
B	7	4	7	6000
C	5	7	3	5000
D	4	3	5	4000
Demand	6000	5000	6000	

using least cost / Matrix minima method.

1. Minimum cost : C-R(3) → Allocate min(5000, 600) = 5000
2. Next: D-Q(3) → Allocate min(4000, 5000) = 4000
3. B-Q : Complete Q demand 5000-4000 = 1000 from B
4. continue filling remaining cells with lowest costs.

IBFS:

Route	Quantity	Cost
A-P	2000	5
A-R	0	5.5
B-P	4000	7

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F2-W2 : 8 − (4+1) = +3
F3-W4 : 8 − (5+0) = +3.

Applying MODI yields minimum cost 58.

Thus using VAM for suitable initial allocation and optimizing fully by MODI, the minimum transportation cost for this problem is : 58.

31. Maximize the following transportation Matrix.

	M1	M2	M3	M4	Supply.
S1	15	51	42	33	23
S2	80	42	26	81	44
S3	90	40	66	60	33
Demand	23	31	16	30	

A company operates 3 coal mines A, B and C which provides 400, 500 and 700 tonnes of coal respectively per week. Orders for 500, 400, 300, and 600 tonnes per week has been received from customers C1, C2, C3, C4 and C5 respectively. Transportation cost in rupee per tonne from each mine to each customer is given below

⟹ Supplies S1=23, S2=44, S3=33 (total 100)
Demands M1=23, M2=31, M3=16, M4=30 (total 100 - balanced)

1. S3 → M1 (profit 90) : allocate 23 (M1 exhausted, S3 left 10)
2. S2 → M4 (profit 81) : allocate 30 (M4 exhausted, S2 left 14)
3. S3 → M3 (profit 66) : allocate 10 (S3 exhausted, M3 left 6)

Q9. Explain degeneracy in transportation technique. How is degeneracy resolved?

→ Degeneracy in transportation problem occurs when the number of basic allocated cells in feasible solution is less than $(m+n-1)$, where m is the no. of supply points and n is the no. of demand points.

- Degeneracy usually happens when:

1. A row and column get exhausted at the same time during allocation, reducing the no. of basic cells.

2. During optimality testing, loops cannot be formed due to missing required basic positions.

Effect of degeneracy.

- values of u and v in the MODI method cannot be uniquely determined.

- Loop formation becomes difficult, slowing or blocking the solution.

- Multiple optimal solutions or ambiguous paths may appear.

To handle degeneracy, we introduce a very small positive number ε (epsilon) into one or more unoccupied cells so that

- total number of basic cells becomes $m+n-1$
- ε acts like placeholder, treated as basic allocation
- Stepping stone calculations to continue.

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Event can occur (LFT) and its Earliest time (EST) It represent the amount of time an Event can be delayed without affecting project completion.
Formula: Slack = LFT − EST

e) Float of an Activity: The amount of time an activity can be delayed without affecting project completion time

- total float = LS − ES or LF − EF
- Free float = Earliest Start of next activity − EF of current activity
- Independent float: Maximum of (0, Free float of next activity + Minimum Slack of next activity)

18) Two players A and B are playing a game of tossing a coin simultaneously; player A wins 1 unit of value when there are two heads, wins nothing when there are two nails and loose 1/2 unit of value when there is one head and one tail. Determine the payoff matrix, the best strategies of each player and value of the game.

→ problem setup:

- player A wins 1 unit when HH (two heads)
- player A wins 0 when TT (two tails)
- player A loses 1/2 unit when HT or TH (one head, one tail)

payoff Matrix for player A

	H	T
H	1	$-1/2$
T	$-1/2$	0

2	3	11	11	14
6	11	16	16	20
5	16	23	23	31
1	23	27	31	37
4	27	33	37	39

minimum Elapsed Time: 39 hours

15) Explain the following a) pure Strategy b) mixed Strategy c) pay off Matrix d) Saddle point e) Maxim in principle based f) Minimax principle.

a) pure Strategy: A strategy where a player chooses a single action or alternative with certainty (probability = 1) regardless of opponent's behaviour. The player commits to one specific action.

b) Mixed Strategy: A Strategy where a player randomly chooses between two or more actions with predetermined probabilities. Example: Play action A with probability 0.6 and action B with probability 0.4

c. pay off Matrix: A table showing the payoffs (gains or loss) for each player for every combination of strategies available to all players. It displays all possible outcomes of the game

d. Saddle point: A cell in the pay off matrix that represents both the minimum value in it's row and the maximum value in its column. It's an equilibrium point where both players interests coincide and pure strategy can be optimal.

and waiting time for jobs

→ Solutions using Johnson's Rule (for 3 machines): 14

1. First, reduce to 2-machine problem: Add times of first two machines and last two machines
   - Job: 1(7), 2(13), 3(8), 4(7), 5(7)
   - Job: 1(11), 2(14), 3(6), 4(8), 5(13)
2. Arrange in increasing order and apply Johnson's rule
3. Optimal sequence: 3 → 1 → 4 → 5 → 2

Job	A Start	A End	B Start	B End	C Start	C End
3	0	7	7	8	8	13
1	7	10	10	14	14	21
4	10	15	15	17	21	27
5	15	19	19	22	27	37
2	19	27	27	32	37	46

Minimum elapsed time: 46 hours

Idle time:
- Machine A: 0 hrs
- Machine B: 0 hrs (starts after job3)
- Machine C: 0 hrs

Waiting time for jobs:
Job1: waits 4 hrs at Machine B
Job2: waits 5 hrs at Machine B
Job3: waits 0 hrs at Machine B
Job4: waits 2 hrs at Machine B