

$$l = \frac{1}{N} \sum_{i=1}^N (y_i - a - bx_i)^2$$

$$\frac{dl}{da} = \frac{1}{N} \sum_{i=1}^N [2(y_i - a - bx_i) * (-1)] = 0$$

$$= \frac{1}{N} \sum_{i=1}^N -y_i + \frac{1}{N} \sum_{i=1}^N a + \frac{1}{N} \sum_{i=1}^N bx_i = 0$$

$$\frac{1}{N} \sum_{i=1}^N y_i = \frac{1}{N} \sum_{i=1}^N a + \frac{1}{N} \sum_{i=1}^N bx_i \quad \text{--- (1)}$$

$$\frac{dl}{db} = \frac{1}{N} \sum_{i=1}^N [2(y_i - a - bx_i) * (-x_i)] = 0$$

$$= \frac{1}{N} \sum_{i=1}^N [-2(x_i y_i - ax_i - bx_i^2)] = 0$$

$$= \frac{1}{N} \sum_{i=1}^N x_i y_i = \frac{1}{N} \sum_{i=1}^N ax_i + \frac{1}{N} \sum_{i=1}^N bx_i^2 \quad \text{--- (2)}$$

Lets use \bar{x} and \bar{y} to denote

$$\bar{x} = \frac{1}{N} \sum_{i=1}^N x_i$$

$$\bar{y} = \frac{1}{N} \sum_{i=1}^N y_i$$

$$\overline{xy} = \frac{1}{N} \sum_{i=1}^N x_i y_i$$

$$\overline{x^2} = \frac{1}{N} \sum_{i=1}^N x_i^2$$

$$\textcircled{1} \rightarrow \bar{y} = a + b\bar{x}$$

$$\textcircled{2} \rightarrow \overline{xy} = a\bar{x} + b\overline{x^2}$$

Divide $\textcircled{2}$ by \bar{x} on both sides $\rightarrow \frac{\overline{xy}}{\bar{x}} = a + \frac{b\overline{x^2}}{\bar{x}} \Rightarrow \textcircled{3}$

Solving $\textcircled{1}$ and $\textcircled{3}$ we get

$$b = \left(\frac{\overline{xy} - \bar{x}\bar{y}}{\overline{x^2} - \bar{x}\bar{x}} \right) \quad a = \bar{y} - b\bar{x}$$

$$a = \frac{1}{N} \sum_{i=1}^N y_i - \frac{b}{N} \sum_{i=1}^N x_i$$

$$b = \frac{\sum_{i=1}^N x_i y_i - \frac{1}{N} \sum_{i=1}^N x_i \sum_{i=1}^N y_i}{\sum_{i=1}^N x_i^2 - \frac{1}{N} \sum_{i=1}^N x_i \sum_{i=1}^N x_i}$$