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S20210010125

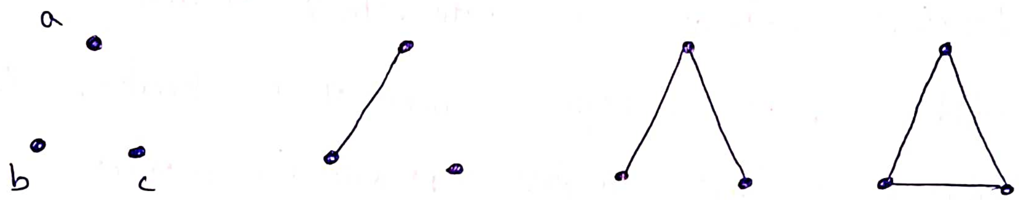
GTA
ASSIGNMENT - 1

1. Draw all simple graphs of one, two, three and four vertices.

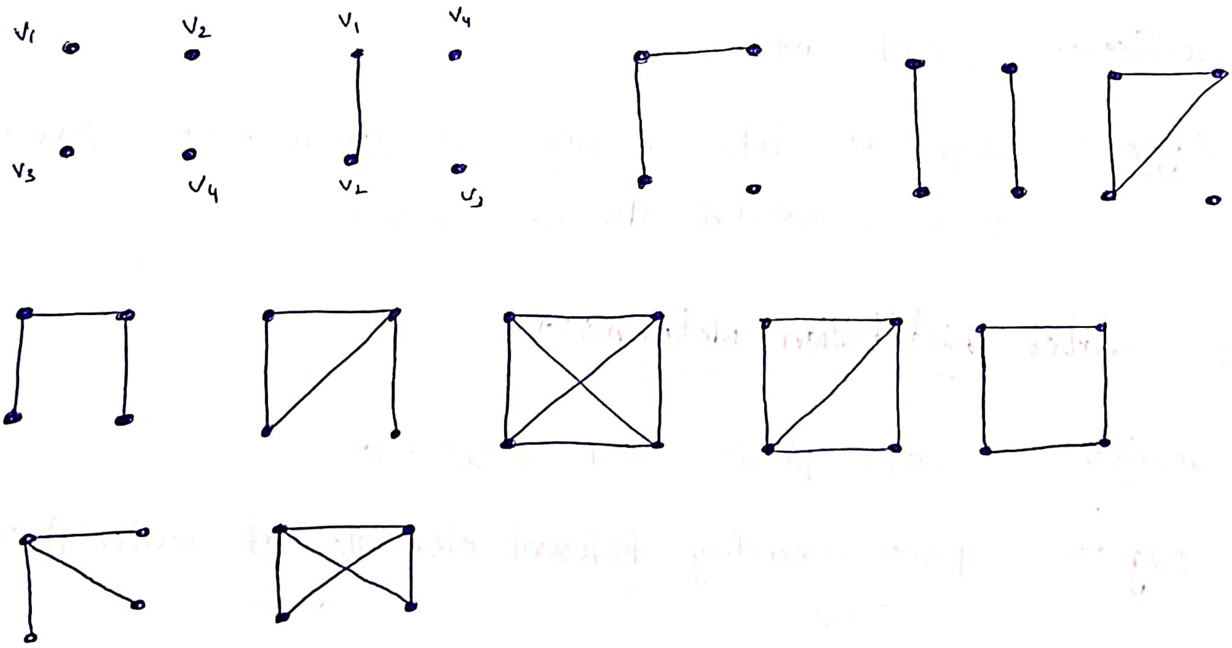
a) simple graph with one vertex

b) simple graph with two vertices.

c) simple graph with three vertices.



d) Simple graph with four vertices.



2 Name 5 situations that can be represented by means of graphs. Explain what the vertices and edges denote.

1. Social Networks:-

Vertices:- individuals in the social network.

edges:- connections & relationships between individuals. if there is a connection from person A to person B. Then there is a corresponding edge between them.

2. Transportation Networks:-

Vertices:- Locations such as cities, intersections & airports.

Edges:- Roads, streets, & flights connecting the locations. The edges could have weights representing distances.

3. world wide web:-

vertices:- web pages.

Edges:- Hyperlinks between pages. if page A links to page B there is directed edge from A to B.

4. water Distribution Networks:-

vertices:- water plants and Reservoirs

edges:- pipes connecting different elements of water distribution systems.

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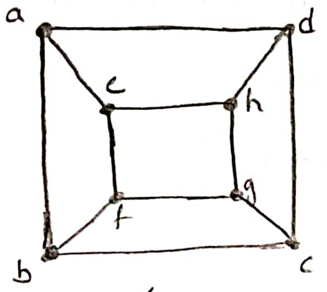
5. Communication Networks:-

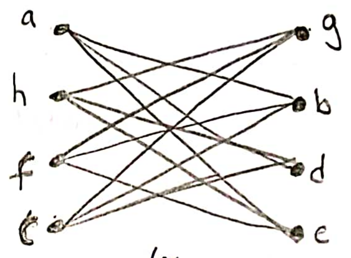
Vertices:- individuals with their mobile

edges:- Communication between any persons. connection an edge.

3 Show that the two graphs are isomorphic.



G_1



G_2

Condition-1:- Two graphs having same number of vertices. i.e 8

Condition-2:- Two graphs having same number of edges $G_1 = G_2 = 12$.

Condition-3:- Degree sequence.

for graph G_1 = {3, 3, 3, 3, 3, 3, 3, 3}

for graph G_2 = {3, 3, 3, 3, 3, 3, 3, 3}

Matrix Representation:-

G_1

	a	b	c	d	e	f	g	h
a	0	1	0	1	1	0	0	0
b	1	0	1	0	0	1	0	0
c	0	1	0	1	0	0	1	0
d	1	0	1	0	0	0	0	1
e	1	0	0	0	0	1	0	1
f	0	1	0	0	1	0	1	0
g	0	0	1	0	0	1	0	1
h	0	0	0	1	1	0	1	0

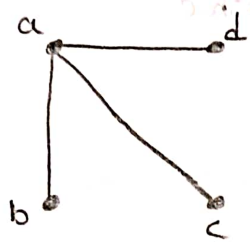
G_2

	a	b	c	d	e	f	g	h
a	0	1	0	1	1	0	0	0
b	1	0	1	0	0	1	0	0
c	0	1	0	1	0	0	1	0
d	1	0	1	0	0	0	0	1
e	1	0	0	0	0	1	0	1
f	0	1	0	0	1	0	1	0
g	0	0	1	0	0	1	0	1
h	0	0	0	1	1	0	1	0

× both graphs G_1 and G_2 have same adjacency matrix

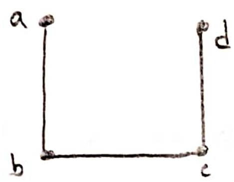
∴ both graphs are isomorphic

4 Draw a connected graph that becomes disconnected when any edge is removed from it.

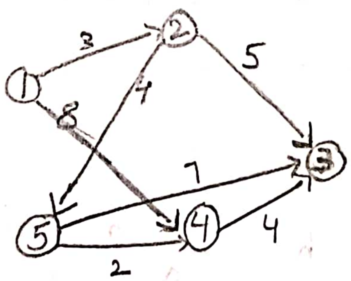


⇒ an edge in a connected graph on one removal it disconnected the graph is called bridge.

bridges :- {(a,b), (a,c), (a,d)}



5 find the shortest path from vertex 5 to all other vertex using Dijkstras algorithm for given directed graph.



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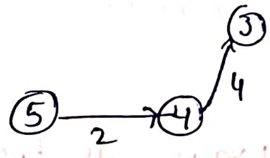
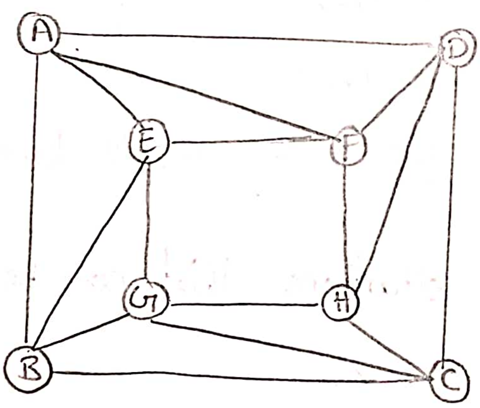
Dijkstra's Algorithm :-

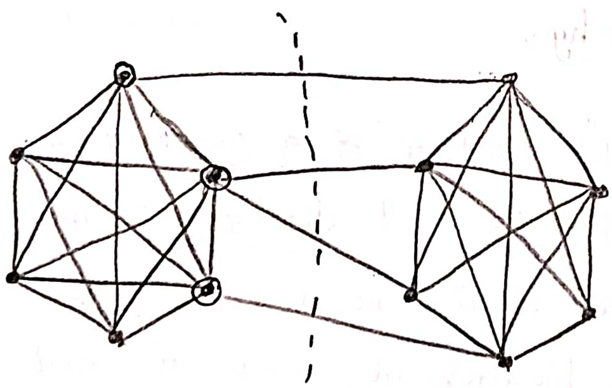
	5	1	2	3	4
5	0	∞	∞	∞	∞
4		∞	∞	7	$\boxed{2}$
3		∞	∞	$\boxed{6}$	
		∞	∞		



$\therefore$ shortest path is $5 \to 4 \to 3$, from vertex 5 to 3.

6. Draw and explain a graph that has edge connectivity is 4. Vertex connectivity is 3 and degree of every vertex ≥ 5.





⇒ from second graph the edge connectivity is 4 and vertex connectivity is 3 and every vertex degree is greater than 5.

edge connectivity:-

minimum number of edges to removal it became disconnected graph.

7 Draw all trees of n labelled vertices for n=1,2,3,4 and 5.

by using cayley's formula - n^{n-2}

→ for 1 vertex has one tree

→ for 2 vertices - one tree

→ for n=3, 3^{3-2} ⇒ 3 trees

→ for n=4, 4^{4-2} ⇒ 16 trees

→ for n=5. 5^{5-2} ⇒ 125 trees uniquly labelled.

10 cite three different situations that can be represented by trees. Explain

1. File system Hierarchy:-

Explanation:- The file system of a computer is often represented as a tree. The root directory is the starting point and each subdirectory or file is a node connected by edges. This tree structure is hierarchical, with the root directory at the top and subdirectories branching down.

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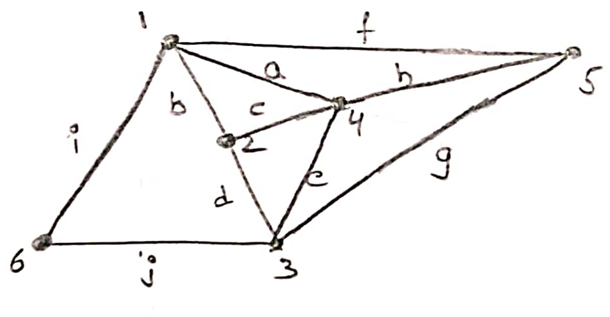
2. project planning :-

Explanation:- In project management, a work Breakdown structure (WBS) can be represented as a tree. The root node represents the entire project and each subsequent nodes breaks down the project into smaller manageable tasks. The leaf nodes represents small work packages or tasks need to be completed.

3. Decision - Making in strategy games:-

Explanation:- In strategy games, such as real-time strategy (RTS) or turn based strategy (TBS) games, decision trees are used to model the decision-making process of computer controlled opponents (AI). Nodes in the tree represent different possible actions or strategies. and edges depict the outcomes or respones to those actions.

11. find the following for the given graph.



a) Eccentricity of vertices

Eccentricity :- maximum distance of one vertex from other vertex.

e(1) :- (eccentricity of 1 is) :- 2

e(2) - 2

eccentricity of 3 - 2
eccentricity of 4 - 2
eccentricity of 5 - 2
eccentricity of 6 - 2

b. centre, radius and diameter of the graph

<u>diameter</u>:- The maximum distance between the pair of vertices.

<u>Radius</u> :- The minimum distance between pair of vertices.

<u>centre</u>:- The centre of graph is comprised of the nodes whose eccentricity is equal to the graph's radius.

c.d All the path between 5 and 6 and distances.

1.

path b/w 5 to 6		Distances
1) 5f1i6	-	2
2) 5h4a1i6	-	3
3) 5h4e2b1i6	-	4
4) 5h4e2d3j6	-	4
5) 5h4e3j6	-	3
6) 5g3j6	-	2
7) 5f1b2d3j6	-	4
8) 5f1b2c4e3j6	-	5
9) 5f1a4e3j6	-	4
10) 5f1a4c2d3j6	-	5

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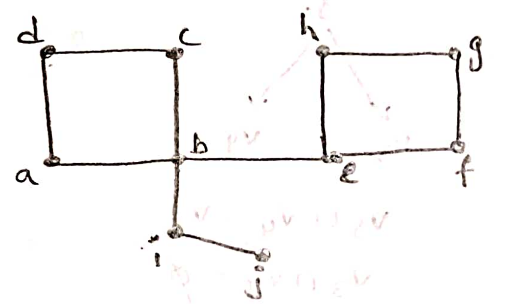
12. Using an Example. explain connectivity and separability of a graph.

Connectivity :- minimum number of edges whose removal reduces rank by one. (it disconnects the graph)

separability :- vertex connectivity - 1

removal of one vertex it disconnects the graph.



separability :-

→ removal of b, e and i it divides the graph (disconnects the graph)

edge connectivity :-

removal of edges {(b,e), (b,i), (i,j)} is minimum edge connectivity of 1.

8 prove that the ringsum of any two cutsets in a graph is either a third cutset or an edge disjoint union of cutsets.

proof :- Let S_1 and S_2 be two cutsets.

S_1 partitioned the vertex set into V_1 & V_2

S_2 partitioned the vertex set into V_3 & V_4

$$\begin{array}{ccc} & S_1 & \\ \swarrow & & \searrow \\ V_1 & & V_2 \end{array} \qquad\qquad \begin{array}{ccc} & S_2 & \\ \swarrow & & \searrow \\ V_3 & & V_4 \end{array}$$

$V_1 \cup V_2 = V$ $\qquad$ $V_3 \cup V_4 = V$

$V_1 \cap V_2 = \phi$ $\qquad$ $V_3 \cap V_4 = \phi$

Let considered V_5 and V_6 as new vertex set.

$$V_5 = (V_1 \cap V_4) \cup (V_2 \cap V_3)$$

$$V_6 = (V_1 \cap V_3) \cup (V_4 \cap V_2)$$

$$V_5 = V_1 \oplus V_3 \quad \text{and} \quad V_6 = V_2 \oplus V_3$$

consider $S_1 \oplus S_2$ (S_1 Ringsum S_2).

Edges belonging to this ringsum are only from V_5 to V_6. and also contains all the edges between V_5 and V_6.

Removal of $S_1 \oplus S_2$ disconnects the G, partitioned

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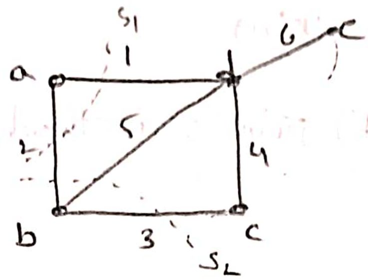
The vertex set into V_5 and V_6 such that.

$V_5 \cup V_6 = V$. and $V_5 \cap V_6 = \phi$.

Example :-



$S_1 = \{1, 2\}$ → $V_1 = \{a\}$
→ $V_2 = \{b, c, d, e\}$

$S_2 = \{2, 5, 3\}$ → $V_3 = \{b\}$
→ $V_4 = \{a, d, e, c\}$

$V_5 = (V_1 \cap V_4) \cup (V_2 \cap V_3) = \{a\} \cup \{b\} = \{a, b\}$

$V_6 = (V_1 \cap V_3) \cup (V_4 \cap V_2) = \{\} \cup \{c, d, e\} = \{c, d, e\}$

let S_3 be cutset divided into vertex set $\{a, b\}$ & $\{c, d, e\}$.

S_3 → $\{a, b\}$
→ $\{c, d, e\}$

$S_1 \oplus S_2 = \{1, 5, 3\}$

⇓

$\{1, 5, 3\} = S_1 \oplus S_2$

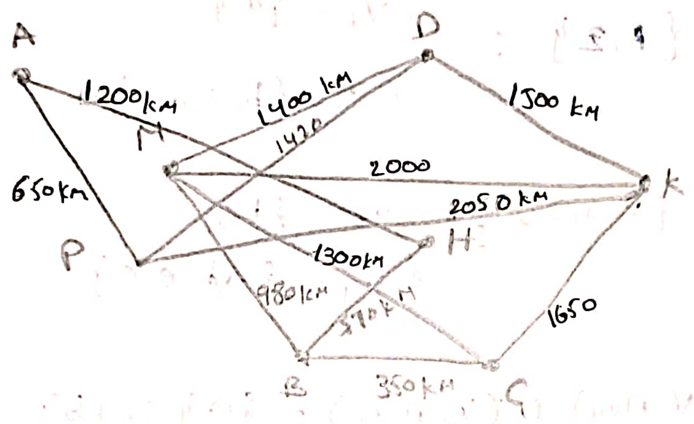
∴ cut set S_3 is ringsum of cutsets S_1 and S_2.

∴ Ringsum of any two cutsets in a graph is either third cut set & an edge disjoint union of cutsets.

Hence, proved.

9 pick any 8 Indian cities and obtain the intercity distances from a map. find the shortest spanning tree connecting these cities by using

a) krushkal's method b) prim's method.



a) krushkal's method :-

Arrange all edges in increasing order.

D. Bengaluru - chennai : 350 km

Bengaluru - Hyderabad : 570 km

Ahmedabad - pune : 650 km

mumbai - Bengaluru : 980 km

Ahmedabad - Hyderabad : 1200 km

mumbai - chennai : 1300 km

mumbai - Delhi : 1400 km

pune - Delhi : 1420 km,

Delhi - kolkata : 1500 km

chennai - kolkata : 1650 km

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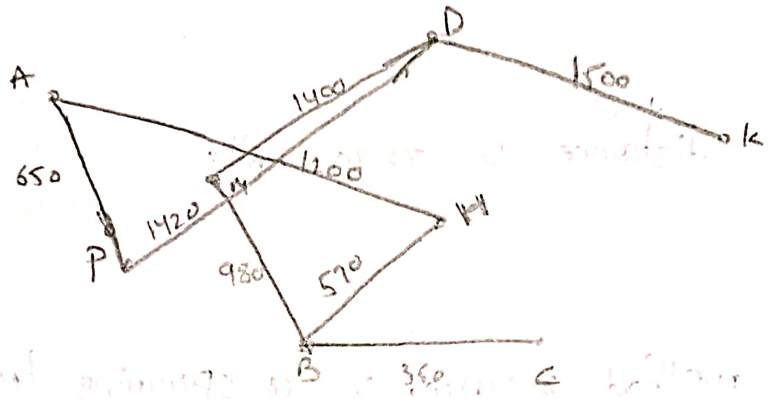
mumbai - kolkata : 2000 KM

pune - kolkata : 2050 KM

* start with smallest edge Bengaluru - chennai (350 KM) adding all edges that don't form a cycle and reaches all vertices.

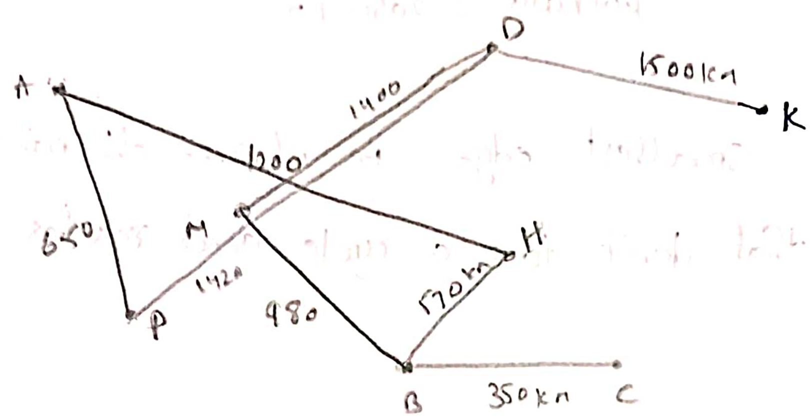


* This is the minimum spanning tree with total distance: 4870 km. using krushkal's method.

b) prim's method :-

1. Starting with any vertex. let's say Bengaluru.
2. Add Bengaluru to the spanning tree.
3. select the edge with the smallest weight connecting Bengaluru to another city, which is Bengaluru to Chennai - 350 km.
4. add chennai to the spanning tree.
5. Repeat this until all cities are included in the spanning tree.

The resulting spanning tree is:



* The total distance is 7870 kms.

Comparison:-

* krushkal's method resulted in a spanning tree with 7870km
* Prim's method resulted in a spanning tree with 7870 km.

so, both methods produce minimum spanning tree. with same shorter distances.

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7 Draw all trees of n labelled vertices for $n = 1, 2, 3, 4$ and 5.

n=1:-

There is only one unique labelled tree.

n=2:-

for n value 2 also only one unique labelled tree.

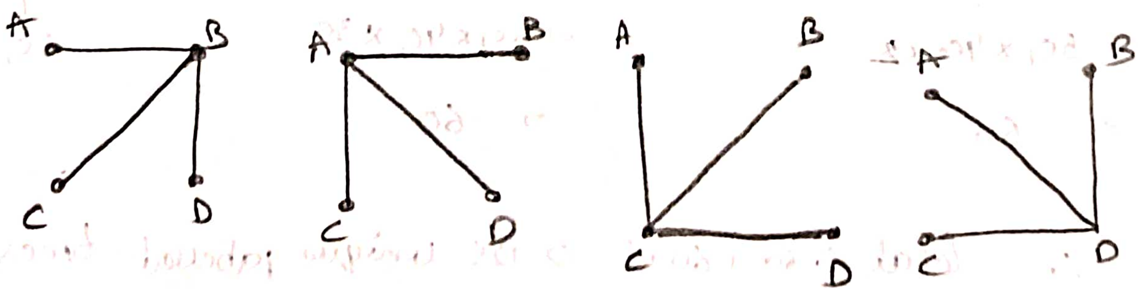
n=3:-

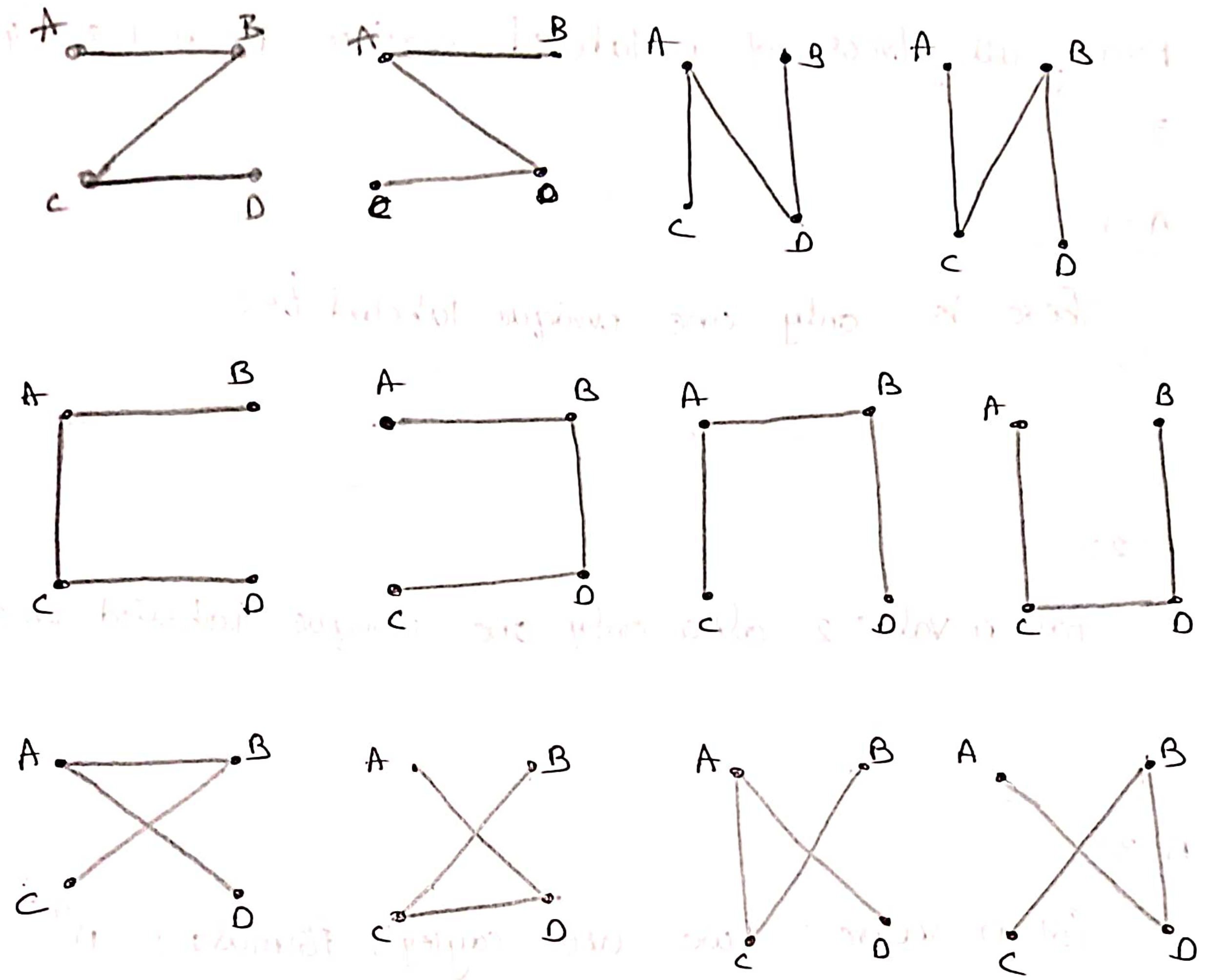
for n value 3 we use cayley's formula : n^{n-2}

$\therefore n = 3 \Rightarrow 3^{3-2} \Rightarrow 3$ unique labelled trees.

n=4:-

There are $n^{n-2} \Rightarrow 4^{4-2} \Rightarrow 16$ unique labelled tree's.

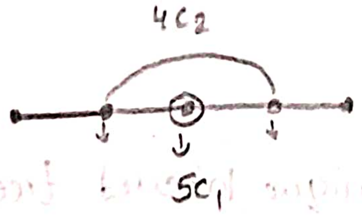




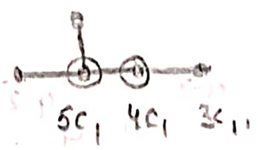
n = 5 :-

There are $n^{n-2} \Rightarrow 5^{5-2} \Rightarrow 125$ possible trees

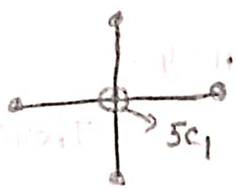
1)



2)



3)



$\Rightarrow {}^5C_1 \times {}^4C_2 \times 2$

$\Rightarrow 60$

$\Rightarrow {}^5C_1 \times {}^4C_1 \times {}^3C_1$

$\Rightarrow 60$

$\Rightarrow {}^5C_1$ ways

$\therefore$ total $\Rightarrow 60 + 60 + 5 \Rightarrow 125$ unique labelled trees.