Homework4

November 24, 2022

1 Question 1

1.0.1 For the prostate data of Chapter 3 (ESL), carry out a best-subset linear regression analysis, as in Table 3.3 (third column from the left). Compute the AIC, BIC, five- and tenfold cross-validation, and bootstrap .632 estimates of prediction error. The data can be obtained from the book website.

[357]:
```
install.packages("ISLR")
install.packages("knitr")
install.packages("printr")
install.packages('leaps')
install.packages('bootstrap')
install.packages('lattice')
install.packages('caret')
library(lattice)
library(caret)
library(bootstrap)
library(leaps)
library(ISLR)
library(knitr)
library(printr)
library(boot)
```

```
The downloaded binary packages are in
/var/folders/ch/lqq67w6x5px6fcc9cnbp457m0000gn/T//Rtmp2yueMP/downloaded_packages

also installing the dependency 'xfun'



The downloaded binary packages are in
/var/folders/ch/lqq67w6x5px6fcc9cnbp457m0000gn/T//Rtmp2yueMP/downloaded_packages

The downloaded binary packages are in
/var/folders/ch/lqq67w6x5px6fcc9cnbp457m0000gn/T//Rtmp2yueMP/downloaded_packages

The downloaded binary packages are in
```

```
/var/folders/ch/lqq67w6x5px6fcc9cnbp457m0000gn/T//Rtmp2yueMP/downloaded_packages

The downloaded binary packages are in
/var/folders/ch/lqq67w6x5px6fcc9cnbp457m0000gn/T//Rtmp2yueMP/downloaded_packages

The downloaded binary packages are in
/var/folders/ch/lqq67w6x5px6fcc9cnbp457m0000gn/T//Rtmp2yueMP/downloaded_packages

The downloaded binary packages are in
/var/folders/ch/lqq67w6x5px6fcc9cnbp457m0000gn/T//Rtmp2yueMP/downloaded_packages
```

```
Warning message in FUN(X[[i]], …):
"unknown type in R_decompress3"
```

```
Error in FUN(X[[i]], …): lazy-load database '/Library/Frameworks/R.framework/
↪Versions/4.2/Resources/library/lattice/data/Rdata.rdb' is corrupt
Traceback:

1. library(boot)
2. checkConflicts(package, pkgname, pkgpath, nogenerics, ns)
3. same.isFn(i)
4. vapply(same, exists, NA, where = where, mode = "function", inherits = FALSE)
5. FUN(X[[i]], …)
```

load the data

[21]:
```
install.packages("ElemStatLearn_2015.6.26.tar.gz", repos = NULL, type =␣
↪"source")
library("ElemStatLearn")
data(prostate)
```

```
Warning message in install.packages("ElemStatLearn_2015.6.26.tar.gz", repos =
NULL, :
"installation of package ‘ElemStatLearn_2015.6.26.tar.gz’ had non-zero exit
status"
```

[22]:
```
str( prostate )
```

```
'data.frame':	97 obs. of  10 variables:
 $ lcavol : num  -0.58 -0.994 -0.511 -1.204 0.751 …
 $ lweight: num  2.77 3.32 2.69 3.28 3.43 …
 $ age    : int  50 58 74 58 62 50 64 58 47 63 …
 $ lbph   : num  -1.39 -1.39 -1.39 -1.39 -1.39 …
 $ svi    : int  0 0 0 0 0 0 0 0 0 0 …
 $ lcp    : num  -1.39 -1.39 -1.39 -1.39 -1.39 …
 $ gleason: int  6 6 7 6 6 6 6 6 6 6 …
 $ pgg45  : int  0 0 20 0 0 0 0 0 0 0 …
 $ lpsa   : num  -0.431 -0.163 -0.163 -0.163 0.372 …
```

```
 $ train  : logi  TRUE TRUE TRUE TRUE TRUE TRUE …
```

Computing best subsets regression

[29]:
```
train_set<-subset(prostate,train=="TRUE")[,1:9]
test_set<-subset(prostate,train=="FALSE")[,1:9]
y_train<-train_set$lpsa
y_test<-test_set$lpsa

regfit.full<-regsubsets(lpsa~.,data=train_set,␣
 ↪nbest=1,nvmax=8,method="exhaustive")
my_sum <- summary(regfit.full)
summary(regfit.full)
```

```
Subset selection object
Call: regsubsets.formula(lpsa ~ ., data = train_set, nbest = 1, nvmax = 8,
    method = "exhaustive")
8 Variables  (and intercept)
        Forced in Forced out
lcavol      FALSE      FALSE
lweight     FALSE      FALSE
age         FALSE      FALSE
lbph        FALSE      FALSE
svi         FALSE      FALSE
lcp         FALSE      FALSE
gleason     FALSE      FALSE
pgg45       FALSE      FALSE
1 subsets of each size up to 8
Selection Algorithm: exhaustive
         lcavol lweight age lbph svi lcp gleason pgg45
1  ( 1 ) "*"    " "     " " " "  " " " " " "     " "
2  ( 1 ) "*"    "*"     " " " "  " " " " " "     " "
3  ( 1 ) "*"    "*"     " " " "  "*" " " " "     " "
4  ( 1 ) "*"    "*"     " " "*"  "*" " " " "     " "
5  ( 1 ) "*"    "*"     " " "*"  "*" " " " "     "*"
6  ( 1 ) "*"    "*"     " " "*"  "*" "*" " "     "*"
7  ( 1 ) "*"    "*"     "*" "*"  "*" "*" " "     "*"
8  ( 1 ) "*"    "*"     "*" "*"  "*" "*" "*"     "*"
```

1.1 Model selection criteria: AIC & BIC

[53]:
```
select = my_sum$outmat
train_error_store<-c()
test_error_store<-c()
AIC_store<-c()
BIC_store<-c()

for(i in 1:8){
```

```r
  temp<-which(select[i,]=="*")
  red_train<-train_set[,c(9,temp)]
  red_test<-test_set[,c(9,temp)]
  red_fit<-lm(lpsa~.,data=red_train)
  AIC<-AIC(red_fit)
  BIC<-BIC(red_fit)
  predict_train<-predict(red_fit,newdata=red_train)
  predict_test<-predict(red_fit,newdata=red_test)
  train_error<-sum((predict_train-y_train)^2)/length(y_train)
  test_error<-sum((predict_test-y_test)^2)/length(y_test)
  train_error_store<-c(train_error_store,train_error)
  test_error_store<-c(test_error_store,test_error)
  AIC_store<-c(AIC_store,AIC)
  BIC_store<-c(BIC_store,BIC)


}

# print('Train error')
# train_error_store
# min(train_error_store)
# print('Test error')
# test_error_store
# min(test_error_store)


print(paste("Subset selected by AIC =",which.min(AIC_store)))
print('AIC values')
AIC_store

print(paste("Subset selected by BIC =",which.min(BIC_store)))
print('BIC values')
BIC_store

print(paste('Test error of 2 variable model:',test_error_store[2]))


upper= max(AIC_store,BIC_store)
lower= min(AIC_store,BIC_store)


plot(AIC_store,type="o",lty=2,col = "blue",ylim = c(lower-5,upper+5),xlab =
  "k",main="AIC & BIC",ylab="Value")
lines(BIC_store,type="o",lty=1,col="red")
legend("topright",c("AIC", "BIC"),lty=c(2,1),col=c("blue","red"))
```

```
plot(train_error_store,type="o",lty=2,col = "blue",ylim = c(0,1),xlab =␣
 ↪"k",ylab="error",main="Error")
lines(test_error_store,type="o",lty=1,col="red")
legend("topright",c("training.error", "test.
 ↪error"),lty=c(2,1),col=c("blue","red"))
```

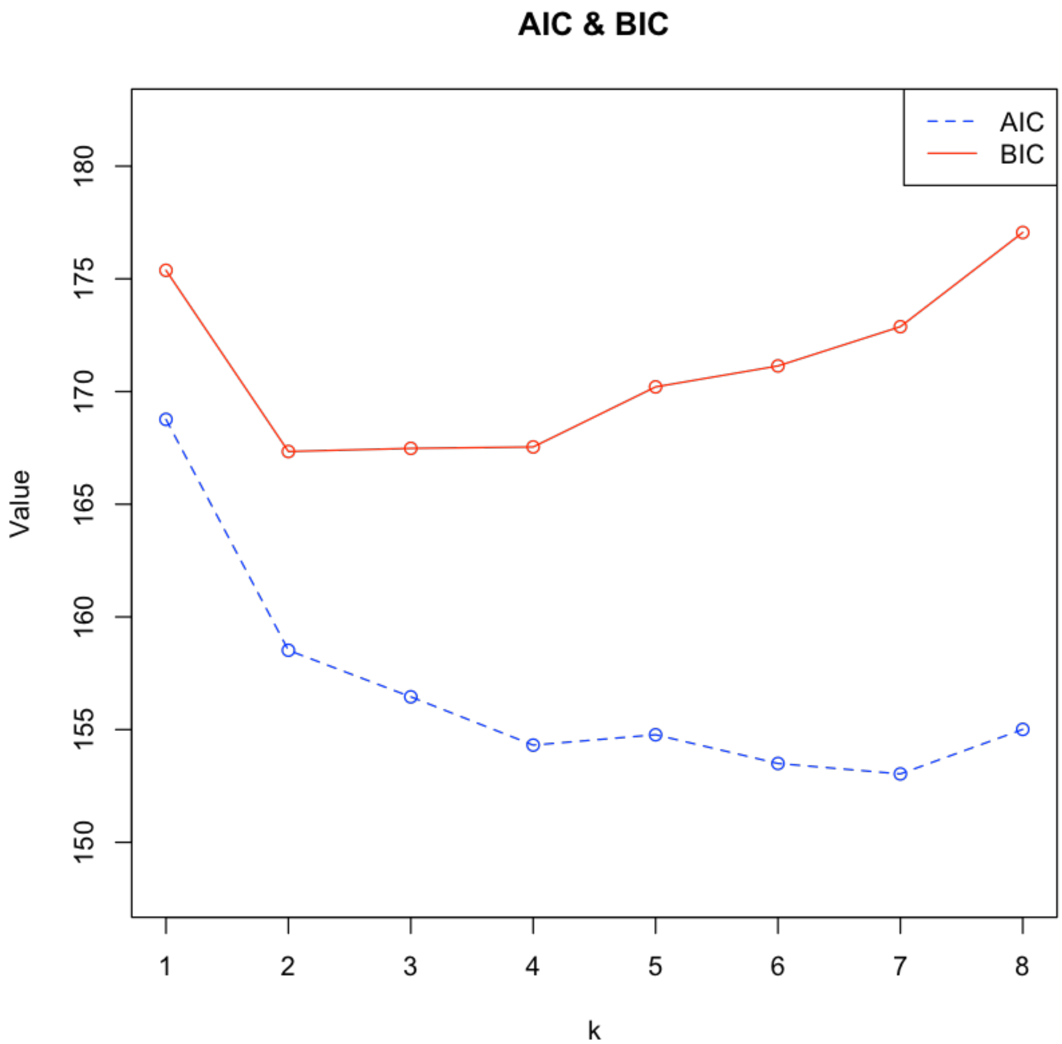
```
[1] "Subset selected by AIC = 7"
[1] "AIC values"
```

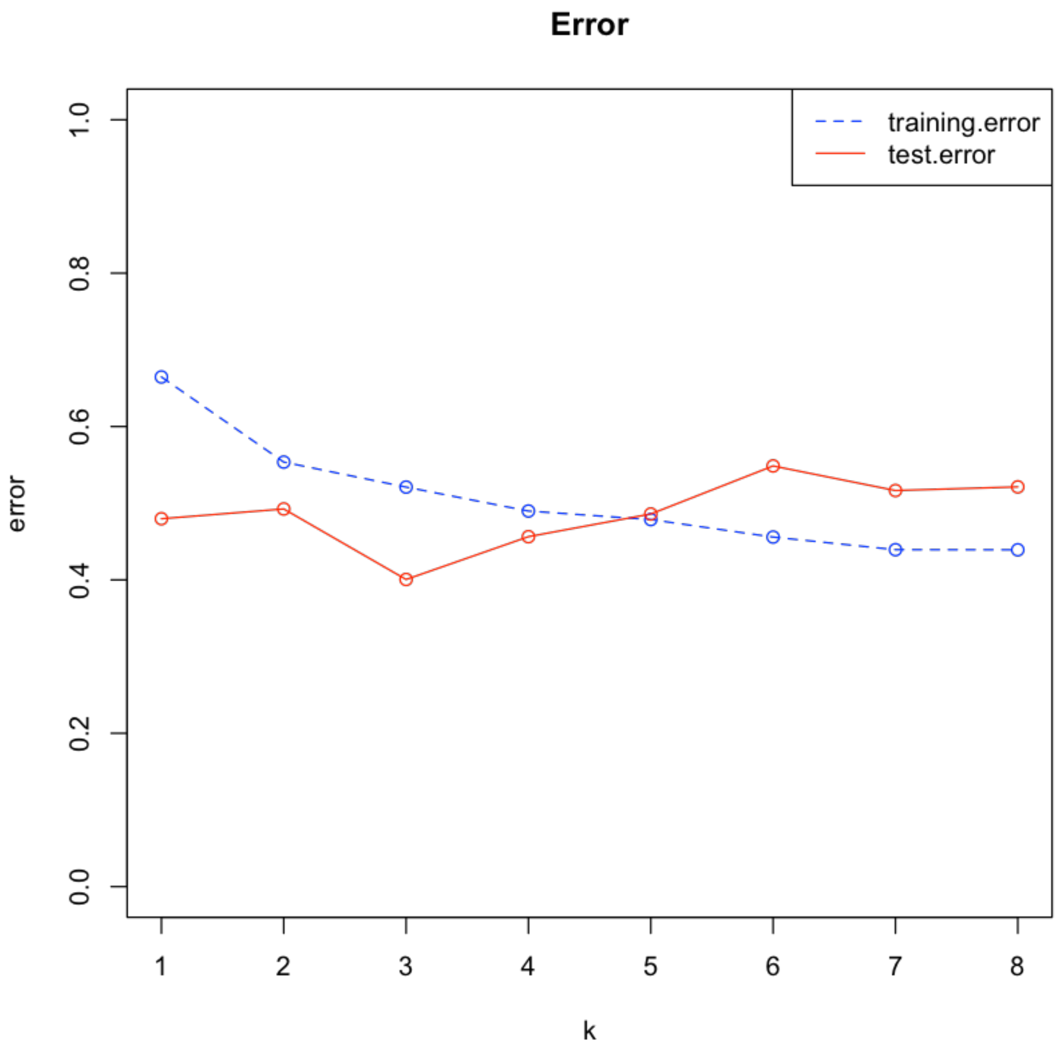
1. 168.764154371389 2. 158.520967079593 3. 156.454849884341 4. 154.31269099965 5. 154.772911236327 6. 153.498370284356 7. 153.03495137847 8. 155.010102017978

```
[1] "Subset selected by BIC = 2"
[1] "BIC values"
```

1. 175.378232229561 2. 167.339737557157 3. 167.478312981296 4. 167.540846715996 5. 170.205759572064 6. 171.135911239484 7. 172.877184952989 8. 177.057028211888

```
[1] "Test error of 2 variable model: 0.492482349035638"
```


AIC & BIC
AIC
BIC
Value
180
175
170
165
160
155
150
k
1
2
3
4
5
6
7
8



The model with the lowest AIC is the best among several. We discover a broader model with seven factors for the AIC (lcavol, lweight, age, lbph, svi, lcp ,pgg45). Only the predictor Gleason was eliminated during this selection process. Since the BIC penalizes the number of parameters more severely, it chooses smaller models. In this instance, we discover that the ideal model includes lweight and lcavol.

We can observe that this model's test error is 0.49.

1.2 Model selection criteria: k-cross validation

1.2.1 k=5

[128]:
```
predict.regsubsets = function(object,newdata,id,...){
      form = as.formula(object$call[[2]]) # Extract the formula used when we␣
 ↪called regsubsets()
      mat = model.matrix(form,newdata)    # Build the model matrix
      coefi = coef(object,id=id)          # Extract the coefficiants of the ith␣
 ↪model
      xvars = names(coefi)                # Pull out the names of the␣
 ↪predictors used in the ith model
      mat[,xvars]%*%coefi               # Make predictions using matrix␣
 ↪multiplication
}
```

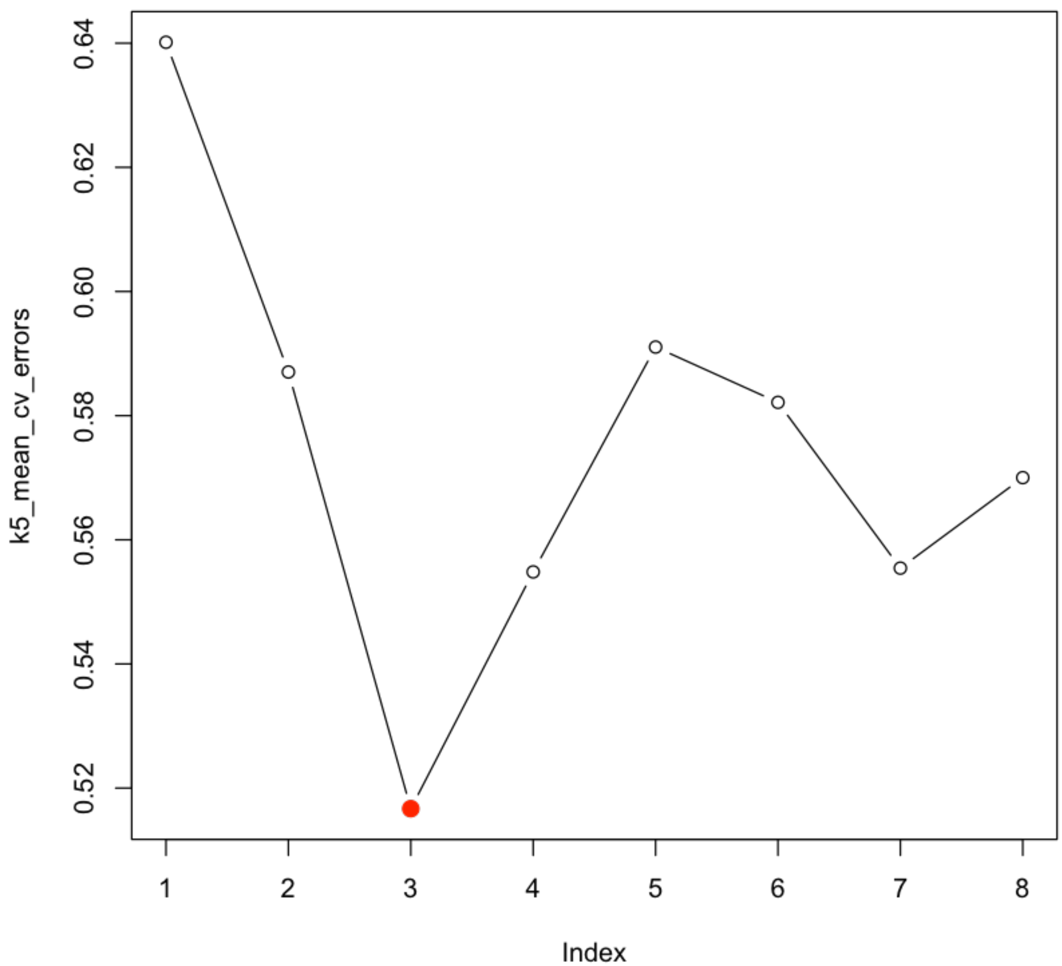
[129]:
```
k = 5        # number of folds
set.seed(1)   # set the random seed so we all get the same results

# Assign each observation to a single fold
folds = sample(1:k, nrow(prostate), replace = TRUE)

# Create a matrix to store the results of our upcoming calculations
cv_errors = matrix(NA, k, 8, dimnames = list(NULL, paste(1:8)))
```

[130]:
```
# Outer loop iterates over all folds
for(j in 1:k){

    # The perform best subset selection on the full dataset, minus the jth fold
    best_fit = regsubsets(lpsa~., data = prostate[folds!=j,], nvmax=8)

    # Inner loop iterates over each size i
    for(i in 1:8){

        # Predict the values of the current fold from the "best subset" model␣
 ↪on i predictors
        pred = predict(best_fit, prostate[folds==j,], id=i)

        # Calculate the MSE, store it in the matrix we created above
        cv_errors[j,i] = mean((prostate$lpsa[folds==j]-pred)^2)
    }
}
```

[149]:
```
# Take the mean of over all folds for each model size
k5_mean_cv_errors = apply(cv_errors, 2, mean)

# Find the model size with the smallest cross-validation error
min = which.min(k5_mean_cv_errors)
```

```
# Plot the cross-validation error for each model size, highlight the min
plot(k5_mean_cv_errors, type='b')
points(min, k5_mean_cv_errors[min][1], col = "red", cex = 2, pch = 20)

print(paste("MSE for k=5: ",min(mean_cv_errors)))
```

```
[1] "MSE for k=5:  0.51668234608548"
```



[150]:
```
reg_best = regsubsets(lpsa~., data = prostate, nvmax = 8)
coef(reg_best, 3)
```

(Intercept) -0.777156641580076 **lcavol** 0.52585188198094 **lweight** 0.661769911594472 **svi**

0.665666562857202

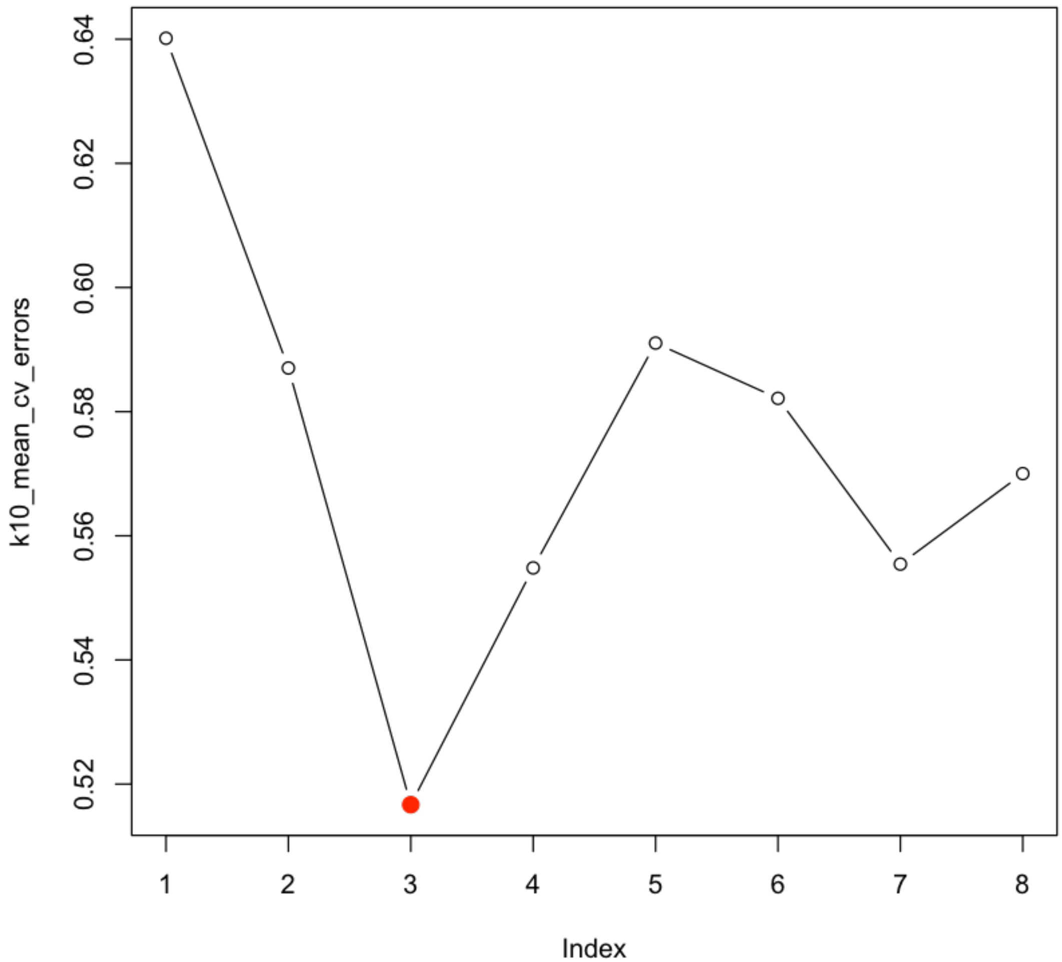
1.2.2 k=10

[151]:
```
k = 10          # number of folds
set.seed(567)   # set the random seed so we all get the same results

# Assign each observation to a single fold
folds = sample(1:k, nrow(prostate), replace = TRUE)

# Create a matrix to store the results of our upcoming calculations
cv_errors = matrix(NA, k, 8, dimnames = list(NULL, paste(1:8)))
```

[152]:
```
# Outer loop iterates over all folds
for(j in 1:k){

    # The perform best subset selection on the full dataset, minus the jth fold
    best_fit = regsubsets(lpsa~., data = prostate[folds!=j,], nvmax=8)

    # Inner loop iterates over each size i
    for(i in 1:8){

        # Predict the values of the current fold from the "best subset" model␣
 ↪on i predictors
        pred = predict(best_fit, prostate[folds==j,], id=i)

        # Calculate the MSE, store it in the matrix we created above
        cv_errors[j,i] = mean((prostate$lpsa[folds==j]-pred)^2)
    }
}
```

[153]:
```
# Take the mean of over all folds for each model size
k10_mean_cv_errors = apply(cv_errors, 2, mean)

# Find the model size with the smallest cross-validation error
min = which.min(k10_mean_cv_errors)

# Plot the cross-validation error for each model size, highlight the min
plot(k10_mean_cv_errors, type='b')
points(min, k10_mean_cv_errors[min][1], col = "red", cex = 2, pch = 20)


print(paste("MSE for k=10: ",min(mean_cv_errors)))
```

```
[1] "MSE for k=10:  0.51668234608548"
```



[154]:
```
reg_best = regsubsets(lpsa~., data = prostate, nvmax = 8)
coef(reg_best, 3)
```

(Intercept) -0.777156641580076 **lcavol** 0.52585188198094 **lweight** 0.661769911594472 **svi** 0.665666562857202

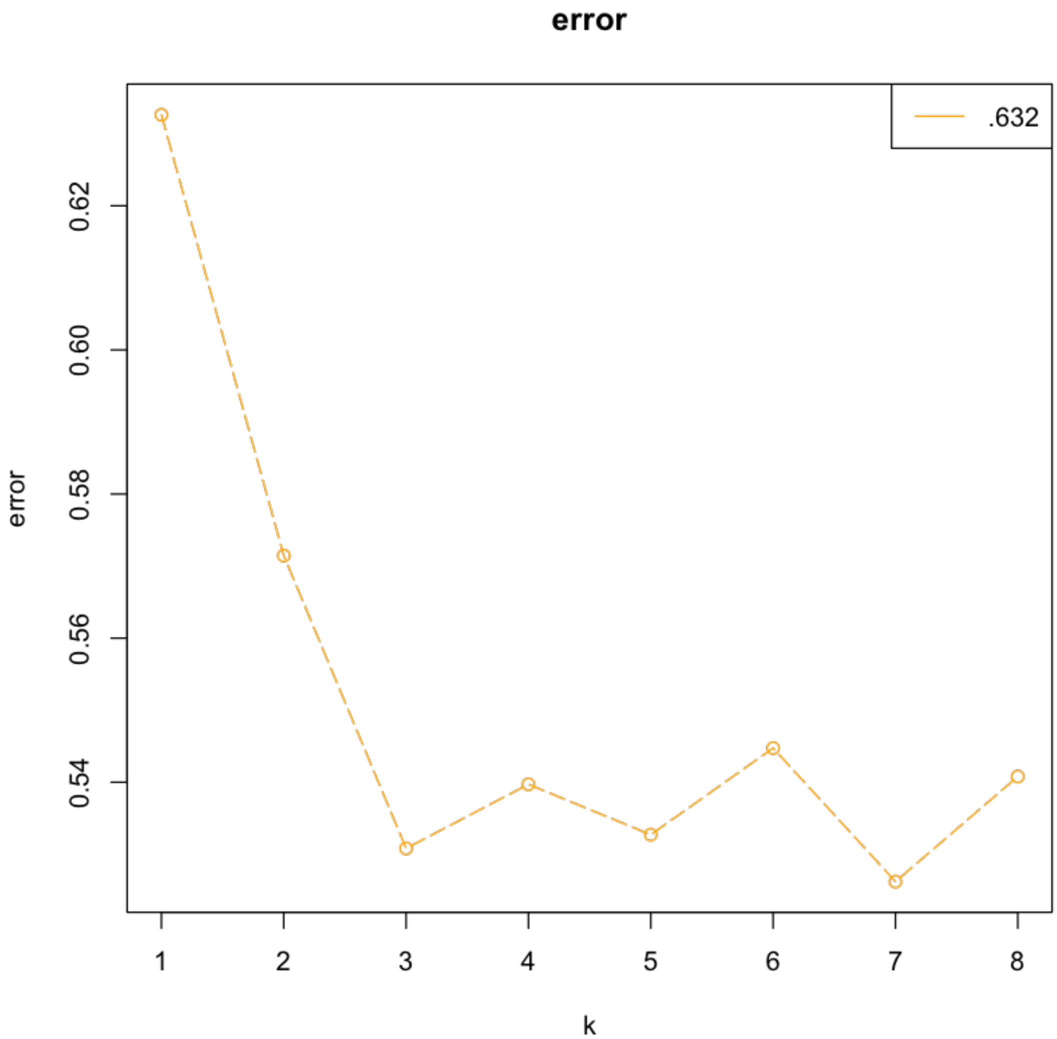
Above, we implemented best-subset cross-validation for linear regression. We call that procedure and then receive the set of predictors from the k=5, k=10 that have the smallest cross-validated estimate of the mean square error. Similar subset with three variables is obtained using 5-fold and 10-fold cross-validation (lcavol,lweight,svi).

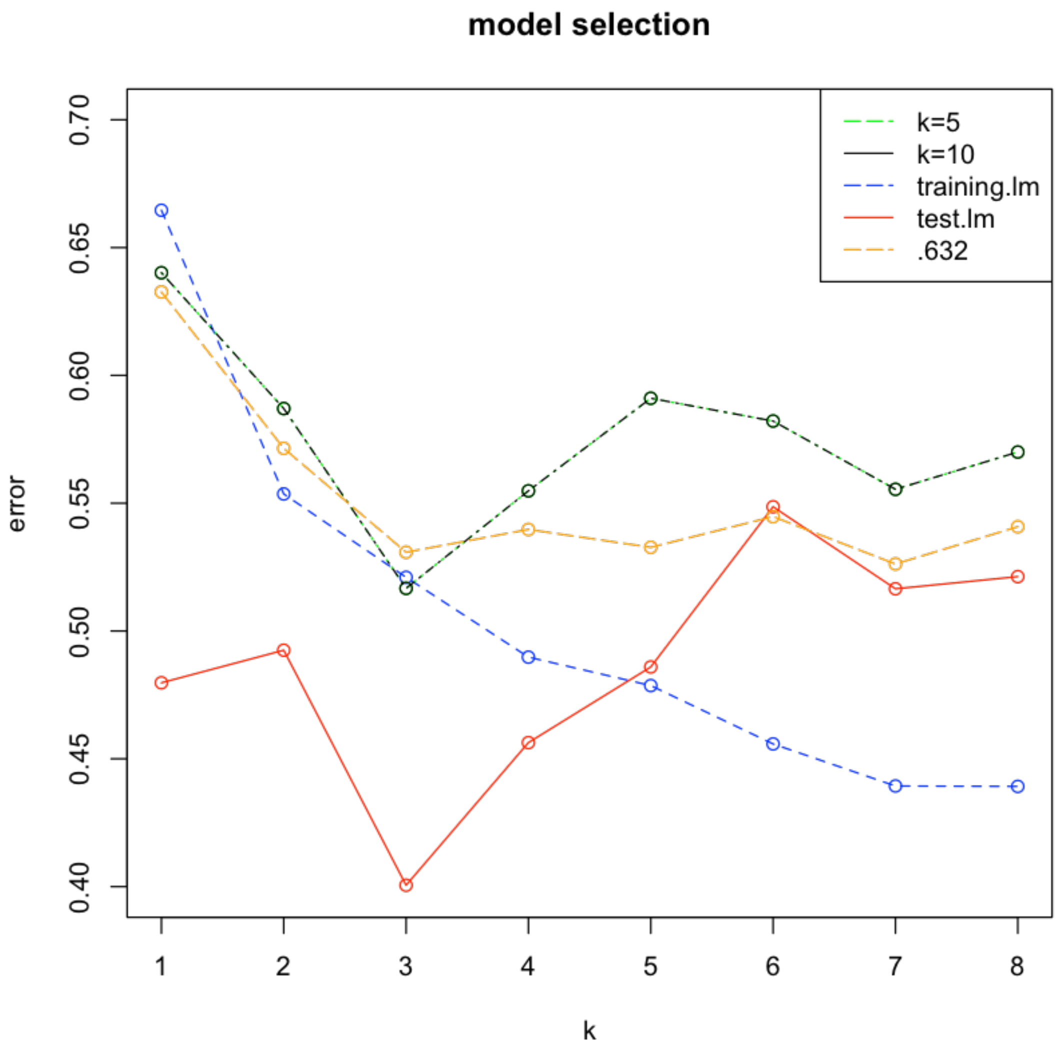
1.3 Model selection criteria: bootstrap.632

[156]:
```
set.seed(1)

x<-prostate[,1:8]
y<-prostate[,9]

theta_fit<-function(x,y){lsfit(x,y)}
theta_predict<-function(fit,x){cbind(1,x)%*%fit$coef}
sq_err<-function(y,yhat){(y-yhat)^2}

bootstrap_632_error_store<-c()
for(i in 1:8){
  temp<-which(select[i,]=="*")
  res<-bootpred(x[,temp],y,nboot = 50,theta_fit,theta_predict,err.meas=sq_err)
  bootstrap_632_error_store<-c(bootstrap_632_error_store,res[[3]])
}

bootstrap_632_error_store
print(paste("Bootstrp.632 error for ",which.
 ↪min(bootstrap_632_error_store),"variabl model␣
 ↪is",min(bootstrap_632_error_store)))

plot(bootstrap_632_error_store,type="o",lty=5,col="orange",main="error",xlab =␣
 ↪"k",ylab="error")
legend("topright",c(".632"),lty=1,col=c("orange"))

plot(train_error_store,type="o",lty=2,col = "blue",ylim = c(0.4,0.7),xlab =␣
 ↪"k",ylab="error",main="model selection")
lines(test_error_store,type="o",lty=1,col="red")
lines(k5_mean_cv_errors,type="o",lty=3,col = "green")
lines(k10_mean_cv_errors,type="o",lty=4,col="black")
lines(bootstrap.632.error.store,type="o",lty=5,col="orange")
legend("topright",c("k=5", "k=10","training.lm", "test.lm",".
 ↪632"),lty=c(5,1),col=c("green","black","blue","red","orange"))
```

1. 0.632630056057999 2. 0.57146399530187 3. 0.530823037892348 4. 0.53971847264767 5. 0.53273057426127 6. 0.544743393174272 7. 0.526214505661281 8. 0.540796707029031

```
[1] "Bootstrp.632 error for  7 variabl model is 0.526214505661281"
```


error
.632
error
0.54
0.56
0.58
0.60
0.62
1
2
3
4
5
6
7
8
k



The model with two variables is the best fit for this data set, according to the subset selection approach, which was based on BIC. The model with three variables appears to have the best accuracy, according to the MSE obtained by running Cross Validation (both k5 and k10) on the same data set. In contrast, the bootstrap.632 resampling approach estimates an error of 0.52 for a model with 7 variables that is similar to the AIC.

The 0.632 estimator uses bootstrap samples to calculate the prediction error. It gives equal weight to a bootstrapped estimate and the training error. The best subset size determined by different methods is 3, and the stated error lies between 0.50 and 0.51.The testing MSE of 0.49 appears to be a bit optimistic based on these figures.

2 Question 2

2.0.1 The Bikeshare data (ISLR2) contains the hourly and daily count of rental bikes between 2011-2012 in a bikeshare program. Other important features such as weather, and seasonal information, are also included. You are asked to construct a regression tree to predict the daily count of rental bikes.

2.0.2 *(a) Divide the data into test and training and perform model selection to determine the optimal tree size. Comment on the performance of the tree. How many test samples are assigned to each terminal region?*

[24]:
```
install.packages("rpart")
install.packages('MASS')
install.packages('ISLR2')
install.packages('rpart.plot')
library(rpart.plot)
library(ISLR2)
library(rpart)
library(MASS)
```

```
The downloaded binary packages are in
/var/folders/ch/lqq67w6x5px6fcc9cnbp457m0000gn/T//Rtmp2yueMP/downloaded_packages

The downloaded binary packages are in
/var/folders/ch/lqq67w6x5px6fcc9cnbp457m0000gn/T//Rtmp2yueMP/downloaded_packages

  There is a binary version available but the source version is later:
      binary source needs_compilation
ISLR2  1.3-1  1.3-2             FALSE


installing the source package 'ISLR2'



The downloaded binary packages are in
/var/folders/ch/lqq67w6x5px6fcc9cnbp457m0000gn/T//Rtmp2yueMP/downloaded_packages
```

[229]:
```
set.seed(2255)

bikeshare_df<-read.csv("/Users/sreeragvenugopalan/Desktop/Sem 1/Statistical␣
↪Data Mining/HW/4/Bike-Sharing-Dataset/day.csv")
head(bikeshare_df,5)
```

		instant	dteday	season	yr	mnth	holiday	weekday	workingday	wea
		<int>	<chr>	<int>	<int>	<int>	<int>	<int>	<int>	<in
	1	1	2011-01-01	1	0	1	0	6	0	2
A data.frame: 5 × 16	2	2	2011-01-02	1	0	1	0	0	0	2
	3	3	2011-01-03	1	0	1	0	1	1	1
	4	4	2011-01-04	1	0	1	0	2	1	1
	5	5	2011-01-05	1	0	1	0	3	1	1

2.0.3 PreProcess Data

We can see that column 'cnt' is the sum of 'casual' and 'registered', So we will remove those two columns.

[230]:
```
bikeshare_df<-subset(bikeshare_df,select=-c(casual,registered))
head(bikeshare_df,5)
```

		instant	dteday	season	yr	mnth	holiday	weekday	workingday	wea
		<int>	<chr>	<int>	<int>	<int>	<int>	<int>	<int>	<in
	1	1	2011-01-01	1	0	1	0	6	0	2
A data.frame: 5 × 14	2	2	2011-01-02	1	0	1	0	0	0	2
	3	3	2011-01-03	1	0	1	0	1	1	1
	4	4	2011-01-04	1	0	1	0	2	1	1
	5	5	2011-01-05	1	0	1	0	3	1	1

Renaming the column titles.

[231]:
```
names(bikeshare_df)<-c('ID','datetime','season','year','month','holiday','weekday','workingday
head(bikeshare_df,5)
```

		ID	datetime	season	year	month	holiday	weekday	workingday	wea
		<int>	<chr>	<int>	<int>	<int>	<int>	<int>	<int>	<in
	1	1	2011-01-01	1	0	1	0	6	0	2
A data.frame: 5 × 14	2	2	2011-01-02	1	0	1	0	0	0	2
	3	3	2011-01-03	1	0	1	0	1	1	1
	4	4	2011-01-04	1	0	1	0	2	1	1
	5	5	2011-01-05	1	0	1	0	3	1	1

Typcast some columns

[232]:
```
bikeshare_df$datetime<- as.Date(bikeshare_df$datetime)
bikeshare_df$year<-as.factor(bikeshare_df$year)
bikeshare_df$month<-as.factor(bikeshare_df$month)
bikeshare_df$season <- as.factor(bikeshare_df$season)
bikeshare_df$holiday<- as.factor(bikeshare_df$holiday)
bikeshare_df$weekday<- as.factor(bikeshare_df$weekday)
bikeshare_df$workingday<- as.factor(bikeshare_df$workingday)
bikeshare_df$weather_condition<- as.factor(bikeshare_df$weather_condition)
head(bikeshare_df,5)
```

A data.frame: 5 × 14

	ID <int>	datetime <date>	season <fct>	year <fct>	month <fct>	holiday <fct>	weekday <fct>	workingday <fct>	wea <fct
1	1	2011-01-01	1	0	1	0	6	0	2
2	2	2011-01-02	1	0	1	0	0	0	2
3	3	2011-01-03	1	0	1	0	1	1	1
4	4	2011-01-04	1	0	1	0	2	1	1
5	5	2011-01-05	1	0	1	0	3	1	1

check for NA values

[233]:
```
any(is.na(bikeshare_df))
```

FALSE

Split the dataset into train and test

[234]:
```
train_index<-sample(1:nrow(bikeshare_df),0.7*nrow(bikeshare_df))
train_data<-bikeshare_df[train_index,]
test_data<-bikeshare_df[-train_index,]
head(train_data,5)
head(test_data,5)
```

A data.frame: 5 × 14

	ID <int>	datetime <date>	season <fct>	year <fct>	month <fct>	holiday <fct>	weekday <fct>	workingday <fct>	w <
29	29	2011-01-29	1	0	1	0	6	0	1
557	557	2012-07-10	3	1	7	0	2	1	2
689	689	2012-11-19	4	1	11	0	1	1	2
117	117	2011-04-27	2	0	4	0	3	1	2
541	541	2012-06-24	3	1	6	0	0	0	1

A data.frame: 5 × 14

	ID <int>	datetime <date>	season <fct>	year <fct>	month <fct>	holiday <fct>	weekday <fct>	workingday <fct>	we <f
2	2	2011-01-02	1	0	1	0	0	0	2
7	7	2011-01-07	1	0	1	0	5	1	2
13	13	2011-01-13	1	0	1	0	4	1	1
20	20	2011-01-20	1	0	1	0	4	1	2
23	23	2011-01-23	1	0	1	0	0	0	1

[235]:
```
train<-subset(train_data,select=c('season','year','month','holiday',␣
↪'weekday','workingday','weather_condition','temp','humidity','windspeed','total_count'))
test<-subset(test_data,select=c('season','year','month','holiday','weekday','workingday','weat
```

One-hot encoding the categorical values usigng dummyVars

[236]:
```
dumy_cat_vals <- dummyVars('~␣
↪season+holiday+workingday+weather_condition+year', data = train)
dumy_cat_vals_df <- data.frame(predict(dumy_cat_vals, newdata = train))
```

create a new subset for train and test; categorical, numerical attributes

[237]:
```
train_cat_attributes<-subset(train,select=c('season','holiday','workingday','weather_condition
test_cat_attributes<-subset(test,select=c('season','holiday','workingday','weather_condition',
train_num_attributes<-subset(train,select=c('weekday','month','temp','humidity','windspeed','t
test_num_attributes<-subset(test,select=c('weekday','month','temp',␣
↪'humidity','windspeed','total_count'))
```

[238]:
```
train_encoded_attributess<-cbind(train_num_attributes,dumy_cat_vals_df)
head(train_encoded_attributess,5)
```

A data.frame: 5 × 19

	weekday <fct>	month <fct>	temp <dbl>	humidity <dbl>	windspeed <dbl>	total_count <int>	season.1 <dbl>	seas… <db…
29	6	1	0.196522	0.651739	0.145365	1098	1	0
557	2	7	0.720833	0.667500	0.151737	6290	0	0
689	1	11	0.380833	0.623333	0.235067	5499	0	0
117	3	4	0.620000	0.835417	0.312200	3872	0	1
541	0	6	0.743333	0.479167	0.145525	6891	0	0

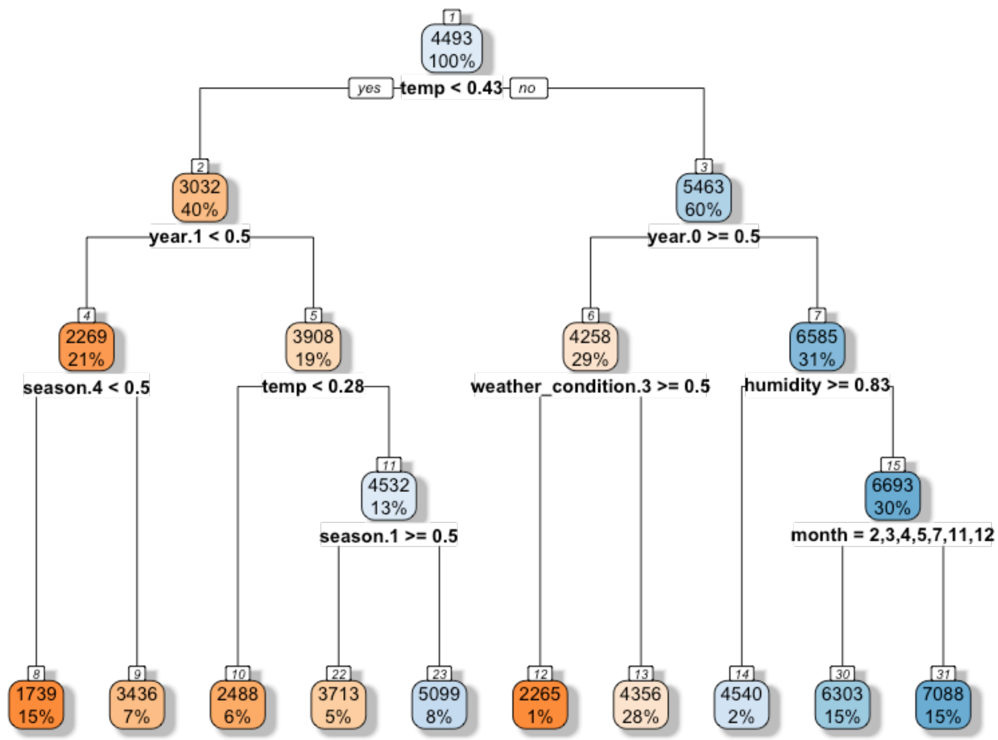
Building and traing the model

[239]:
```
rpart.control<-rpart.control(minbucket = 2,minsplit = 4, xval = 10, cp = 0.01)
original_fit_bshare<-rpart(train_encoded_attributess$total_count~.
↪,data=train_encoded_attributess[,-c(6)],control=rpart.
↪control,method='anova',cp=0.01)
original_fit_bshare
```

```
n= 511

node), split, n, deviance, yval
      * denotes terminal node

 1) root 511 1882115000.0 4492.544
   2) temp< 0.432373 204  449964900.0 3032.240
     4) year.1< 0.5 109  114334800.0 2268.734
       8) season.4< 0.5 75   29791150.0 1739.467 *
       9) season.4>=0.5 34   17190340.0 3436.235 *
     5) year.1>=0.5 95  199185000.0 3908.263
      10) temp< 0.2804165 29   14609140.0 2488.448 *
      11) temp>=0.2804165 66  100428300.0 4532.121
        22) season.1>=0.5 27   20318370.0 3713.000 *
        23) season.1< 0.5 39   49452270.0 5099.205 *
   3) temp>=0.432373 307  708048800.0 5462.909
     6) year.0>=0.5 148  109593600.0 4257.520
      12) weather_condition.3>=0.5 7     858316.9 2264.857 *
      13) weather_condition.3< 0.5 141   79560490.0 4356.447 *
     7) year.0< 0.5 159  183255500.0 6584.906
      14) humidity>=0.8322915 8    2719668.0 4539.750 *
      15) humidity< 0.8322915 151  145301700.0 6693.258
        30) month=2,3,4,5,7,11,12 76   65508410.0 6303.447 *
```

```
31) month=6,8,9,10 75   56542530.0 7088.267 *
```

[240]:
```
rpart.plot(original_fit_bshare, box.palette="OrBu", shadow.col="gray",
  nn=TRUE,roundint=FALSE)
```



[275]:
```
min_cp = which.min(original_fit_bshare$cptable[,4])
pruned_fit_bshare <- prune(original_fit_bshare, cp =
  original_fit_bshare$cptable[min_cp, 1])
pruned_fit_bshare
```

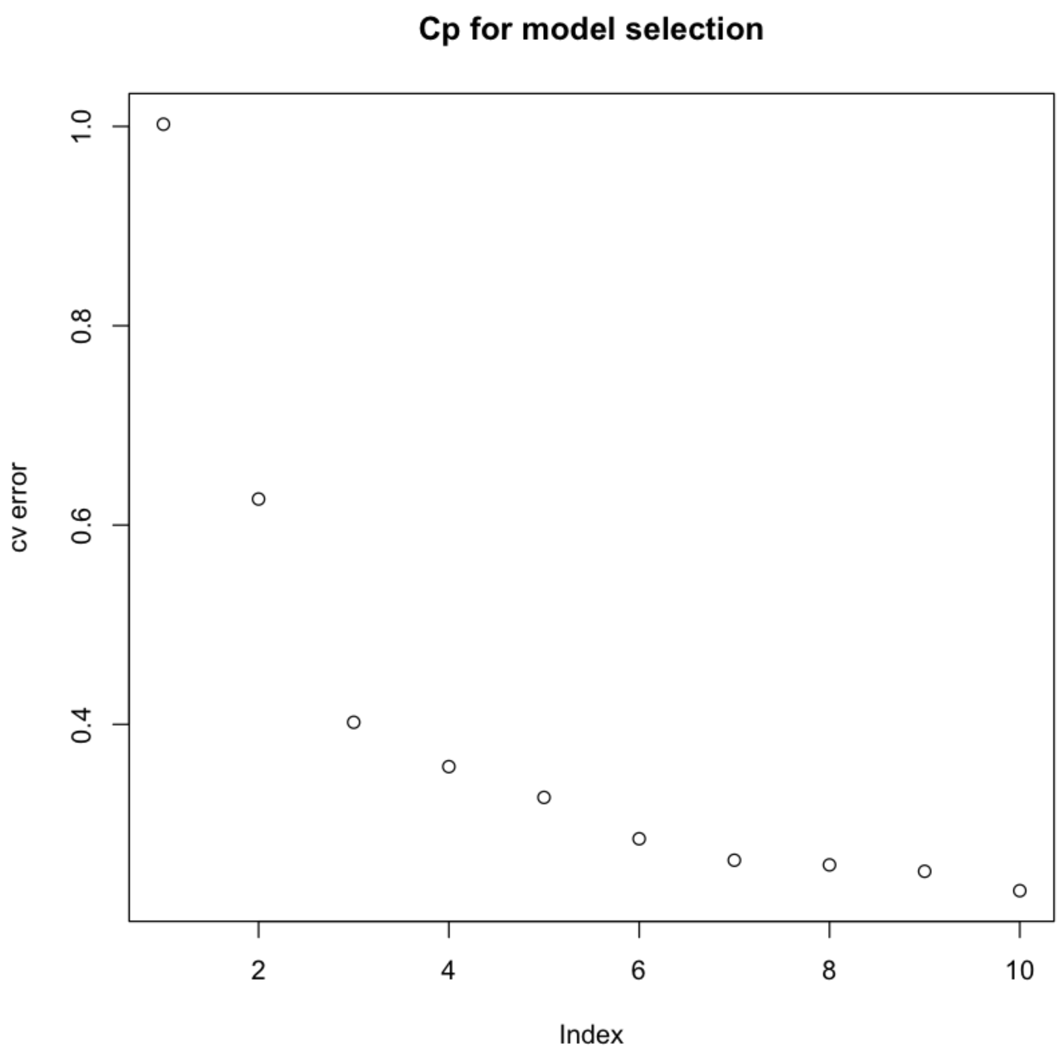
```
n= 511

node), split, n, deviance, yval
```

```
      * denotes terminal node

 1) root 511 1882115000.0 4492.544
   2) temp< 0.432373 204  449964900.0 3032.240
     4) year.1< 0.5 109  114334800.0 2268.734
       8) season.4< 0.5 75   29791150.0 1739.467 *
       9) season.4>=0.5 34   17190340.0 3436.235 *
     5) year.1>=0.5 95  199185000.0 3908.263
      10) temp< 0.2804165 29   14609140.0 2488.448 *
      11) temp>=0.2804165 66  100428300.0 4532.121
        22) season.1>=0.5 27   20318370.0 3713.000 *
        23) season.1< 0.5 39   49452270.0 5099.205 *
   3) temp>=0.432373 307  708048800.0 5462.909
     6) year.0>=0.5 148  109593600.0 4257.520
      12) weather_condition.3>=0.5 7     858316.9 2264.857 *
      13) weather_condition.3< 0.5 141   79560490.0 4356.447 *
     7) year.0< 0.5 159  183255500.0 6584.906
      14) humidity>=0.8322915 8    2719668.0 4539.750 *
      15) humidity< 0.8322915 151  145301700.0 6693.258
        30) month=2,3,4,5,7,11,12 76   65508410.0 6303.447 *
        31) month=6,8,9,10 75   56542530.0 7088.267 *
```

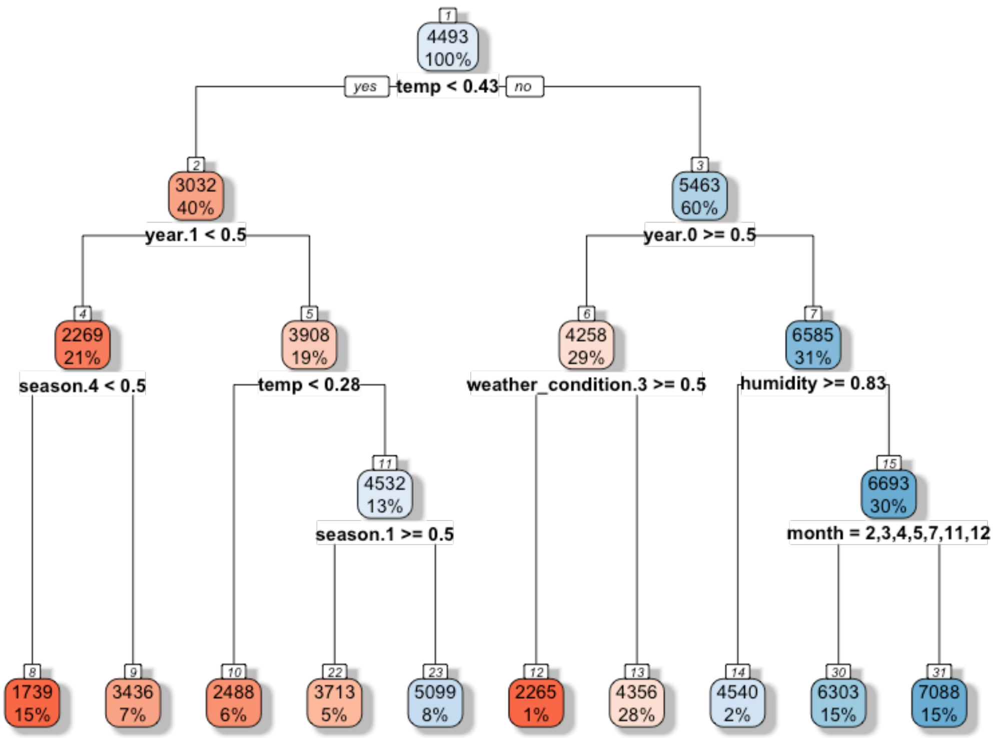
[276]:

```
plot(original_fit_bshare$cptable[,4], main = "Cp for model selection", ylab = "cv error")

print(paste('Optimal tree size is:',min_cp))
```

```
[1] "Optimal tree size is: 10"
```

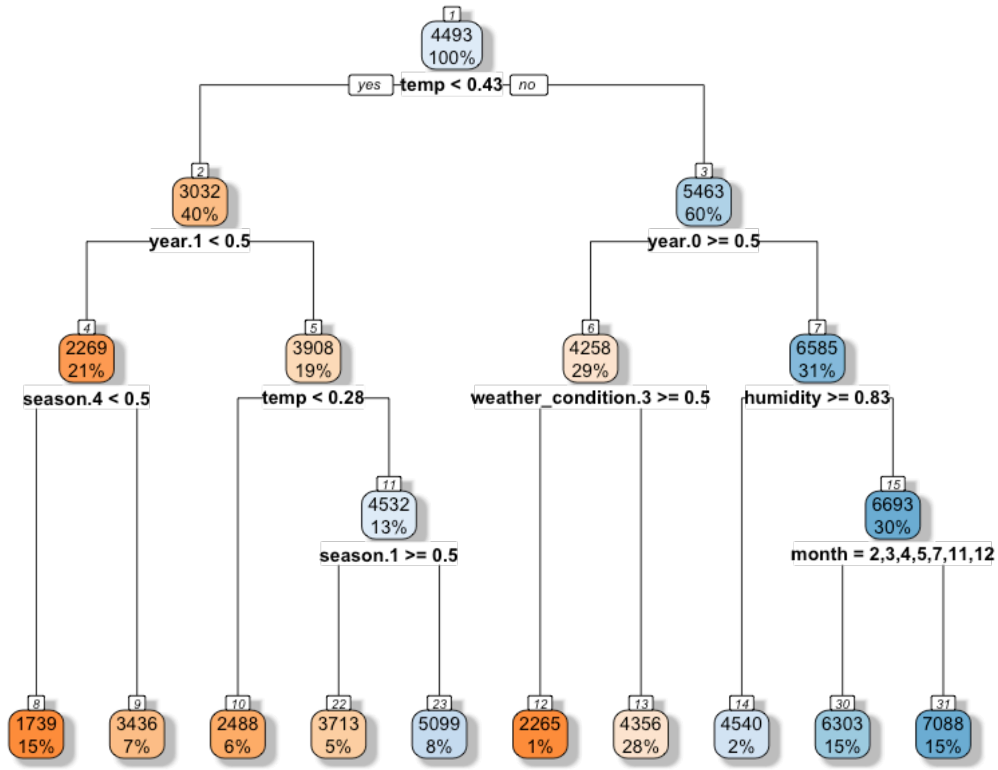


[277]:
```
rpart.plot(pruned_fit_bshare, box.palette="RdBu", shadow.col="gray",
    nn=TRUE,roundint=FALSE)
```

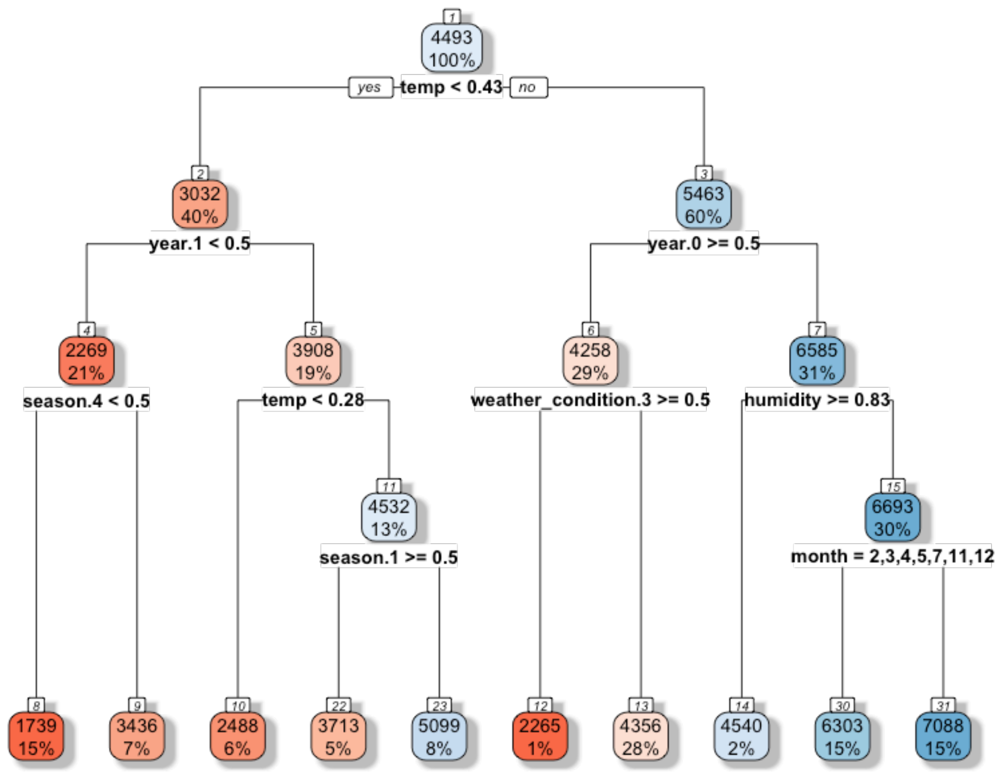


[244]:
```
rpart.plot(original_fit_bshare, box.palette="OrBu", shadow.col="gray",
 ↪nn=TRUE,roundint=FALSE, main='Full Tree')


rpart.plot(pruned_fit_bshare, box.palette="RdBu", shadow.col="gray",
 ↪nn=TRUE,roundint=FALSE,main='Pruned Tree')
```

Full Tree


1
4493
100%
yes
temp < 0.43
no
2
3032
40%
year.1 < 0.5
3
5463
60%
year.0 >= 0.5
4
2269
21%
season.4 < 0.5
5
3908
19%
temp < 0.28
6
4258
29%
weather_condition.3 >= 0.5
7
6585
31%
humidity >= 0.83
11
4532
13%
season.1 >= 0.5
15
6693
30%
month = 2,3,4,5,7,11,12
8
1739
15%
9
3436
7%
10
2488
6%
22
3713
5%
23
5099
8%
12
2265
1%
13
4356
28%
14
4540
2%
30
6303
15%
31
7088
15%

Pruned Tree



As you can see, pruning had no impact on the original tree. I believe this is because the tree was already in extremely good condition to begin with. Pruning is the process of removing leaves that do not considerably improve accuracy, i.e., to avoid overfitting. Since we deleted a few columns that seemed insignificant during the feature selection and preprocessing, the data is optimally balanced.

We can see that almost all termincal node have less than 10% of distribution of test samples. Only one node with decision 'no' has the distribtion of 28%.

2.0.4 *(b) The board wants to identify good times to do "repair" and "tune-ups". However, they want to minimize disruption of usage. You are asked to describe the "low yield" settings when the bikes are not being utilized (counts are low). Use your tree to advise on this matter.*

From the above pruned tree it is clear that the bikers count are low at the left most node of the tree. A warm summer day with minimal humidity would be ideal for the highest bike rental rates. We can infer that the quantity of bicycle rentals every day depends on a variety of parameters, including seasonal and weather-related ones. On the other hand, the fall (seaosn 4) season, which runs from September through December, has the lowest bike rental rates, indicating that the two most significant factors influencing demand for bike sharing rentals are the season and the temperature. So the low yeilds can be maximized durinf these seasons.

3 Question 3

3.0.1 The Wage data (ISLR2) contains information related to demographics and earnings of males in the Mid-Atlantic area. Discretize "Wage" into three categories (low, med, high). You are asked to construct a classification tree to predict "Wage".

3.0.2 *a) Divide the data into test and training and perform model selection to determine the optimal tree size. Comment on the performance of the tree. How many test samples are assigned to each terminal region?*

loadding libraries

[180]:
```
install.packages('ISLR')
install.packages('ggplot2')
install.packages('caret')
install.packages('tidyverse')
library(tidyverse)
library(ISLR)
library(ggplot2)
library(caret)
```

```
The downloaded binary packages are in
/var/folders/ch/lqq67w6x5px6fcc9cnbp457m0000gn/T//Rtmp2yueMP/downloaded_packages

The downloaded binary packages are in
/var/folders/ch/lqq67w6x5px6fcc9cnbp457m0000gn/T//Rtmp2yueMP/downloaded_packages

The downloaded binary packages are in
/var/folders/ch/lqq67w6x5px6fcc9cnbp457m0000gn/T//Rtmp2yueMP/downloaded_packages

The downloaded binary packages are in
/var/folders/ch/lqq67w6x5px6fcc9cnbp457m0000gn/T//Rtmp2yueMP/downloaded_packages
```

```
  Attaching packages                                tidyverse
1.3.2
```

```
tibble  3.1.8      dplyr   1.0.10
tidyr   1.2.1      stringr 1.4.1
readr   2.1.3      forcats 0.5.2
purrr   0.3.4
 Conflicts
tidyverse_conflicts()
 dplyr::filter() masks stats::filter()
 dplyr::lag()    masks stats::lag()
 purrr::lift()   masks caret::lift()
 dplyr::select() masks MASS::select()
```

[888]:
```
data(Wage)
head(Wage,5)
```

A data.frame: 5 × 11

	year <int>	age <int>	maritl <fct>	race <fct>	education <fct>	region <fct>
231655	2006	18	1. Never Married	1. White	1. < HS Grad	2. Middle Atl
86582	2004	24	1. Never Married	1. White	4. College Grad	2. Middle Atl
161300	2003	45	2. Married	1. White	3. Some College	2. Middle Atl
155159	2003	43	2. Married	3. Asian	4. College Grad	2. Middle Atl
11443	2005	50	4. Divorced	1. White	2. HS Grad	2. Middle Atl

Now that we have seen that the variable “region” only has the value “Middle Atlantic,” we will remove it as it is unable to distinguish between any observations. Additionally, the predictor “logwage” will be removed because it has no significance with the dependent variable. We shall also eliminate that variable as we will discretize the “wage.” We can combine “3. Widowed”, “4. Divorced”, and “5. Separated” into a more populous level known as “3. Other” to increase the robustness of the models to be built because they all have somewhat similar meanings.

[889]:
```
Wage = Wage %>%
  mutate(wage_category = case_when(wage < quantile(wage, 0.25) ~ "Low",
                                   wage >= quantile(wage, 0.25) & wage <=
 ↪quantile(wage, 0.75) ~ "Medium",
                                   wage > quantile(wage, 0.75) ~ "High"))
head(Wage,5)
```

A data.frame: 5 × 12

	year <int>	age <int>	maritl <fct>	race <fct>	education <fct>	region <fct>
231655	2006	18	1. Never Married	1. White	1. < HS Grad	2. Middle Atl
86582	2004	24	1. Never Married	1. White	4. College Grad	2. Middle Atl
161300	2003	45	2. Married	1. White	3. Some College	2. Middle Atl
155159	2003	43	2. Married	3. Asian	4. College Grad	2. Middle Atl
11443	2005	50	4. Divorced	1. White	2. HS Grad	2. Middle Atl

[890]:
```
Wage$region <- NULL
Wage$wage <- NULL
Wage$logwage <- NULL
levels(Wage$maritl)[3:5] <- "3. Other"
```

Checking for missing values

[891]:
```
any(is.na(Wage))
```

FALSE

Split the dataset into train and test

[892]:
```
set.seed(2021)
partition <- createDataPartition(y = as.factor(Wage$wage_category),p = .7,list␣
↪= FALSE)
data.train <- Wage[partition, ]
data.test <- Wage[-partition, ]
head(data.train,5)
```

A data.frame: 5 × 9

	year <int>	age <int>	maritl <fct>	race <fct>	education <fct>	jobclass <fct>
86582	2004	24	1. Never Married	1. White	4. College Grad	2. Information
161300	2003	45	2. Married	1. White	3. Some College	1. Industrial
11443	2005	50	3. Other	1. White	2. HS Grad	2. Information
376662	2008	54	2. Married	1. White	4. College Grad	2. Information
450601	2009	44	2. Married	4. Other	3. Some College	1. Industrial

[893]:
```
data.train$wage_category <- factor(data.train$wage_category)
```

[894]:
```
head(data.train,5)
```

A data.frame: 5 × 9

	year <int>	age <int>	maritl <fct>	race <fct>	education <fct>	jobclass <fct>
86582	2004	24	1. Never Married	1. White	4. College Grad	2. Information
161300	2003	45	2. Married	1. White	3. Some College	1. Industrial
11443	2005	50	3. Other	1. White	2. HS Grad	2. Information
376662	2008	54	2. Married	1. White	4. College Grad	2. Information
450601	2009	44	2. Married	4. Other	3. Some College	1. Industrial

[895]:
```
original_fit_wage =  rpart(wage_category ~. , data = data.train, method =␣
↪"class", control = rpart.control(minbucket=5, cp=0.0005, maxdepth=7), parms␣
↪= list(split = "gini"))

original_fit_wage
```

```
n= 2102

node), split, n, loss, yval, (yprob)
      * denotes terminal node

  1) root 2102 1038 Medium (0.24881066 0.24500476 0.50618459)
    2) education=4. College Grad,5. Advanced Degree 795  427 High (0.46289308 0.
↪10440252 0.43270440)
```

```
    4) age>=33.5 632  300 High (0.52531646 0.09177215 0.38291139)
      8) education=5. Advanced Degree 259   88 High (0.66023166 0.06177606 0.
↪27799228)
       16) health_ins=1. Yes 209   61 High (0.70813397 0.02870813 0.26315789)
         32) maritl=2. Married,3. Other 185   46 High (0.75135135 0.02702703 0.
↪22162162)
           64) health=2. >=Very Good 149   30 High (0.79865772 0.02013423 0.
↪18120805) *
           65) health=1. <=Good 36   16 High (0.55555556 0.05555556 0.38888889)
            130) year>=2006.5 15    4 High (0.73333333 0.06666667 0.20000000) *
            131) year< 2006.5 21   10 Medium (0.42857143 0.04761905 0.
↪52380952) *
         33) maritl=1. Never Married 24   10 Medium (0.37500000 0.04166667 0.
↪58333333)
           66) jobclass=2. Information 19    9 Medium (0.47368421 0.00000000 0.
↪52631579)
            132) age< 49 13    5 High (0.61538462 0.00000000 0.38461538) *
            133) age>=49 6    1 Medium (0.16666667 0.00000000 0.83333333) *
           67) jobclass=1. Industrial 5    1 Medium (0.00000000 0.20000000 0.
↪80000000) *
       17) health_ins=2. No 50   27 High (0.46000000 0.20000000 0.34000000)
         34) age>=37.5 44   23 High (0.47727273 0.22727273 0.29545455)
           68) age< 62 38   19 High (0.50000000 0.26315789 0.23684211)
            136) race=2. Black,3. Asian 6    2 Low (0.33333333 0.66666667 0.
↪00000000) *
            137) race=1. White 32   15 High (0.53125000 0.18750000 0.28125000)␣
↪*
           69) age>=62 6    2 Medium (0.33333333 0.00000000 0.66666667) *
         35) age< 37.5 6    2 Medium (0.33333333 0.00000000 0.66666667) *
      9) education=4. College Grad 373  203 Medium (0.43163539 0.11260054 0.
↪45576408)
       18) health_ins=1. Yes 295  153 High (0.48135593 0.07118644 0.44745763)
         36) age< 64.5 289  147 High (0.49134948 0.06920415 0.43944637)
           72) maritl=2. Married 227  108 High (0.52422907 0.06167401 0.
↪41409692)
            144) year< 2008.5 194   88 High (0.54639175 0.06185567 0.39175258)␣
↪*
            145) year>=2008.5 33   15 Medium (0.39393939 0.06060606 0.
↪54545455) *
           73) maritl=1. Never Married,3. Other 62   29 Medium (0.37096774 0.
↪09677419 0.53225806) *
         37) age>=64.5 6    1 Medium (0.00000000 0.16666667 0.83333333) *
       19) health_ins=2. No 78   40 Medium (0.24358974 0.26923077 0.48717949)
         38) year< 2005.5 38   22 Low (0.23684211 0.42105263 0.34210526)
           76) age< 39.5 9    4 High (0.55555556 0.33333333 0.11111111) *
           77) age>=39.5 29   16 Low (0.13793103 0.44827586 0.41379310)
```

```
            154) health=1. <=Good 11    4 Low (0.09090909 0.63636364 0.
↪27272727) *
            155) health=2. >=Very Good 18    9 Medium (0.16666667 0.33333333 0.
↪50000000) *
         39) year>=2005.5 40   15 Medium (0.25000000 0.12500000 0.62500000)
           78) age>=55.5 7    4 High (0.42857143 0.28571429 0.28571429) *
           79) age< 55.5 33   10 Medium (0.21212121 0.09090909 0.69696970) *
    5) age< 33.5 163   61 Medium (0.22085890 0.15337423 0.62576687)
     10) race=3. Asian 23   11 High (0.52173913 0.08695652 0.39130435) *
     11) race=1. White,2. Black,4. Other 140   47 Medium (0.17142857 0.
↪16428571 0.66428571)
       22) age< 24.5 15    7 Low (0.06666667 0.53333333 0.40000000)
         44) year< 2004.5 7    2 Low (0.00000000 0.71428571 0.28571429) *
         45) year>=2004.5 8    4 Medium (0.12500000 0.37500000 0.50000000) *
       23) age>=24.5 125   38 Medium (0.18400000 0.12000000 0.69600000)
         46) year>=2004.5 87   31 Medium (0.25287356 0.10344828 0.64367816)
           92) age>=32.5 15    8 High (0.46666667 0.06666667 0.46666667)
            184) maritl=2. Married 8    3 High (0.62500000 0.00000000 0.
↪37500000) *
            185) maritl=1. Never Married 7    3 Medium (0.28571429 0.14285714␣
↪0.57142857) *
           93) age< 32.5 72   23 Medium (0.20833333 0.11111111 0.68055556) *
         47) year< 2004.5 38    7 Medium (0.02631579 0.15789474 0.81578947) *
  3) education=1. < HS Grad,2. HS Grad,3. Some College 1307  587 Medium (0.
↪11859220 0.33052793 0.55087988)
    6) age< 25.5 134   26 Low (0.01492537 0.80597015 0.17910448)
     12) maritl=1. Never Married,3. Other 115   14 Low (0.00000000 0.87826087␣
↪0.12173913) *
     13) maritl=2. Married 19    9 Medium (0.10526316 0.36842105 0.52631579)
       26) education=1. < HS Grad,2. HS Grad 13    7 Low (0.15384615 0.
↪46153846 0.38461538) *
       27) education=3. Some College 6    1 Medium (0.00000000 0.16666667 0.
↪83333333) *
    7) age>=25.5 1173  477 Medium (0.13043478 0.27621483 0.59335038)
     14) health_ins=2. No 400  211 Medium (0.06000000 0.46750000 0.47250000)
       28) education=1. < HS Grad 81   23 Low (0.00000000 0.71604938 0.
↪28395062)
         56) year< 2005.5 36    6 Low (0.00000000 0.83333333 0.16666667)
          112) race=1. White,2. Black 31    3 Low (0.00000000 0.90322581 0.
↪09677419) *
          113) race=3. Asian,4. Other 5    2 Medium (0.00000000 0.40000000 0.
↪60000000) *
         57) year>=2005.5 45   17 Low (0.00000000 0.62222222 0.37777778)
          114) age< 35 11    1 Low (0.00000000 0.90909091 0.09090909) *
          115) age>=35 34   16 Low (0.00000000 0.52941176 0.47058824)
            230) race=2. Black,3. Asian,4. Other 12    2 Low (0.00000000 0.
↪83333333 0.16666667) *
```

```
          231) race=1. White 22    8 Medium (0.00000000 0.36363636 0.63636364) *
       29) education=2. HS Grad,3. Some College 319  153 Medium (0.07523511 0.40438871 0.52037618)
         58) race=2. Black,3. Asian,4. Other 63   28 Low (0.07936508 0.55555556 0.36507937)
          116) year>=2003.5 51   20 Low (0.05882353 0.60784314 0.33333333) *
          117) year< 2003.5 12    6 Medium (0.16666667 0.33333333 0.50000000) *
         59) race=1. White 256  113 Medium (0.07421875 0.36718750 0.55859375)
          118) health=1. <=Good 93   48 Medium (0.04301075 0.47311828 0.48387097)
            236) year< 2003.5 9    1 Low (0.11111111 0.88888889 0.00000000) *
            237) year>=2003.5 84   39 Medium (0.03571429 0.42857143 0.53571429) *
          119) health=2. >=Very Good 163   65 Medium (0.09202454 0.30674847 0.60122699) *
     15) health_ins=1. Yes 773  266 Medium (0.16688228 0.17723157 0.65588616)
       30) education=3. Some College 288   92 Medium (0.24305556 0.07638889 0.68055556)
         60) maritl=2. Married 205   72 Medium (0.29756098 0.05365854 0.64878049) *
         61) maritl=1. Never Married,3. Other 83   20 Medium (0.10843373 0.13253012 0.75903614)
          122) year< 2004.5 24   11 Medium (0.16666667 0.29166667 0.54166667)
            244) race=2. Black,4. Other 6    3 Low (0.33333333 0.50000000 0.16666667) *
            245) race=1. White,3. Asian 18    6 Medium (0.11111111 0.22222222 0.66666667) *
          123) year>=2004.5 59    9 Medium (0.08474576 0.06779661 0.84745763) *
       31) education=1. < HS Grad,2. HS Grad 485  174 Medium (0.12164948 0.23711340 0.64123711)
         62) maritl=2. Married,3. Other 419  145 Medium (0.14081146 0.20525060 0.65393795) *
         63) maritl=1. Never Married 66   29 Medium (0.00000000 0.43939394 0.56060606)
          126) jobclass=1. Industrial 41   20 Low (0.00000000 0.51219512 0.48780488)
            252) health=2. >=Very Good 26   10 Low (0.00000000 0.61538462 0.38461538) *
            253) health=1. <=Good 15    5 Medium (0.00000000 0.33333333 0.66666667) *
          127) jobclass=2. Information 25    8 Medium (0.00000000 0.32000000 0.68000000)
            254) age>=42 13    6 Low (0.00000000 0.53846154 0.46153846) *
            255) age< 42 12    1 Medium (0.00000000 0.08333333 0.91666667) *
```

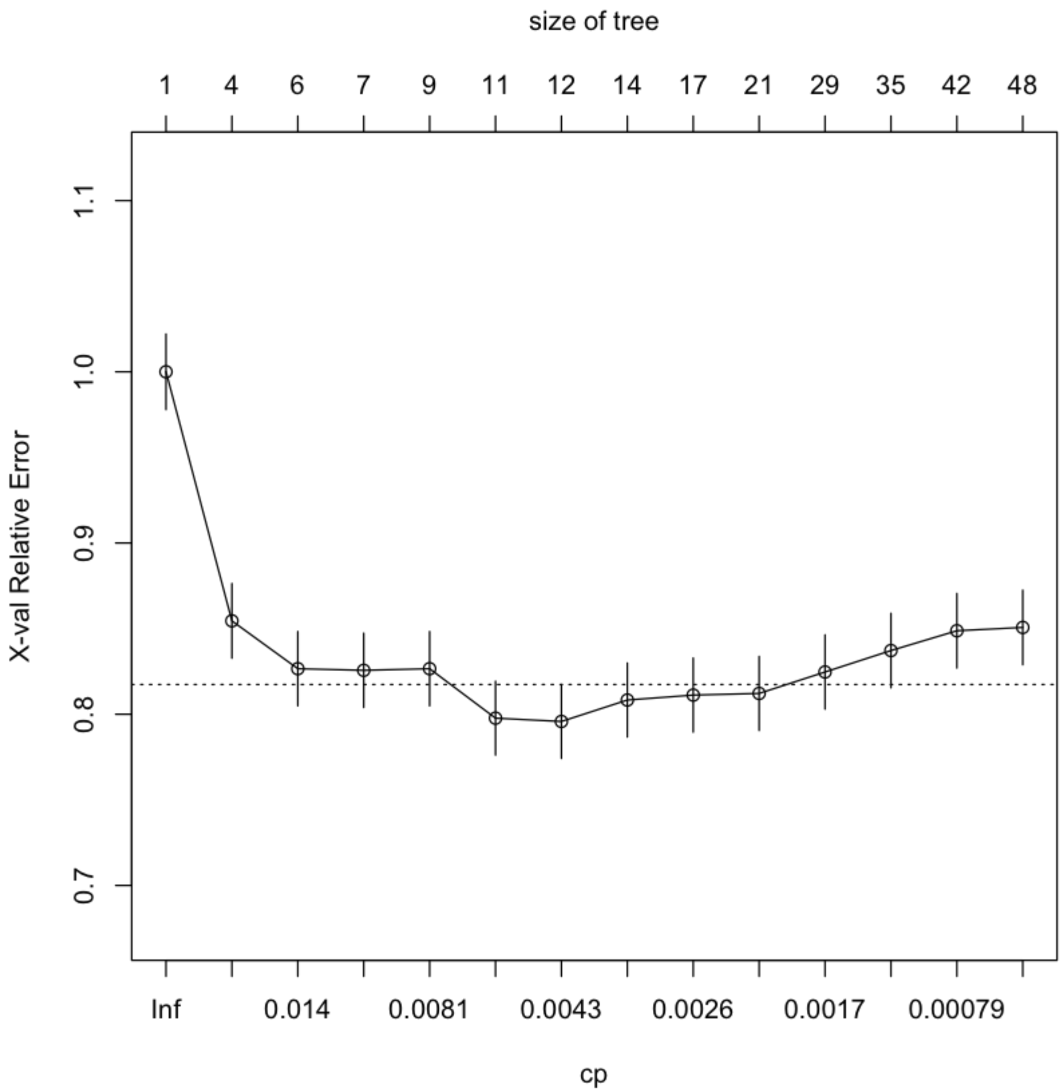
[896]:
```
prop.table(table(data.train$wage_category))
```

```
     High       Low    Medium
0.2488107 0.2450048 0.5061846
```

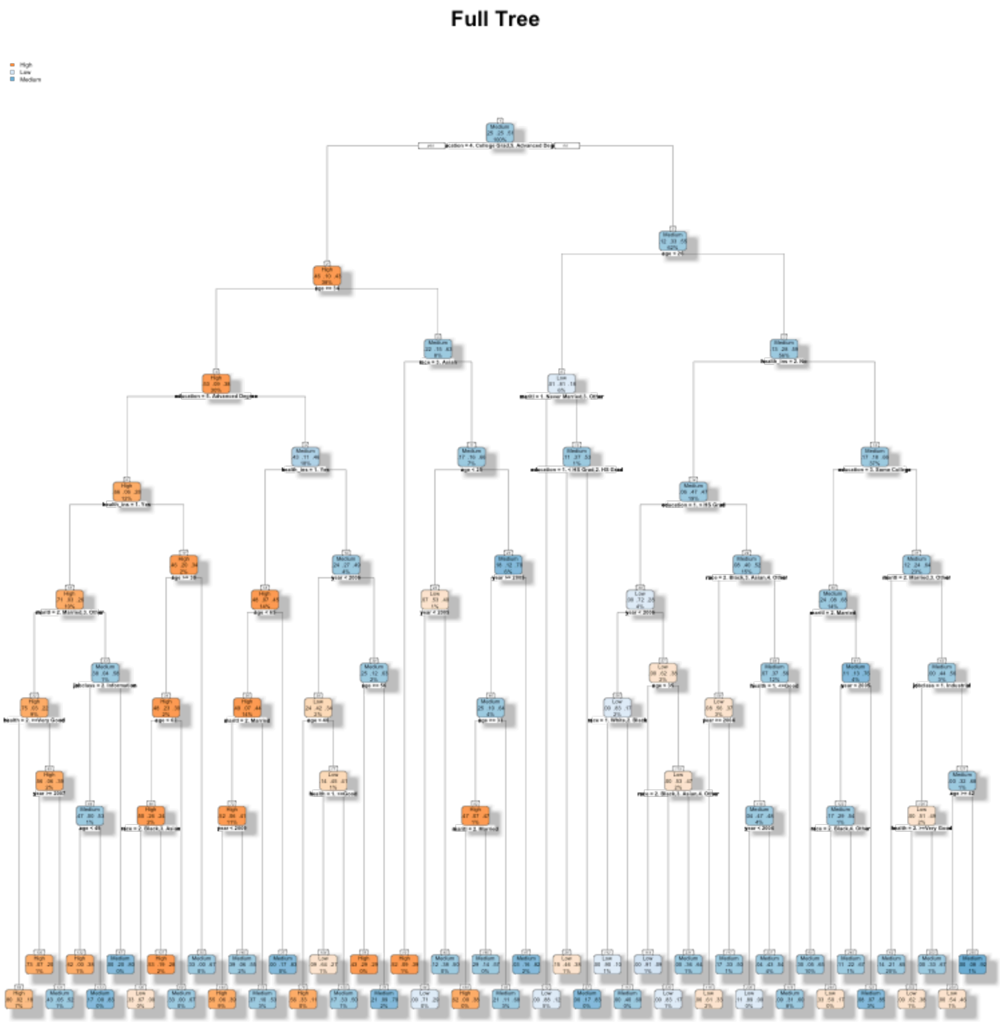
[897]:
```
original_fit_wage$cptable
plotcp(original_fit_wage)
```

A matrix: 14 × 5 of type dbl

	CP	nsplit	rel error	xerror	xstd
1	0.0558766859	0	1.0000000	1.0000000	0.02208288
2	0.0168593449	3	0.8323699	0.8545279	0.02181405
3	0.0115606936	5	0.7986513	0.8265896	0.02170902
4	0.0091522158	6	0.7870906	0.8256262	0.02170508
5	0.0072254335	8	0.7687861	0.8265896	0.02170902
6	0.0048169557	10	0.7543353	0.7976879	0.02158173
7	0.0038535645	11	0.7495183	0.7957611	0.02157256
8	0.0028901734	13	0.7418112	0.8082852	0.02163062
9	0.0024084778	16	0.7331407	0.8111753	0.02164351
10	0.0019267823	20	0.7235067	0.8121387	0.02164776
11	0.0014450867	28	0.7080925	0.8246628	0.02170112
12	0.0009633911	34	0.6994220	0.8371869	0.02175092
13	0.0006422608	41	0.6926782	0.8487476	0.02179376
14	0.0005000000	47	0.6888247	0.8506744	0.02180060

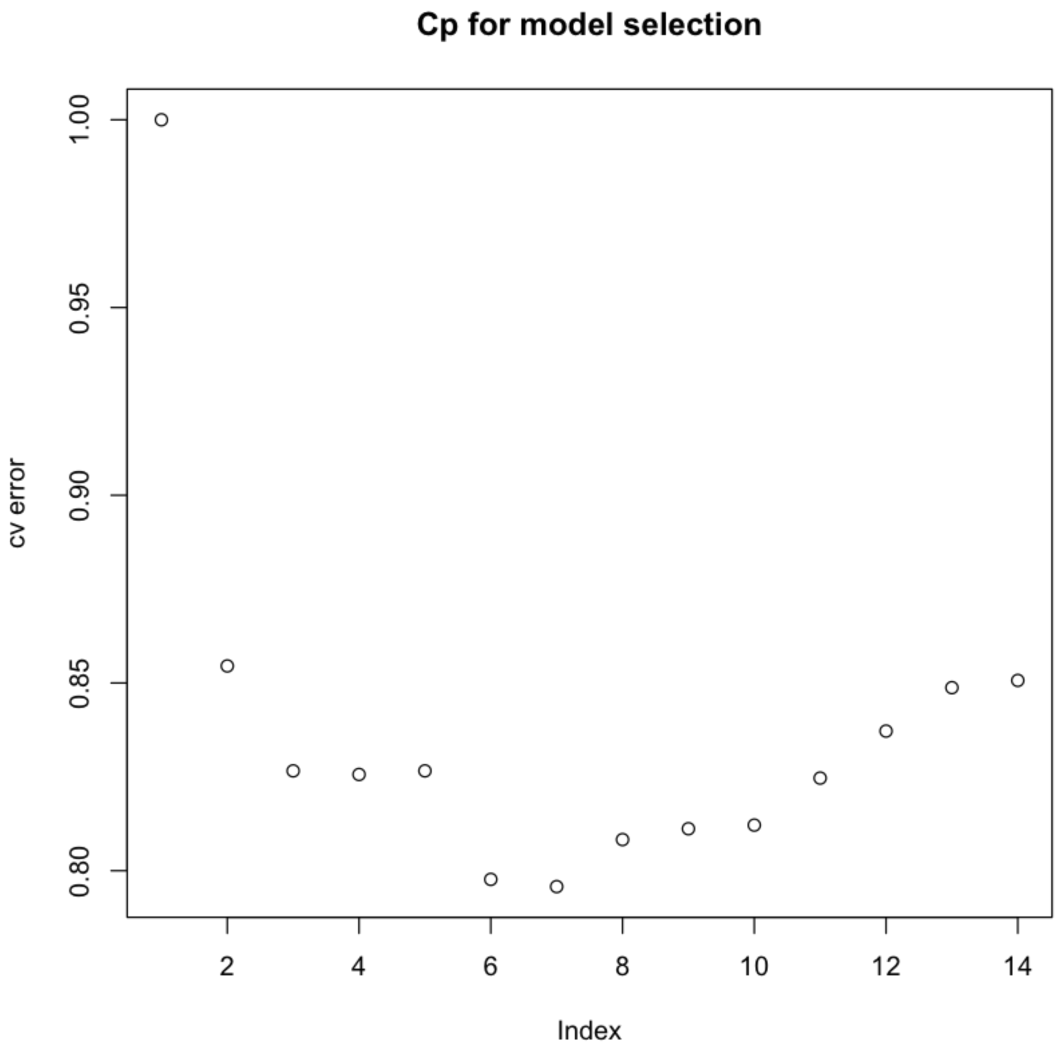


[898]:
```
rpart.plot(original_fit_wage, box.palette="OrBu", shadow.col="gray",␣
↪nn=TRUE,roundint=FALSE, main='Full Tree')
```

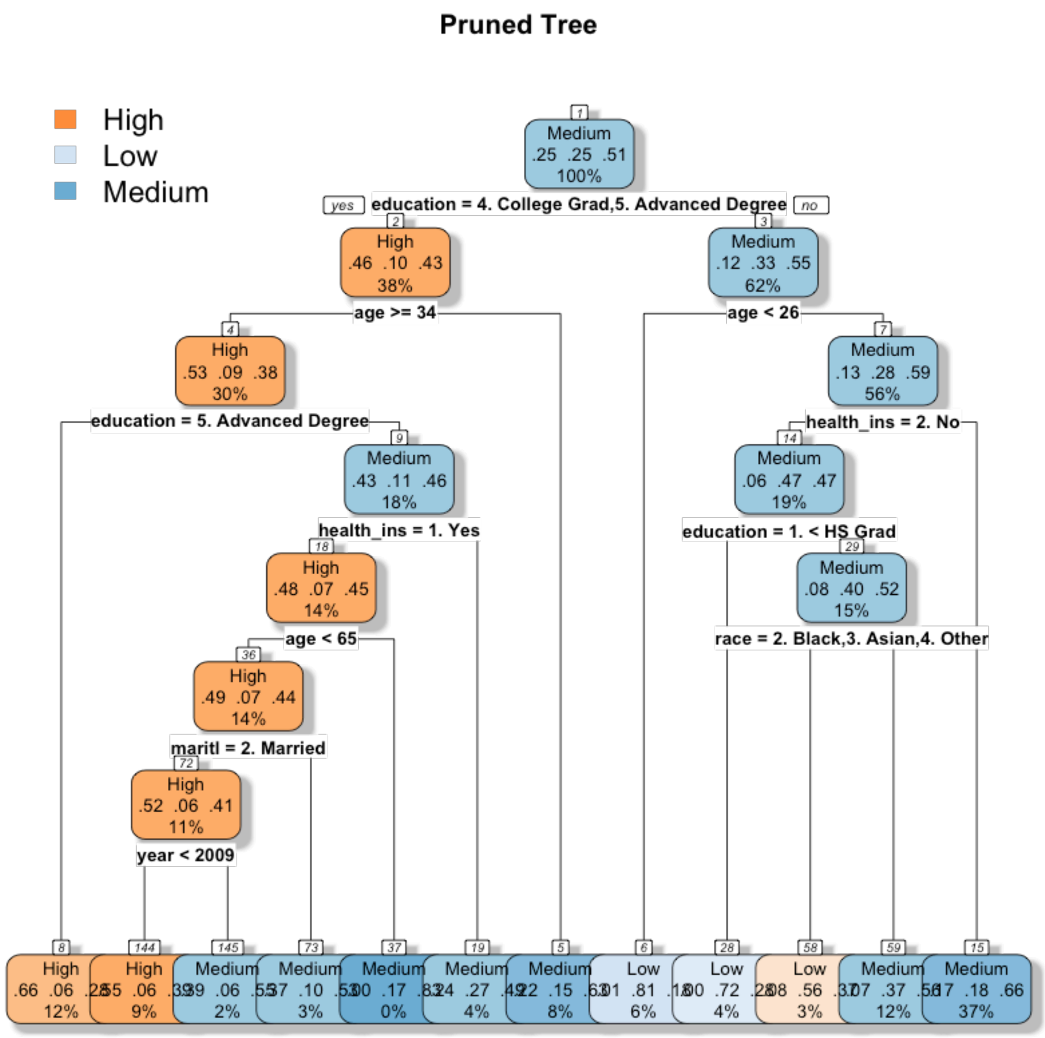


[899]:
```
plot(original_fit_wage$cptable[,4], main = "Cp for model selection", ylab = "cv␣
↪error")

print(paste('Optimal tree size is:',which.min(original_fit_wage$cptable[,4])))
```

```
[1] "Optimal tree size is: 7"
```



[902]:
```
min_cp = original_fit_wage$cptable[which.
  ↪min(original_fit_wage$cptable[,"xerror"]),"CP"]
pruned_fit_wage <- prune(original_fit_wage,cp = min_cp)
rpart.plot(pruned_fit_wage,tweak=1.5, box.palette="OrBu", shadow.col="gray",␣
  ↪nn=TRUE,roundint=FALSE, main='Pruned Tree')
```



The tree shows that the most important predictor of whether a worker will make a high income is education. The factors that follow schooling in terms of relevance are health indicators, age, and marital status. The pruned tree has 12 terminal nodes and 11 splits. Although it appears simpler than the original tree, the pruned tree is nonetheless complex.

3.0.3 *b) Model interpretation – you are asked to characterize the high wage earners. What can you say about this group (based on the tree)?*

The pruned tree has 12 terminal nodes and 11 splits. Tree is considerably easier to understand and comprehend than the original tree. The first division is made based on education, which can be thought of as the main predictor. The following divisions include health insurance, marital status, age, year, and education once again. The first division separates areas with high education levels to the left and relatively low education levels to the right. Low/medium earners are the anticipated class for those with low levels of education. High earners are the anticipated class for those with high education levels. The race of Black, Asian, and others is used as the cutoff level for further partitioning of Node 9. Workers of these races are categorized as low earners. Similar to this, Node 8 is further partitioned using a cutoff level of 65 years old. Workers that are younger than this age threshold are categorized as high earners.

```
[ ]:
```