

Homework_3

October 25, 2022

1 Homework 3

1.1 Question 1

1. We have seen that as the number of features used in a model increase, the training error will necessarily decrease, but the test error may not. Let's examine this in simulation.

1.1.1 (a) Generate a data set with $p = 25$ features, $n = 1,000$ observations, and an associated quantitative response vector generated according to the model: $Y = X + \epsilon$, where ϵ has some elements that are exactly equal to zero. (be sure to use "set.seed")

```
[2]: install.packages('tidyverse')
library(tidyverse)
install.packages('corrplot')
library(corrplot)
install.packages('leaps')
install.packages('ggplot2')
install.packages('dplyr')
install.packages('ggthemes')
library(leaps)
library(ggplot2)
library(dplyr)
library(ggthemes)
```

The downloaded binary packages are in
/var/folders/ch/lqq67w6x5px6fcc9cnbp457m0000gn/T//RtmpFB0bGM/downloaded_packages

```
Attaching packages: tidyverse
1.3.2
ggplot2 3.3.6 purrr 0.3.4
tibble 3.1.8 dplyr 1.0.10
tidyr 1.2.1 stringr 1.4.1
readr 2.1.3 forcats 0.5.2
Conflicts:
tidyverse_conflicts()
dplyr::filter() masks stats::filter()
dplyr::lag() masks stats::lag()
```

The downloaded binary packages are in
/var/folders/ch/lqq67w6x5px6fcc9cnbp457m0000gn/T//RtmpFB0bGM/downloaded_packages
corrplot 0.92 loaded

The downloaded binary packages are in
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The downloaded binary packages are in
/var/folders/ch/lqq67w6x5px6fcc9cnbp457m0000gn/T//RtmpFB0bGM/downloaded_packages
installing and loading the libraries

[206]: `set.seed(201)`

```
p=25
n=1000

X = matrix(rnorm(n*p), n, p)

beta <- rnorm(25)
beta[1]=beta[6]=beta[9]=beta[16]=beta[19]=beta[3]=beta[23]=0
```

Created dataset with 25 features and 1000 observations with normal distribution. Defined a ($p \times 1$) vector beta with normal distribution with mean 0 and s.d 1. And assign few values to zero as mentioned.

[207]: `epsilon <- rnorm(1000)`

defined another ($n \times 1$) vector epsilon for the model.

[208]: `y <- X%*%beta + epsilon`

created the dataset model with $Y = X +$

1.1.2 (b) *Split your data set into a training set containing 500 observations and a test set containing 500 observations.*

```
[209]: train = sample(seq(n), 500, replace=FALSE)
       test = (-train)

       x.train = X[train,]
       x.test = X[test,]
       y.train = y[train]
       y.test = y[test]
```

converted the matrix into dataframe. Splitted the dataset into train and test by 500 each and the response variable for each.

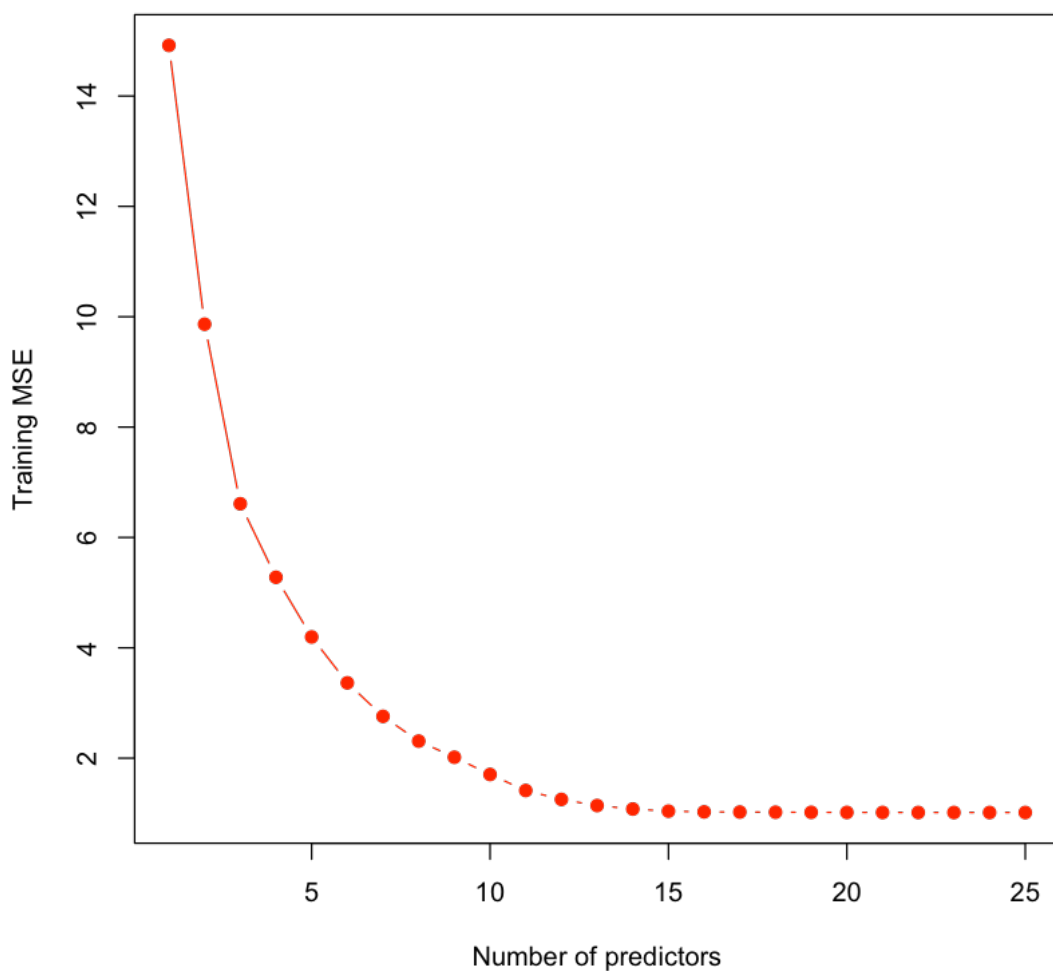
1.1.3 (c) *Perform subset selection (best, forward or backwards) on the training set, and plot the training and test MSE associated with the best model of each size.*

2 Training MSE

```
[225]: data.train = data.frame(y = y.train, x = x.train)
       regfit.model = regsubsets(y ~ ., data = data.train, nvmax = p,
       ↪) #method='backward')
       train.matrix = model.matrix(y ~ ., data = data.train, nvmax = p)
       val.errors = rep(NA, p)

       for (i in 1:p) {
         coefi = coef(regfit.model, id = i)
         pred = train.matrix[, names(coefi)] %*% coefi
         val.errors[i] = mean((pred - y.train)^2)
       }

       plot(val.errors, xlab = "Number of predictors",
       ylab = "Training MSE", pch = 19, type = "b", col="red")
```



```
[226]: which.min(val.errors)
```

25

The training MSE decreases with the addition of any new variable, even after 20 variables. Here the model selected 25 predictors for the model with least MSE.

3 Test MSE

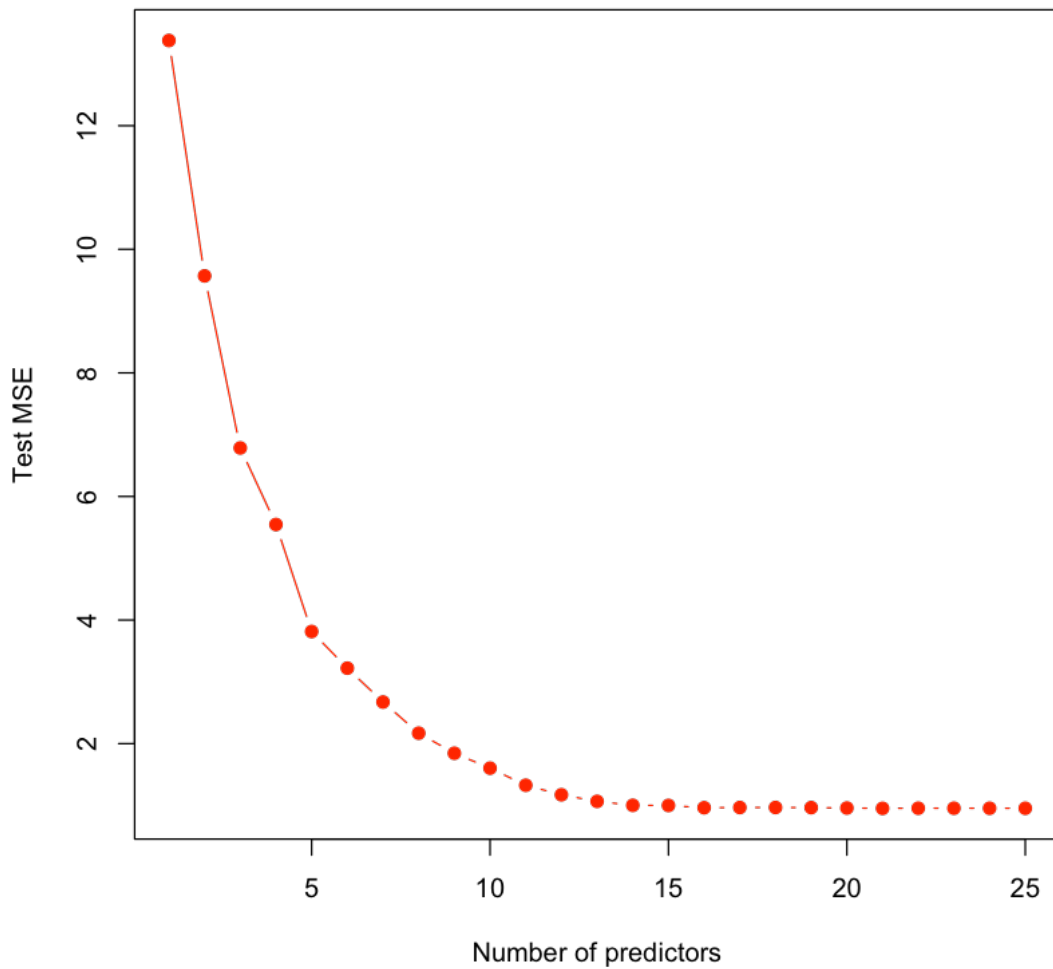
```
[229]: data.test = data.frame(y = y.test, x = x.test)
test.matix = model.matrix(y ~ ., data = data.test, nvmax = p)
val.errors = rep(NA, p)
```

```

for (i in 1:p) {
  coefi = coef(regfit.model, id = i)
  pred = test.matix[, names(coefi)] %*% coefi
  val.errors[i] = mean((pred - y.test)^2)
}

plot(val.errors, xlab = "Number of predictors",
     ylab = "Test MSE", pch = 19, type = "b", col="red")

```



```
[230]: which.min(val.errors)
```

21

Here we can see the test MSE is the least at 21 predictors which is using almost all predictors.

But the values of the test error is slightly reduced.

3.0.1 (d) *For which model size does the test set MSE take on its minimum value? Comment on your results. If it takes on its minimum value for a model containing only an intercept a model containing all the features, then play around with the way that you are generating the data in (a) until you come up with a scenario in which the test set MSE is minimized for an intermediate model size*

As we can see above for both the test and training MSE is minimised for model with 20 predictors. So we will generate data so that we can get test set MSE minimized for an intermediate model size.

```
[286]: set.seed(300)
```

```
train = sample(seq(n), 900, replace=FALSE)
test = (-train)

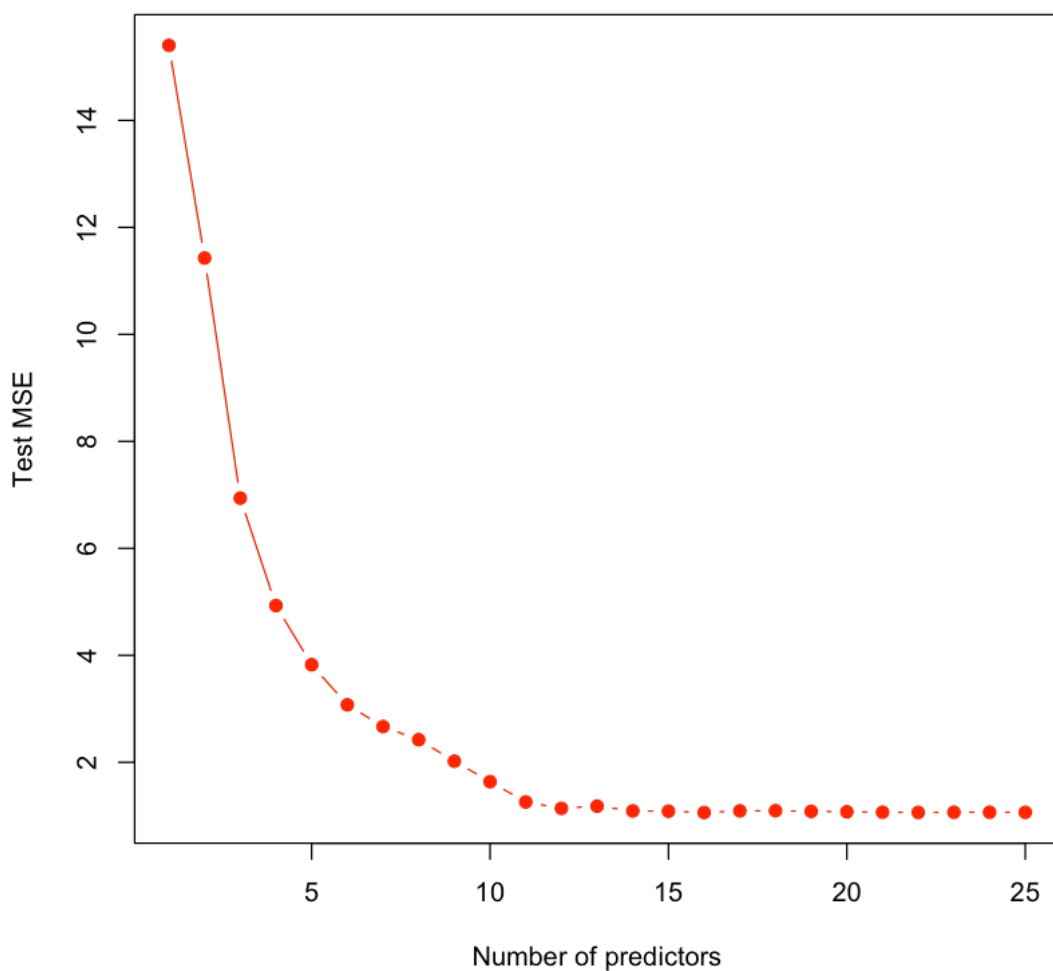
x.train = X[train,]
x.test = X[test,]
y.train = y[train]
y.test = y[test]
```

```
[287]: data.train = data.frame(y = y.train, x = x.train)
regfit.inter.model = regsubsets(y ~ ., data = data.train, nvmax = p)

data.test = data.frame(y = y.test, x = x.test)
test.matix = model.matrix(y ~ ., data = data.test, nvmax = p)
val.errors = rep(NA, 25)

for (i in 1:p) {
  coefi = coef(regfit.model, id = i)
  pred = test.matix[, names(coefi)] %*% coefi
  val.errors[i] = mean((y.test-pred)^2)
}

plot(val.errors, xlab = "Number of predictors",
     ylab = "Test MSE", pch = 19, type = "b", col="red")
```



```
[288]: which.min(val.errors)
```

16

So here when we split the dataset in 3:4 ratio, the test MSE got minimized to generate model with 16 predictors.

3.0.2 (e) *How does the model at which the test set MSE is minimized compare to the true model used to generate the data? Comment on the coefficient values.*

```
[289]: (coef(regfit.inter.model, which.min(val.errors)) )
```

```
(Intercept)    0.00411463481057264 x.2    -0.253076755431062 x.4    0.540708115976977 x.5
-1.0892715133691 x.7 0.174275064609131 x.8 -0.883093203242521 x.10 -0.419843673860151 x.11
```

0.706641896154916 **x.12** 2.27413060171697 **x.13** -0.151759916888031 **x.14** -1.21432378327401
x.15 0.784565295722092 **x.17** -0.331673170004355 **x.18** 2.3575066889276 **x.21** -1.71656070698095
x.24 -0.558039216051135 **x.25** -0.535777755826469

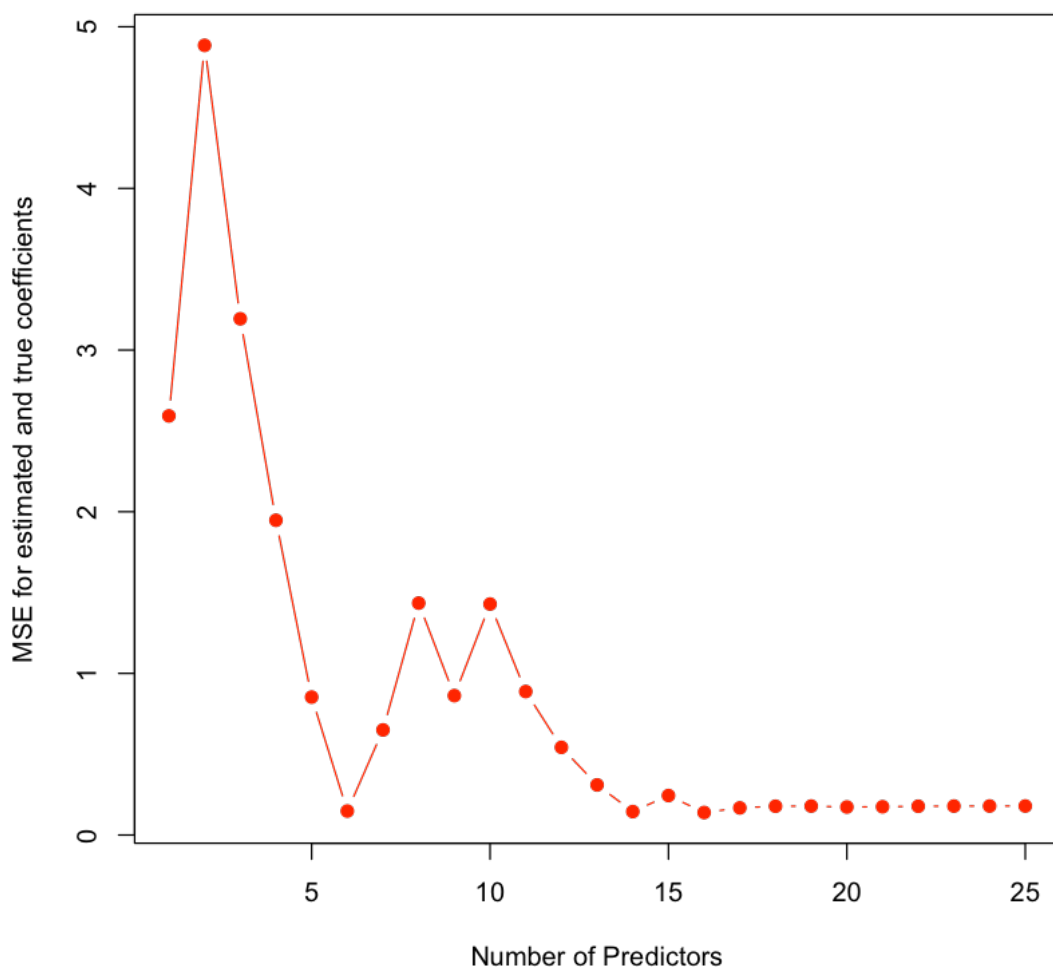
In the true model, we assigned 5 parameters zero. More specifically, we put the variables 1 = 3 = 6 = 9 = 16 = 19 = 23 = 0, so here and the best subset model find those variables and removed it from the model. For this model, the test MSE is minimal with 16 variables (ie, the no. of predictors - predictors with beta0).

3.0.3 (f) Create a plot containing $pi=1(j - \hat{jr})^2$ for a range of values r , where \hat{jr} is the j th coefficient estimate for the best model containing r coefficient. Comment on what you observe. How do these results compare to part D.

```
[290]: val.errors = rep(NA, p)
x_cols = colnames(X, do.NULL = FALSE, prefix = "x.")
for (i in 1:p) {
  coefi = coef(regfit.inter.model, id = i)
  val.errors[i] = sqrt(sum((beta[x_cols %in% names(coefi)] -
    ↪coefi[names(coefi) %in% x_cols])^2)
+ sum(beta[!(x_cols %in% names(coefi))])^2)
}
which.min(val.errors)
```

16

```
[291]: plot(val.errors, xlab = "Number of Predictors",
ylab = "MSE for estimated and true coefficients",
pch = 19, type = "b", col="red")
```

```
[292]: coef(regfit.model,which.min(val.errors))
```

```
(Intercept)    0.0102984447598383 x.2    -0.247911320704428 x.4    0.524086784820448 x.5
-1.08985909199452 x.7    0.158488762607049 x.8    -0.876496556504852 x.10    -0.423320725539383
x.11    0.692891028395666 x.12    2.2784928357215 x.13    -0.15036124827506 x.14    -1.20661824272458
x.15    0.782050822685462 x.17    -0.306849546461202 x.18    2.35453172445457 x.21
-1.72494868451755 x.24    -0.562621258410586 x.25    -0.562569636969656
```

Here also the error is minimized for 16 variables. The model that provides parameter estimates (part d) and this true parameter estimate provides the least test MSE. But when we chnage the seed value above we will see that the model that provides parameter estimates that are closest to the true parameter estimate are not the same model that provides the least test MSE as in (part d).

3.1 Question 2

Consider the Diabetes dataset (posted with assignment). Assume the population prior probabilities are estimated using the relative frequencies of the classes in the data.

3.1.1 (a) *Produce pairwise scatterplots for all five variables, with different symbols or colors representing the three different classes. Do you see any evidence that the classes may have different covariance matrices? That they may not be multivariate normal?*

```
[293]: install.packages('klaR')
install.packages('ggplot2')
install.packages('GGally')
library(GGally)
library(ggplot2)
library(klaR)
```

```
The downloaded binary packages are in
/var/folders/ch/lqq67w6x5px6fcc9cnbp457m0000gn/T//RtmpFB0bGM/downloaded_packages
```

```
The downloaded binary packages are in
/var/folders/ch/lqq67w6x5px6fcc9cnbp457m0000gn/T//RtmpFB0bGM/downloaded_packages
```

```
The downloaded binary packages are in
/var/folders/ch/lqq67w6x5px6fcc9cnbp457m0000gn/T//RtmpFB0bGM/downloaded_packages
```

```
Registered S3 method overwritten by 'GGally':
  method from
+.gg      ggplot2
```

```
Loading required package: MASS
```

```
Attaching package: 'MASS'
```

```
The following object is masked from 'package:dplyr':
```

```
select
```

```
[294]: setwd("/Users/sreeragvenugopalan/Desktop/Sem 1/Statistical Data Mining/HW/3")
data = load('Diabetes.RData')
```

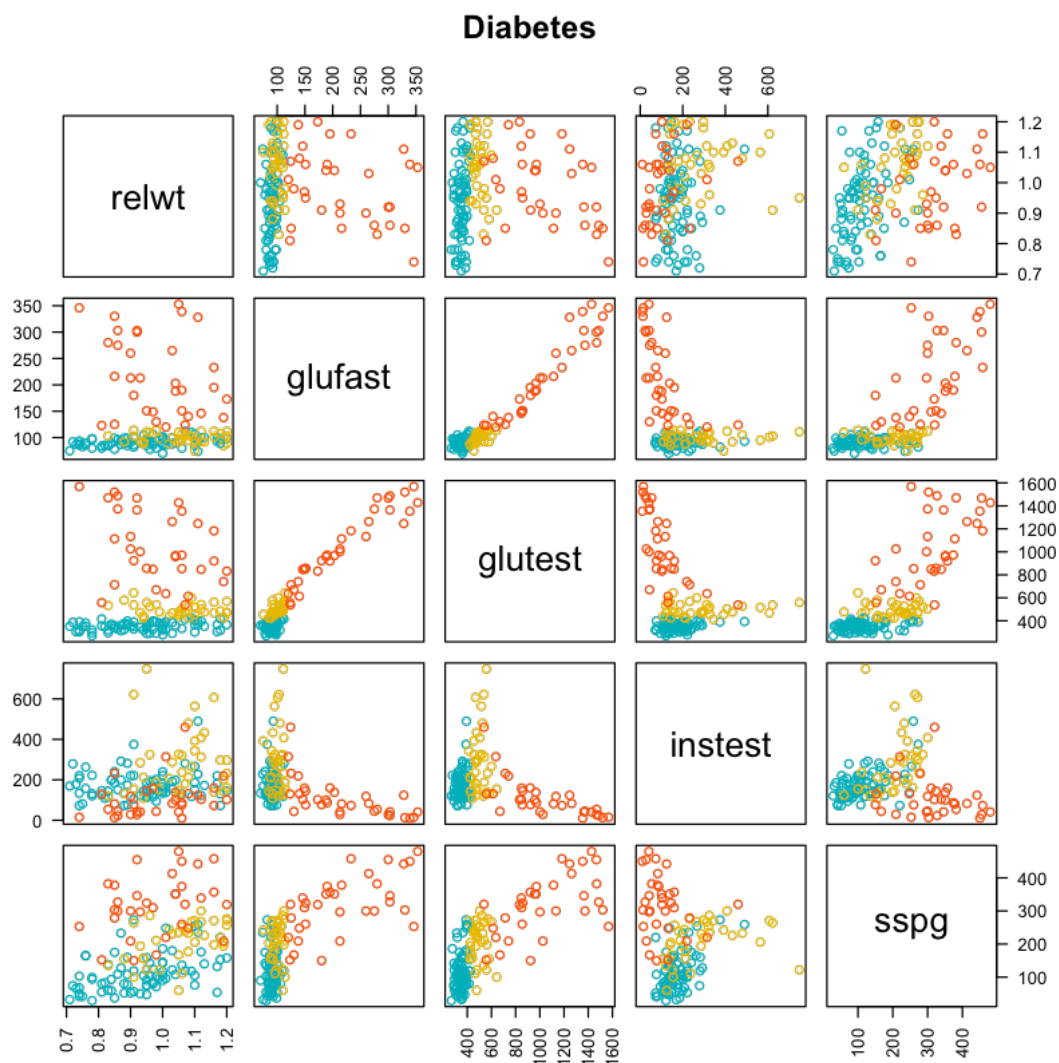
```
[295]: head(Diabetes,5)
```

		relwt <dbl>	glufast <int>	glutest <int>	instest <int>	sspg <int>	group <fct>
A data.frame: 5 × 6	1	0.81	80	356	124	55	Normal
	2	0.95	97	289	117	76	Normal
	3	0.94	105	319	143	105	Normal
	4	1.04	90	356	199	108	Normal
	5	1.00	90	323	240	143	Normal

```
[299]: summary(Diabetes$group)
```

```
Normal          76 Chemical\_Diabetic          36 Overt\_Diabetic          33
```

```
[300]: my_cols <- c("#00AFBB", "#E7B800", "#FC4E07")
pairs(Diabetes[c('relwt', 'glufast', 'glutest', 'instest', 'sspg')], col = my_cols[Diabetes$group], las=2, main='Diabetes')
```



```
[302]: ggpairs(Diabetes, columns = 1:5, aes(color = group, alpha = 0.5),
           upper = list(continuous = wrap("cor", size = 1.75)))
```



Yes, from the above plot it is clear that the classes have different covariance matrices. The *Normal* group shows the smallest variance and the *Overt Diabetic* shows the largest covariance. There seems to be a direct development from normal to chemical to overt, in the glufast - glutest, sspg - glufast and sspg - glutest. However, for others, we can observe that the group with chemical diabetes differs from the controls in one way, whereas the group with overt diabetes differs in a different direction and has an intragroup correlation with the opposite sign to the others.

3.1.2 (b) *Apply linear discriminant analysis (LDA) and quadratic discriminant analysis (QDA). How does the performance of QDA compare to that of LDA in this case?*

3.2 LDA Analysis

```
[304]: # Create a test and training set
set.seed(2255)
indis <- sample(1:nrow(Diabetes), round(2/3*nrow(Diabetes)), replace = FALSE)

train.data <- Diabetes[indis, ]
test.data <- Diabetes[-indis, ]
dim(train.data)
dim(test.data)
```

```
1. 97 2. 6
```

```
1. 48 2. 6
```

```
[305]: lda.fit <- lda(group~., data = train.data)
lda.fit

symbol <- c(1,2,4)[train.data$group]
color <- c('green','orange','blue')[train.data$group]
plot(lda.fit, col=color, pch = symbol)
legend(x='topright', legend=c('Normal','Chemical_Diabetic','Overt_Diabetic'),
      col=c('green','orange','blue'), pch=c(1,2,4))
```

Call:

```
lda(group ~ ., data = train.data)
```

Prior probabilities of groups:

	Normal	Chemical_Diabetic	Overt_Diabetic
	0.5773196	0.1855670	0.2371134

Group means:

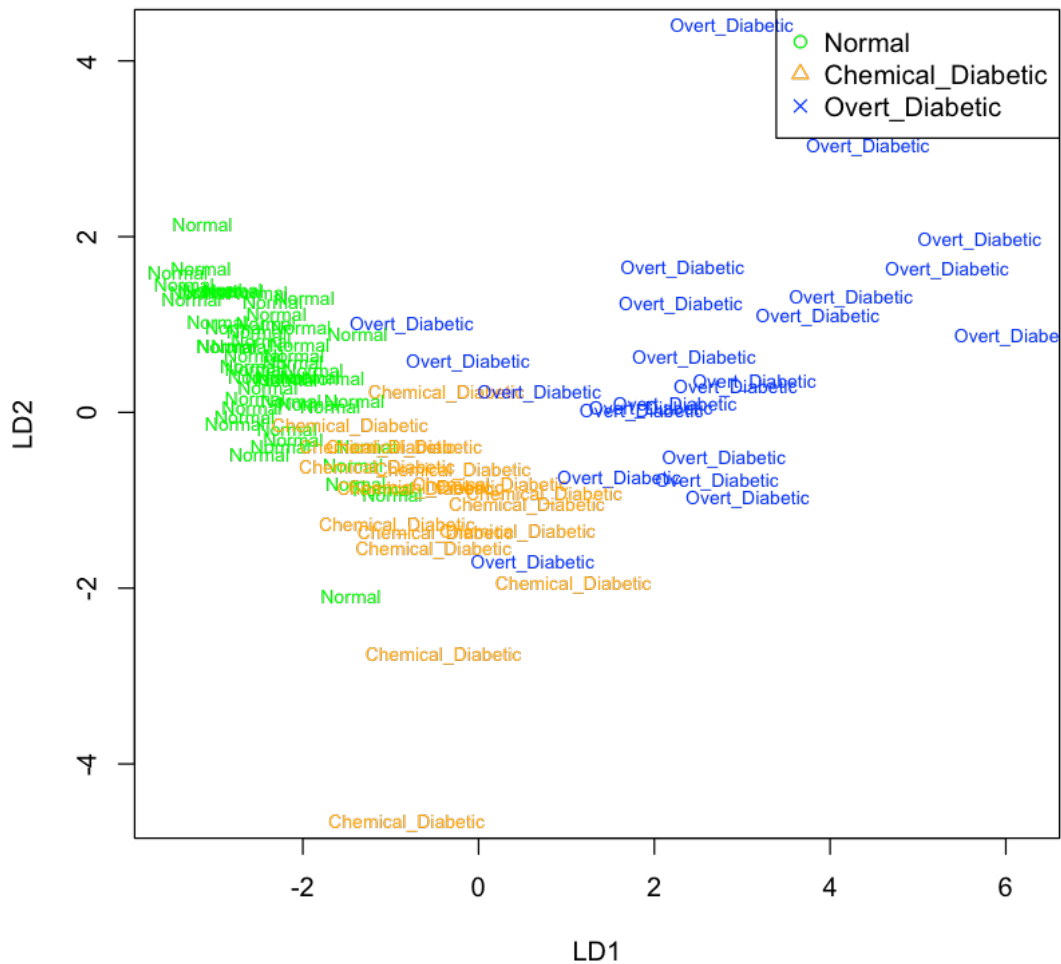
	relwt	glufast	glutest	instest	sspg
Normal	0.9457143	90.94643	349.2679	171.30357	112.4643
Chemical_Diabetic	1.0450000	101.44444	506.8889	262.72222	197.6667
Overt_Diabetic	0.9695652	203.82609	1002.7826	99.43478	308.9130

Coefficients of linear discriminants:

	LD1	LD2
relwt	0.743436274	-4.046036046
glufast	-0.037139505	0.046139316
glutest	0.012922477	-0.009713514
instest	-0.000825485	-0.007125650
sspg	0.003630909	0.004464830

Proportion of trace:

LD1	LD2
0.9009	0.0991



```
[324]: # make predictions for the training.
lda_train_pred <- predict(lda.fit, newdata = train.data)
head(data.frame(lda_train_pred$class, lda_train_pred$posterior,
  ↪ lda_train_pred$x), 5)
```

		lda_train_pred.class <fct>	Normal <dbl>	Chemical_Diabetic <dbl>	Overt_Diabetic <dbl>	LD1 <dbl>
A data.frame: 5 × 6	29	Normal	9.862200e-01	1.377866e-02	1.319578e-06	-1.5362
	45	Normal	9.998612e-01	1.387751e-04	2.652104e-08	-2.3664
	117	Overt_Diabetic	2.060416e-08	1.521519e-05	9.999848e-01	4.6339
	67	Normal	7.108025e-01	2.890426e-01	1.548966e-04	-0.5025
	145	Overt_Diabetic	8.629813e-10	6.968141e-08	9.999999e-01	5.2043

```
[323]: # make predictions for the test
lda_test_pred <- predict(lda.fit, newdata = test.data)
head(data.frame(lda_test_pred$class, lda_test_pred$posterior,
  ↪lda_test_pred$x),5)
```

		lda_test_pred.class <fct>	Normal <dbl>	Chemical_Diabetic <dbl>	Overt_Diabetic <dbl>	LD1 <dbl>
A data.frame: 5 × 6	3	Normal	0.9997449	2.551165e-04	1.392365e-08	-2.479569
	6	Normal	0.9849702	1.499372e-02	3.610714e-05	-0.900246
	9	Normal	0.9841154	1.588191e-02	2.723117e-06	-1.394524
	10	Normal	0.9999244	7.559219e-05	7.129466e-09	-2.628654
	25	Normal	0.9796169	2.038168e-02	1.381927e-06	-1.519330

```
[325]: # compute the train error rates
train_error <- (1/length(train.data$group))*length(which(train.data$group !=
  ↪lda_train_pred$class))
train_error
```

0.123711340206186

```
[333]: # compute the test error rates
test_error <- (1/length(test.data$group))*length(which(test.data$group !=
  ↪lda_test_pred$class))
test_error
```

0.125

3.3 QDA Analysis

```
[327]: qda.fit <- qda(group~., data = train.data)
qda.fit
```

Call:

```
qda(group ~ ., data = train.data)
```

Prior probabilities of groups:

Normal	Chemical_Diabetic	Overt_Diabetic
0.5773196	0.1855670	0.2371134

Group means:

relwt	glufast	glutest	instest	sspg
-------	---------	---------	---------	------

Normal	0.9457143	90.94643	349.2679	171.30357	112.4643
Chemical_Diabetic	1.0450000	101.44444	506.8889	262.72222	197.6667
Overt_Diabetic	0.9695652	203.82609	1002.7826	99.43478	308.9130

```
[328]: qda_train_pred <- predict(qda.fit, newdata = train.data)
qda_test_pred <- predict(qda.fit, newdata = test.data)
```

```
[329]: y_hat_train <- qda_train_pred$class
y_hat_test <- qda_test_pred$class

y_true_train <- train.data$group
y_true_test <- test.data$group
```

```
[334]: train_err <- (1/length(y_hat_train))*length(which(y_true_train != y_hat_train))
test_err <- (1/length(y_hat_test))*length(which(y_true_test != y_hat_test))
```

```
train_err
test_err
```

0.0412371134020619

0.04166666666666667

From the error rate it is clear that QDA performed the best with an Accuracy of almost 95% where as LDA of 87% without

3.3.1 (c) Suppose an individual has (glucose test/intolerance= 68, insulin test=122, SSPG = 544. Relative weight = 1.86, fasting plasma glucose = 184). To which class does LDA assign this individual? To which class does QDA?

3.3.2 LDA Prediction

```
[335]: X.data = data.frame(1.86,184,68,122,544,NA)
colnames(X.data) = c('relwt','glufast','glutest','instest','sspg')
X.data.lda_pred = predict(lda.fit, newdata = X.data)
X.data.lda_pred$class
```

Normal Levels: 1. 'Normal' 2. 'Chemical_Diabetic' 3. 'Overt_Diabetic'

LDA model predicted this data as Normal.

3.3.3 QDA Prediction

```
[336]: X.data = data.frame(1.86,184,68,122,544,NA)
colnames(X.data) = c('relwt','glufast','glutest','instest','sspg')
X.data.qda_pred = predict(qda.fit, newdata = X.data)
X.data.qda_pred$class
```

Overt_Diabetic Levels: 1. 'Normal' 2. 'Chemical_Diabetic' 3. 'Overt_Diabetic'

QDA model predicted this data as Over Diabetic.

3.3.4 (d) Apply RDA (regularized discriminant analysis). What is the optimal value of α in this case? Does this support your observations about the covariance matrices in (a)

```
[337]: #rda_fit <- rda(group~., data = train.data, regularization = c(gamma=0, lambda=.
      ↪5))

#rda_fit$error.rate

alpha = seq(from = 0, to = 1, by = .1)

err_store <- c()

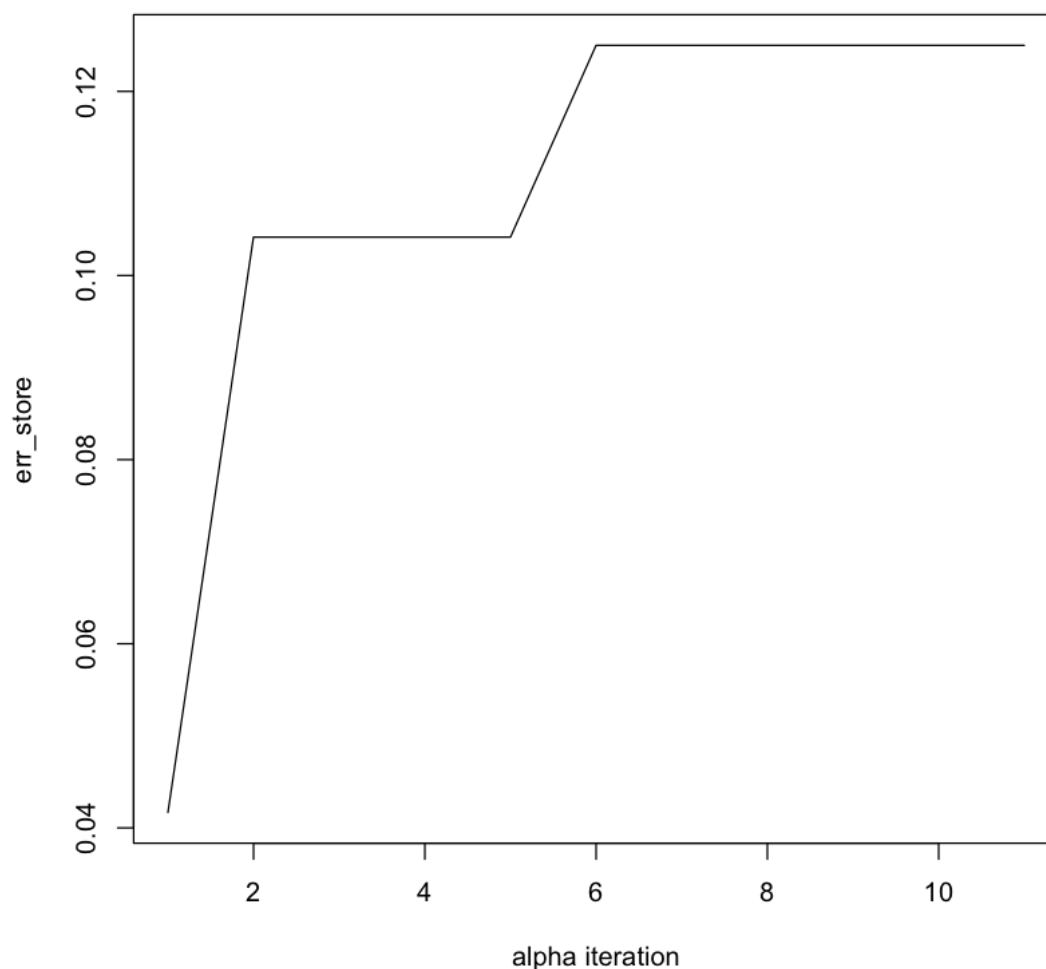
for (i in 1:length(alpha)){
  rda_fit <- rda(group~., data = train.data, regularization = c(gamma=0, ↪
    ↪lambda=alpha[i]))

  y_hat_test <- predict(rda_fit, newdata=test.data)$class

  err <- (1/length(y_hat_test))*length(which(y_hat_test != test.data$group))

  err_store <- c(err_store,err)
}

plot(err_store, type = "l", xlab = "alpha iteration")
```



[338]: `err_store`

1. 0.041666666666667 2. 0.104166666666667 3. 0.104166666666667 4. 0.104166666666667
5. 0.104166666666667 6. 0.125 7. 0.125 8. 0.125 9. 0.125 10. 0.125 11. 0.125

for this data set we can see that QDA dis the best here with least error rate than the LDA. The error rate is the same for both from halfway of alpha iteration with last one before dropping. So the simplistic QDA is the best in this case. We cannot ignore the quadratic terms, hence the QDA covariance matrix may differ for each class. QDA typically fits the data better since it gives the covariance matrix more flexibility. However, there will be more parameters to estimate at that point. With QDA, there are a lot more parameters. Because each class will have its own covariance matrix with QDA,

4 Question 3

This problem concerns the Boston data set (ISLR2 package).

4.0.1 (a) *Fit classification models in order to predict whether a given census tract has a high or low crime rates. Explore logistic regression, LDA, QDA and KNN models using various subsets of the predictors. Describe your findings.*

```
[339]: library(MASS)
library(caret)
install.packages('ISLR2')
install.packages('MMST')
install.packages('caret')
install.packages("corrplot")
library(corrplot)
```

Loading required package: lattice

Attaching package: 'caret'

The following object is masked from 'package:purrr':

lift

The downloaded binary packages are in
/var/folders/ch/lqq67w6x5px6fcc9cnbp457m0000gn/T//RtmpFB0bGM/downloaded_packages

Warning message:

"package 'MMST' is not available for this version of R

A version of this package for your version of R might be available elsewhere,
see the ideas at
<https://cran.r-project.org/doc/manuals/r-patched/R-admin.html#Installing-packages>"

The downloaded binary packages are in
/var/folders/ch/lqq67w6x5px6fcc9cnbp457m0000gn/T//RtmpFB0bGM/downloaded_packages

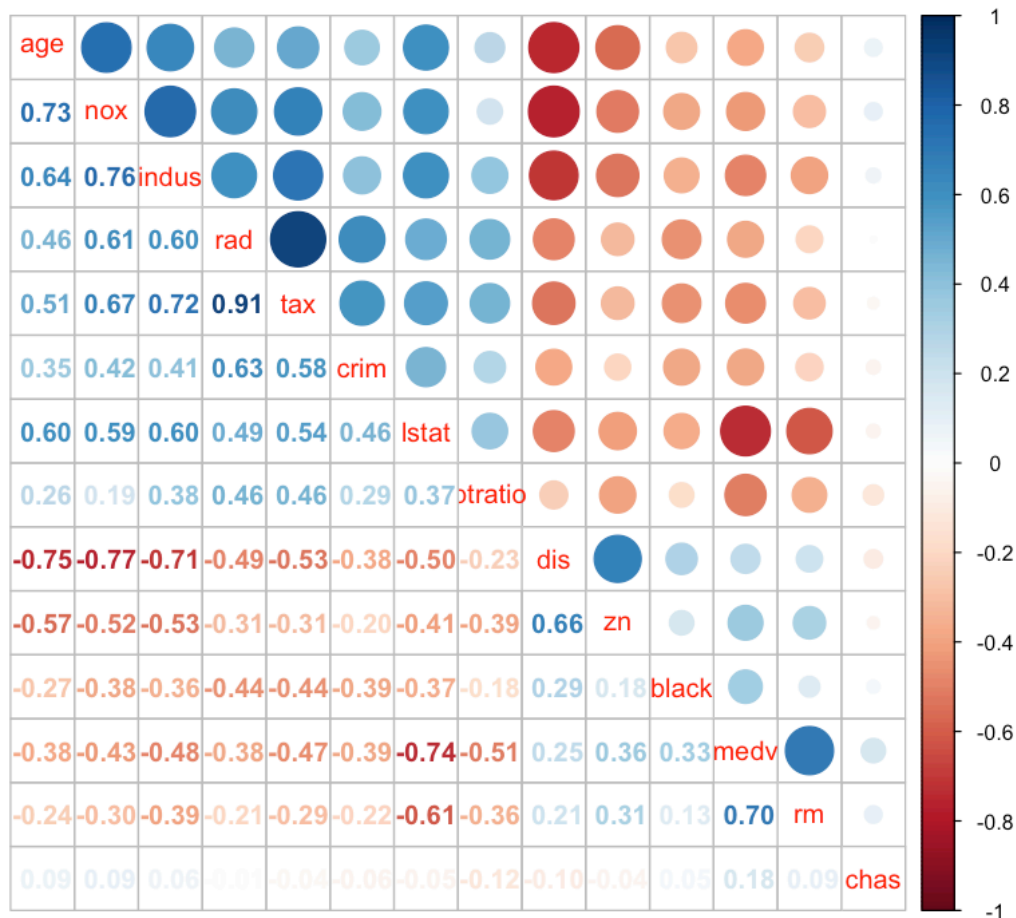
The downloaded binary packages are in
/var/folders/ch/lqq67w6x5px6fcc9cnbp457m0000gn/T//RtmpFB0bGM/downloaded_packages

```
[340]: data(Boston)
class(Boston$crim)
```

'numeric'

So here we need to predict the crime is high or low when a census tract given. but here the crim varibale is numerical so we need to convert it into factor of two level; 'high' and 'low'.

```
[341]: cor_data = cor(Boston)
corrplot.mixed(cor_data, order = 'AOE')
```



from this correlation matrix it is clear that variables age, nox, indus, rad, tax, and lstat are more correlated with crim.

```
[342]: Boston$crim = factor(ifelse(Boston$crim < median(Boston$crim), "Low", "High"))
```

we factorised the crim variable as high and low based on the median. So anything below median is set to low and high otherwise.

```
[343]: summary(Boston$crim)
       head(Boston,1)
```

	High	253 Low			253						
		crim	zn	indus	chas	nox	rm	age	dis	rad	tax
		<fct>	<dbl>	<dbl>	<int>	<dbl>	<dbl>	<dbl>	<dbl>	<int>	<dbl>
A data.frame: 1 × 14	1	Low	18	2.31	0	0.538	6.575	65.2	4.09	1	296

```
[344]: set.seed(123)
       indis <- sample(1:nrow(Boston), round(2/3*nrow(Boston)), replace = FALSE)
       train.data <- Boston[indis,]
       test.data <- Boston[-indis,]

       y_true_train <- as.numeric(train.data$crim)-1
       y_true_test <- as.numeric(test.data$crim)-1
```

splitting data into train adn test.

4.0.2 Logistic Regression with 5 variable subset

```
[345]: glm.fit <- glm(crim ~nox+indus+rad+tax+lstat, data = train.data, family = "binomial")
       #glm.fit <- glm(crim ~zn+nox+age+rad+prratio+black+medv, data = train.data, family = "binomial")
       summary(glm.fit)
```

Call:

```
glm(formula = crim ~ nox + indus + rad + tax + lstat, family = "binomial",
    data = train.data)
```

Deviance Residuals:

	Min	1Q	Median	3Q	Max
	-2.52603	-0.01085	0.05006	0.30545	2.00287

Coefficients:

	Estimate	Std. Error	z value	Pr(> z)
(Intercept)	19.57250	3.18556	6.144	8.04e-10 ***
nox	-35.44264	6.50805	-5.446	5.15e-08 ***
indus	0.03219	0.05341	0.603	0.546715
rad	-0.51661	0.14568	-3.546	0.000391 ***
tax	0.00751	0.00290	2.590	0.009611 **
lstat	-0.05618	0.03626	-1.549	0.121293

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

(Dispersion parameter for binomial family taken to be 1)

Null deviance: 467.04 on 336 degrees of freedom
Residual deviance: 178.94 on 331 degrees of freedom
AIC: 190.94

Number of Fisher Scoring iterations: 8

All of the coefficients are significant or close to being significant.

```
[346]: glm.probs.train <- predict(glm.fit, newdata = train.data, type = "response")
y_hat_train <- round(glm.probs.train)
glm.probs.test <- predict(glm.fit, newdata = test.data, type = "response")
y_hat_test <- round(glm.probs.test)

train_err <- sum(abs(y_hat_train - y_true_train))/length(y_true_train)
test_err <- sum(abs(y_hat_test - y_true_test))/length(y_true_test)

print(paste('Train Accuracy:', 1 - train_err))
print(paste('Test Accuracy:', 1 - test_err))
```

```
[1] "Train Accuracy: 0.872403560830861"
```

```
[1] "Test Accuracy: 0.887573964497041"
```

Logistic Regression performed good with 88% accuracy having subset of 5 significant variables.

4.0.3 Logistic Regression with 6 variable subset

```
[347]: glm.fit <- glm(crim ~ age + nox + indus + rad + tax + lstat, data = train.data, family = "binomial")
#glm.fit <- glm(crim ~ zn + nox + age + rad + ptratio + black + medv, data = train.data, family = "binomial")
summary(glm.fit)
```

Call:

```
glm(formula = crim ~ age + nox + indus + rad + tax + lstat, family = "binomial",
    data = train.data)
```

Deviance Residuals:

Min	1Q	Median	3Q	Max
-2.66986	-0.01222	0.04462	0.31111	1.91988

Coefficients:

	Estimate	Std. Error	z value	Pr(> z)
(Intercept)	18.585591	3.261470	5.699	1.21e-08 ***
age	-0.012407	0.010853	-1.143	0.252964
nox	-32.364504	6.936218	-4.666	3.07e-06 ***
indus	0.027860	0.053402	0.522	0.601871

```
rad          -0.511580    0.144910   -3.530 0.000415 ***
tax           0.007762    0.002953    2.629 0.008575 **
lstat        -0.039756    0.039847   -0.998 0.318414
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

(Dispersion parameter for binomial family taken to be 1)

```
Null deviance: 467.04  on 336  degrees of freedom
Residual deviance: 177.60  on 330  degrees of freedom
AIC: 191.6
```

Number of Fisher Scoring iterations: 8

```
[348]: glm.probs.train <- predict(glm.fit, newdata = train.data, type = "response")
y_hat_train <- round(glm.probs.train)
glm.probs.test <- predict(glm.fit, newdata = test.data, type = "response")
y_hat_test <- round(glm.probs.test)

train_err <- sum(abs(y_hat_train- y_true_train))/length(y_true_train)
test_err <- sum(abs(y_hat_test- y_true_test))/length(y_true_test)

print(paste('Train Accuracy:',1-train_err))
print(paste('Test Accuracy:',1-test_err))
```

```
[1] "Train Accuracy: 0.875370919881306"
[1] "Test Accuracy: 0.905325443786982"
```

As we can see here that we we added one more significant variable to the new model the accuracy has increased to 90%.

4.0.4 LDA with 5 variable subset

```
[349]: lda.fit <- lda(crim~nox+indus+rad+tax+lstat, data = train.data)

# make predictions for the training.
train_pred <- predict(lda.fit, newdata = train.data)

# make predictions for the test
test_pred <- predict(lda.fit, newdata = test.data)
```

```
[350]: # compute the train error rates
train_error <- (1/length(train.data$crim))*length(which(train.data$crim !=
  ↪ train_pred$class))

# compute the test error rates
```

```
test_error <- (1/length(test.data$crim))*length(which(test.data$crim !=  
  ↪test_pred$class))

# compute Accuracy
print(paste('Train Accuracy:',1-train_error))
print(paste('Test Accuracy:',1-test_error))
```

```
[1] "Train Accuracy: 0.845697329376855"
```

```
[1] "Test Accuracy: 0.857988165680473"
```

LDA performed worse than the Logistic regression with an accuracy of 85%.

4.0.5 LDA with 6 variable subset

```
[351]: lda.fit <- lda(crim~age+nox+indus+rad+tax+lstat, data = train.data)

# make predictions for the training.
train_pred <- predict(lda.fit, newdata = train.data)

# make predictions for the test
test_pred <- predict(lda.fit, newdata = test.data)

# compute the train error rates
train_error <- (1/length(train.data$crim))*length(which(train.data$crim !=  
  ↪train_pred$class))

# compute the test error rates
test_error <- (1/length(test.data$crim))*length(which(test.data$crim !=  
  ↪test_pred$class))

# compute Accuracy
print(paste('Train Accuracy:',1-train_error))
print(paste('Test Accuracy:',1-test_error))
```

```
[1] "Train Accuracy: 0.836795252225519"
```

```
[1] "Test Accuracy: 0.840236686390533"
```

Here surprisingly the model's accuracy got decreased when one significant variable was added.

4.0.6 QDA with 5 variable subset

```
[352]: qda.fit <- qda(crim~nox+indus+rad+tax+lstat, data = train.data)
```

```
[353]: # make predictions for the training and test.
train_pred <- predict(qda.fit, newdata = train.data)
test_pred <- predict(qda.fit, newdata = test.data)
```



```
[354]: # defining the dependent variable.
y_hat_train <- train_pred$class
y_hat_test <- test_pred$class
```

```
y_true_train <- train.data$crim
y_true_test <- test.data$crim
```

```
[355]: # compute train and test error.
train_err <- (1/length(y_hat_train))*length(which(y_true_train != y_hat_train))
test_err <- (1/length(y_hat_test))*length(which(y_true_test != y_hat_test))
```

```
# compute Accuracy
print(paste('Train Accuracy:', 1-train_err))
print(paste('Test Accuracy:', 1-test_err))
```

```
[1] "Train Accuracy: 0.875370919881306"
```

```
[1] "Test Accuracy: 0.85207100591716"
```

Voila! here also QDA performed better than the LDA, and much closer to the logistic model. Still the logistic regression is better among all.

4.0.7 QDA with 6 variable subset

```
[356]: qda.fit <- qda(crim~age+nox+indus+rad+tax+lstat, data = train.data)
```

```
# make predictions for the training and test.
train_pred <- predict(qda.fit, newdata = train.data)
test_pred <- predict(qda.fit, newdata = test.data)
```

```
# defining the dependent variable.
y_hat_train <- train_pred$class
y_hat_test <- test_pred$class
```

```
y_true_train <- train.data$crim
y_true_test <- test.data$crim
```

```
# defining the dependent variable.
y_hat_train <- train_pred$class
y_hat_test <- test_pred$class
```

```
y_true_train <- train.data$crim
y_true_test <- test.data$crim
```

```
# compute train and test error.
train_err <- (1/length(y_hat_train))*length(which(y_true_train != y_hat_train))
test_err <- (1/length(y_hat_test))*length(which(y_true_test != y_hat_test))
```

```
# compute Accuracy
```

```
print(paste('Train Accuracy:',1-train_err))
print(paste('Test Accuracy:',1-test_err))
```

```
[1] "Train Accuracy: 0.875370919881306"
```

```
[1] "Test Accuracy: 0.875739644970414"
```

QDA haven't performed noticeably with 6 feature.

4.0.8 kNN with 5 varibale subset.

```
[357]: install.packages("class")
install.packages('ggplot2')
install.packages('gganimate')
library(class)
library(ggplot2)
library(gganimate)
```

The downloaded binary packages are in
/var/folders/ch/lqq67w6x5px6fcc9cnbp457m0000gn/T//RtmpFB0bGM/downloaded_packages

The downloaded binary packages are in
/var/folders/ch/lqq67w6x5px6fcc9cnbp457m0000gn/T//RtmpFB0bGM/downloaded_packages

The downloaded binary packages are in
/var/folders/ch/lqq67w6x5px6fcc9cnbp457m0000gn/T//RtmpFB0bGM/downloaded_packages

No renderer backend detected. gganimate will default to writing frames to
separate files

Consider installing:

- the `gifski` package for gif output
 - the `av` package for video output
- and restarting the R session

```
[358]: set.seed(122)
k_vals = c(1:20)
sig_variables <- c('nox','indus','rad','tax','lstat')

x_train <- train.data[,sig_variables]
y_train <- train.data$crim
x_test <- test.data[,sig_variables]
y_test <- test.data$crim
```

```
[359]: ''' # error rates for different k values
for (i in 1:20){
    knn_pred_vals <- knn(x_train, x_test, y_train, k=i)
    knn_error_rate <- mean(knn_pred_vals!=y_test)
```

```

        print(paste("kNN error rate when K_
↵=",k_vals[i],"is",knn_error_rate))
    }
    ...

```

```

Error in parse(text = x, srcfile = src): <text>:1:3: unexpected string constant
6: }
7: '
    ^
Traceback:

```

```

[361]: # accuracy for different k values
accu_store <- matrix(rep(NA, 20))
for (i in 1:20){
    knn_pred_vals <- knn(x_train, x_test, y_train, k=i)
    knn_error_rate <- mean(knn_pred_vals!=y_test)
    print(paste("kNN Accuracy when K_
↵=",k_vals[i],"is",1-knn_error_rate))
    accu_store[i] = 1-knn_error_rate
}

```

```

[1] "kNN Accuracy when K = 1 is 0.952662721893491"
[1] "kNN Accuracy when K = 2 is 0.952662721893491"
[1] "kNN Accuracy when K = 3 is 0.964497041420118"
[1] "kNN Accuracy when K = 4 is 0.946745562130177"
[1] "kNN Accuracy when K = 5 is 0.946745562130177"
[1] "kNN Accuracy when K = 6 is 0.923076923076923"
[1] "kNN Accuracy when K = 7 is 0.93491124260355"
[1] "kNN Accuracy when K = 8 is 0.923076923076923"
[1] "kNN Accuracy when K = 9 is 0.93491124260355"
[1] "kNN Accuracy when K = 10 is 0.917159763313609"
[1] "kNN Accuracy when K = 11 is 0.917159763313609"
[1] "kNN Accuracy when K = 12 is 0.917159763313609"
[1] "kNN Accuracy when K = 13 is 0.899408284023669"
[1] "kNN Accuracy when K = 14 is 0.905325443786982"
[1] "kNN Accuracy when K = 15 is 0.899408284023669"
[1] "kNN Accuracy when K = 16 is 0.869822485207101"
[1] "kNN Accuracy when K = 17 is 0.869822485207101"
[1] "kNN Accuracy when K = 18 is 0.875739644970414"
[1] "kNN Accuracy when K = 19 is 0.887573964497041"
[1] "kNN Accuracy when K = 20 is 0.887573964497041"

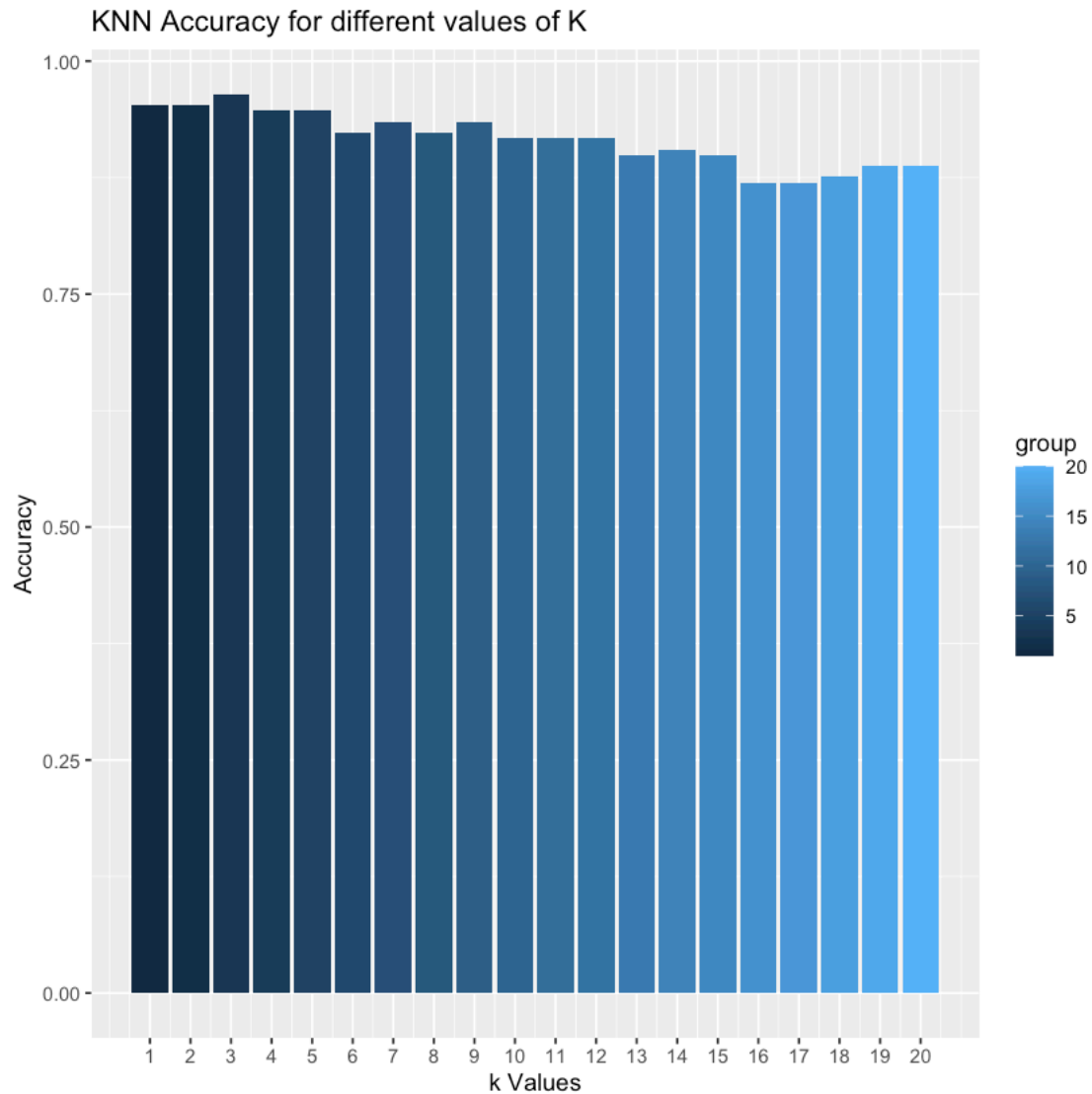
```

```

[362]: data <- data.frame(group=c(1:20), values=accu_store, frame=rep('a',20))

```

```
ggplot(data, aes(x=group, y=values, fill=group, lab='k Values')) +
  geom_bar(stat='identity')+xlab("k Values") + ggtitle('KNN Accuracy for
different values of K')+ylab("Accuracy")+scale_x_continuous(breaks = 1:20)
```



kNN magic!! the KNN achieves the best accuracy of 96% with $K = 3$.

4.0.9 kNN with 6 varibale subset

```
[363]: set.seed(368)
sig_variables <- c('age', 'nox', 'indus', 'rad', 'tax', 'lstat')
x_train <- train.data[,sig_variables]
y_train <- train.data$crim
x_test <- test.data[,sig_variables]
```

```

y_test <- test.data$crim

accu_store <- matrix(rep(NA, 20))
for (i in 1:20){
    knn_pred_vals <- knn(x_train, x_test, y_train, k=i)
    knn_error_rate <- mean(knn_pred_vals!=y_test)
    print(paste("kNN Accuracy when K_
↵=",k_vals[i], "is", 1-knn_error_rate))
    accu_store[i] = 1-knn_error_rate
}

data <- data.frame(group=c(1:20), values=accu_store, frame=rep('a',20))

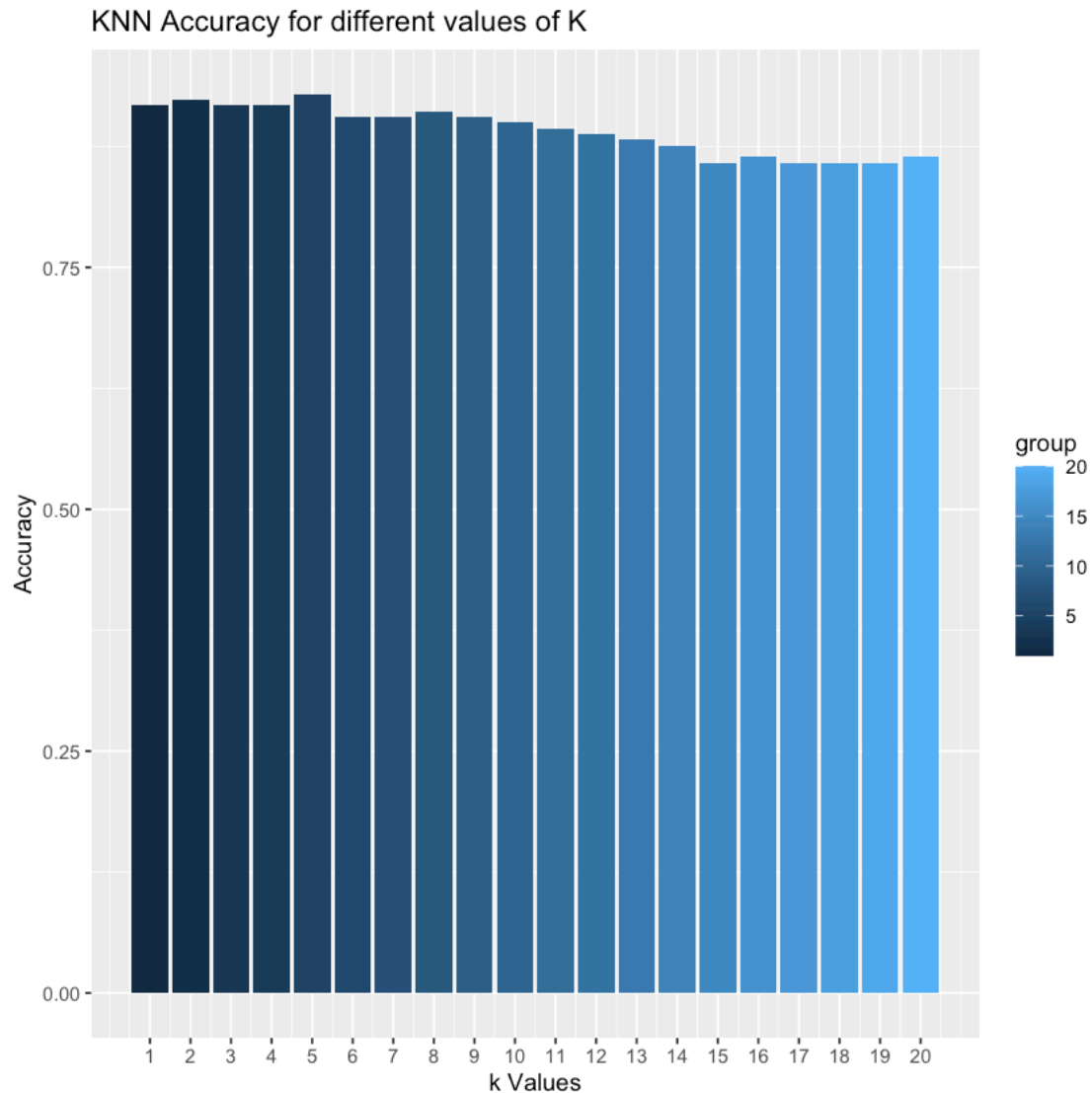
ggplot(data, aes(x=group, y=values, fill=group,lab='k Values')) +
  geom_bar(stat='identity')+xlab("k Values") + ggtitle('KNN Accuracy for_
↵different values of K')+ylab("Accuracy")+scale_x_continuous(breaks = 1:20)

```

```

[1] "kNN Accuracy when K = 1 is 0.917159763313609"
[1] "kNN Accuracy when K = 2 is 0.923076923076923"
[1] "kNN Accuracy when K = 3 is 0.917159763313609"
[1] "kNN Accuracy when K = 4 is 0.917159763313609"
[1] "kNN Accuracy when K = 5 is 0.928994082840237"
[1] "kNN Accuracy when K = 6 is 0.905325443786982"
[1] "kNN Accuracy when K = 7 is 0.905325443786982"
[1] "kNN Accuracy when K = 8 is 0.911242603550296"
[1] "kNN Accuracy when K = 9 is 0.905325443786982"
[1] "kNN Accuracy when K = 10 is 0.899408284023669"
[1] "kNN Accuracy when K = 11 is 0.893491124260355"
[1] "kNN Accuracy when K = 12 is 0.887573964497041"
[1] "kNN Accuracy when K = 13 is 0.881656804733728"
[1] "kNN Accuracy when K = 14 is 0.875739644970414"
[1] "kNN Accuracy when K = 15 is 0.857988165680473"
[1] "kNN Accuracy when K = 16 is 0.863905325443787"
[1] "kNN Accuracy when K = 17 is 0.857988165680473"
[1] "kNN Accuracy when K = 18 is 0.857988165680473"
[1] "kNN Accuracy when K = 19 is 0.857988165680473"
[1] "kNN Accuracy when K = 20 is 0.863905325443787"

```



for this subset KNN achieves the best accuracy of 93% with $K = 5$ which is also our subset size. It is clear that the best model for this particular dataset at this time is a kNN model with 5 feature subset.

4.0.10 (b) *Fit classification models in order to predict whether a given census tract has a high, medium or low crime rates. Explore logistic regression, LDA, QDA, and KNN models using various subsets of the predictors. Describe your findings.*

we will do the above same predictoin for new dataset with 3 types crim rates.

```
[365]: data(Boston)
class(Boston$crim)
summary(Boston$crim)
```

```

sd(Boston$crim)
mean(Boston$crim)

#Boston$crim[Boston$crim<sd(Boston$crim)- mean(Boston$crim)]= "Low"
#Boston$crim[Boston$crim>sd(Boston$crim)+ mean(Boston$crim)]= "High"

Boston$crim =cut(Boston$crim, breaks = c(-Inf,sd(mean(Boston$crim) -
↪Boston$crim),mean(Boston$crim) + sd(Boston$crim) ,Inf), labels =
↪c("Low", "Medium", "High"))

```

'numeric'

	Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
	0.00632	0.08204	0.25651	3.61352	3.67708	88.97620

8.60154510533249

3.61352355731225

using cut() function to categorize numerical data based on mean and sd of data.

```

[366]: set.seed(4455)
indis <- sample(1:nrow(Boston), round(2/3*nrow(Boston)), replace = FALSE)
train.data <- Boston[indis,]
test.data <- Boston[-indis,]

y_true_train <- as.numeric(train.data$crim)-1
y_true_test <- as.numeric(test.data$crim)-1

```

splitting train and test data.

4.0.11 Logistic Regression with 5 varibales

```

[367]: glm.fit <- glm(crim ~nox+indus+rad+tax+lstat, data = train.data, family =
↪"binomial")
#glm.fit <- glm(crim ~zn+nox+age+rad+prratio+black+medv, data = train.data,
↪family = "binomial")

glm.probs.train <- predict(glm.fit, newdata = train.data, type = "response")
y_hat_train <- round(glm.probs.train)
glm.probs.test <- predict(glm.fit, newdata = test.data, type = "response")
y_hat_test <- round(glm.probs.test)

train_err <- sum(abs(y_hat_train- y_true_train))/length(y_true_train)
test_err <- sum(abs(y_hat_test- y_true_test))/length(y_true_test)

print(paste('Train Accuracy:',1-train_err))

```

```
print(paste('Test Accuracy:',1-test_err))
```

```
[1] "Train Accuracy: 0.830860534124629"
```

```
[1] "Test Accuracy: 0.875739644970414"
```

4.0.12 Logistic Regression with 6 varibales

```
[368]: glm.fit <- glm(crim ~age+nox+indus+rad+tax+lstat, data = train.data, family =  
  ↪"binomial")  
#glm.fit <- glm(crim ~zn+nox+age+rad+ptratio+black+medv, data = train.data,  
  ↪family = "binomial")  
  
glm.probs.train <- predict(glm.fit, newdata = train.data, type = "response")  
y_hat_train <- round(glm.probs.train)  
glm.probs.test <- predict(glm.fit, newdata = test.data, type = "response")  
y_hat_test <- round(glm.probs.test)  
  
train_err <- sum(abs(y_hat_train- y_true_train))/length(y_true_train)  
test_err <- sum(abs(y_hat_test- y_true_test))/length(y_true_test)  
  
print(paste('Train Accuracy:',1-train_err))  
print(paste('Test Accuracy:',1-test_err))
```

Warning message:

```
"glm.fit: fitted probabilities numerically 0 or 1 occurred"
```

```
[1] "Train Accuracy: 0.830860534124629"
```

```
[1] "Test Accuracy: 0.869822485207101"
```

the model accuracy increased with the number of variables.

4.0.13 LDA with 5 varibales

```
[369]: lda.fit <- lda(crim~nox+indus+rad+tax+lstat, data = train.data)  
  
# make predictions for the training.  
train_pred <- predict(lda.fit, newdata = train.data)  
  
# make predictions for the test  
test_pred <- predict(lda.fit, newdata = test.data)  
  
# compute the train error rates  
train_error <- (1/length(train.data$crim))*length(which(train.data$crim !=  
  ↪train_pred$class))  
  
# compute the test error rates
```



```
test_error <- (1/length(test.data$crim))*length(which(test.data$crim !=  
  ↪test_pred$class))

# compute Accuracy
print(paste('Train Accuracy:',1-train_error))
print(paste('Test Accuracy:',1-test_error))
```

```
[1] "Train Accuracy: 0.86053412462908"
[1] "Test Accuracy: 0.857988165680473"
```

4.0.14 LDA with 6 Variables

```
[374]: lda.fit <- lda(crim~age+nox+indus+rad+tax+lstat, data = train.data)

# make predictions for the training.
train_pred <- predict(lda.fit, newdata = train.data)

# make predictions for the test
test_pred <- predict(lda.fit, newdata = test.data)

# compute the train error rates
train_error <- (1/length(train.data$crim))*length(which(train.data$crim !=  
  ↪train_pred$class))

# compute the test error rates
test_error <- (1/length(test.data$crim))*length(which(test.data$crim !=  
  ↪test_pred$class))

# compute Accuracy
print(paste('Train Accuracy:',1-train_error))
print(paste('Test Accuracy:',1-test_error))
```

```
[1] "Train Accuracy: 0.863501483679525"
[1] "Test Accuracy: 0.857988165680473"
```

Amazingly LDA perform the same accuracy for both different subsets.

4.0.15 QDA with 5 Variables

```
[375]: qda.fit <- qda(crim~age+nox+indus+lstat+rad, data = train.data)

# make predictions for the training and test.
train_pred <- predict(qda.fit, newdata = train.data)
test_pred <- predict(qda.fit, newdata = test.data)

# defining the dependent variable.
y_hat_train <- train_pred$class
```

```

y_hat_test <- test_pred$class

y_true_train <- train.data$crim
y_true_test <- test.data$crim

# defining the dependent variable.
y_hat_train <- train_pred$class
y_hat_test <- test_pred$class

y_true_train <- train.data$crim
y_true_test <- test.data$crim

# compute train and test error.
train_err <- (1/length(y_hat_train))*length(which(y_true_train != y_hat_train))
test_err <- (1/length(y_hat_test))*length(which(y_true_test != y_hat_test))

# compute Accuracy
print(paste('Train Accuracy:',1-train_err))
print(paste('Test Accuracy:',1-test_err))

```

```

Error in qda.default(x, grouping, ...): rank deficiency in group Medium
Traceback:

```

```

1. qda(crim ~ age + nox + indus + lstat + rad, data = train.data)
2. qda.formula(crim ~ age + nox + indus + lstat + rad, data = train.data)
3. qda.default(x, grouping, ...)
4. stop(gettextf("rank deficiency in group %s", lev[i]), domain = NA)

```

These data cannot be used to train this classifier. The issue is that the training set is rank insufficient for at least one class. The estimates in Medium cannot be obtained by inverting one or more covariance matrices because some variables are collinear. We select the most correlated variables for the forecasts, for this reason.

4.0.16 QDA with 6 Variables

```

[376]: qda.fit <- qda(crim~., data = train.data)

# make predictions for the training and test.
train_pred <- predict(qda.fit, newdata = train.data)
test_pred <- predict(qda.fit, newdata = test.data)

# defining the dependent variable.
y_hat_train <- train_pred$class
y_hat_test <- test_pred$class

y_true_train <- train.data$crim

```

```

y_true_test <- test.data$crim

# defining the dependent variable.
y_hat_train <- train_pred$class
y_hat_test <- test_pred$class

y_true_train <- train.data$crim
y_true_test <- test.data$crim

# compute train and test error.
train_err <- (1/length(y_hat_train))*length(which(y_true_train != y_hat_train))
test_err <- (1/length(y_hat_test))*length(which(y_true_test != y_hat_test))

# compute Accuracy
print(paste('Train Accuracy:',1-train_err))
print(paste('Test Accuracy:',1-test_err))

```

Error in qda.default(x, grouping, ...): rank deficiency in group Medium
Traceback:

```

1. qda(crim ~ ., data = train.data)
2. qda.formula(crim ~ ., data = train.data)
3. qda.default(x, grouping, ...)
4. stop(gettextf("rank deficiency in group %s", lev[i]), domain = NA)

```

[377]: `summary(train.data$crim)`

Low	286 Medium	18 High	33
-----	------------	---------	----

4.0.17 kNN with 5 variables

```

[378]: set.seed(122)
k_vals = c(1:20)
sig_variables <- c('nox','indus','rad','tax','lstat')

x_train <- train.data[,sig_variables]
y_train <- train.data$crim
x_test <- test.data[,sig_variables]
y_test <- test.data$crim

accu_store <- matrix(rep(NA, 20))
for (i in 1:20){
    knn_pred_vals <- knn(x_train, x_test, y_train, k=i)
    knn_error_rate <- mean(knn_pred_vals!=y_test)
    print(paste("kNN Accuracy when K_",
    ↵=" ,k_vals[i], "is", 1-knn_error_rate))
}

```

```

        accu_store[i] = 1-knn_error_rate
    }

data <- data.frame(group=c(1:20), values=accu_store, frame=rep('a',20))

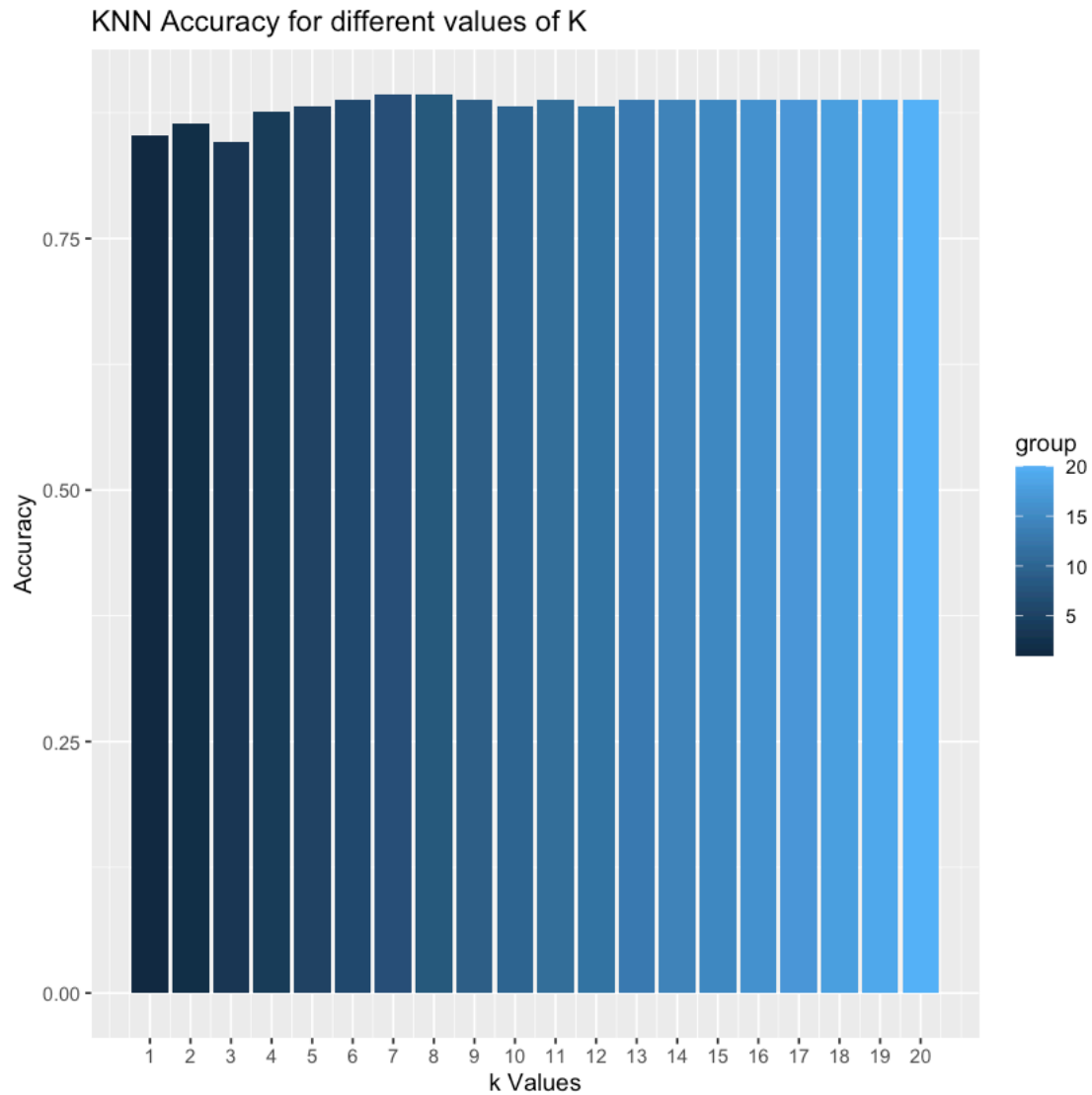
ggplot(data, aes(x=group, y=values, fill=group,lab='k Values')) +
  geom_bar(stat='identity')+xlab("k Values") + ggtitle('KNN Accuracy for_
↳different values of K')+ylab("Accuracy")+scale_x_continuous(breaks = 1:20)

```

```

[1] "kNN Accuracy when K = 1 is 0.85207100591716"
[1] "kNN Accuracy when K = 2 is 0.863905325443787"
[1] "kNN Accuracy when K = 3 is 0.846153846153846"
[1] "kNN Accuracy when K = 4 is 0.875739644970414"
[1] "kNN Accuracy when K = 5 is 0.881656804733728"
[1] "kNN Accuracy when K = 6 is 0.887573964497041"
[1] "kNN Accuracy when K = 7 is 0.893491124260355"
[1] "kNN Accuracy when K = 8 is 0.893491124260355"
[1] "kNN Accuracy when K = 9 is 0.887573964497041"
[1] "kNN Accuracy when K = 10 is 0.881656804733728"
[1] "kNN Accuracy when K = 11 is 0.887573964497041"
[1] "kNN Accuracy when K = 12 is 0.881656804733728"
[1] "kNN Accuracy when K = 13 is 0.887573964497041"
[1] "kNN Accuracy when K = 14 is 0.887573964497041"
[1] "kNN Accuracy when K = 15 is 0.887573964497041"
[1] "kNN Accuracy when K = 16 is 0.887573964497041"
[1] "kNN Accuracy when K = 17 is 0.887573964497041"
[1] "kNN Accuracy when K = 18 is 0.887573964497041"
[1] "kNN Accuracy when K = 19 is 0.887573964497041"
[1] "kNN Accuracy when K = 20 is 0.887573964497041"

```



4.0.18 kNN with 6 varibales

```
[379]: set.seed(12)
k_vals = c(1:20)
sig_variables <- c('age','nox','indus','rad','tax','lstat')

x_train <- train.data[,sig_variables]
y_train <- train.data$crim
x_test <- test.data[,sig_variables]
y_test <- test.data$crim

accu_store <- matrix(rep(NA, 20))
for (i in 1:20){
```

```

        knn_pred_vals <- knn(x_train, x_test, y_train, k=i)
        knn_error_rate <- mean(knn_pred_vals!=y_test)
        print(paste("kNN Accuracy when K_␣
↵=",k_vals[i],"is",1-knn_error_rate))
        accu_store[i] = 1-knn_error_rate
    }

data <- data.frame(group=c(1:20), values=accu_store, frame=rep('a',20))

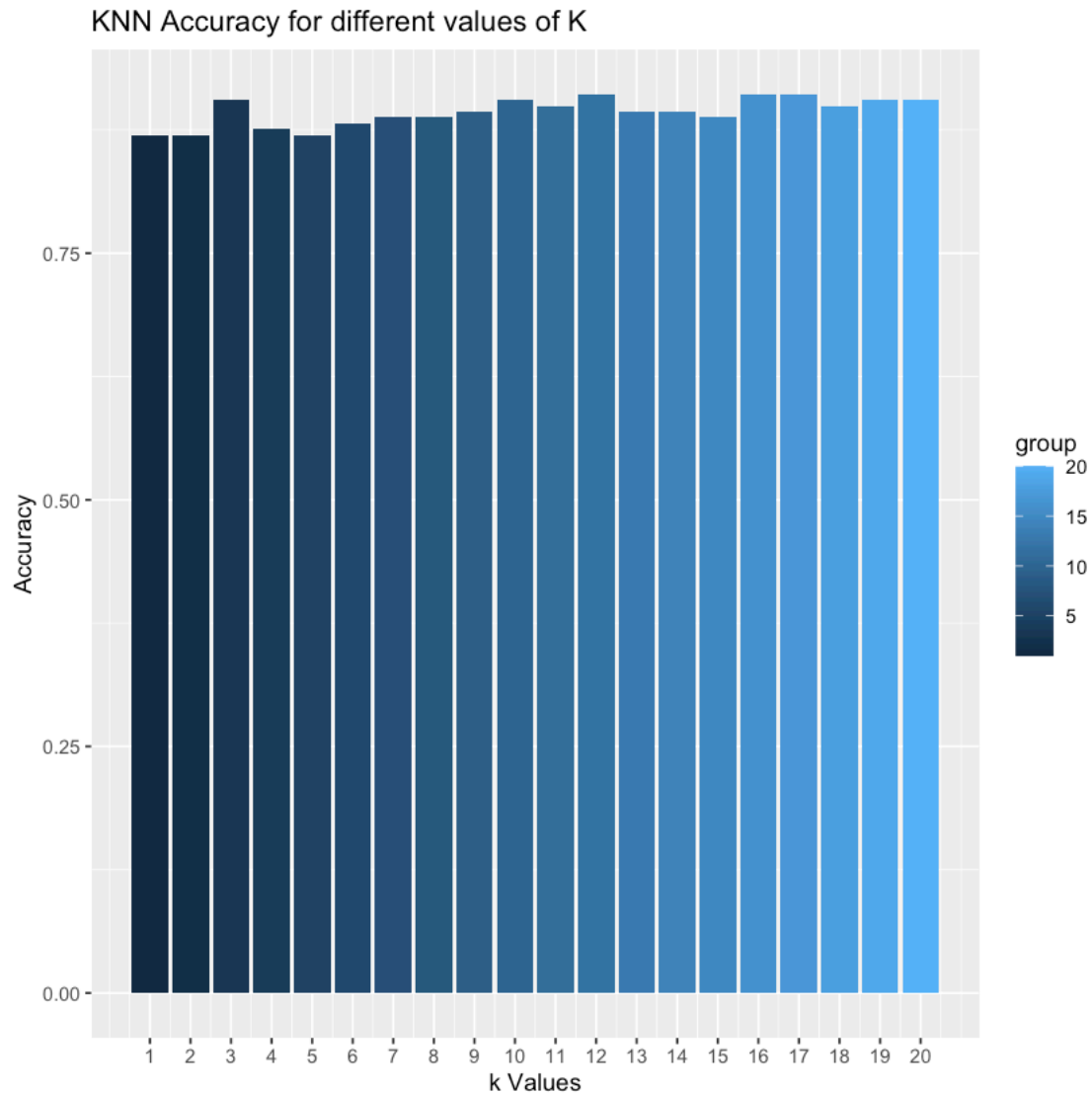
ggplot(data, aes(x=group, y=values, fill=group,lab='k Values')) +
  geom_bar(stat='identity')+xlab("k Values") + ggtitle('KNN Accuracy for_␣
↵different values of K')+ylab("Accuracy")+scale_x_continuous(breaks = 1:20)

```

```

[1] "kNN Accuracy when K = 1 is 0.869822485207101"
[1] "kNN Accuracy when K = 2 is 0.869822485207101"
[1] "kNN Accuracy when K = 3 is 0.905325443786982"
[1] "kNN Accuracy when K = 4 is 0.875739644970414"
[1] "kNN Accuracy when K = 5 is 0.869822485207101"
[1] "kNN Accuracy when K = 6 is 0.881656804733728"
[1] "kNN Accuracy when K = 7 is 0.887573964497041"
[1] "kNN Accuracy when K = 8 is 0.887573964497041"
[1] "kNN Accuracy when K = 9 is 0.893491124260355"
[1] "kNN Accuracy when K = 10 is 0.905325443786982"
[1] "kNN Accuracy when K = 11 is 0.899408284023669"
[1] "kNN Accuracy when K = 12 is 0.911242603550296"
[1] "kNN Accuracy when K = 13 is 0.893491124260355"
[1] "kNN Accuracy when K = 14 is 0.893491124260355"
[1] "kNN Accuracy when K = 15 is 0.887573964497041"
[1] "kNN Accuracy when K = 16 is 0.911242603550296"
[1] "kNN Accuracy when K = 17 is 0.911242603550296"
[1] "kNN Accuracy when K = 18 is 0.899408284023669"
[1] "kNN Accuracy when K = 19 is 0.905325443786982"
[1] "kNN Accuracy when K = 20 is 0.905325443786982"

```



kNN performed well with 6 variables when $k=3$. We can see that the accuracy has been increased as one more classifier has been introduced. So with three target classes kNN performed best with 6 variables with an accuracy of 90% which followed by LR 87% which not much difference.

4.0.19 (c) Reflect on the results from (a) and (b). Is this within your expectation, why or why not?

We can see that when the number of classes in the target variable increased the accuracy decreased, but not that much. But 90% accuracy for 3 target classes is more better than 93% for the 2 target class. Also the data was also unbalanced for the 'Medium' and 'High' targets. Replicating the data helps to lower noise, but it has little effect on improving numerical rank. So let's say you just have two data points. A distinct quadratic model cannot be estimated from the points. Even if you reproduce each point a million times, you will still only be able to fit a straight line through what are still essentially just a pair of points. Replication essentially does not increase the content

of information. It only reduces noise in areas where you already have information.

[]: