CS577
Homework #7, Problem #3
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#3

In the residual graph G_f with original weights it is not possible for $u \in S$ and $v \notin S$. Since, the flow at this edge $(f_e) \leq w'_e$ and $w'_e = w_e - 1$. We derive that $f_e \leq w_e - 1$. This equation means that the flow at this edge in the original graph is never equal to the capacity of the edge in the original graph. This means we can conclude that in G_f there will always be an edge (u, v) with at least one unit of flow available to send from u to v. If such an edge exists, it is not possible for $u \in S$ and $v \notin S$ simultaneously. Additionally, we note that the flow at this edge is left unaffected. Thus, given a maximum flow f that contains (u, v), a min-cut of the graph will result with the same capacity in both G and G'. Thus, if both graph result in the same value for capacity of maximum flow, then the initial flow found (f) is equal to the flow in G'. Therefore, f is still a maximum flow.