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#9a:

In order for $x^m = y^n$ to be true, the length of x^m must be equal to the length of y^n. In the context of the problem the power operation of x^m means x is repeated m times. Let x have c_1 amount of characters and y have c_2 amount of characters. If x^m means x is repeated the length of y times, it results in $c_1 * c_2$ characters. By the same logic y^n results in $c_2 * c_1$ characters. By the commutative property of multiplication $c_1 * c_2 = c_2 * c_1$ and the length of the two strings are the same. Therefore, the two strings can only be the same when $m = |y|$ and $n = |x|$.

```
IntegerExistence(x,y){
      lengthX = x.length;
      lengthY = y.length;
      originalY = y;
      originalX = x;

      while(lengthX != lengthY){
            if(lengthX < lengthY){
                  x += originalX;
                  lengthX = x.length;
            } else if(lengthY < lengthX){
                  y += originalY;
                  lengthY = y.length;
            }
      }

      return YES;
}
```

#9b:

If the length of the string *y (m)* is less than or equal to the length of the string *x (n)* and $x^m = y^n$ then we can look at each string for the length of *y* and compare it to *x*. Since $x^m = y^n$, it must be true that at every point until length *m* the two strings share the same characters. This means that regardless of what characters *x* has after *m* characters, *y* is a prefix to *x*. By the same note, the fact $x^m = y^n$, means that the two strings share the same characters for the entirety of the extended string. Therefore, the last *m* characters of x^m are equal to the characters of *y*. Thus, *y* is also a suffix of *x*.

#9c:

If *xy = yx* it means that either *x* or *y* must be a repeated string of the other. If *x* is an arbitrary string of size *n* and *y* is an arbitrary string of size m. Then the combination of the two in the

order *xy* cannot equal *yx* unless for each character they are equal and if one is not the multiple of the other then it is impossible for the two strings to be the same in combination. Note that the GCD of the two strings must then be the same number as the size of the smaller string. Since, the larger string is a multiple of the smaller string. If we raise the larger string to the GCD of the length of the two strings, this length will also be a multiple of the smaller string. In fact, if we take the smaller string and raise it to the length of the longer string, the length of the two strings will be the same. That is $x^n = y^m$ where n is the length of y and m is the length of x.

From the other direction if $x^m = y^n$, then the $|x^m|=|y^n|$. In order for this to be true, the length of either x or y has to be a multiple of the other. If one string is a multiple of the other, it must also be true that the longer string is already the smaller string repeated any amount of times. This means that taking the either string and concatenating at either the front of the other string or at the end, will result in the same string. Additionally, since one must be a multiple of the other, it will create the same string, as you are simply adding the smaller string one more time to either the beginning or the end. Thus, *xy = yx.*

```
IntegerExistence(x,y){
      if((x+y).equals(y+x)){
            return “YES”;
      }
      return “NO”;
}
```