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Assignment 3.2

Assignment: Section 2.2: 2, 4, 12, 16, 18, 20 (7th edition)

2.

a) $A \cap B$

b) $A - B$

c) $A \cup B$

d) $\overline{A} \cup \overline{B}$

4.

a) {a, b, c, d, e, f, g, h}

b) {a, b, c, d, e}

c) ϕ

d) {f, g, h}

12.

Let's prove $A \cup (A \cap B) \subseteq A$: Let's assume $x \in A \cup (A \cap B)$, then there are three options: $x \in A$, $x \in (A \cap B)$, $x \in A$ and $x \in (A \cap B)$. In the case of the first option, $x \in A$, we immediately get that $A \cup (A \cap B) \subseteq A$, In the case of the third option, we also get $A \cup (A \cap B) \subseteq A$ immediately, and in the case of the second option, $x \in (A \cap B)$, by definition of intersection of two sets, we get that $x \in A$ and $x \in B$, which again confirms that $A \cup (A \cap B) \subseteq A$.

16.

a) For $x \in (A \cap B)$, $x \in A$ and $x \in B$. So, $x \in A$. Since $x \in (A \cap B) \Rightarrow x \in A$, we have $(A \cap B) \subseteq A$.

b) Suppose $x \in A$. Then certainly $(x \in A) \vee (x \in B)$. So, $x \in A \cup B$. Since $x \in A \Rightarrow x \in A \cup B$, we have $A \subseteq A \cup B$

c) Suppose $x \in A \setminus B$. Then $(x \in A) \wedge (x 6\in B)$. So $x \in A$. Thus $(x \in (A \setminus B)) \Rightarrow x \in A$, so $A \setminus B \subseteq A$.

d) Suppose $x \in A \cap (B \setminus A)$. So $x \in A$ and $x \in B \setminus A$. Since $x \in B \setminus A$, $x \in B$ and $x 6\in A$. But $x \in A$! Contradiction, so our assumption $x \in A \cap (B \setminus A)$ must be wrong. Thus $A \cap (B \setminus A)$ must be empty.

e) Suppose $x \in A \cup (B \setminus A)$. Then $x \in A$ or $x \in B \setminus A$. If $x \in B \setminus A$, $(x \in B) \vee (x 6\in A)$, so $x \in B$. So $x \in A$ or $x \in B$, which implies $x \in A \cup B$. Thus, $x \in A \cup (B \setminus A) \Rightarrow x \in A \cup B$, or $A \cup (B \setminus A) \subseteq A \cup B$.

18.

a) Let's say $x \in (A \cup B)$. By the definition of union, $x \in A$ or $x \in B$. It follows
that $x \in A$, $x \in B$, or $x \in C$ holds. Using the definition of union, we conclude that

$x \in (A \cup B \cup C)$.

b) Let's say $x \in (A \cap B \cap C)$. By the definition of intersection, $x \in A$, $x \in B$, and $x \in C$. Therefore, it is apparent that $x \in A$ and $x \in B$ holds. By the definition of intersection, $x \in A \cap B$. So we conclude that $(A \cap B \cap C) \subseteq (A \cap B)$.

c) Let's say $x \in (A - B) - C$. By the definition of difference, $x \in A$, $x \notin B$, and $x \notin C$. Therefore, $x \in A$ and $x \notin C$. By the definition of difference, $x \in A - C$.

d) Let's say that $(A - C) \cap (C - B) = \phi$. Then there exits an x such that $x \in (A - C) \cap (C - B)$. By the definition of intersection, $x \in (A - C)$ and $x \in (C - B)$. By the definition of difference, it follows that $x \in A$ and $x \notin C$ and that $x \in C$ and $x \notin B$. This leads to the contradiction that $x \in C$ and $x \notin C$. Hence, our assumption is false and we conclude that $(A - C) \cap (C - B) = \phi$.

e)

A	B	C	C --- A	$B - A$	$B \cup C$	$(C - A) \cup (B - A)$	$(B \cup C) - A$
1	1	1	0	0	1	0	0
1	1	0	0	0	1	0	0
1	0	1	0	0	1	0	0
1	0	0	0	0	0	0	0
0	1	1	1	1	1	1	1
0	1	0	0	1	1	1	1
0	0	1	1	0	1	1	1
0	0	0	0	0	0	0	0

Membership table proves identity.

20.

A B	$(A \cap B)$	$\overline{B}$	$(A \cap \overline{B})$	$(A \cap B) \cup (A \cap \overline{B})$
1 1	1	0	0	1
1 0	0	1	1	1
0 1	0	0	0	0
0 0	0	1	0	0

Membership table proves identity