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Assignment: Section 5.1: 6 (see definition of n! pg. 151), 8, 14, 18, 28, 38, 40 ( 7th Edition)

6.

Using mathematical induction in the basis step, for n = 1, the equation states that 1 · 1! = (1 + 1)! − 1, and this is true because both sides of the equation evaluate to 1. For the inductive step, we assume that 1 · 1! + 2 · 2! + … + k · k! = (k + 1)! − 1 for some positive integer k. We add (k + 1)(k + 1)! to the left hand side to find that 1 · 1! + 2 · 2! + · · · + (k + 1) · (k + 1)! = (k + 1)! − 1 + (k + 1)(k + 1)! The right hand side equals (k + 1)!(k + 2) − 1 = (k + 2)! − 1. This establishes the desired equation also for k + 1, and we are done by the principle of mathematical induction.

8.

Base Case: P(0) is that 2(−7)0 = 2 = (1 − (−7))/4 = 8/4.

Assume that P(n) is true

Inductive Step: We show P(n + 1) is true.

2 − 2 · 7 + ... + 2(−7)n + 2(−7)n+1 = (1 − (−7)^n+1)/4 + 2(−7)^n+1

= (1 − (−7)^n+1 + 8(−7)^n+1)/4

= (1 − (1 − 8)(−7)^n+1)/4

= (1 − (−7)^n+2)/4

14.

We will show this by induction on n.

Base: Consider when n = 1: $\sum$1 k=1 k2^k = 1 · 2^1 = 2 and (1 − 1)2^1+1 + 2 = 2. Since these are equal, the formula holds for n = 1.

Induction: Suppose that the claim holds for n = i. That is, there is an i ∈ Z^+ such that $\sum$i k=1 k2^k = (i−1)2^i+1 +2. We need to show: $\sum$i+1 k=1 k2^k = ((i+1)−1)2(i+1)+1 +2 = i2^i+2 +2 By pulling a term out of the summation, we can write $\sum$i+1 k=1 k2^k as $\sum$i k=1 k2^k + (i + 1)2^i+1 . Now we can substitute the k case from the induction hypothesis:

$\sum$ i k=1 k2^k + (i + 1)2^i+1 = (i − 1)2^i+1 + 2 + (i + 1)2^i+1

= i · 2^i+1 – 2^i+1 + 2 + i · 2^i+1 + 2^i+1

= i(2^i+1 + 2^i+1) + 2

= i(2 · 2^i+1) + 2

$= i2^{i+2} + 2$

18.

a) P(2) is $2! < 2^2$.

b) True because 2 < 4

c) P(k) is $k! < k^k$

d) You need to show that assuming the inductive hypothesis(part c), we can show $(k + 1)! < (k + 1)^{k+1}$.

e) Multiply (k + 1) to both sides of the inequality asserted by P(k). Here we have:
$k! \cdot (k + 1) < k^k \cdot (k + 1) < (k + 1)^k \cdot (k + 1) = (k + 1)^{k+1}$

f)Since the basis and inductive step are completed, using the principle of mathematical induction, this statement is true for every integer greater than 1.

28.

Basis: Let n = 3. Then $n 2 - 7n + 12 = 32 - 7 \cdot 3 + 12 = 9 - 21 + 12 = 0$.

Inductive hypothesis: Assume for some integer $k \geq 3$ that $k 2 - 7k + 12$ is nonnegative. Inductive step:

$(k + 1)^2 - 7(k + 1) + 12 = k^2 + 2k + 1 - 7k - 7 + 12$

$= (k^2 - 7k + 12) + (2k + 1 - 7) \geq 0 + 2k + 1 - 7$

$= 2k - 6 \geq 2 \cdot 3 - 6$

$= 0$

38.

Basis: P(1) asserts that $A1 \subseteq B1$, which directly implies that $1 \cup j=1\ Aj \subseteq 1 \cup j=1\ Bj$.

Inductive: The inductive hypothesis is the statement that P(k) is true, where k is a positive integer. That is, P(k) is the statement that if $Aj \subseteq Bj$ for j = 1, 2,..., k, then $k+1 \cup j=1\ Aj \subseteq k+1 \cup j=1\ Bj$. If x is an element of $k+1 \cup j=1\ Aj$ then we can anticipate that $x \in A \vee k+1$. If $x \in A_{k+1}$, we know from the given fact that $A_{k+1} \subseteq B_{k+1}$ that $x \in B_{k+1}$. Therefore, we have shown that if the inductive hypothesis P(k) is true, then P(k + 1) must also be true. This completes the inductive argument. So based on the basis step and the inductive step, we have used mathematical induction to prove P(n) is true for all positive integers.

40.

$(A1 \cap A2) \cup B = (A1 \cup B) \cap (A2 \cup B)$........distributive Property of union over intersection

Assume that the statement is true for k, where k > 2, then:

$(A1 \cap A2 \cap \cdots \cap Ak+1) \cup B = ((A1 \cap A2 \cap \cdots \cap Ak) \cap Ak+1) \cup B$ ............Associative property for intersection

Assume that $(A1 \cap A2 \cap \cdots \cap Ak) = C$

$=(A_1 \cap A_2 \cap \cdot\cdot\cdot \cap A_{k+1}) \cup B = ((A_1 \cap A_2 \cap \cdot\cdot\cdot \cap A_k) \cap A_{k+1}) \cup B$

$=(C \cap A_{k+1}) \cup B$

$= (C \cup B) \cap (A_{k+1} \cup B)$by distributive property of union of intersection

$=((A_1 \cap A_2 \cap \cdot\cdot\cdot \cap A_k) \cup B) \cap (A_{k+1} \cup B)$

But $(A_1 \cap A_2 \cap \cdot\cdot \cap A_k) \cup B = (A_1 \cup B) \cap (A_2 \cup B) \cap \cdot\cdot \cap (A_k \cup B)$ The statement is true for k

$=((A_1 \cap A_2 \cap \cdot\cdot\cdot \cap A_k) \cup B) \cap (A_{k+1} \cup B) = (A_1 \cup B) \cap (A_2 \cup B) \cap \cdot\cdot \cap (A_k \cup B) \cap (A_{k+1} \cup B)$

= The statement is true for k+1

=By mathematical induction, the statement is true for any positive integer n.