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Assignment 2.1

Section 1.7: 2, 4, 6, 14, 16 (use the contrapositive), 18a

2.

$a + b$ even integers

$$a = 2x$$

$$b = 2y$$

$$a + b = 2x + 2y$$

$$a + b = 2(x + y)$$

Since $2(x + y)$ is an even number we can conclude that $a + b$ is even as well.

4.

$-x$ is the additive inverse of x . Thus $-x = -(2a) = 2(-a)$ for some integer a . Now simply let $b = -a$. Thus b is also an integer, and we have $-x = 2b$ for some integer b . Then $-x$ is even by definition of an even number.

6.

We can show odd numbers in the form $2n + 1$, where n is an integer.

$$(2a + 1) \cdot (2b + 1) = 4ab + 2a + 2b + 1$$

$$= 2(ab + a + b) + 1, \text{ where } n = ab + a + b$$

$$= 2n + 1$$

14.

Definition of numbers states any number that can be put in the form of p/q where p and q are integers and q is not $=0$ is a rational number. So $x = p/q$ is rational. Therefore, $1/x = q/p$ and as long as p is not equal to 0 it is rational. Therefore, $1/x$ is rational.

16.

If m and n are given to be odd, then we know the product is odd since the product of any two odd integers is always odd. Since we have proven that the contrapositive is valid, the original equivalent statement is also valid and If the product mn is even then m is even or n is even.

18a.

Let n be an arbitrary integer. Assume that n is odd, i.e., $n = 2k + 1$ for some integer k . We must show that $3n + 2$ is odd. Since $n = 2k + 1$, $3n + 2 = 3(2k + 1) + 2 = 6k + 5 = 2(3k + 2) + 1$, where $3k + 2 \in \mathbb{Z}$ since $k \in \mathbb{Z}$. So $3n + 2$ is odd.