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Section 5.3: 12, 26c ("5 | a+b" meas that 5 divides a+b), 43, 44; (7th edition)

12.

Basis: f^2_1 below = 1^2 = $f_1 f_2$.

Inductive case: Suppose $f^2_1 + f^2_2 + \cdots + f^2_n = f_n f_n+1$. Then

$$f^2_1 + f^2_2 + \cdots + f^2_n + f^2_{n+1} = f_n f_{n+1} + f^2_{n+1}$$

$$= (f_n + f_{n+1}) f_{n+1}$$

$$= f_{n+1} f_{n+2}$$

So $f^2_1 + f^2_2 + \cdots + f^2_n = f_n f_{n+1}$, when n is a positive integer.

26.

c) Basis: Holds because 5 |(0 + 0 = 0)

Recursive: Suppose a + b = 5k for some integer k. Then 5|(a+2)+(b+3), because (a+2)+(b+3) = a+b+5 = 5k+5 = 5(k+1), where k+1 is also an integer. Similarly, 5|(a+3)+(b+2), because (a+3)+(b+2) = a+b+5 = 5k+5 = 5(k+1), where k + 1 is also an integer.

43. Basis: n(T) = 1 and h(T) = 0, and 1 >= 2* 0 + 1

Recursive: Assume result holds for all binary trees smaller than T. Need to show that m(T) >= 2h(T) + 1 for the binary tree T. Based on the recursive definition of a full binary tree, T is formed by two subtrees T_1 and T_2, where T_1 and T_2 are smaller than T. By the induction hypothesis, we know that the inductive hypothesis holds for T_1 and T_2, i.e. $n(T_1) >= 2h(T_1)+1$ and $n(T_2) >= 2h(T_2) + 1$. By the recursive definition of n(T) and h(T), we have $n(T) = 1 + n(T_1) + n(T_2)$ and $h(T) = 1 + \max(h(T_1), h(T_2))$. We can then show that:

$n(T) = 1 + n(T_1) + n(T_2)$
$>= 1 + 2h(T_1) + 1 + 2h(T_2) + 1$
$>= 1 + 2\max(h(T_1), h(T_2)) + 2$
$= 1 + 2(\max(h(T_1), h(T_2)) + 1)$
$= 1 + 2h(T)$

44.

The base binary tree has a single node: its root. As it has no descendants, it is thought of as a leaf, giving # of leaves − # of internal nodes = 1.

More generally, suppose that this is true for a given pair of binary trees T_1 and T_2, and form a new tree T from these by attaching them to a new root node. This preserves all the old leaf nodes and adds one new internal node (the new root), giving:

(# of leaves of T) − (# of I.N.s of T)
= (# of leaves of T_1 + # of leaves of T_2) − (# of I.N.s of T_1 + # of I.N.s of T_2 + 1)
= (# of leaves of T_1 − # of I.N.s of T_1) + (# of leaves of T_2 − # of I.N.s of T_2) – 1
= 1 + 1 − 1 = 1,

using the inductive assumptions. By structural induction, this demonstrates the equality for all full binary trees.