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Section 10.1: 24a, 28; Section 10.2: 6, 16, 18 (think about Pigeons), 26 a, b, c (think about colorings)

Section 10.4: 12 (a,b,c), 18

10.1

24.

a) You can model it as a directed graph where each email address is represented by a vertex. An edge can be used at the start of the email address represented by x and ends at the email address y only if there's a message sending from x to y. Multiple edges should be allowed because you can send multiple emails to the same address. Loops should be allowed because you can email yourself.

28.

Like above, you can model it as a directed graph. Each subway station can be represented by a vertex. An edge starts at station stop x and ends at y if there's a subway that goes between the 2 stations. Multiple edges should be allowed because multiple subways can go from one station to another. Loops should not be allowed since we do not need to have the station going to itself.

10.2

6.

Let the vertices of a graph be the people at the party, with an edge between two people if they shake hands. Then the degree of each vertex is the number of people that person shakes hands with.

The sum of the degree is even (2 * edges).

16.

The in degree is the number of pages that have a link pointing to a. The out degree is the number of pages that a has a link pointing to it.

18.

A graph with 2 vertices has either 0 or 1 edges. And in either case, the two nodes have the same degree. Assume that the theorem holds for k vertices and there are two or more of the same degree. The existing vertices have at most k – 1 different degrees. The lowest degree of a vertex of k is 0. If all vertices have different degrees k could be 0 in this situation. We can also assume that in this situation that j is k – 1. If j has a vertex j – 1, then there's an edge between j and k thus making k = 0 an impossibility. So now we know that the possible degrees of vertices in the graph are from 1 to k – 1. And since there's k vertices in the graph, there must be at least 2 vertices that have the same degree.

26.

a) For $n = 1$ and $n = 2$, K_n is bipartite (K_1 is trivially so, and K_2 is two-colorable). For $n \geq 3$, though, there exist cycles of odd length; for instance, (a, b, c, a) for any three distinct points a, b, c in the graph, so when $n \geq 3$ K_n is not bipartite.

b) C_n is bipartite for n even, and not bipartite for n odd. This is because all cycles are of even length when n is even, but there exist cycles of odd length for n odd.

c) W_n is not bipartite for any n, since it is not twocolorable: if the center point were, say blue, then all other points would be forced to be red, and then there would be adjacent reds.

10.4

12.

a) Not strongly connected. It is weakly connected.

b) It is strongly connected

c) It is not strongly or weakly connected.

18.

Assume there is a directed path from a to b. a and b are both nodes s in the strongly connected component, visiting vertices a, $v_1, \ldots, v_n$, b. We need to show that v_i, $i = 1, \ldots, n$ is also in the strongly connected component. There is a path from v_i to u and from u to v_i. Consider any vertex v_i on the path and any node u in the strongly connected component. From v_i we can reach b, from which we know that we can reach u, since b and u are in the strongly connected component. Second, from u we can reach a, since they are in the same component and then from a we can also reach v_i by the directed path a, $v_1, \ldots, v_i$. Since we show that there is path from v_i to u and from u to v_1, where u is any node in the strongly connected component, we show that v_i is also in the same strongly connected component.