Question 1

10 / 10 pts

Skip to question text.

Let P, Q, and R be the propositions

P : Tom is smart.

Q : Tom is lazy.

R : Tom is successful.

Express each of the following compound propositions as an English sentence.

a) $P \rightarrow R$

b) $R \leftrightarrow (P \wedge \neg Q)$

c) $\neg P \wedge \neg R$

d) $R \rightarrow (P \vee \neg Q)$

Your Answer:

a) If Tom is smart then he is successful

b) Tom is successful if and only if he is smart and not lazy

c) Tom isn't smart and he isn't successful

d) If Tom is successful then he is either smart or he is not lazy

Question 2

3 / 5 pts

Skip to question text.

Based on the diagram, indicate which of the following relationships are true and which are false - numberline.pdf

Your Answer:

a) True

b) False

c) False

d) True

e) False

Question 3

10 / 10 pts

Answer the following question -

Let A and B be sets. Prove that $A - B \subseteq A$.

Your Answer:

A minus B is a subset of A. Means that every element of A - B is an element of A. Since A = A subtracting B would still make A - B a subset of A because you need A in order to determine this subset. So if you assume any integer as A and any integer as B the only conclusion is that A - B is a subset of A

Ds

Question 4

10 / 10 pts

Skip to question text.

Let B(x), W(x), and S(x) be the predicates
B(x) : x is a good basketball player
W(x) : x is a good weight lifter
S(x) : x is strong
Express each of the following English sentences in terms of B(x), W(x), S(x), quantifiers, and logical connectives. Assume the domain is all people.

(You may need to use these symbols : : ≥ ≤ ≠ ¬ ∧ ∨ ⊕ ≡ → ↔ ∃ ∀)

a) All good weight lifters are strong.

b) Some good weight lifters are not a good basketball player.

c) If someone is strong then he is a good basketball player or a good weight lifter.

d) Every person who is a good basketball player is also a good weight lifter.

Your Answer:
∀ = for all

∃ = there exists

a) ∀x(W(x) ∧ S(x))

b) ∃x(W(x) ∨¬B(x))

c) ∃x(S(x) --> B(x) ∨ W(x))

d) ∀x(B(x) --> W(x))

DS

Question 5

10 / 10 pts

Skip to question text.

Use truth tables to show that the two compound propositions $((P \rightarrow Q) \rightarrow R)$ and $((P \wedge \neg Q) \vee R)$ are logically equivalent.

You may start by copying the following table to fill in:

Your Answer:

P	Q	R	$P \rightarrow Q$	$P \wedge \neg Q$	$(P \rightarrow Q) \rightarrow R$	$(P \wedge \neg Q) \vee R$
T	T	T	T	F	T	T
T	T	F	F	T	F	F
T	F	T	T	F	T	T
T	F	F	T	F	T	T
F	T	T	T	F	T	T
F	T	F	F	T	F	F
F	F	T	T	F	T	T
F	F	F	T	F	F	F

Based on truth table, they are logically equivalent.

Question 6

8 / 10 pts

Use a direct proof to show that the sum of two rational numbers is rational. (Recall that a number is rational if and only if it can be expressed as the ratio of integers.)

Your Answer:

The sum of two rational numbers will always be rational. Let's assume A and B are rational numbers. If we take the sum of A and B by definition of mathematical law the result will be a rational number as you can not get an irrational number from adding 2 irrational numbers. Say you take 1 and 2. You will get 3 every time. This works for any ratio of integers.

Question 7

9 / 10 pts

Use a proof by contraposition to show that if n - 5m is odd, then n and m are of opposite parity. (Hint : Parity indicates whether a number is odd or even. Use both cases.)

Your Answer:

Contrapositive proof: Negate both terms and reverse the direction of the inference . AKA if A, then B ---> if not B, then not A

A = n - 5m is odd

B = n and m are the opposite of parity

Using this definition we say that n and m are not the opposite of parity, when means that n - 5m is even. If n - 5m is even we can assume that 2k = 2(n- 5m). By mathematical law anything multiplied by a 2 will always give an even number. Because of this and using contrapositive we know that n - 5m is odd.

Question 8

9 / 15 pts

Skip to question text.

Compute the value of the following sums.

You do not need to do all the arithmetic to obtain a final number.
It is sufficient for you to produce a closed form expression for the answer that could be easily evaluated with a calculator such as: **6 * 2^16 + 5**.
The notes included near the top of this exam includes summation formulae that must be used to help you compute the sums for this question.
1) sigma 3 <= j <= 6;   (2* j^2 + $(-1)^j$)
2) sigma 2 <= i <= 8 ; 3^{i+2}

Your Answer:

1) 2* 3^2 + (-1)3 ...2* 6^2 + (-1)6 = 2*3^2 -1 + 2* 4^2 + 1 + 2*5^2 -1 + 2*6^2 + 1 = 17 + 33 + 49 + 73 = 172

2) 3^2^{+2} + 3^3^{+2} + ... + 3^8^{+2} = 3^4 + 3+5 + 3^6 + 3^7 + 3^8 + 3^9 + 3^10

Note for number 1 the math was done in my head but it should be properly laid out and for number 2 I can't compute that in my head without a calculator, hopefully this is sufficient

Question 9

9 / 10 pts

Skip to question text.

Use mathematical induction to prove that for any positive integer n,
3^0 + 3^1 + ...+ 3^n = 1/2 (3^{n+1} - 1)

Your Answer:

Basis: P(0) is true because 3^0 = 1 and 1/2(3^1 - 1) = 1

Inductive: The inductive hypothesis is the statement that P(k) is true, where k is a positive integer up to n that is P(k) is the statement 3^0 = 1/2(3^1 -1)

To complete the inductive step we must show that if P(k) is true then P(k+ 1) is also true. To do this we need to express P(k+1) in terms of P(k). That means if we express (n+1)^0 = 1/2(3^n+1 - 1) and show the difference is not a negative then we prove that (n+1)^0 = 1/2(3^n+1 - 1) since 3^0 is not negative by inductive hypothesis. Because the difference is not negative whenever n>= 0 it also means that 1/2(3^n+1 - 1) must also be a non negative number. This completes the inductive argument and thus proves the basis step and makes it true for all P(n).

Base step is wrong (-1)

Question 10

7 / 10 pts

Use strong induction to prove that every amount of postage of 12 cents or more can be formed using just 4-cent and 5-cent stamps.

Your Answer:

Let P(n) be tbe statement that we can form in cents of postage using just 4 and 5 cent stamps. We want to prove that P(n) is true for all n >= 12

Basis:

12 = 4 + 4 + 4 (3 4 cent stamps)

13 = 4 + 4 + 5 (2 4 cent stamps and 1 5 cent stamp)

14 = 4 + 5 + 5 (1 4 cent stamp and 2 5 cent stamps)

Assume we can form any cost of postage by the inductive hypothesis.

Inductive: for any value of k + 1 greater than or equal to 14 we can form k + 1 cents worth of postage from only 4 and 5 cent stamps.

P(K-3) + one 4 cent stamp should be used to show P(K+1).