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Assignment: Section 5.2: 2, 4, 12, 30 (7th edition)

2.

Basis: We are told the first 3 dominoes fall, so P(1) is true.

Inductive: Assuming the inductive hypothesis that P(j) is true for any positive integer j < k, we must show that P(k+1) is true. If k = 2, because the first domino falls, we know that the domino three farther down in the arrangement also falls. Namely, the domino in position 4 falls. Since we are told that the dominoes at 2 and 3 fall, we know that P(2) is true. If k > 2, then the inductive hypothesis told us that P(k-2) is true, which means that the dominoes at n - 2, n - 1 and n position fall. Because when a domino falls, the domino three farther down in the arrangement also falls, we know that the dominoes at the n + 1, n + 2, n + 3, which shows that P(n) is true. So we have proved that P(n) is true for all positive integers. Because far all possible integer n, the dominoes at the n, n+1 and n+2 position in an infinite arrangement fall, we show that all dominoes in the infinite arrangement fall.

4.

a) P(18) is true using one 4 cent stamp and 2 7 cent stamps. P(19) is true using one 7 cent stamp and 3 4 cent stamps. P(20) is true using 5 4 cent stamps. P(21) is true using 3 7 cent stamps.

b) The inductive hypothesis P(k) is that just use 4-cent and 7-cent stamps, we can form i cents postage for all i with 18 <= i <= k, where we assume that k >= 18.
c) Need to prove that k+1 cents postage using only 4 and 7 cent stamps
d) Because k >=21 is true you can deduce that k+1 cents of postage is true because we know that P(k-3) is true
e)Since the basis and inductive step are completed, the statement for every int over 18 is true

12.
Basis: P(1) implies that 1 = 2^0 and P(2) implies that 2 = 2^1
Inductive: Let k ≥ 1, and assume the claim holds for all n with 1 ≤ n ≤ k. We wish to show that the claim holds for k + 1.
Case 1: k + 1 is even. If k + 1 is even, then (k + 1)/2 is an integer. Moreover, it is an integer between 1 and k. By strong inductive hypothesis, the claim holds for (k + 1)/2. This lets us write k + 1/2 = 2^a1 + 2^a2 + ... + 2^am, where a_1, ..., a_m are all distinct. Multiplying both sides by 2 yields
k + 1 = 2(2^a1 + 2^a2 + ... + 2^am) = 2^a1+1 + 2^a2+1 + ... + 2^am+1 . Since a_1 + 1, ..., a_m + 1 are all distinct (because a1, ..., am) were all distinct, we have written k + 1 as a sum of distinct powers of two. Therefore, this is true.
Case 2: k + 1 is odd. Using the strong inductive hypothesis, we may write k = 2^b1 + 2^b2 + ... + 2^bt , where b^1, ..., b^t are all distinct. Since k + 1 is odd, we know k is even. This implies that none of the b_t's are equal to 0: if one were, then we would have:
k = 20 + (higher powers of 2) = 1 + (an even number),
meaning k would be odd, but it's not. Thus we can write: k + 1 = 1 + 2^b1 + 2^b2 + ... + 2b^t = 2^0 + 2^b1 + 2b^2 + ... + 2b^t , where 0, b_1, ..., b_t are all distinct. This means we have written k + 1 as a sum of distinct powers of two, so the claim is true (in the case where k + 1 is odd). Having proven the inductive step for the cases of k + 1 even and k + 1 odd, we have that the inductive step holds Since the base case

holds, and since the inductive step holds, the claim is true for all positive integers n; that is, any positive integer n can be written as a sum of distinct powers of 2.

30.

Base: P(0) is equivalent to a^0 = 1, which is true by definition of a^0.

Inductive: By induction hypothesis, a^k = 1 for all k ∈ N such that k ≤ n. But then a^n+1 = a^n x a^n / a^n-1 = 1 x 1 / 1 = 1 which implies that P(n + 1) holds. It follows by induction that P(n) holds for all n ∈ N, and in particular, a^n = 1 holds for all n ∈ N.

Solution: The flaw comes in the inductive step, where we implicitly assume n ≥ 1 in order to talk about a^n−1 in the denominator (otherwise the exponent is not a nonnegative integer, so we cannot apply the inductive hypothesis). We checked the base case only for n = 0, so we are not justified in assuming that n ≥ 1 when we try to prove the statement for n + 1 in the inductive step. And of course the proposition first breaks precisely at n = 1.