Question 1

Not yet graded / 10 pts

Use a proof by contraposition to show that if *m* "and" *n* are integers and $m \cdot n$ is even, then *m* is even or *n* is even.

Your Answer:

Assume not Q and prove not P

Taking the definition of proof by contraposition, we assume that m and n are integers and m * n is odd, then m is odd or n is odd. If one number is odd and the other is even and you multiply them together it will always produce an even number by laws of mathematics. So, therefore if m and n are integers where one of which is odd and you multiply them together you will get an even number which by contraposition proves this theorem.

Question 2

Not yet graded / 10 pts

For any set A, B and C,

prove that $(A - B) - C \subseteq A - C$

Your Answer:

(A - B) - C must be a subset of A - C because by laws of subtraction anything within A and C will not be changed because you are subtracting from B. Even if B removes everything from A and C, A and C still exist within that original set and therefore A - C would be a subset of (A -B) - C

Question 3

7 / 10 pts

Skip to question text.

Use mathematical induction to prove that for every integer n greater than 1, $n! < n^n$

Your Answer:

Basis step: n = 2. For n = 2, 2! < 22

Inductive step: We assume that n = 2 will hold true for $n! < n^{n.}$ The laws of mathematics dictate that taking the power of a number n will always be greater than the factorial of that same number. Therefore, it is true that all numbers n greater than 1 will be n! < nn

induction step is wrong (-3)

Question 4

Not yet graded / 10 pts

Give a recursive definition of the sequence $\{a_n\}$ "where" $a_n = (n+1)^2$.

Your Answer:
Basis: Specify that a0 = $(0 + 1)^2 = 1^2 = 1$

Recursive: $a_n = (n + 1)^2$ so therefore $a_{n+1} = (n+ 1 + 1)^2 = (n+ 2)$ 2 = n2 + 4n + 4 so a_{n+1} = a0 + 4n + 4

Question 5

10 / 10 pts

Skip to question text.

Consider the following recursive definition of a set *S* of ordered pairs of integers.

Base Case: $(0,0) \in S$

Recursive Case: "If" $(a,b) \in S$,"then" $(a+2, b+1) \in S$ *and* $(a+1, b+2) \in S$.

Use structural induction to prove that for any $(a,b) \in S$, it is the case that 3|a + b.

Your Answer:
Basis step: This holds for the basis because (0, 0) holds for 3|a+b.

Recursive step: Need to show that if 3|a+b holds, then this also holds for the elements obtained from (a,b). Therefore, we must consider both cases.

Case 1: (a+2, b+1)

If 3|a+b then 3|a+2 + b+ 1. Since these are still elements with 3|a+b the hypothesis holds for this case

Case 2: (a+ 1, b+ 2)

If 3|a+b then 3|a+1 + b+ 2. Since these are still elements with 3|a+b the hypothesis holds for this case

Question 6

10 / 10 pts

Skip to question text.

Answer the following questions -

a) Show that if seven integers are selected from the first 10 positive integers, there must be at least two pairs of these integers with the sum 11.

b) How many 7-digit binary strings (0010100, 1010101, etc.) have an even number of 1's ?

Your Answer:
a) If you divide the first ten positive integers into the following five groups: {1, 10}, {2, 9}, {3, 8}, {4, 7}, {5, 6} you will notice that the sum of the two numbers in any of these groupings is equal to 11. Since we choose 7 numbers and there's 5 groups, by Dirichlet's principle at least two numbers are in the same

group. Their sum is equal to 11. Besides the 2 numbers in one group, we chose 5 other number from 4 other groups. There are more numbers than groups so therefore at least two numbers are in one group. There two numbers form another pair with the sum equal to 11.

b) An even number of 1's would mean that the binary string either has 0 1's, 2 1's, 4 1's or 6 1's. Therefore the total is C(7,6) + C(7, 4) + C(7, 2) + C(7, 0)

Question 7

Not yet graded / 10 pts

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15 people on a softball team show up for the game. Of the 15 people who show up, 4 are women. How many ways are there to choose 7 players to take the field if at least one of these players must be a woman?

Your Answer:

4 C(15, 7)

Question 8

Not yet graded / 10 pts

How many solutions does the equation have, x1 + x2 + x3 + x4 = 15, where x1, x2, x3 and x4 are non-negative integers?

Your Answer:

To count the number of solutions, you must note that a solution corresponds to selecting 15 items from a set with 4 elements so that x1 items of type one, x2 items of type two, etc. Therefore, the number of solutions is equal to the number of 15 combinations with repetitions allowed from a set with 4 elements. Using Theorem 2, the equation has: C(4 + 15, 1, 15) = C(18, 15) possible solutions.

Question 9

10 / 10 pts

Prove that if a graph has a Euler circuit, then all of its vertices must have even degree.

Your Answer:

A Euler circuit is a circuit that uses every edge of a graph exactly once. A Euler circuit starts and ends at the same vertex. Suppose that we have a graph A and it has a Euler circuit of C. For every vertex in A, each edge has a as an endpoint shows up exactly once in C. Since C enters a the same numbers of times that is leaves, say x times, we know that a has degree of 2x and therefore by laws of multiplcation anything multiplied by 2 with be even and therefore all the vertices will have an even degree.

Question 10

Not yet graded / 10 pts

Skip to question text.

For the following graph consider running Dijkstra's algorithm to find the shortest path between vertices a and z. Give the order that the vertices will be added to the "solved vertex set" S during during the run of the algorithm. Also give the labels for each of the vertices when they are added to the set S. (You do not need to give the label of each vertex in the graph at each iteration, just the vertex that is added to S.)

Your Answer:
Let's start with S(a=0)

1st iteration: S(a =0, b =2)

2nd iteration: S(a =0, b =2, e =2)

3rd iteration: S(a =0, b =2, e = 2, d = 1)

4th iteration: S(a =0, b =2, e = 2, d = 1, z = 2)

Since z is now solved, this completes the algorithm. The shortest path is a b e d z with a length of 7