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Assignment: Section 1.7: 8 (use contradiction), 22, 24, 26, 30; Section 1.8: 8, 30, 36

1.7:

8. Suppose that n = x^2 and n + 2 = y^2 , with both x and y nonnegative naturals. We know a formula for the difference of squares: (n + 2) − (n) = x 2 − y 2 2 = (x − y)(x + y). Since 2 is a prime number, one of these factors must be 2 and the other must be 1. Since both x and y are assumed nonnegative, it must be that x +y > x −y, so x −y = 1 and x +y = 2. This system does not have a solution over the naturals: the first equation begets the substitution x = 1+y, which transforms the second into x +y = (1+y)+y = 1 + 2y = 2. Whenever y is an integer, this middle expression is an odd number, and hence can never be equal to 2. There can be no pair of perfect squares whose difference is 2.

22. Let A be the number of blue socks, and A the number of black socks. Suppose you have neither a pair of blue socks nor a pair of black socks. Then A ≤ 1 and B ≤ 1. Thus the total number of socks you've chosen is A + B ≤ 2. Thus, if you have a pair of their blue nor black socks, then you've drawn fewer than 3. Taking the contrapositive, if you take three (or more), you must have either a pair of blue socks or a pair of black socks.

24. Assume that we find a way to choose 25 days such that no more than 2 days fall on the same day of the week. Given that there are 7 days in the week, we have chosen no more than 2X7=14 days. This contradicts the assumption that we have chosen 25 days (25>14). So the initial assumption is false, if we choose 25 days, than at least 3 of them will fall on the same day of the week.

26. We prove first the direct implication. Assume n is even. Then n = 2k for some integer k. Then 7n + 4 = 14k + 4 = 2(7k + 2), which is even. For the converse, which is "if 7n + 4 is even, then n is even", we use a proof by contrapositive. The contrapositive is: "if n is not even (that is, odd), then 7n + 4 is not even (that is, odd)". If n is odd, then n = 2k + 1, for some integer k. Then 7n + 4 = 14k + 11 = 2(7k + 5) + 1, which is odd.

30i., that a < b. Dividing by 2, we get a 2 < b 2 . Adding a 2 to both sides of the inequality, we get a 2 + a 2 < a 2 + b 2 = a + b 2 . Therefore a < a+b 2 , or equivalently, a+b 2 > a.

30ii. Assume that a+b 2 is not less than b (this is the negation of (iii)). So a+b 2 > a and a+b 2 ≥ b. Adding both inequalities, we conclude that a + b > a + b, which is a contradiction. (If x > y and s ≥ t, then x + s > y + t.)

30iii. So a+b 2 < b. Multiplying by 2, we get a + b < 2b. Subtracting b gives a < b.

1.8:

8. 1 + 2 = 3. Therefore, an integer like this exists. This is constructive.

30. If we solve for y, we have $y = \pm\sqrt{\frac{14-2x^2}{5}}$. This will only be a real number if $7 - x^2 \geq 0$. The only possible integer solutions for a real value of y are x=0,1,2. Testing these in the equation, we have possible y values of $y = \pm\sqrt{\frac{14}{5}}, \pm\sqrt{\frac{12}{5}}, \pm\sqrt{\frac{6}{5}}$. None of which are themselves integers.

36. Let a be a rational number and b be an irrational number. We construct b+a/2 and prove that b+a/2 is an irrational number by contradiction. Suppose b+a/2 is a rational number. By the definition of rational number, we have b+a/2 = s/t and r = p/q, where s, t, p and q are integers and t≠ 0, q≠ 0. Then b = 2 ∗ b + a/2 − a = 2s/t – p/q = 2sq – pt/qt and 2sq − pt, qt are integers, qt≠ 0. By the definition of rational number, b is rational, which contradicts with the proposition that b is irrational. So we proved that b+a/2 is an irrational number.