

Given an array of  $[4, 2, 5, 3, 10, -5, 2, 8, -3, 6, 7, -4, 1, 9, -1, 0, -6, -8, 11, -9]$  integers, find the maximum and minimum product that can be obtained by multiplying two integers from the array.

Given array is  $[4, -2, 5, 3, 10, -5, 2, 8, -3, 6, 7, -4, 1, 9, -1, 0, -6, -8, 11, -9]$

we need to consider the largest and smallest products that can be formed by selecting the numbers from the array.

1. Sort the array.

Sorted array is  $[-9, -8, -6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11]$

2. Identify possible candidates for maximum product

3. Identify possible candidates for minimum product.

Calculating maximum product:-

→ The two largest positive integers are 10 and 11  
 $10 \times 11 = 110$ .

→ The two smallest negative integers are -9 and -8  
 $-9 \times -8 = 72$ ,

→ The maximum product is 110.

Calculating minimum products

→ largest positive and negative number is 11 and -9  
 $11 \times -9 = -99$

→ The smaller negative numbers are  
 $-9 \times -8 = 72$ .

-99 is smaller than 72 so,

maximum product = 110.

and minimum product = -99.

(12) Demonstrate the binary search method to search for the  $= 23$  from the array  $= \{2, 5, 8, 12, 16, 23, 38, 56, 72, 91\}$

Sol: Given Key  $= 23$  and array  $= \{2, 5, 8, 12, 16, 23, 38, 56, 72, 91\}$ .

1. Initialise pointers

low  $= 0$  and high  $= 9$

Calculate  $mid = \left\lfloor \frac{low + high}{2} \right\rfloor = \left\lfloor \frac{0 + 9}{2} \right\rfloor = 4$

Compare  $arr[mid]$  with key:

$arr(4) = 16$

Since  $16 < 23$  update  $low = mid + 1 = 5$

Calculate  $mid = \left\lfloor \frac{low + high}{2} \right\rfloor = \left\lfloor \frac{5 + 9}{2} \right\rfloor = 7$

Compare  $arr[mid]$  with key:

$arr(7) = 56$

Since  $56 > 23$  update  $high = mid - 1 = 6$

$mid = \left\lfloor \frac{5 + 6}{2} \right\rfloor = 5$

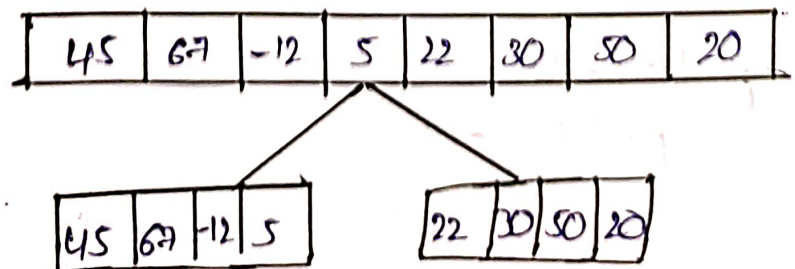
$arr[mid] = arr(5) = 23$

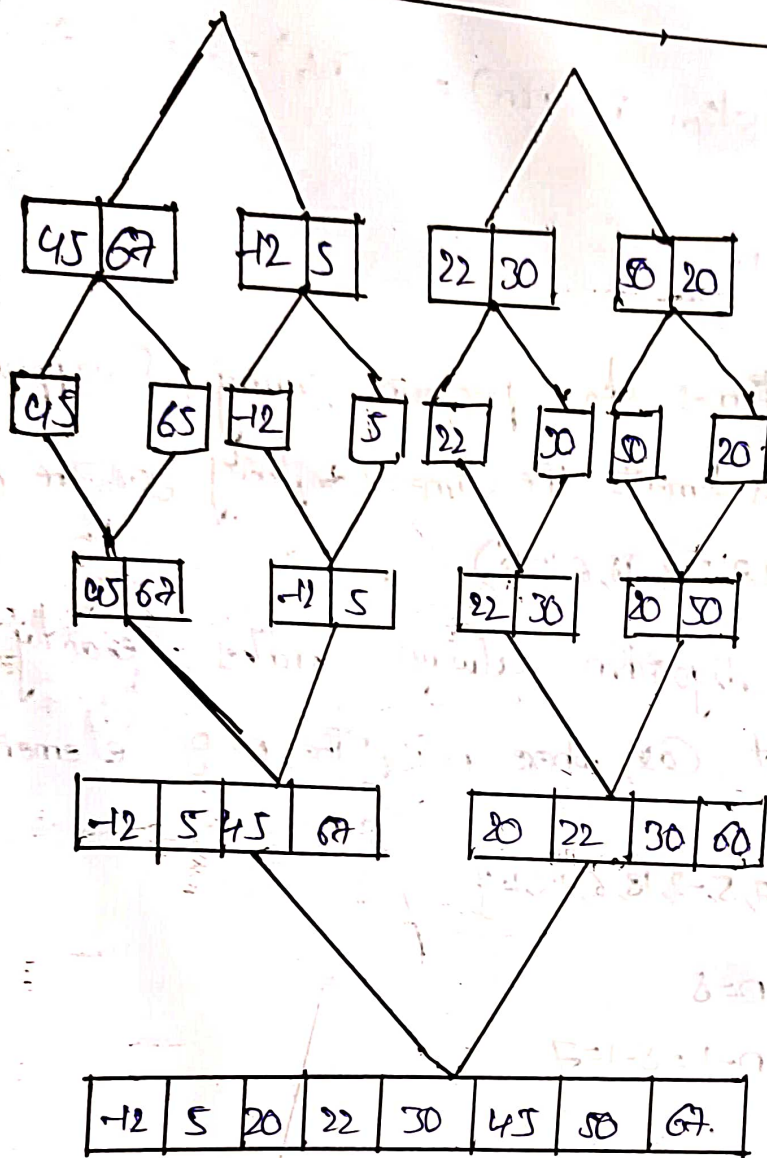
$23 == 23$  the key is found.

$\therefore$  the key  $= 23$  is found at index 5.

(13) Apply merge sort and other list of 8 elements, data  $d = (45, 67, -12, 5, 22, 30, 50, 20)$  set up Recurrence Relation for the number of key comparisons made by merge.

Sol:





∴ the sorted list =  $[-12, 5, 20, 22, 30, 45, 50, 67]$

Recurrence Relation for Comparisons:

$$T(n) = 2 + (n/2) + O(n)$$

If  $n=1$ ,  $T(1)=0$  base case.

→ At each level of recursion we make at most  $n-1$  comparisons to merge two halves of size  $n/2$  so it becomes.

$$T(n) = 2T + (n/2) + (n-1)$$

Solving Recurrence Relation we get

$$T(n) = n \log_2 n - n + 1$$



$$T(n) = O(n \log n)$$

∴ The Recurrence Relation is  $T(n) = 2T(n/2) + O(n)$  or more precisely.

$$T(n) = n \log_2 n - n + 1.$$

(14) find the no. of times to perform swapping for selection sort also ascertains the time complexity for the order of relation sets (12, 7, 5, 2, 18, 6, 13, 4).

Sol: The selection sort algorithm always makes exactly  $n-1$  swaps in the worst case, where  $n$  is the no. of elements in the list.

$$\text{Given } S = \{12, 7, 5, 2, 18, 6, 13, 4\}$$

$$\text{No. of elements, } n = 8$$

$$\text{No. of swaps} = n - 1 = 8 - 1 = 7.$$

Time complexity :-

The time complexity of selection sort in Big O notation is  $O(n^2)$ .

So, the number of swaps is 7, and the time complexity is  $O(n^2)$ .

(15) Find the index of the target value 10 using binary search from the following list of elements [2, 4, 6, 8, 10, 12, 14, 16, 18, 20].

Given list = {2, 4, 6, 8, 10, 12, 14, 16, 18, 20} and value = 10.

low = 0 and high

$$\text{mid} = \frac{\text{low} + \text{high}}{2} = \frac{0 + 9}{2} = 4.$$

$$\text{list}[4] = \text{mid} = 10 \quad \text{mid} = \text{value}.$$

Since  $10 = 10$  The target is found at index.

∴ The target value = 10 is found at index.