1. Big Omega Notation prove that $g(n) = n^3+2n^2+4n$ is $\Omega(n^3)$.

$g(n) \geq cn^3$

$g(n) = n^3+2n^2+4n$

for finding constants c and n_0

$$n^3+2n^2+4n \geq cn^3$$

Divide both sides with n^3

$$1+\frac{2n^2}{n^3}+\frac{4n}{n^3} \geq c$$

$$1+\frac{2}{n}+\frac{4}{n^2} \geq c$$

Here $\frac{2}{n}$ and $\frac{4}{n^2}$ approaches 0

$$1+\frac{2}{n}+\frac{4}{n^2} \simeq 1$$

Example $c=\frac{1}{2}$

$$1+\frac{2}{n}+\frac{4}{n^2} \geq \frac{1}{2}$$

$$1+\frac{2}{n}+\frac{4}{n^2} \geq 1 \qquad (1 \geq \tfrac{1}{2}, n \geq 1)$$

$$1+\frac{2}{n}+\frac{4}{n^2} \geq \frac{1}{2} \qquad (n \geq 1, n_0 = 1)$$

Thus, $g(n) = n^3+2n^2+4n$ is indeeded $\Omega(n^3)$

2. Big Theta Notation determine where $h(n) = 4n^2+3n$ is $\Theta(n^2)$ or not.

$$c_1n^2 \leq h(n) \leq c_2n^2$$

In upper bound $h(n)$ is $O(n^2)$

In lower bound $h(n)$ is $\Omega(n^2)$

upper bound $(O(n^2))$

$$h(n) = 4n^2+3n$$

$$h(n) \leq c_2n^2$$

$$4n^2+3n \leq c_2n^2 \Rightarrow 4n^2+3n \leq cn^2$$

Let $C_2 = 5$

divide both sides by n^2

$4 + 3/n \leq 5$

$h(n) = 4n^2 + 3n$ is $O(n^2)$ $(C_1 = 4, n_0 = 1)$

$1 + \frac{n}{n \log n} \leq 2$ simplify

$C_2 = 2$

$1 + \frac{1}{\log n} \leq C_2$ $((C_2 = 2, n_0 = 2)$

$1 + \frac{1}{\log n} \leq 2$

Then $h(n)$ is $O(n \log n)$

lower bound

$h(n) \geq C_1 (n \log n)$

$h(n) = n \log n + n$

$n \log n + n \geq C_1 n \log n$

divide both sides by $n \log n$

$1 + \frac{n}{n \log n} \geq C_1$ $C_1 = 1$

$1 + \frac{1}{\log n} \geq 5,$

$\frac{1}{\log n} \geq 0$ for all $n > 1$

$h(n) = n \log n + n$ is $\theta(n \log n)$

3. solve the following recurrence relations and find the growth of solution $T(n) = 4T(n/2) + n_2 + T(1) = 1$

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$f(n) \geq C_1 g(n)$

subtituting $f(n)$ & $g(n)$ into this inequality we get

$n^3 - 2n^2 + n \geq c(-n^2)$

A and C and n_0 holds $n \geq n_0$

$n^3 - 2n^2 + n \geq cn^2$

$n^3 + c(-2)n^2 + n \geq 0$

$n^3 + (1-2)n^2 + n = n^3 - n^2 + n \geq 0$

$T(n) = n^3 - 2n^2 + n$ is $\Omega(g(n_1)) = \Omega(-n^2)$

$\therefore$ The statement $f(n) = \Omega g(n)$ is true

4. Determine whether $h(n) = n(\log n + n$ is $O(n \log n)$ prove a rigorous proof for your conclusion.

$C_1 n \log n \leq h(n) \leq C_2 n \log n$

upper bound :-

$h(n) \leq C_2 . n\log n$

$h(n) = n \log n + n$

$n \log n + n \leq C_2 n \log n$

divide both sides, with $n \log n$

$$1 + \frac{n}{n \log n} \leq C$$

$$1 + \frac{n}{\log n} \leq C_2$$

$$1 + \frac{1}{\log n} \leq 2$$

Then $h(n)$ is $O(n \log n)$

lower bound:-

$h(n) \geq c_1 n \log n$

$h(n) = n \log n + n$

$n \log n + n \geq c_1 n \log n$

divide bothsides with $n \log n$

$$1 + \frac{n}{n \log n} \geq c_1$$

$$1 + \frac{1}{\log n} \geq c_1$$

$$1 + \frac{1}{\log n} \geq 1$$

$$\frac{1}{\log n} \geq 0$$

$h(n)$ is $\Omega(n \log n)$ $(c_1 = 1, n_0 = 1)$

$h(n) = n \log n + n$ is $\Theta(n \log n)$

5. Solve the following recurrence relations and find the order of growth of solutions.

$T(n) = 4T(n/2) + n^2,\ T(1) = 1$

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$T(n) = aT(n/b) + f(n)$

$a = 4 \quad b = 2 \quad f(n) = n^2$

Applying master's theorem

$T(n) = aT(n/b) + f(n)$

$f(n) = O(n^{\log_b^a - \epsilon})$

$f(n) = \Theta(n^{\log_b^a})$, then $T(n) = \Theta(n \log_b^a \log n)$

$f(n) = \Omega(n^{\log_b a + \epsilon})$, Then $T(n) = f(n)$

calculating $\log_b^a$

$\log_b a = \log_2 4 = 2$

$f(n) = n^2 = \Theta(n^2)$ (comparing $f(n)$ with $n^{\log_b a}$)

$f(n) = \Theta(n^2) = \Theta(n^{\log_b a})$

$T(n) = 4T(n/2) + n^2$

$T(n) = \Theta(n^{\log_b a} \log n) = \Theta(n^2 \log n)$

Order of growth

$T(n) = 4T(n/2) + n^2$ with $T(1) = 1$

$\Rightarrow O(n^2 \log n)$