Adaptive Mesh Refinement for the Conjugate Heat Transfer Equations Using Firedrake

The conjugate heat transfer equations (CHT) are often used to model microfluidic cooling for microchips. In this scenario there are two layers: The thermal fluid layer and the substrate layer.

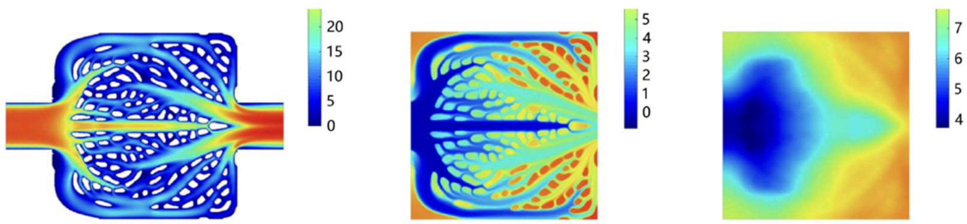

Figure 1: An optimised microfluidic design. The left plot is the velocity magnitude, the middle the temperature in the channel layer, and the right the temperature in the substrate layer. In the left plot, white zones correspond to solid regions

The general procedure of solving the CHT equations goes as follows:

1. Solve the steady-state Navier-Stokes equation to get the fluid flow in a given channel topology.
2. Using the velocity field from above compute the substrate temperature and the channel temperature. This can be done using the advection-diffusion equation.

I will first implement a solution to the CHT equations using firedrake without adaptive mesh refinement (AMR). Then I will try to implement it with AMR. To do this I will first need to find a suitable error indicator for the CHT equations. To implement AMR I will use the library Netgen/NGS-Solve[i]. This library has already successfully used together with firedrake to use AMR for other equations[ii].

I will then compare my normal implementation with my AMR implementation and using example channel topologies and substrates.

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[i] https://docu.ngsolve.org/latest/

[ii] https://www.firedrakeproject.org/demos/netgen_mesh.py.html