CT-Assignment 3

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(1)

a)

$$\theta(t) = \theta(0) + 2\pi k_f \int_0^t m(\tau)\, d\tau$$

As there is no offset in the plot $\phi(0) = 0$

$$\phi(t) = 2\pi k_f \int_0^t m(\tau)\, d\tau$$

As $m(t) \rightarrow$ sinusoidal

So, $\boxed{\phi(t) = \beta \sin(2\pi f_m t)}$

So, $\beta = A = 540^\circ \Rightarrow 3\pi$ (approx)

$\boxed{\beta = 3\pi} \rightarrow$ modulation index.

$\boxed{f_m = 5 \times 10^3} \rightarrow$ from plot.

(c) Carson's formula,

$$B_{fm} = 2B(\beta + 1)$$

$$= 2B(3\pi + 1)$$

$\boxed{B_{fm} = 104.24\ \text{KHz}}$ (approx)

(2)

$m(t) \rightarrow$ message signal.

$\boxed{k_f = 1}$

$$M(f) = \begin{cases} j2\pi f & |f|<1 \\ 0 & \text{o.w} \end{cases}$$

output of fm modulator

$$\boxed{v(t) = A\cos(2\pi f_c t + \phi(t))}$$

(a)

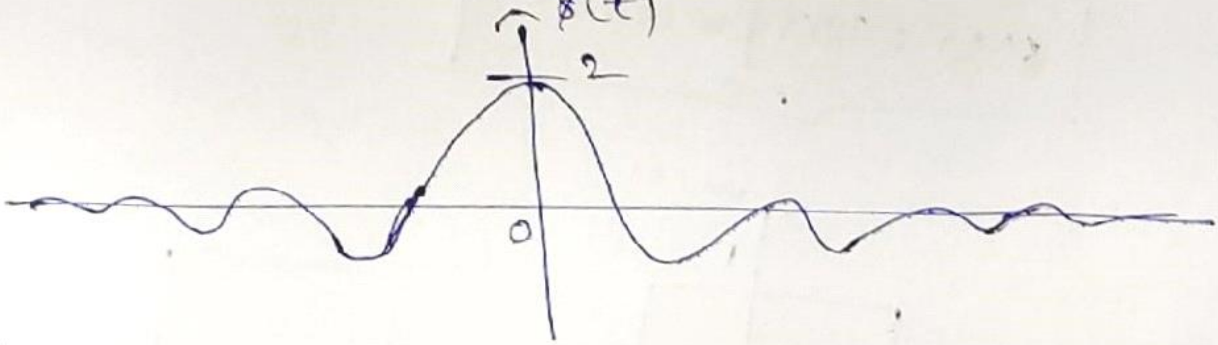
$$\frac{d\phi(t)}{dt} = 2\pi f(t)$$
$$= 2\pi k_f \cdot m(t)$$

taking FT,

$$j(2\pi f)\cdot\Phi(f) = 2\pi k_f \cdot M(f)$$

$$\Rightarrow \Phi(f) = \begin{cases} 2\pi & |f|<1 \\ 0 & \text{o.w} \end{cases} \rightarrow \text{rectangular pulse.}$$

So, $\boxed{\phi(t) = \frac{2\pi \sin(2\pi t)}{\pi t}}$



(b)

$$f(t) = \frac{1}{2\pi}\frac{d\phi(t)}{dt}$$

$$= \frac{1}{2\pi^2}\left[\frac{t\cdot\cos(2\pi t)\cdot 2\pi - \sin(2\pi t)}{t^2}\right]\Bigg|_{t=1/4}$$

$$= \frac{1}{2\pi^2}\left[\frac{\frac{1}{4}\cos(\pi/2)\cdot 2\pi - \sin\pi/2}{1/16}\right] = \frac{-8}{\pi^2}\times 2\pi = \frac{-16}{\pi}$$

So, $\boxed{f(t=1/4) = -\frac{16}{\pi^2}}$

$\boxed{|f(t=1/4)| = 16/\pi^2}$

(c) $B_{FM} = 2B(\beta+1)$

$= 2B + 2\Delta f_{max}$

$\boxed{B_{FM} = 2 + \frac{32}{\pi^2}}$

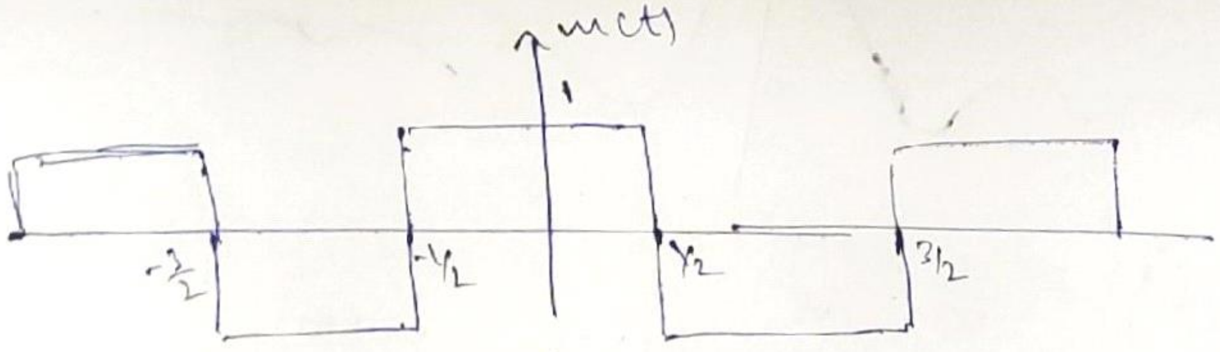
(3) $p(t) = I_{[-1/2,\,1/2]}(t)$

$m(t) = \sum_{n=-\infty}^{\infty} (-1)^n p(t-n)$

output of fm modulator

$\boxed{v(t) = 20\cos(2\pi f_c t + \phi(t))}$

$\boxed{\phi(t) = 20\pi \int_{-\infty}^{t} m(\tau)\, d\tau}$



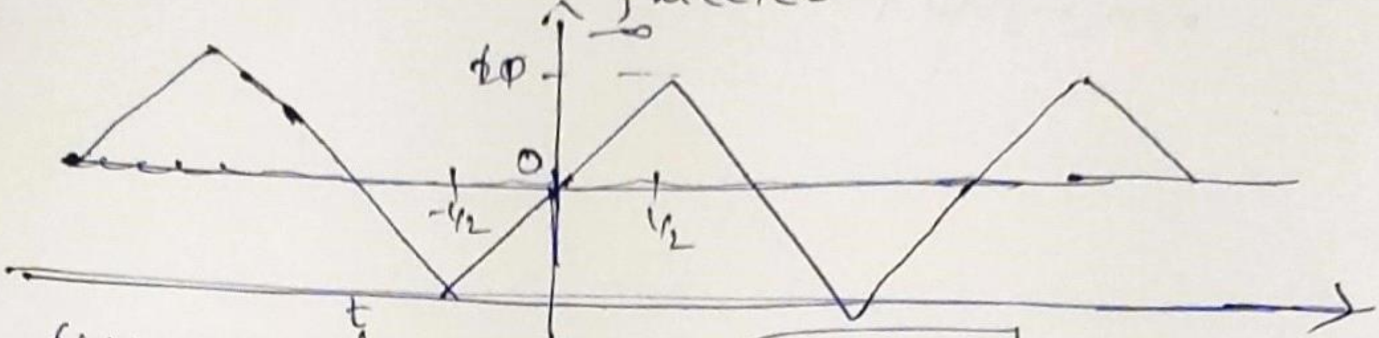
~~As $m(t)$ is even, $\int_{-\infty}^{0} m(t) = 0$.~~

$\int_{-\infty}^{t} m(\tau)\,d\tau$ is integrating $m(t)$ from $-\infty$ to t

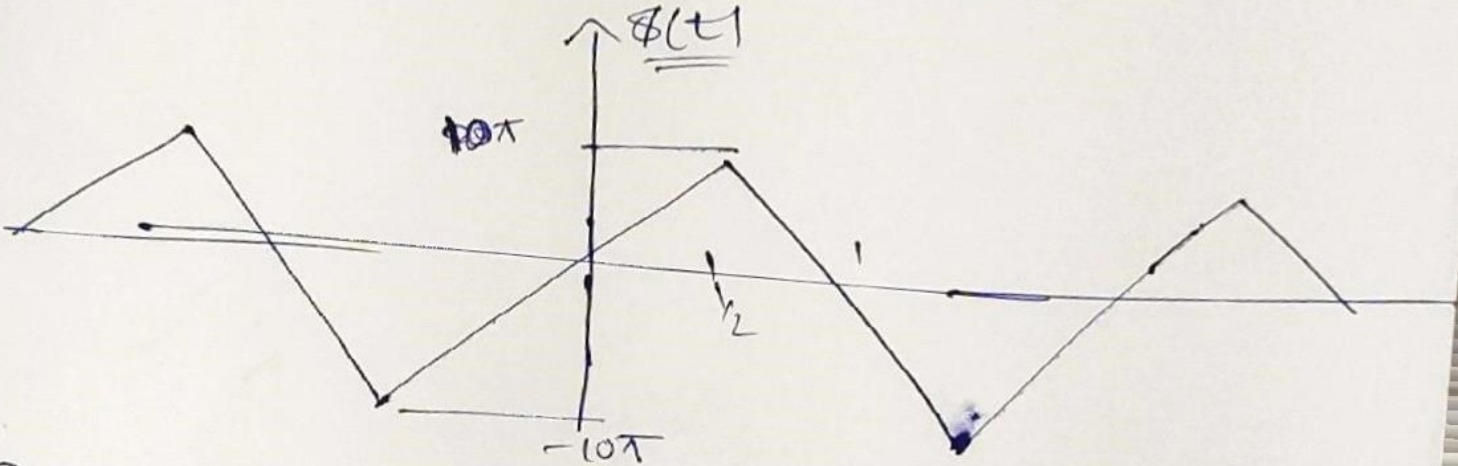
$-\infty < t < \infty$

So, the rectangular pulses becomes triangular pulses.

So, we get-



$$\phi(t) = 20\pi \int_{-\infty}^{t} m(\tau)\,d\tau + a \qquad \boxed{a = -10\pi}$$



(b) $B_{FM} = 2B + 2\Delta f_{max}$

$= 2(2+10)$

$\boxed{B_{FM} = 24\ Hz}$

(c) $|\overline{v(t)}|^2 \rightarrow$ power

$V(f) * V(f)$

$v(t) = e^{j\theta(t)} \rightarrow$ sinusoidal

$\theta(t) \rightarrow$ periodic with period 2,

$$v(t) = \sum_{n=-\infty}^{\infty} v[n]\, e^{j(2\pi n)(\frac{1}{2})(t)}$$

$$v(f) = \sum_{n=-\infty}^{\infty} \sigma J_n(\beta)\, \delta\left(f - \frac{n}{2}\right)$$

$\therefore$ impulses at $\left(f_c - \frac{n}{2}\right)$

So, for $\alpha = 0.5, 1$ there will be non-zero power.