CT- ASSIGNMENT -1

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(1)

$$x(t) = \begin{cases} e^{-at} & t \geq 0 \\ 0 & t < 0 \end{cases}$$

$$X(f) = \frac{1}{a + i(2\pi f)}$$

$$|X(f)| = \frac{1}{\sqrt{a^2 + 4\pi^2 f^2}}$$

$$|X(f)|^2 = \frac{1}{a^2 + 4\pi^2 f^2}$$

To find the bandwidth required to transmit 95% of signal we have.

$$\int_{-\omega}^{\omega} |X(f)|^2 df = \alpha \int_{-\infty}^{\infty} |x(t)|^2 dt$$

$$\Rightarrow \int_{-\omega}^{\omega} \frac{df}{a^2 + 4\pi^2 f^2} = (0.95) \int_{0}^{\infty} e^{-2at} dt$$

$$\Rightarrow 2\int_{0}^{\omega} \frac{df}{a^2 + 4\pi^2 f^2} = \frac{(0.95)}{2a}$$

$$\Rightarrow \int_{0}^{\omega} \frac{1}{a^2 + 4\pi^2 f^2} df = \frac{0.95}{4a}$$

$$\Rightarrow \frac{1}{4\pi^2} \int_{0}^{\omega} \frac{1}{f^2 + (a/2\pi)^2} df = \frac{0.95}{4a}$$

$$\Rightarrow \frac{\cancel{2\pi}}{\cancel{a}} \tan^{-1}\left(\frac{\omega \times 2\pi}{a}\right) = \frac{0.95\pi^2}{\cancel{a}}$$

$$\frac{2\pi\omega}{a} = \tan\left(\frac{0.95\pi^2}{2}\right)$$

$$\omega = \frac{a}{2\pi} \tan(0.95\pi^2)$$

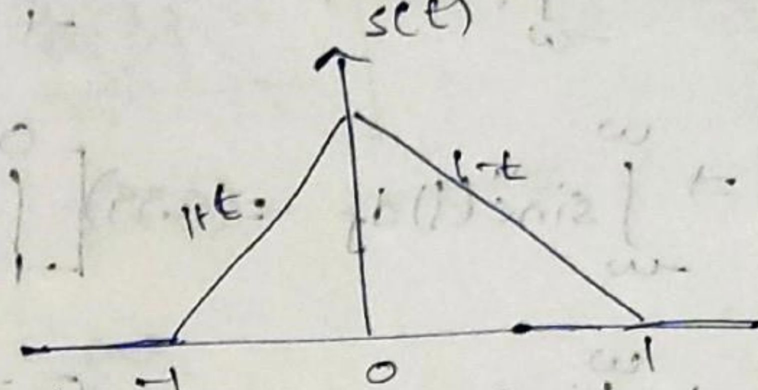
$$\Rightarrow \boxed{\omega = 0.0262\, a}$$

(2) $s(t) = (1-|t|)\, I_{[-1,1]}(t)$

$$s(t) = \begin{cases} 1+t & -1 \le t \le 0 \\ 1-t & 0 \le t \le 1 \\ 0 & \text{o.w} \end{cases}$$

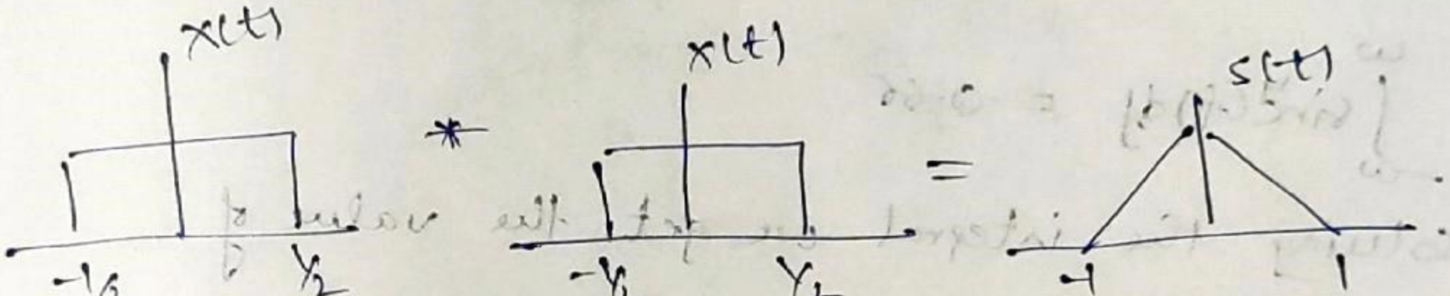
a)

$$S(f) = \int_{-\infty}^{\infty} s(t)\, e^{-(2\pi f)jt}\, dt$$

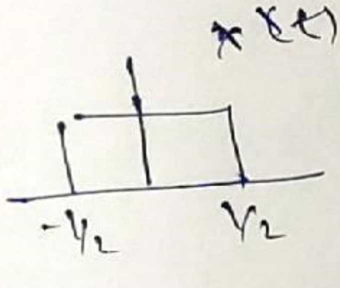


But we can observe that

s(t) is the convolution of two box functions $(x(t))$



So, in frequency domain its multiplication,



$\xrightarrow{F.T}$ $\underline{X(f)}$ sinc(f)

So,

$S(f) = X^2(f)$

$$\boxed{S(f) = \text{sinc}^2(f)}$$

$\left[\because \text{sinc}^2(f) = \dfrac{\sin^2(\pi f)}{\pi^2 f^2}\right]$

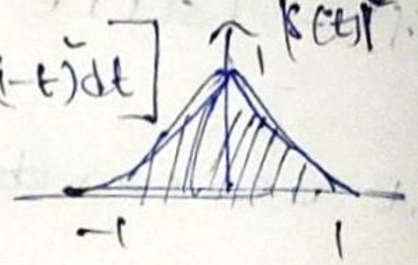
b) To find the 99% energy containment bandwidth.

$$\int_{-W}^{W} |U(f)|^2 \, df = \alpha \int_{-\infty}^{\infty} |u(t)|^2 \, dt$$

$$\Rightarrow \int_{-W}^{W} |S(f)|^2 \, df = (0.99) \int_{-1}^{1} |s(t)|^2 \, dt$$

→ Area of $|s(t)|^2$ by x-axis.

$$\Rightarrow \int_{-W}^{W} \text{sinc}^4(f) \, df = (0.99)\left[\int_{-1}^{0} (1+t)^2 \, dt + \int_{0}^{1} (1-t)^2 \, dt\right]$$



$$\Rightarrow \int_{-W}^{W} \text{sinc}^4(f) \, df = (0.99)\left[2/3\right]$$

$$\Rightarrow \int_{-W}^{W} \text{sinc}^4(f) \, df = 0.66.$$

solving the integral we get the value of W which is the bandwidth in kHz

We get this value, $W = 0.649$ kHz

(3) $x(t)$; $y(t)$ → two periodic signals with period T_0 and x_n, y_n are the fourier series coefficients respectively.

(a)

$$x_n = \frac{1}{T_0}\int_{\alpha}^{\alpha+T_0} x(t)\cdot e^{-j\left(\frac{2\pi n t}{T_0}\right)} \cdot dt$$

$$y_n = \frac{1}{T_0}\int_{\alpha}^{\alpha+T_0} y(t)\cdot e^{-j\left(\frac{2\pi n t}{T_0}\right)} \cdot dt$$

so,

$$y_n^* = \frac{1}{T_0}\int_{\alpha}^{\alpha+T_0} y^*(t)\, e^{j\left(\frac{2\pi n t}{T_0}\right)} dt$$

$$\boxed{y^*(t) = \sum_{n=-\infty}^{\infty} y_n^* e^{-j\left(\frac{2\pi n t}{T_0}\right)}}$$

Now, we have,

$$\sum_{n=-\infty}^{\infty} x_n \cdot y_n^* = \sum_{n=-\infty}^{\infty} \frac{1}{T_0}\int_{\alpha}^{\alpha+T_0} x(t)\cdot e^{-j\left(\frac{2\pi n t}{T_0}\right)} dt \cdot y_n^*$$

$$= \frac{1}{T_0}\int_{\alpha}^{\alpha+T_0} x(t) \cdot \underbrace{\sum_{n=-\infty}^{\infty} e^{-j\left(\frac{2\pi n t}{T_0}\right)} \cdot y^*(n)}_{y^*(t)}\, dt$$

$$= \frac{1}{T_0}\int_{\alpha}^{\alpha+T_0} x(t)\cdot y^*(t)\, dt$$

∴ Hence proved.

→ To get rayleigh's equation we replace $y(t)$ with $x^*(t)$ to get.

$$\frac{1}{T_0}\int_{\alpha}^{\alpha+T_0} x(t).\,x^*(t)\,dt = \sum_{n=-\infty}^{\infty} x_n.\,x_n^*$$

$$\boxed{\frac{1}{T_0}\int_{\alpha}^{\alpha+T_0} |x(t)|^2 dt = \sum_{n=-\infty}^{\infty} |x_n|^2}$$

(b) From the equation;

$$\frac{1}{T_0}\int_{\alpha}^{\alpha+T_0} x(t).\,x^*(t)\,dt = \sum_{n=-\infty}^{\infty} |x_n|^2$$

As $T_0 \to \infty$ LHS becomes the power of the function $x(t)$ and is finite as RHS is finite

$$\sum_{n=-\infty}^{\infty} |x_n|^2 = \text{finite}$$

$$\boxed{\sum_{n=-\infty}^{\infty} |x_n|^2 < \infty}$$

By convergence theorem, we can say that

$$\boxed{\lim_{n\to\infty} x_n = 0}$$

(c) Consider $f(x) = x^2$ $\quad x \in [-\pi, \pi]$

$T_0 = 2\pi$, $\alpha = -\pi$

$$a_n = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x).\,e^{-jnx}\,dx$$

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} x^2 dx = \frac{1}{2\pi}\left[\frac{2\pi^3}{3}\right] = \frac{\pi^2}{3}$$

$$a_n = \frac{1}{2\pi}\int_{-\pi}^{\pi} x^2 e^{-jnx} dx$$

$$= \frac{1}{2\pi}\left[\frac{x^2 e^{-jnx}}{-jn} - \frac{\int 2x.\,e^{-jnx} dx}{-jn}\right]$$

$$= \frac{1}{2\pi}\left(\frac{x^2 e^{-jnx}}{-jn} + \frac{2}{jn}\left(\frac{x e^{-jnx}}{-jn} - \frac{e^{-jnx}}{(jn)^2}\right)\right)\Bigg|_{-\pi}^{\pi}$$

$$= \frac{1}{2\pi}\left[\frac{2\pi^2}{jn}\cancel{\left(-\sin n\pi\right)} + \frac{2\cdot 2\pi}{n^2}\left[\cos n\pi\right] + \frac{2\times 2}{jn^3}\cancel{\left[\sin n\pi\right]}\right]$$

$$= \frac{2(-1)^n}{n^2}$$

So,

$$\sum_{n=-\infty}^{\infty} |x_n|^2 = 2\sum_{n=1}^{\infty} |x_n|^2 + |x_0|^2$$

$$\boxed{\sum_{n=-\infty}^{\infty} |x_n|^2 = 2\sum_{n=1}^{\infty} \frac{4}{n^4} + \frac{\pi^4}{9}}$$

Passeval's identity:

$$\frac{1}{T_0}\int_{\alpha}^{\alpha+T_0} x(t)\cdot x^*(t)\,dt = \sum_{n=-\infty}^{\infty} |x_n|^2$$

$$\Rightarrow \frac{1}{2\pi}\int_{-\pi}^{\pi} x^4\,dx = 2\sum_{n=1}^{\infty}\frac{4}{n^4} + \frac{\pi^4}{9}$$

$$\Rightarrow \frac{1}{2\pi}\cdot\frac{2\pi^5}{5} = 8\sum_{n=1}^{\infty}\frac{1}{n^4} + \frac{\pi^4}{9}$$

$$\Rightarrow \frac{\pi^4}{5} - \frac{\pi^4}{9} = 8\sum_{n=1}^{\infty} 1/n^4$$

$$\Rightarrow \boxed{\sum_{n=1}^{\infty} 1/n^4 = \frac{\pi^4}{90}}$$

divide it into odd and even terms,

$$\sum_{k=1}^{\infty}\frac{1}{(2k+1)^4} + \sum_{k=0}^{\infty}\frac{1}{(2k)^4} = \frac{\pi^4}{90}$$

$$\Rightarrow \sum_{k=1}^{\infty} \frac{1}{(2k+1)^4} + \frac{1}{16} \sum_{k=0}^{\infty} \frac{1}{k^4} = \frac{\pi^4}{90}$$

$$\Rightarrow \sum_{k=1}^{\infty} \frac{1}{(2k+1)^4} = \frac{\pi^4}{90}\left(\frac{15}{16}\right)$$

$$\Rightarrow \boxed{\sum_{k=1}^{\infty} \frac{1}{(2k+1)^4} = \frac{\pi^4}{96}}$$

$\therefore$ Hence proved

(4)

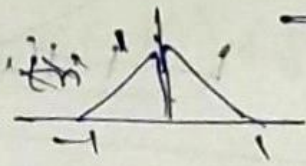
(a) $\mathrm{sinc}^3(t) = x(t)$

This can be written as product of

$$\mathrm{sinc}^3(t) = \mathrm{sinc}(t) \times \mathrm{sinc}^2(t)$$

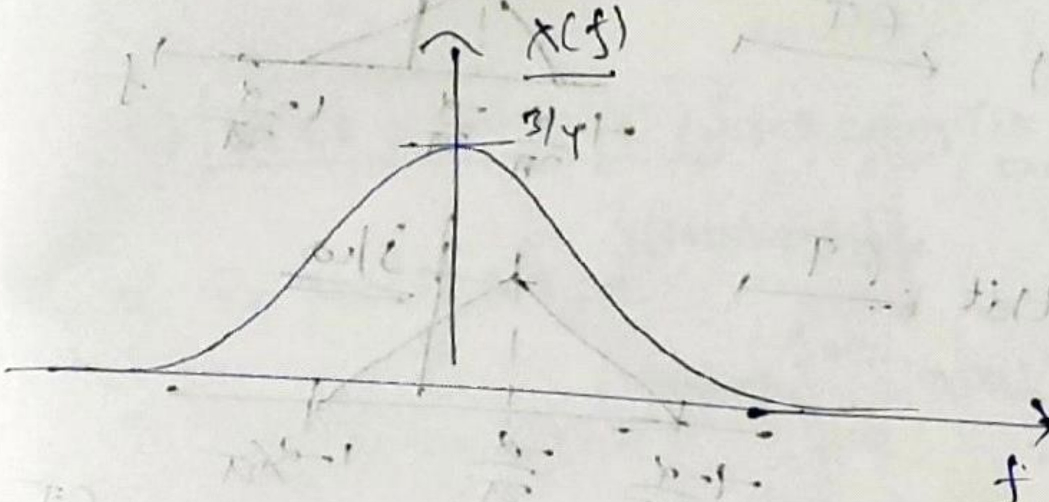
Applying fourier transform

$$X(f) = \frac{1}{2\pi}\left[I_{[-1/2, 1/2]} * \right]$$



Gnvolving rectangular & triangular functions we get,

$$X(f) = \begin{cases} 0 & f > 3/2 \\ \frac{1}{8}(2f-3)^2 & \frac{1}{2} < f < 3/2 \\ 3/4 - f^2 & -\frac{1}{2} < f < 1/2 \\ \frac{1}{8}(2f+3)^2 & -3/2 < f < -1/2 \\ 0 & f < -3/2 \end{cases}$$



(b) $t\,\mathrm{sinc}(t)$

$$= t\,\frac{\sin(\pi t)}{\pi t}$$

$$= \frac{1}{\pi}(\sin(\pi t))$$

$$= \frac{1}{2\pi j}\left[\delta(f - 1/2) - \delta(f + 1/2)\right]$$

$$\left[\sin(\omega_0 t) \xrightarrow{F.T} \frac{1}{2j}\left[\delta\left(f - \frac{\omega_0}{2\pi}\right) - \delta\left(f + \frac{\omega_0}{2\pi}\right)\right]\right]$$

4(c) $t e^{-\alpha t} \cdot \cos(\beta t) = g(t)$

let $x_1(t) = t e^{-\alpha t} \xrightarrow{F.T} \left(\frac{1}{\alpha + j\omega}\right)^2$

$x_2(t) = \cos(\beta t) \xrightarrow{F.T} \frac{1}{2}\left[\delta(\omega - \beta) + \delta(\omega + \beta)\right]$

$$G(\omega) = X_1(f) * X_2(f)$$

$$= \left(\frac{1}{a+j\omega}\right)^2 * \left(\frac{1}{2}\left(\delta(\omega-\beta) + \delta(\omega+\beta)\right)\right)$$

$$= \frac{1}{2}\left[\left(\frac{1}{a+j(\omega-\beta)}\right)^2 + \frac{1}{(a+j(\omega+\beta))^2}\right]$$

$$= \frac{1}{2}\left[\frac{1}{a^2 + 2a(\omega-\beta)j - (\omega-\beta)^2} + \frac{1}{a^2 + 2a(\omega+\beta)j - (\omega+\beta)^2}\right]$$

$$= \frac{1}{2}\left[\frac{2a^2 + 2a(2\omega)j - 2(\omega^2+\beta^2)}{\left[(a+j(\omega-\beta))(a+j(\omega+\beta))\right]^2}\right]$$

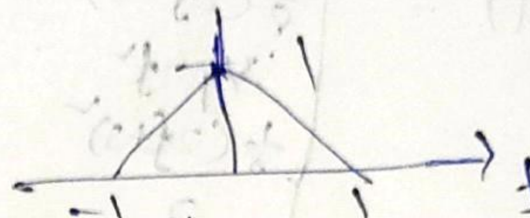
$$= \left[\frac{a^2 + 2a\omega j - (\omega^2+\beta^2)}{(a^2 + 2a\omega j - (\omega^2-\beta^2))^2}\right]$$

$$\boxed{G(\omega) = \left[\frac{a^2 + 2a\omega j - (\omega^2+\beta^2)}{((a^2 + 2a\omega j - (\omega^2-\beta^2))^2}\right]}$$

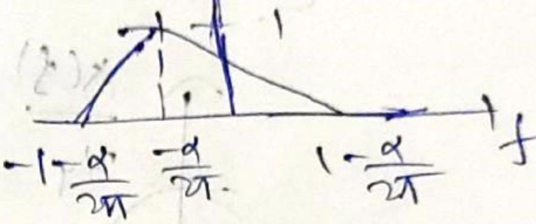
(5)

(a) $\int_0^{\infty} e^{-\alpha t}\,\text{sinc}(t)\,dt$

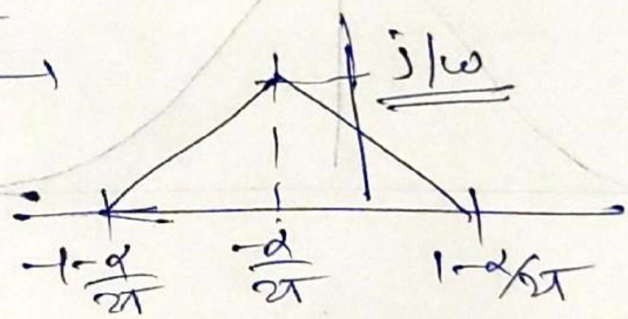
$\text{sinc}(t) \xleftrightarrow{F.T}$



$e^{-\alpha t}\,\text{sinc}(t) \xleftrightarrow{F.T}$



$\int_0^{\infty} e^{-\alpha t}\,\text{sinc}(t)\,dt \xleftrightarrow{F.T}$



$\left[\because \int x\,dt \xleftrightarrow{F.T} \frac{1}{j\omega}X(f)\right.$

$\left. x(t)e^{-j\omega t} \xleftrightarrow{F.T} \cancel{X(f)}\ X\left(f+\frac{\omega}{2\pi}\right)\right]$

(b) $\int_0^\infty e^{-\alpha t} \cos(\beta t)\, dt$

Assume $G(\omega) = \int_0^\infty e^{-\alpha t} \cos(\beta t)\, e^{-j\omega t}\, dt$.

Let $e^{-\alpha t} = x_1(t) \xrightarrow{FT} \frac{1}{\alpha + j\omega}$ $\qquad \omega = 2\pi f$

$\cos(\beta t) = x_2(t) \xrightarrow{FT} \frac{1}{2}[\delta(\omega-\beta) + \delta(\omega+\beta)]$

$$G(\omega) = X_1(f) * X_2(f)$$

$$= \left(\frac{1}{\alpha + j\omega}\right) * \left(\frac{\delta(\omega-\beta) + \delta(\omega+\beta)}{2}\right)$$

$$= \frac{1}{2}\left[\frac{1}{\alpha + j(\omega-\beta)} + \frac{1}{\alpha + j(\omega+\beta)}\right]$$

$$= \frac{1}{2}\left[\frac{2(\alpha + j\omega)}{\alpha^2 - (\omega^2-\beta^2) + 2\alpha j\omega}\right]$$

$$G(\omega) = \frac{\alpha + j\omega}{\alpha^2 - (\omega^2-\beta^2) + 2\alpha j\omega}$$

$$\int_0^\infty e^{-\alpha t} \cos(\beta t)\, e^{-j\omega t}\, dt = \frac{\alpha + j\omega}{\alpha^2 - (\omega^2-\beta^2) + 2\alpha j\omega}$$

put $\omega = 0$

$$\boxed{\int_0^\infty e^{-\alpha t} \cdot \cos(\beta t)\, dt = \frac{\alpha}{\alpha^2 + \beta^2}}$$

(6) $u_p(t) = \text{sinc}(2t)\cdot\cos(100\pi t)$

$v_p(t) = \text{sinc}(t)\cdot\sin(101\pi t + \pi/4)$

$f_c = 50\,Hz$

$\boxed{\omega_c = 100\pi}$

(a) $u_p(t) = \text{Re}\{\text{sinc}(2t)\, e^{j(100\pi t)}\}$

$\Rightarrow \boxed{u(t) = \text{sinc}(2t)}$ → complex envelop.

$v_p(t) = \text{Re}\{j\,\text{sinc}(t)\cdot e^{j(101\pi t+\pi/4)}\}$

$= \text{Re}\{-j\cdot \text{sinc}(t)\, e^{j(\pi t+\pi/4)} \cdot e^{j(100\pi t)}\}$

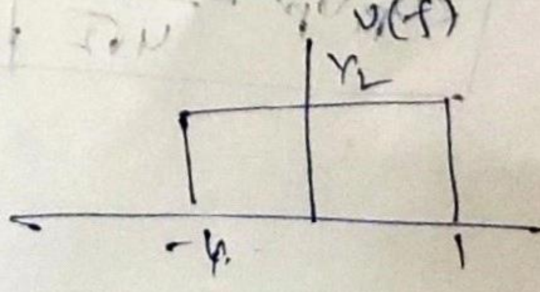
$\Rightarrow v(t) = j\,\text{sinc}(t)\, e^{j(\pi t+\pi/4)}$

$\Rightarrow \boxed{v(t) = \text{sinc}(t)\, e^{j(\pi t-\pi/4)}}$ → Complex envelop

(b) Bandwidth of $u_p(t)$:

$u(t) = \text{sinc}(2t) \xleftrightarrow{F.T} \frac{1}{2} I_{[-1,1]}(t)$

So, two sided bandwidth $= \underline{\underline{2}}$

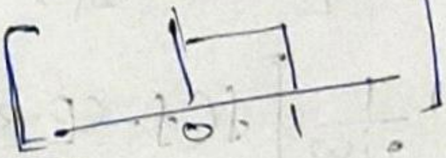


Bandwidth of $V_p(t)$:

$$V(t) = \operatorname{sinc}(t)\, e^{j(\pi t - \pi/4)} \qquad \left[x(t) e^{j\omega t} \xrightarrow{FT} X\left(f - \frac{\omega}{\pi}\right) \right]$$

$$= e^{-\pi/4 j}\left[\operatorname{sinc}(t) \cdot e^{j\pi t}\right]$$

$$V(f) = \left(e^{-j\pi/4}\right)\left[\;\right]$$



$$\boxed{V(f) = e^{-j\pi/4} \cdot I_{[0,1]}(t)}$$

Bandwidth $= 1$

(c) Inner Product $\langle U_p, V_p \rangle$:

$\cancel{\langle U_p, V_p \rangle = \frac{1}{2}}$ $\langle U_p, V_p \rangle = \frac{1}{2} \operatorname{Re}\{\langle U, V \rangle\}$

$$\langle U, V \rangle = \int U(f) \cdot V^*(f)\, df$$

$$= \frac{1}{2} \int I_{[-1,0]} \cdot I_{[0,1]}\, e^{j\pi/4}\, df$$

$$\langle U, V \rangle = \frac{e^{j\pi/4}}{2}$$

$$\operatorname{Re}\{U, V\} = \frac{\cos \pi/4}{2}$$

$$\frac{1}{2} \operatorname{Re}\{U, V\} = \frac{1}{4} \cos \pi/4$$

$$= \frac{1}{4\sqrt{2}}$$

$$\Rightarrow \boxed{\langle U_p, V_p \rangle = \frac{1}{4\sqrt{2}}}$$

(d) $y_p(t) = (u_p * v_p)(t)$

$y_p(t) = \frac{1}{2}(u(t) * v(t))$

$Y_p(f) = \frac{1}{2}(U(f) \times V(f))$

$= \frac{1}{2}\left[\frac{1}{2} I_{[-1,1]} \times I_{[0,1]} e^{j\pi/4}\right]$

$Y_p(f) = \frac{1}{4} e^{j\pi/4}$

$\boxed{y_p(t) = \frac{1}{4} e^{j\pi/4} \delta(t)}$

(7) $u_p(t) = I_{[-1,1]}(t) \cos(100\pi t)$

$h_p(t) = I_{[0,3]}(t) \sin(100\pi t)$

$y_p(t) = u_p(t) * h_p(t)$

$h_p(t) = \mathrm{Re}\{-j I_{[0,3]} e^{j(100\pi t)}\}$

$u_p(t) = \mathrm{Re}\{ I_{[-1,1]} e^{j(100\pi t)} \}$

$\boxed{h(t) = -j I_{[0,3]}}$

$\boxed{u(t) = I_{[-1,1]}}$

$y_p(t) = \frac{1}{2}[u(t) * h(t)]$

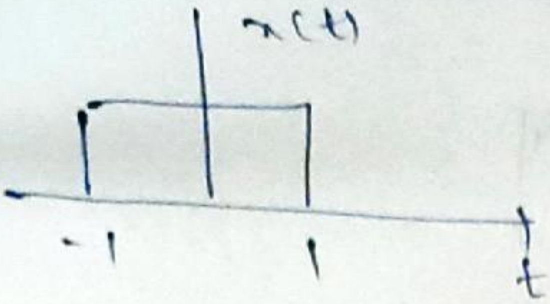
$= \frac{1}{2}\left[I_{[-1,1]} * -j \cdot I_{[0,3]}\right]$

$= \frac{-j}{2}\underbrace{\left[I_{[-1,1]} * I_{[0,3]}\right]}_{s(t)}$

$y_p(t) = \frac{-j\, s(t)}{2}$

$\boxed{y(t) = \frac{-j\, s(t)}{2} \cdot \sin(100\pi t)}$

Now, $s(t) = \underbrace{I_{[-1,1]}}_{x(t)} * \underbrace{I_{[0,3]}}_{y(t)}$



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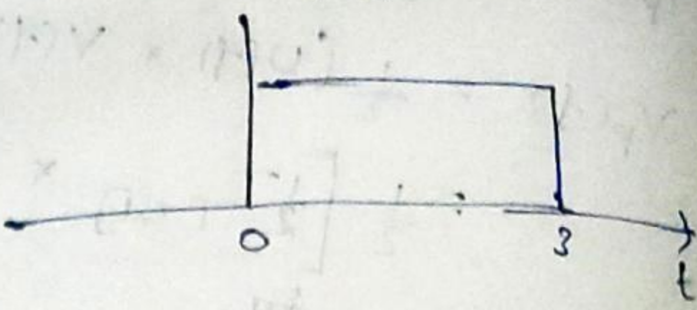

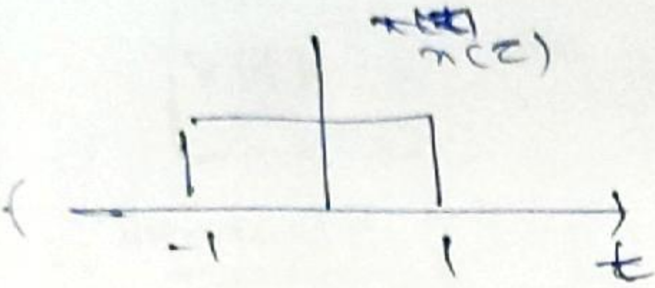

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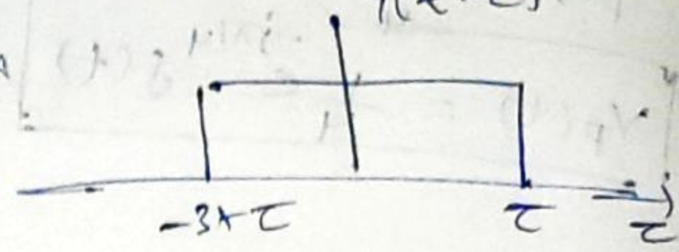
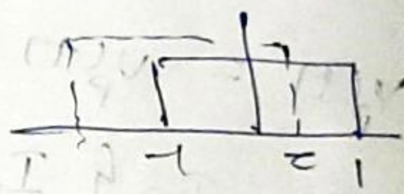


for $\boxed{\tau < -1}$ $\int_{-\infty}^{\infty} x(\tau) \cdot y(t-\tau)\, d\tau = 0 = s(t)$

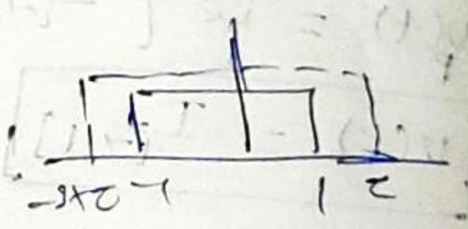
$\tau - 3 > 1$ $\int_{-\infty}^{\infty} x(\tau) \cdot y(t-\tau)\, d\tau = 0 = s(t)$

$\boxed{\tau > 4}$

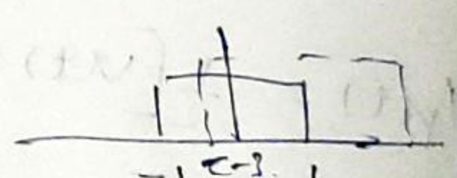
for $-1 < \tau < 1$: $s(\tau) = \tau + 1$



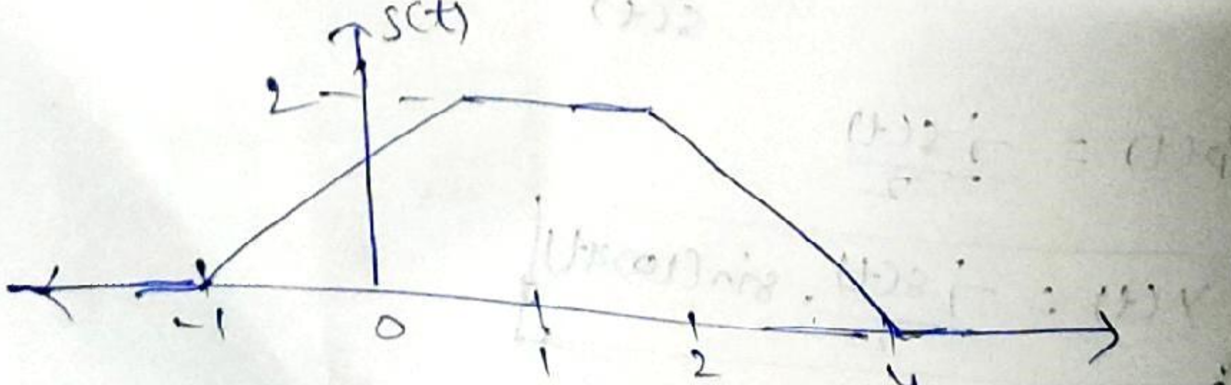
$1 < \tau < 2$ $s(\tau) = 2$



$2 < \tau < 4$ $s(\tau) = 4 - \tau$



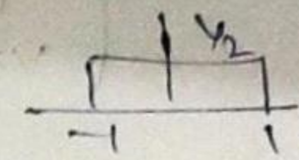
So $s(t) = \begin{cases} 0 & t < -1 \\ t+1 & -1 \le t \le 1 \\ 2 & 1 \le t \le 2 \\ 4-t & 2 \le t \le 4 \end{cases}$



(8) $u_p(t) = a(t)\cos(200\pi t)$

$a(t) = \text{sinc}(2t)$

$A(f) =$



(a) frequency band by $u_p(t)$:

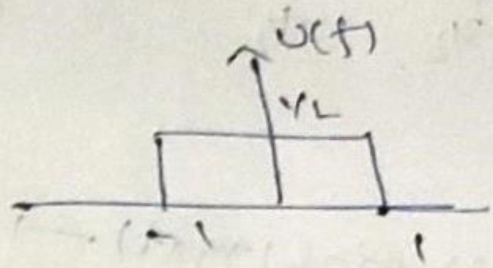
$u_p(t) = \text{Re}\{a(t) \cdot e^{j(200\pi t)}\}$

$u(t) = a(t)$

$\boxed{u(t) = \text{sinc}(2t)}$

$\boxed{U_p(f) = \frac{1}{2}\left[I_{[-101,-99]}(f) + I_{[99,101]}\right]}$

$\boxed{U(f) = \frac{1}{2} I_{[-1,1]}(f)}$



So, two-sided bandwidth $= 2$ MHz and band of frequencies a

$\boxed{-101 \text{ to } -99 \text{ MHz and } 99 \text{ to } 101 \text{ MHz}}$

(b) $u_p(t) \cdot \cos(199\pi t) \longrightarrow \text{LPF} \longrightarrow b(t)$

so, $\boxed{b(t) = \frac{1}{2} u_c(t)}$

$u_p(t) = \text{Re}\{a(t)\, e^{j(200\pi t)}\}$

$= \text{Re}\{a(t)\, e^{j\pi t} \cdot e^{199 j\pi t}\}$

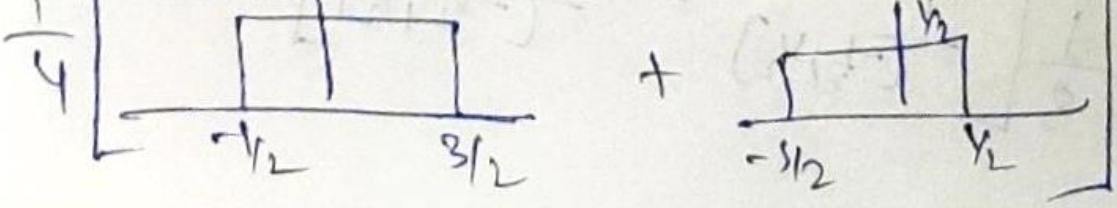
$\Rightarrow \boxed{u(t) = a(t)\, e^{j\pi t}}$

$u_c(t) = a(t) \cdot \cos(\pi t)$

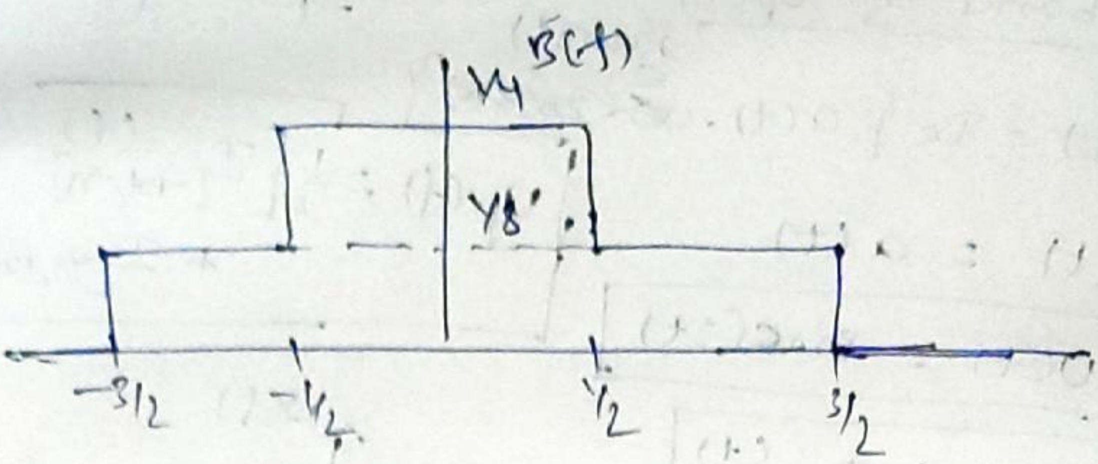
so, $\boxed{b(t) = \frac{1}{2} a(t) \cos(\pi t)}$

$B(f) = \frac{1}{2}\left[A(f) * \frac{1}{2}\left(\delta(f - 1/2) + \delta(f + 1/2)\right)\right]$

$= \frac{1}{4}\left[A(f - 1/2) + A(f + 1/2)\right]$



$$\Rightarrow B(f) = \begin{cases} 1/8 & -3/2 \le f \le -1/2 \\ 1/4 & -\frac{1}{2} \le f \le 1/2 \\ 1/8 & \frac{1}{2} \le f \le 3/2 \end{cases}$$



(c) $v_p(t) \cdot \sin(199\pi t) \rightarrow \text{LPF} \rightarrow c(t)$

$$\boxed{c(t) = -\frac{v_s(t)}{2}}$$

$$v_p(t) = Re\left\{-j \cdot a(t)\, e^{j(200\pi t)}\right\}$$

$$= Re\left\{-j\,a(t)\, e^{j\pi t} \cdot e^{j(199\pi t)}\right\}$$

$$\Rightarrow \cancel{v(t) = a(t)}$$

$$\boxed{v(t) = -j\,a(t)\, e^{j\pi t}}$$

$$\boxed{v_s(t) = a(t) \cdot \sin(\pi t)}$$

$$\boxed{c(t) = -\frac{1}{2} a(t) \sin(\pi t)}$$

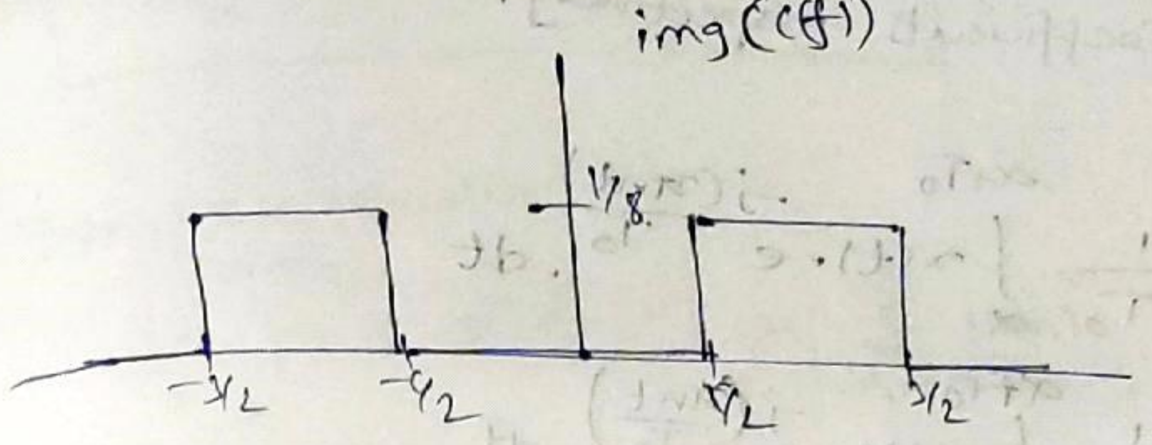
$$C(f) = -\frac{1}{2}\left[A(f) * \frac{1}{2j}\left[\delta(f - 1/2) - \delta(f + 1/2)\right]\right]$$

$$= -\frac{1}{4j}\left[A(f - 1/2) - A(f + 1/2)\right]$$

$$= -\frac{1}{4j}\left[\frac{1}{2} I_{[-\frac{1}{2}, 3/2]} - \frac{1}{2} I_{[-3/2, 1/2]}\right]$$

$$= \frac{j}{8}\left[I_{[-\frac{1}{2}, 3/2]} - I_{[-1/2, 1/2]}\right]$$

$$\Rightarrow c(f) = \begin{cases} 1/8 & -3/2 \le f \le -1/2 \\ 0 & -1/2 \le f \le 1/2 \\ 1/8 & 1/2 \le f \le 3/2 \end{cases}$$



(d) $b(t) = \frac{a(t)}{2} \cos(\pi t)$

$c(t) = -\frac{a(t)}{2} \sin(\pi t)$

Block diagram: