Communication Theory

Assignment - 2

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(1)

$$v(t) = \left[20 + 2\cos(3000\pi t) + 10\cos(6000\pi t)\right]\cos 2\pi f_c t$$

$$f_c = 10^5\,Hz$$

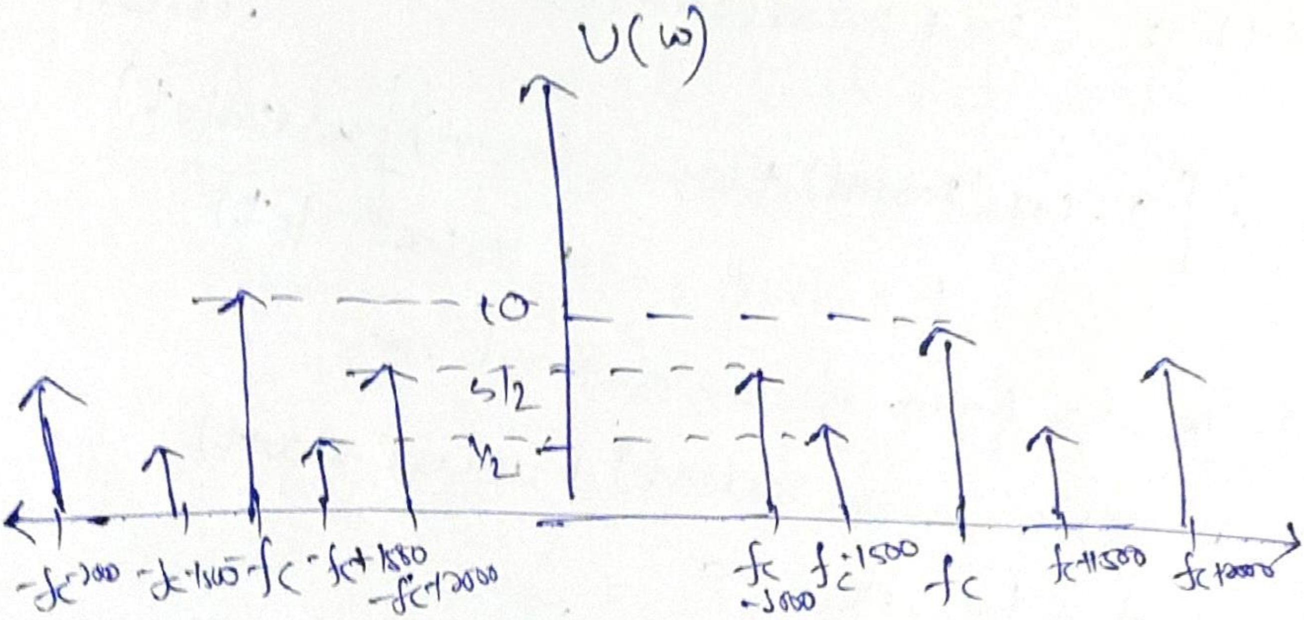
(a)

$$v(t) = m(t)\cdot\cos(2\pi f_c t)$$

$$\boxed{V(f) = M(f) * \left[\tfrac{1}{2}\left(\delta(f+f_c) + \delta(f-f_c)\right)\right]}$$

$$m(t) = 20 + 2\cos(3000\pi t) + 10\cos(6000\pi t)$$

$$M(f) = 20\delta(f) + \left[\delta(f+1500) + \delta(f-1500)\right] + 5\left[\delta(f+3000) + \delta(f-3000)\right].$$



(b) For any term with form,

$$(a \cos \omega) \xrightarrow{\text{power}} \frac{a^2}{2}$$

Power at $f = \pm f_c$ ~~is given by~~ for $20\cos(2\pi f_c t)$

$$P = (100)(2)$$
$$= 200W$$

at $f = \pm(f_c \pm 1500)$
$$\Rightarrow P = 4(1/2)^2 = 1W$$

at $f = \pm(f_c \pm 3000)$
$$\Rightarrow P = 4(5/2)^2 = 25W$$

(c)
$$u(t) = (20 + 2\cdot\cos(3000\pi t) + 10\cdot\cos(6000\pi t))\cdot\cos(2\pi f_c t)$$

$$u(t) = [2\cdot\cos(3000\pi t) + 10\cos(6000\pi t)]\cos(2\pi f_c t) + 20\cos(2\pi f_c t)$$

This is in form of conventional AM

$$m(t) = 2\cos(3000\pi t) + 10\cos(6000\pi t)$$
$$A = 1; \quad A_c = 20$$

$$a_{mod} = \frac{A\,|\min_t (m(t))|}{A_c}$$

message signal

$$m(t) = 2\cos x + 10\cos 2x$$

$$m(x) = 2\cos x + 10(2\cos^2 x - 1)$$

$$m(x) = 20\cos^2 x + 2\cos x - 10$$

$m'(x) = 40\cos x(-\sin x) - 2\sin x = 0$

$\cos x = -1/20$ (or) $\sin x = 0$ $\cos x = \pm 1$

$\boxed{\cos 2x = -\frac{199}{200}}$ (or) $\boxed{\cos 2x = 1}$

$m(x) = 2\cos x + 10\cos 2x$

$= \frac{-2}{20} + \frac{(-199)}{20}$

$= -\frac{201}{20}$

$m(x) = 2\cos x + 10\cos 2x$

$m(x) = 10 \pm 2$

now $\left|\min_t (m(t))\right| = \left|-\frac{201}{20}\right|$

$a_{mod} = \frac{201}{20 \cdot 20}$

$\boxed{a_{mod} = 0.5025}$

d) Power in sidebands will not include Power of carrier signals,

Power of LSB = Power of RSB

$= \frac{25}{2} + \frac{1}{2}$

$= \cancel{26}\ 13.$

Power of signal $= 200 + 1 + 25$

$= \underline{226}$

Power ratio $= \frac{P(LSB + RSB)}{P(\text{signal})}$

$= \frac{\cancel{26}\ 13}{\cancel{226}\ 113} = \frac{13}{113} = \underline{0.115}$

(2) Let $g(t) = m(t) \cdot s(t)$

$s(t) \longrightarrow$ periodic rectangular pulse

$\therefore$ $s(t)$ is periodic odd & antisymmetric about 0,

$s(t)$ is in form of $\boxed{\sum_{n=-\infty}^{\infty} b_n \sin(n\omega_0 t)}$

Given, $f_c = 1/T_p$

$$\boxed{\omega_p = \omega_0 = \frac{2\pi}{T_p}}$$

From fourier series we have

$$b_n = \frac{2}{T_p}\left[\int_0^{T_p/2} \sin(n\omega_0 t)\,dt - \int_{T_p/2}^{T_p} \sin(n\omega_0 t)\,dt\right]$$

$$= \frac{2}{T_p}\left[\frac{-\cos n\omega_0 t}{n\omega_0}\Bigg|_0^{T_p/2} + \frac{\cos(n\omega_0 t)}{n\omega_0}\Bigg|_{T_p/2}^{T_p}\right]$$

$$= \frac{2}{T_p}\left[\frac{2}{n\omega_0} - \frac{2\cos(n\pi)}{n\omega_0}\right]$$

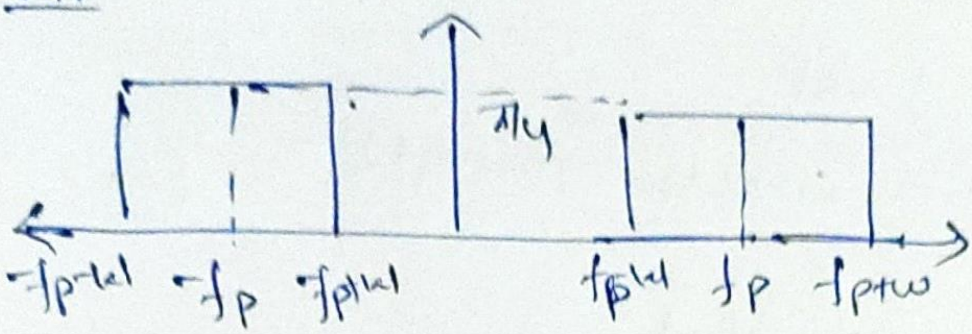
$$\Rightarrow b_n = \begin{cases} 0 & n \to \text{even} \\ \dfrac{8}{T_p \cdot 2\pi n} & n \to \text{odd} \end{cases}$$

Now,

$$g(t) = m(t).\left(\sum_{k=-\infty}^{\infty} \frac{4}{\pi(2k+1)} \cdot \sin((2k+1)\omega_0 t)\right).$$

passing this through band pass filter we have,

BPF



This passes signal b/w $\left[f_p - \frac{|\omega|}{2}, f_p + \frac{|\omega|}{2}\right]$

From $g(t)$ the frequencies that satisfy this are, for $k = 0$,

i.e, $g(t) = \left(m(t) \cdot \frac{4}{\pi} \sin(\omega_0 t)\right) \pi/4$

$\Rightarrow \boxed{v(t) = m(t) . \sin(\omega_0 t)}$

∴ Hence proved.

(3)

$$s(t) = \sum_{n=-\infty}^{\infty} s_n e^{jn\omega_0 t}$$

now,

$$g(t) = m(t) . s(t)$$

$$= \sum_{n=-\infty}^{\infty} c_n . m(t) e^{jn\omega_0 t}$$

$c_n \to$ fourier coefficients.

passing this through BPF frequencies around $\pm f_p$ will only be passed.

$$v(t) = \left(c_{-1} m(t) e^{j\omega_p t} + c_1 m(t) e^{-j\omega_p t}\right)$$

$s(t) \rightarrow$ real-valued periodic signal.

if $s(t) \rightarrow$ even,

$$u(t) = c_1 m(t) \cdot 2\cos(w_p t) \quad \cdots (1) \quad \boxed{c_1 = c_{-1}}$$

if $s(t) \rightarrow$ odd

$$u(t) = c_1 m(t) \cdot 2\sin(w_p t) \quad \cdots (2) \quad \boxed{c_1 = -c_{-1}}$$

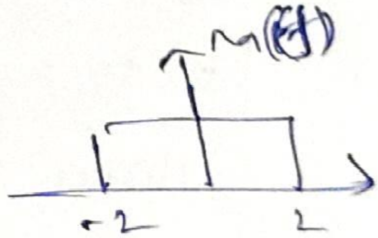
So, we can say that

$$\boxed{u(t) = k\, m(t) \cdot \sin(\theta(t))}$$

We can substitute any periodic any periodic signal for $s(t)$.

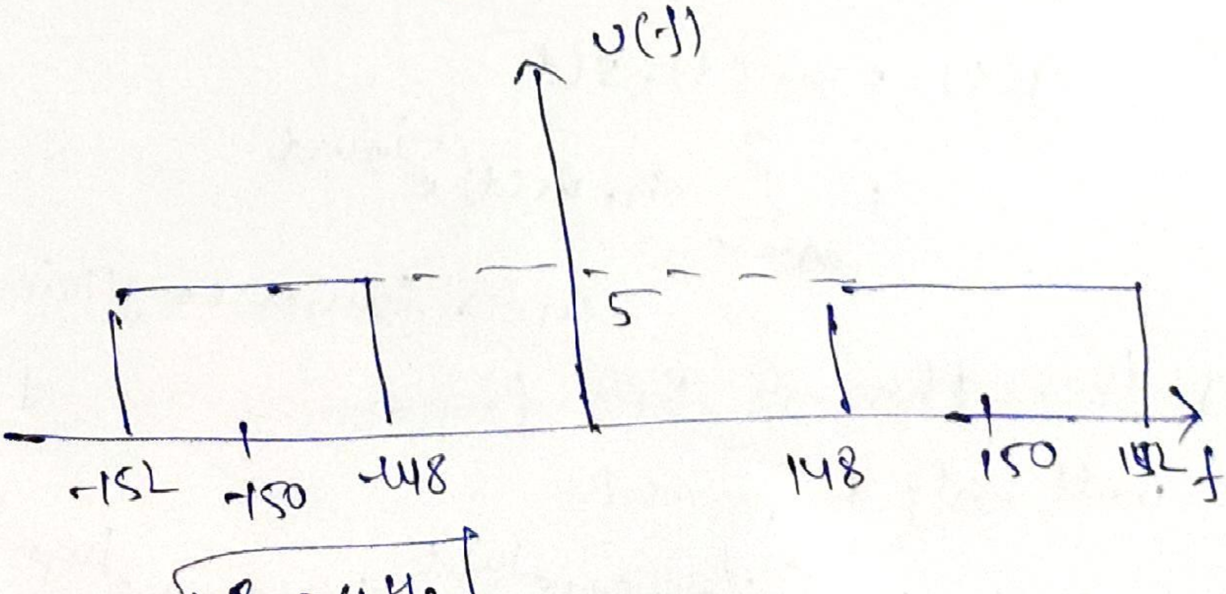
(4) $m(t) \rightarrow$ message signal

$$\boxed{M(f) = I_{[-2,2]}(f)}$$



a) $u(t) = 10\, m(t) \cdot \cos(300\pi t)$

$$U(f) = 5\left[M(f-150) + M(f+150)\right]$$



$$\boxed{B = 4\,Hz}$$

$m(t) = 4\,\text{sinc}(4t)$

$= 4\,\dfrac{\sin(4\pi t)}{4\pi t}$

$$\boxed{m(t) = \frac{\sin(4\pi t)}{\pi t}}$$

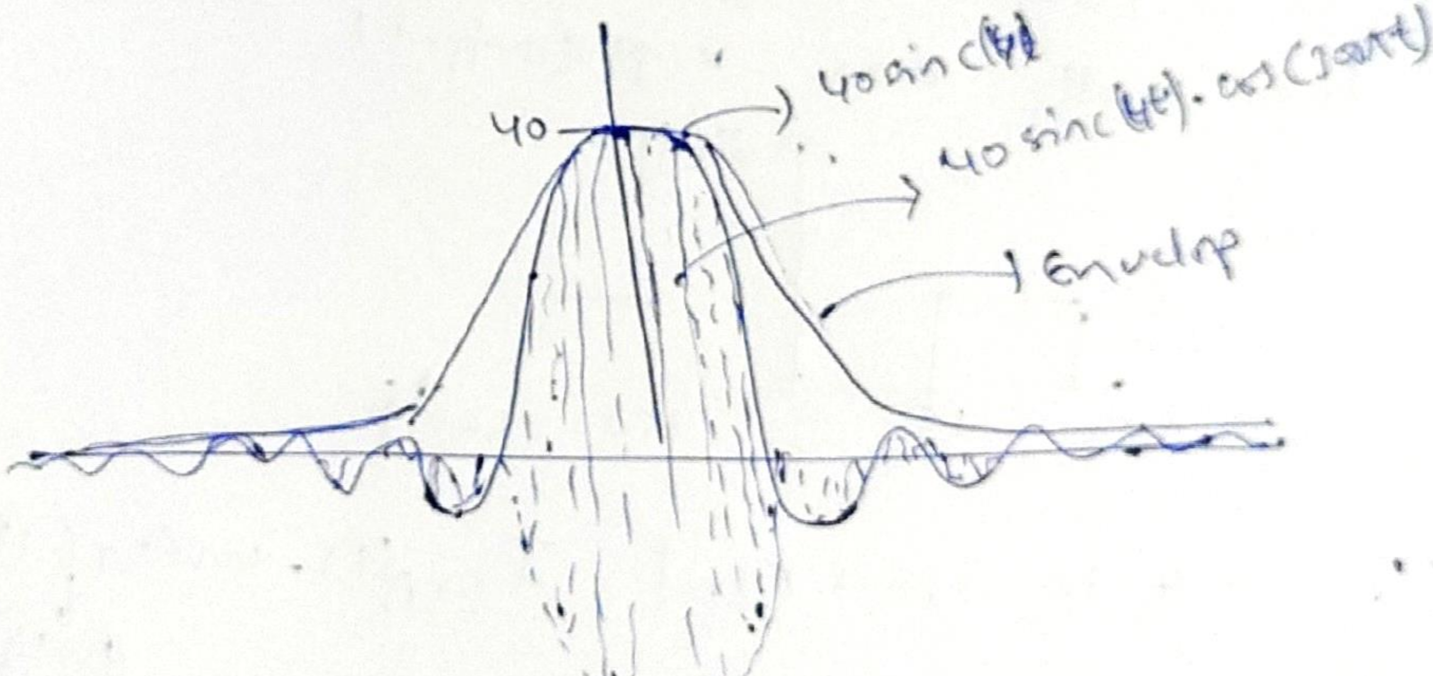
<u>Energy</u> $= \displaystyle\int_{-\infty}^{\infty} |m(t)|^2\,dt = \int_{-\infty}^{\infty} |M(f)|^2\,df$

$= \displaystyle\int_{-2}^{2} df$

$= \underline{\underline{4 \text{ units}}}$

$\boxed{\text{Power} = 0}$

(b) $v(t) = 40\,\text{sinc}(4t) \cdot \cos(300\pi t)$



Envelop is exponentially decreasing graph formed by the peaks of <u>40sinc(4t)</u>

(c) $v_{AM} = (A + m(t)) \cos(300\pi t)$

Conventional AM

$a_{mod} \le 1$ - for signal to be recovered

$A_c = A$; $A' = 1$

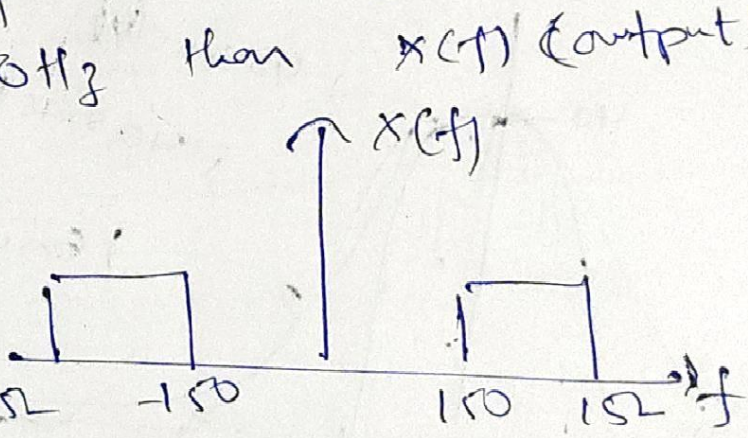
$|\min_t (m(t))| = 0.217$ (calculated)

$\therefore a_{mod} \le 1$

$$\frac{0.217}{A} \le 1$$

So, $\boxed{A \geq 0.217}$

(d) (a) is passed through HPF of cut off freq. 150 Hz then $X(f)$ (output)



This is a band pass signal $I_{[-1,1]}(t) \cdot \cos(2\pi f_c t)$

151 Hz

$x(t) = I_{[-1,1]} \cos(2\pi(150)t + 2\pi t)$

$\Rightarrow x(t) = I_{[-1,1]} \left[\cos(2\pi t) \cdot \cos(2\pi(150)t) - \sin(2\pi t) \cdot \sin(2\pi(150)t)\right]$

Sol/

$$\boxed{\begin{aligned} v_c(t) &= I_{[-1,1]} \cos(2\pi t) \\ v_s(t) &= I_{[-1,1]} \sin(2\pi t) \end{aligned}}$$

e