Lab 9 – FIR filter design

Objectives: In this lab we will study various window functions, design FIR filters using the method of windows, and explore the `filterDesigner` graphical tool in Matlab.

9.1 Window functions (script)

In this task we consider various N-length window functions $w[n], n = 0,1, \dots, N-1$, and their DTFT $W(e^{j\omega})$. Repeat the tasks below for the window functions:

(1) Rectangular (2) Bartlett (3) Hamming (4) Hanning (5) Blackman

(a) For each window generate a figure with 3x3 subplots. In first column, plot the window function $w[n]$ for window lengths N = 51,75,101. In the second and third columns plot the corresponding magnitude and phase of $W(e^{j\omega})$. You can use inbuilt Matlab functions (`bartlett, hann, hamming, blackman`) to generate the windows $w[n]$. Compute $W(e^{j\omega})$ over a grid of 500 points using `fft()` and while plotting the DTFT use `fftshift()` to shift zero-frequency component to the center. Normalize the magnitude spectrum so that its maximum value is 1. Display the magnitude spectrum in decibels after normalization, i.e. plot $20\log_{10}\left|W_{norm}\left(e^{j\omega}\right)\right|$. For the decibel plot, use `ylim()` to restrict y-axis range between [-100, 10].

(b) For any fixed window, what is the effect of changing N on magnitude spectrum?

(c) What can you say about the phase? Is the phase linear?

(d) For each window, visually compute the main-lobe width when N = 51 and note this in a tabular form. Note the width in samples and convert it to frequency as well.

(e) For the rectangular window, visually compute the difference (in dB) between the heights of the main-lobe peak and the immediate next side-lobe. Do this for all values of N. Verify these with the numbers given in the textbook.

9.2 Low-pass filter design using windows (script)

(Proakis problem #10.1) In this task we will design an FIR linear-phase filter approximating the ideal frequency response given by

$$H_d(\omega) = \begin{cases} 1, & |\omega| \le \dfrac{\pi}{6} \\ 0, & \dfrac{\pi}{6} \le |\omega| < \pi. \end{cases}$$

(a) Determine the coefficients of a 51-tap (N = 51) filter based on the window method using a rectangular window.

(b) Plot the filter coefficients $h[n]$ and its magnitude and phase response, all in the same figure using subplots. For magnitude response, perform normalization and plot in decibel scale as described in **9.1**. Is the phase linear for this filter?

Note: if you are manually constructing the sinc signal, make sure the divide by zero problem is correctly addressed.

(c) Repeat (a) and (b) using the Blackman window.

(d) Compare and comment on the transition regions and side-lobe levels of the filters obtained in (a) and (c). Can you correlate these observations with your observations about the corresponding window functions in **9.1?**

(e) Generate 201 samples of the signal $x[n] = \cos\left(\frac{\pi n}{16}\right) + 0.25 \sin\left(\frac{\pi n}{2}\right)$. Pass this signal through the filter designed in (a) by using `conv()` command to implement the filter. In a single figure with two subplots, plot the original signal and the filtered signal. Use `ylim()` to restrict y-axis range between [-1.5, 1.5] for both subplots. Repeat this for the filter designed in (c). Repeat for the signal $x_1[n] = \cos\left(\frac{\pi n}{16}\right) + 0.25\ \mathrm{randn}\ (1{,}201)$.

(f) From the filter $h[n]$ obtained from (a) and (b), construct a new filter $h_1[n] = (-1)^n h[n]$. By plotting the magnitude response (normalized, in decibels) of this new filter, comment on the nature of this filter.

(g) **(Optional) (Proakis problem #10.2)** Repeat (a)-(f) to design an FIR linear-phase filter approximating the band-stop ideal frequency response given by

$$H_d(\omega) = \begin{cases} 1, & |\omega| \leq \frac{\pi}{6} \\ 0, & \frac{\pi}{6} < |\omega| < \frac{\pi}{3} \\ 1, & \frac{\pi}{3} \leq |\omega| < \pi. \end{cases}$$

9.3 Filter design using filterDesigner

This exercise is meant to familiarize you with the filter design tool available in Matlab. Type `filterDesigner` in the command prompt and press enter to open its graphical interface. Explore the various FIR filter design parameters available and relate them to the parameters discussed in the class.

(a) Choose settings to design the exact filter in **9.2 (a)** and compare with your answers in **9.2**. Note that for an N-length filter, chose filter-order to be N-1.

(b) Navigate the Toolbar at the top of the interface to visualize magnitude response, phase response, impulse response, pole-zero plots, etc.

(c) Design a low-pass FIR Equiripple filter with the following specifications: Passband attenuation 1 dB, Stopband attenuation 80 dB, normalized passband frequency of 0.2 normalized stopband frequency 0.5.

(d) Change the design method to least-squares and compare the obtained filter with the one obtained using Equiripple method.