Lab 7 – Z-Transform

Objectives: In this lab we will use Z-Transform to analyse LTI systems

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7.1 ROC, Causality, and Stability (function + script)

Consider LTI systems with transfer (system) function of the form $H(z) = \frac{N(z)}{D(z)}$, i.e. ratio of polynomials in z. Zeros and poles are obtained by solving for roots of polynomials $N(z)$ and $D(z)$ respectively. For this part assume that the numerator is constant $N(z) = 1$, i.e. system has no zeros in the finite z-plane. Recall that there can be many different region of convergence (ROC) associated with the same $H(z)$ expression.

(a) Write a function of the form `[N,ROC,C,S] = roc_cs(p)` with the input vector p containing location of poles in the Z plane (can be complex in general). These are nothing but the roots of $D(z)$. The outputs should be as follows:

- N – number (a positive integer) of unique ROC possible for this $H(z)$
- ROC – Nx2 matrix (of non-negative real numbers) with each row [r1, r2] indicating an ROC of the form $r_1 < |z| < r_2$
- C – length N binary vector (1 if the corresponding system is causal, else 0)
- S – length N binary vector (1 if the corresponding system is stable, else 0)

>> Assume there is at least one pole in the input. The input vector has no particular ordering. There could be multiple poles at the same location. Make sure your code can deal with the various possible inputs given in part (b).

>> In the output above, r1 could be 0 and r2 could be infinity (use Inf to denote this).

(b) Write a script which calls the above function with the following inputs

- p = 3 (expected output: N = 2, ROC = $\begin{bmatrix} 0 & 3 \\ 3 & \mathrm{Inf} \end{bmatrix}$, C = $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$, S = $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$)
- p = 0.1
- p = 0
- p = [0, 0.5]
- p = [2, -0.5]
- p = [0.5, -0.5]
- p = [2, 2, 2]
- p = [0,1,2]
- p = [-0.5, j]
- p = [0, j, -j]
- p = [0.5, -0.5, 2+j, 2-j]
- p = [1+j, 1+2j, 1+3j, 2+j]

Verify your answers by sketching pole-zero plots in your notebook.

7.2 LTI system with a single pole (script)

Consider the LTI system given by transfer function

$$H(z) = \frac{z}{z+p} = \frac{1}{1+pz^{-1}}, \qquad p \in (-1,1)$$

(a) Use the matlab function `zplane()` to get pole-zero plot of this system for p = 0.8. Read documentation of this function and use it in the form `zplane(b,a)`. Note that 'b' and 'a' are coefficients of the polynomial in z^{-1}.

(b) Read documentation of the matlab function `freqz()`. It computes the DTFT of the filter corresponding to the given transfer function. Use the form `freqz(b,a,n)` with n = 1001 for the above system. Plot obtained magnitude response of this filter. Note that it returns DTFT over frequencies in the range $[0, 2\pi]$.

(c) Read documentation of the matlab function `impz()`. Use it to obtain and plot the impulse response of this filter. How many different impulse responses are possible for given system function? Which one is returned by `impz()`? Is this an FIR or IIR filter?

(d) Repeat above for p = - 0.8 and p = 0.1. How does the frequency response and impulse response change with p?

(e) Repeat the above for the system function given by and p = 0.5.

$$H(z) = \frac{z - p^{-1}}{z - p}, \qquad p \in (0,1)$$

What is the kind of filter is represented by this system?

(f) Correlate your observations with the geometrical intuition developed in class.

7.3 LTI system with complex poles (script)

Consider the transfer function given by

$$H(z) = \frac{z^2 - (2\cos\theta)\, z + 1}{z^2 - (2r\cos\theta)z + r^2}, \qquad r \in (0,1), \theta \in [0,\pi]$$

(a) Where are the poles and zeros of this transfer function? Verify your answer in matlab using the `zplane()` function for various r and θ values.

(b) Can this system be both causal and stable simultaneously? What will be ROC?

(c) For fixed $r\ (= 0.95)$, analyse using `freqz()` the behaviour of the system as θ is changed from 0 to π.

(d) For fixed $\theta\ (= 60^o)$, how does the filter frequency response change with r?

(e) Correlate your observations with the geometrical intuition developed in class.

7.4 LTI system with multiple poles (script)

Consider the transfer function given as

$$H(z) = \frac{1}{1 - 0.5\,z^{-1} + 0.2\,z^{-2} - 0.1\,z^{-3} + 0.007\,z^{-4} + 0.14\,z^{-5} + 0.15z^{-6}}$$

(a) Find its frequency response using `freqz()`. Plot the magnitude response. From this plot what can you say about the pole locations? Sketch them roughly.

(b) Use the `zplane()` command to find the true locations of poles and zeros and compare with your sketch.