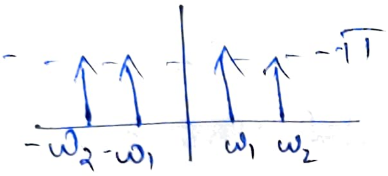
Theory

8.1

a) Given $p(t) = \cos(\omega_1 t) + \cos(\omega_2 t)$

Fourier Transform $P(\omega) = \pi\left(\delta(\omega - \omega_1) + \delta(\omega + \omega_1) + \delta(\omega - \omega_2) + \delta(\omega + \omega_2)\right)$



b) Now we know in discrete domain

$$p[n] = p(nT_s) = \cos(\omega_1 n T_s) + \cos(\omega_2 n T_s)$$

Now $P(e^{j\omega}) = \pi\left(\delta(\omega - \omega_1 T_s) + \delta(\omega - \omega_2 T_s) + \delta(\omega + \omega_1 T_s) + \delta(\omega + \omega_2 T_s)\right)$

It is periodic series of Impulses.

c) Now we have performed windowing using $\omega[n]$

$\Rightarrow x[n] = p[n] \cdot \omega[n]$

By multiplication property

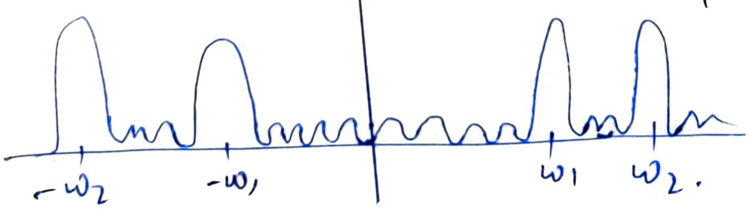
$$X(e^{j\omega}) = \frac{1}{2\pi} \int_{\langle 2\pi \rangle} P(e^{j\theta}) W(e^{j(\omega - \theta)}) \, d\theta.$$

$$|X(e^{j\omega})| = \frac{1}{2\pi} \int_{\langle 2\pi \rangle} |P(e^{j\theta})| \, |W(e^{j(\omega-\theta)})| \, d\theta$$

$\hookrightarrow$ Sinc

So graph now is

Even with peaks at ω_1, ω_2



d) Now we see it is obliging to our theoritical Calculations as L increases sharper curves are obtained. Peaks contain most of the energy.

g) N = 100

we get the peaks at Values 12.5, 37.5, 62.5, 87.5
which can be seen as 12.5, 37.5, (100 - 37.5), (100 - 12.5)
The locations of peaks on x-axis ~~to total~~. Now can be considered these.

We know from above 12.5, 87.5 belong to ω_1
& 37.5, 62.5 belong to ω_2.

$$\omega_1 = \frac{12.5}{100} \quad \omega_s = \omega_s/8 = 1000 \text{rad/s}$$

$$\omega_2 = \frac{37.5}{100} \omega_s = \frac{3\omega_s}{8} = 3000 \text{ rad/s}.$$

Similar are obtained in 1000 by scaling values with factor 10.

h) for 100

the values of x are. 15, 12.5, 85, 87.5.
So similar approach to above is followed.

$$\omega_1 = \frac{125}{1000} = \frac{\omega_s}{8} = 1000 \text{ rad/s}$$

$$\omega_2 = \frac{150}{1000} = \frac{3\omega_s}{20} = 1200 \text{ rad/s}$$

2)

a) <u>Reasoning for Complexity using FFT</u>

We know each butterfly requires

One complex multiplication &

two complex additions.

=> Total $N/2$ butterflies per stage & there are $\log N$ stages.

Therefore order of complexity is $O(N \log N)$

b) <u>Reasoning for complexity using Time domain.</u>

We know for a fixed n:

$$y(n) = \sum_{k=0}^{N-1} x[k] * h(n-k)$$

=> There are N real Multiplications &

$N-1$ real additions.

=> For all n there are $N \cdot N = N^2$ real Multiplications

& $(N-1)N \Rightarrow N^2$ real additions.

So overall complexity is $O(N^2)$ $(\because N^2 + N^2)$