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A Concise Method of Pole Placement to Stabilize the Linear Time Invariant MIMO System

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Abstract— **Stabilization of linear time-invariant multi-input-multi-output (LTI-MIMO) systems is presented distinctively and efficiently in this paper. The idea is to decouple the system state matrix depending on different inputs and outputs using the special canonical transformation proposed. Due to the decoupled form of the observer based controller system, it's possible to use separate transformation for observer and controller design. Since the decoupled state matrix resembles the single-input-single-output (SISO) case, the generalized equation for the system is first obtained, and then it's extended to the MIMO system. The computational complexity in getting the controller and observer gain matrix coefficients are further reduced due to the special form of gain matrices taken.**

Index Terms— **controllability, cyclic subspace, Hurwitz, observability, similarity transformation, stability, state feedback.**

I. Introduction

Controlling of a system becomes more simple when the system is in linear form. In most of the problems, a non-linear system is linearised near the operating point. Stabilization of a linear system is straightforward and is more general whereas nonlinear analysis uses numerical methods. Location of the closed-loop poles of the system determines the performance and stability[1]. Generic approach towards the stabilization of single input single output (SISO) linear time-invariant (LTI) systems using pole placement technique are already mentioned in the literature. But when it comes to the multiple input multiple output (MIMO) case, the design is not anymore straight forward. Some of the approaches mentioned in the literature include conversion of the system to Brunovsky canonical form[2], pole placement after decoupling using Luenberger canonical form[3][4][5]. Brasch explained dynamic pole placement of compensator's with an arbitary transformation [6] and Chen described obtaining state feedback matrix using Lyapnov equation without revealing the structure of the resulting feedback system subjected to the condition that state and feedback coefficient matrix has no common eigenvalues[7].

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This paper proposes a generic approach towards the stabilization of LTI MIMO systems. The objective is to obtain a particular form of input and output matrices by using suitable similarity transformation. These transformation matrices are obtained by slightly modifying the controllability and observability matrices. These transformations also decouple the system matrix to a block upper or block lower triangular form corresponding to different inputs or outputs. A generic system consists of a plant, which is to be stabilized with the associated controllers and observers connected to it. The dynamics of the whole system obtained has the decoupled form, corresponding to the controller and observer. The approach allows taking two different transformations so that the controller and the observer have the independent stable dynamics. The stabilization of the MIMO system using controller has been already described by Wonham[8][9]. Here the approach is extended towards the observer design, where the transformation based on the observability matrix is used.

The transformation used decomposes the MIMO system to multiple SISO systems. Hence initially, the approach is applied to the SISO system to get a generalized view of the components later; these generalized equations are used to the diagonal blocks of the augmented system matrix of MIMO case. The calculation of the controller and observer gain matrices are further simplified by taking the special form of these matrices which are related to the mentioned transformations.

The special forms of input and output matrices allow simplifying the complexity in calculating the controller and observer gain matrices.

Definition 1: In the special form of the input and output matrices, each non-zero entity corresponds to the input and output for the corresponding block of the augmented system matrix of controller and observer, respectively.

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