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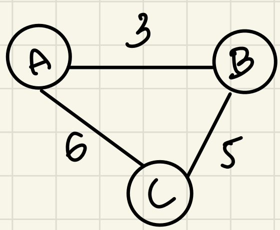
Assignment chapter 4

2.a). True

when all cost positive, by make cost to square cost, it still maintain same manner when we Kruskal algorithm.

b). False



shortest path p is 6

now, square the cost

shortest path p is $3^2 + 5^2 = 34$

not 36.

There for, the statement is false.

8. prove that G has a unique minimum spanning tree.

- Assume that graph G has two different MSTs, T_1 and T_2.

They are different, there must be at least one edge that in T_1 but not in T_2. vice versa.

- identify unique edge, consider the smallest edge with smallest cost that is in T_1 but not in T_2. This edge name e.

Since T_2 is a spanning tree, there must be a path in T_2 connecting the vertices of e. Adding e to T_2 creates a cycle.

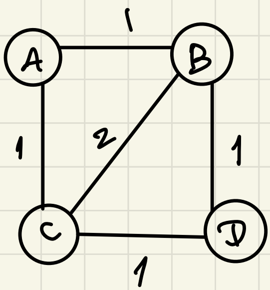
. substitute edge within this cycle there must be an edge that is not in T_1 (otherwise, T_1 and T_2 would be the same. Let's call this e'. The cost of e' must be higher than e (since e is smallest cost edge not in T_2, and all edge costs are distinct.

. create better spanning Tree: Replace e' in T_2 with e. This new tree is still a spanning tree (since we removed one edge from a cycle and added another connection); and the its total cost is lower that of T_2 (since e has a lower cost than e').

. Contradiction : This is contradiction, as T_2 was assumed to be an MST Thus, our assumption that there are two different MST is false.

22. in spanning tree, it is considered minimum-cost if the sum of its edge costs is as low as posible. Even if each edge in given tree T is part of an MST, this does not necessarily mean that T itself is an MST. The key is to cosider the total costs of edges in T

Counter example



consider spanning tree T 's total cost containing of AB, CD, BC $= 1+2+1 = 4$

each edge in T belongs to some MST:

, AB and CD are in MST, AB, AC, CD (total cost $= 3$)

. BC is in MST : AC, BC, BD (total cost $= 3$)

However, T is not a MST since there are spanning tree with lower cost (AC, BC, BC) or (AC, BC, BD) with cost $= 3$.

Therefor, even if every edge of a spanning tree T is part of some minimum-cost spanning tree of G, it does not imply that T itself is an minimum-cost spanning tree. The total cost of T could be higher that that actual MST.