



## University of Kerala

|                |   |                     |                      |           |                         |
|----------------|---|---------------------|----------------------|-----------|-------------------------|
| Discipline     | Mathematics   |                     |                      |           |                         |
| Course Code    | UK1DSCMAT100  |                     |                      |           |                         |
| Course Title   | Foundations of Mathematics  |                     |                      |           |                         |
| Type of Course | DSC   |                     |                      |           |                         |
| Semester       | I   |                     |                      |           |                         |
| Academic Level | 100-199   |                     |                      |           |                         |
| Course Details | Credit  | Lecture<br>per week | Tutorial<br>per week | Practical | Total<br>Hours per week |
|                | 4   | 3                   | -                    | 2         | 5                       |
| Pre-requisites | 1. Basic understanding on the concepts and operations on matrices<br>2. Understanding on the concept of a system of linear equations.<br>3. Basic knowledge of the set number system. |                     |                      |           |                         |
| Course Summary | This course includes set theory, determinants and matrices, number theory and solution of system of equations using matrices and number theory  |                     |                      |           |                         |

## Detailed Syllabus

| Module   | Unit | Contents   | Hrs      |
|----------|------|--|----------|
| <b>I</b> |      | <b>Fundamental Terminology in Proof Techniques</b>   | <b>9</b> |
|          | 1    | Statements, statements with quantifiers, negation of statements with quantifiers, statements involving multiple quantifiers, compound statements, negation of compound statements, implications, converse, if and only if, contrapositive of implications, Proofs in Mathematics (Indirect proofs: proof by contradiction, proof by Induction, counter examples can be omitted) (Chapter 1: Sections 1.1 to 1.5 Text[1]) |          |



| Module     | Unit | Contents   | Hrs      |
|------------|------|--|----------|
| <b>II</b>  |      | <b>Proof techniques and elementary number theory</b>   | <b>9</b> |
|            | 2    | Mathematical induction, The division algorithm, Pigeonhole principle, divisibility relation, inclusion-exclusion principle, prime and composite numbers, infinitude of primes– Sections 1.3, 2.1 (exclude the topics marked as optional), 2.5 (Topics Primes and Pi and exclude the rest in this section) from Text [3])   |          |
| <b>III</b> |      | <b>Primes and composites</b>   | <b>9</b> |
|            | 3    | GCD, linear combination of integers, pairwise relatively prime integers, the Euclidean algorithm for finding GCD, The fundamental theorem of arithmetic, canonical decomposition of an integer into prime factors Section 3.1 (excluding Lemma 3.1, Theorem 3.2, Theorem 3.3 ), section 3.2 (Excluding the topics marked as optional, excluding Lemma 3.2, Theorem 3.12), Section 3.3 (Excluding Theorem 3.14) from Text [3])                            |          |
| <b>IV</b>  |      | <b>System of linear equations and solutions</b>  | <b>9</b> |
|            | 4    | Systems of Linear Equations, Gaussian Elimination, Elementary Matrices and a Method for Finding $A^{-1}$ , Number of Solutions of a Linear System, Applications of Linear Systems –Network Analysis (Chapter 1: Section 1.1, 1.2, 1.5, 1.9 (Network analysis only) of Text[2])   |          |
| <b>V</b>   |      | <b>Suggestions for the teacher designed module</b>   | <b>9</b> |
|            | 5    | Relation: Product sets, Relations, Types of Relations, Equivalence Relations, Partial Ordering Relations Functions: Functions, One-to-One, Onto and Invertible Functions., Different types of matrices – diagonal, triangular, symmetric, Solving system of equations using Cramer's rule, Solving Linear Systems by Matrix Inversion, Applications of linear systems - balancing chemical equations, polynomial interpolation, LCM and related problems |          |

## Practical sessions – 30 hours

All the topics (including those in the suggestions for the teacher designed module) can be used for practical sessions.

### Problems for the practical examination

1. Constructing sets, operations on them like union, intersection, complements
2. Demonstrating basic arithmetic operators  $+$ ,  $-$ ,  $*$ ,  $^$ ,  $,$ , modulo operator  $\%$
3. Forming matrices of different orders
4. Forming identity, zero, scalar matrices
5. Operations on matrices (multiplication, inverses, transposes, cofactor, adjoint)



6. Forming systems of linear equations using symbolic variables
7. Forming matrices for systems, forming augmented matrices
8. Row reduction operations on matrices
9. Finding remainder and quotient
10. Problems using inclusion-exclusion principle
11. Determining if a number is a prime, finding list of primes
12. Using Euler  $\phi$  function
13. Canonical decomposition

A record should be maintained with atleast 7 problems from the above. Each problem in the record must have a description of the problem, algorithm (step by step procedure), commands used, input given and output obtained accordingly. For the ESE, from the list of above 10 problems, the student should be able to answer two selected (from the 7 available in the record) by the examiner.

## Textbooks

1. Ajit Kumar, S Kumaresan, B K Sarma, A foundation course in Mathematics, Narosa Publications, New Delhi 2018
2. H Anton, C Rorres. Elementary linear algebra, 11th Edition, John Wiley & Sons.
3. Thomas Koshy, Elementary Number Theory with Applications, 2nd Edition, Academic Press, 2007.

## References

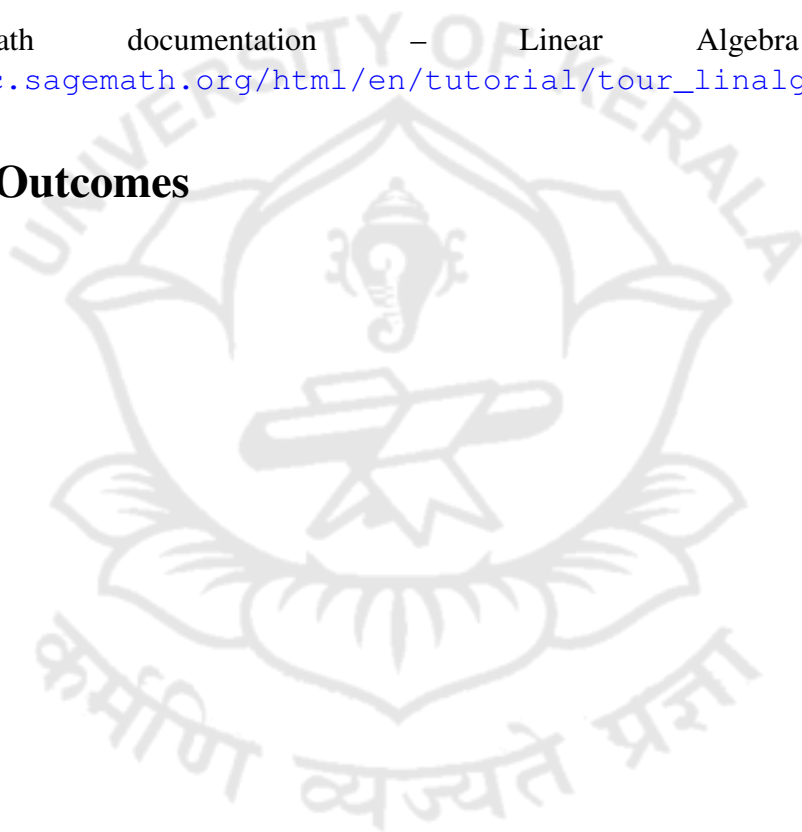
1. David M. Burton, Elementary Number Theory, Seventh Edition, McGraw-Hill, 2011.
2. Gilbert Strang, Introduction to Linear Algebra , 5th Edition, 2005.
3. G A Jones, J M Jones, Elementary Number Theory, Springer, 1998.
4. Lee W. Johnson, R Dean Riess, Jimmy T. Arnold, Introduction to Linear Algebra, Fifth Edition, Addison Wesley, 2019.
5. Seymour Lipschutz. Set Theory and Related Topics, 3rd Edition, Schaum's outline, 1998.



## Resources for practical sessions

- P1. Sagemath documentation – Introductory Sage Tutorial <https://doc.sagemath.org/html/en/prep/Intro-Tutorial.html>
- P2. Saskia Roos, Michael Jung, *An Introductory Course on Sage, Lecture Notes* [https://www.math.uni-potsdam.de/fileadmin/user\\_upload/An\\_Introductory\\_Course\\_on\\_Sage.pdf](https://www.math.uni-potsdam.de/fileadmin/user_upload/An_Introductory_Course_on_Sage.pdf)
- P3. Sagemath documentation – Symbolic variables <https://doc.sagemath.org/html/en/reference/calculus/sage/calculus/var.html>
- P4. P. Zimmermann *et al*, Computational Mathematics with SageMath, <https://www.sagemath.org/sagebook/english.html>
- P5. Gregory V. Bard, Sage for Undergraduates <http://www.people.vcu.edu/~clarson/bard-sage-for-undergraduates-2014.pdf>
- P6. Robert A. Beezer, *A First Course in Linear Algebra* <http://linear.ups.edu/html/sage.html>
- P7. Sagemath documentation – Linear Algebra [https://doc.sagemath.org/html/en/tutorial/tour\\_linalg.html](https://doc.sagemath.org/html/en/tutorial/tour_linalg.html)

## Course Outcomes



| CO No. | Upon completion of the course the graduate will be able to                         | PO/PSO                      | Cognitive Level | Knowledge Category | Lecture(L)<br>Tutorial (T) | Practical (P) |
|--------|--|-----------------------------|-----------------|--------------------|----------------------------|---------------|
| CO 1   | Describe the basic concept of set theory, determinants, Matrices and numbers       | PSO1, PO1, 2, 4, 8          | U               | F,C                | L                          |               |
| CO 2   | Solve system of linear equations using determinants, Matrices                      | PSO2, PO1, 2, 3, 4, 7, 8    | Ap              | P                  | L                          |               |
| CO 3   | Illustration of Mathematical Induction, Division Algorithm and Euclidean Algorithm | PSO1, PO1, 2, 3, 4, 6, 7, 8 | U               | F,C                | L                          |               |
| CO 4   | Categorise functions based on the properties                                       | PSO4, PO1                   | An              | F,C                | L                          |               |

(R-Remember, U-Understand, Ap-Apply, An-Analyse, E-Evaluate, C-Create)  
(F-Factual, C-Conceptual, P-Procedural, M-Metacognitive)

## Mapping of CO with PSOs and POs

|     | PSO1 | PSO2 | PSO3 | PSO4 | PSO5 | PSO6 | PO1 | PO2 | PO3 | PO4 | PO5 | PO6 | PO7 | PO8 |
|-----|------|------|------|------|------|------|-----|-----|-----|-----|-----|-----|-----|-----|
| CO1 | 3    | 2    | 2    | 1    | 1    | 2    | 2   | 2   | 1   | 3   | 1   | 1   | 2   | 2   |
| CO2 | 2    | 3    | 2    | 2    | 1    | 1    | 2   | 3   | 1   | 2   |     |     | 1   | 2   |
| CO3 | 3    | 2    | 2    | 1    | 1    | 1    | 3   | 2   | 1   | 3   |     |     | 1   | 1   |
| CO4 | 2    | 2    | 1    | 3    | 1    | 1    | 3   | 1   | 1   | 1   |     | 1   | 1   | 1   |

( - -Nil, 1-Slightly/Low, 2-Moderate/Medium, 3-Substantial/High)

## Assessment Rubrics

- Quiz/Assignment/Discussion/Seminar
- Midterm Exam

