Rossby Waves

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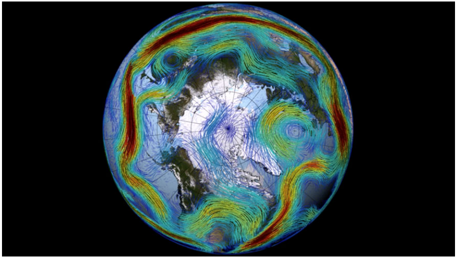
Overview

- Introduce Rossby waves and the Shallow Water Equations (SWE). (Manu)
- Potential Vorticity. (Philipp)
- Linearisation. (Sam)
- (Edward)
- Application of Rossby Waves. (Cathie)



Introduction - Rossby Waves

- Identified by Carl-Gustad Arvid Rossby.
- Large meanders in high altitude winds.
- Responsible for the weather at mid latitudes.
- Caused by the Earth's rotation.
- Two types of Rossby wave

Barotropic	**Baroclinic**
Free wave	Forced wave
Progress eastward and fast moving	Slow moving (cm/s)
Free travelling	Quasi stationary
No significant vertical changes	Vertical changes

- Caused by conservation of **potential vorticity**.

How do we get to the Rossby Waves?

- Navier-Stokes ➔Shallow Water Equations (SWE) ➔Rossby Waves

The momentum equation -

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}.\nabla \mathbf{u} + f\mathbf{z} \times \mathbf{u} = -g\nabla h \tag{1}$$

The mass conservation -

$$\frac{Dh}{Dt} + (h\nabla \mathbf{u}) = 0. \tag{2}$$

What are the Shallow Water Equations?

- A set of hyperbolic PDEs governing fluid flow in the oceans, coastal regions, rivers and channels.
- Consider the vertical length scale to be significantly smaller than the horizontal length scale.
- The SWE are derived from the Navier-Stokes equations.
- The Navier-Stokes equations are themselves derived from the equations for conservation of momentum (1) and the conservation of mass (2).

$$\rho(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}.\nabla\mathbf{u}) = -\nabla p + \mu\nabla^2\mathbf{u} + \rho\mathbf{g}, \tag{3}$$

where $u =$ velocity, $p =$ pressure, $\rho =$ density and $\mu =$ viscosity.

$$\frac{\partial \rho}{\partial t} + (\nabla.\rho\mathbf{u}) = 0. \tag{4}$$

Deriving the Shallow Water Equations

- Derive the Navier-Stokes equations from the conservation laws (1) and (2).
- Specify boundary conditions for the Navier-Stokes equations for a water column.
- Use the boundary conditions to depth integrate the Navier-Stokes equations.

The Shallow Water Equation -

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}.\nabla \mathbf{u} + f\mathbf{z} \times \mathbf{u} = -g\nabla h \tag{5}$$

- Linearise the system of equations to perform analysis on.

Vorticity Equation

Rewrite shallow water equations in terms of *relative vorticity* ζ

- take the curl $\nabla \times (...)$
- $\frac{\partial \zeta}{\partial t} + \boldsymbol{u}\nabla(\zeta + f) = (\zeta + f)\nabla \boldsymbol{u}$

Inserting the mass conservation $\frac{\mathrm{D}h}{\mathrm{D}t} + h\nabla \boldsymbol{u} = 0$, we can restate the equations as

- $\frac{\mathrm{D}}{\mathrm{D}t}\left(\frac{\zeta+f}{h}\right) = 0$

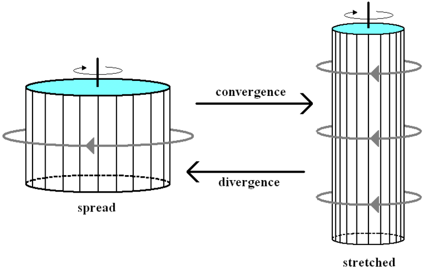
We found a conserved quantity! It is called **potential vorticity**.

Potential Vorticity

What is potential vorticity $q = \frac{\zeta + f}{h}$, $\frac{\mathsf{D}}{\mathsf{D}t} q = 0$?

- Continuum equivalent of *angular momentum* conservation

In f-plane approximation, $f = f_0$ =const. :



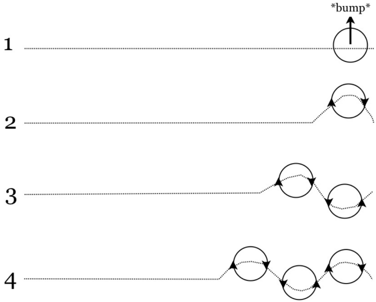
- Change in ζ has to be compensated by change in hight h

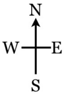
Potential Vorticity

In β-plane approximation $f = f_0 + \beta y$ is not constant.

$$\boxed{q = \frac{\zeta + f}{h}}$$

Fluid parcels can move north-/southward when changing vorticity ζ.





- Additionally induce movement on neighboured fluid particles

A simple model

In order to obtain a model for the propogation of Rossby waves, we make the following simplifying assumptions:

- The height of the free surface is fixed at $h = H$.
- The Coriolis parameter varies linearly with latitude as $f = f_0 + \beta y$ (beta plane approximation)

Then the potential vorticity equation (TODO ref) reduces to

$$\frac{D}{Dt}\left(\zeta + f_0 + \beta y\right) = 0$$

which simplifies to

$$\frac{D\zeta}{Dt} + \beta v = 0$$

A simple model

Since h is assumed constant, the continuity equation (TODO ref) implies that $\nabla \cdot \mathbf{u} = 0$. Then there exists a stream function ψ such that $\mathbf{u} = \left(-\frac{\partial \psi}{\partial y}, \frac{\partial \psi}{\partial x}\right)$. By expanding the material derivative and writing u and v in terms of ψ, we get

$$\frac{\partial \zeta}{\partial t} - \frac{\partial \psi}{\partial y}\frac{\partial \zeta}{\partial x} + \frac{\partial \psi}{\partial x}\frac{\partial \zeta}{\partial y} + \frac{\partial \psi}{\partial x} = 0$$

Then using the fact that $\zeta = \nabla^2 \psi$, we obtain an equation for ψ:

$$\frac{\partial \left(\nabla^2 \psi\right)}{\partial t} - \frac{\partial \psi}{\partial y}\frac{\partial \left(\nabla^2 \psi\right)}{\partial x} + \frac{\partial \psi}{\partial x}\frac{\partial \left(\nabla^2 \psi\right)}{\partial y} + \frac{\partial \psi}{\partial x} = 0$$

Linearization

We see that a steady uniform flow with velocity $\mathbf{u_0} = (U, 0)$ and stream function $\psi_0 = -Uy$ satisfies equation (TODO ref).
We now consider a small perturbation $\psi = \psi_0 + \epsilon\psi_1 + O(\epsilon^2)$. By substituting this into (TODO ref) and discarding higher order terms we obtain the following equation for ψ_1:

$$\frac{\partial\left(\nabla^2\psi\right)}{\partial t} + U\frac{\partial\left(\nabla^2\psi\right)}{\partial x} + \beta\frac{\partial\psi}{\partial x} = 0$$

Now we seek a trial solution of the form $\psi_1 = e^{i(kx+ly-\omega t)}$.
Substituting this into the above equation, we obtain the following dispersion relation:

$$\omega = Uk - \frac{\beta k}{k^2 + l^2}$$

Plot

Quasi stationary synoptic Rossby waves high amplitude during weather events

by Petoukhov, Rahmstorf, Pteri and Schellnhuber 2013

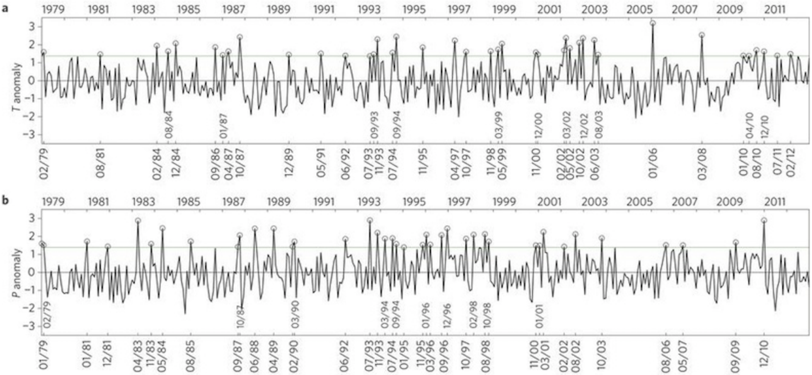
- Investigated physical model of quasi resonance effect.
- Forced wave trapping a free wave.
- Amplification of pressure system.
- Extreme weather results.

Amplified mid-latitude planetary waves favour particular regional weather extremes

by Screen and Simmonds

- Looked at distribution of quasi resonance events.
- Examined links between temperature and precipitation anomalies and abnormal quasi stationary wave amplitude from 1979 to 2012.

40 most extreme temperature and precipitation events in the mid latitudes between 1979 and 2012.



Anomalies: Prolonged periods over a large area

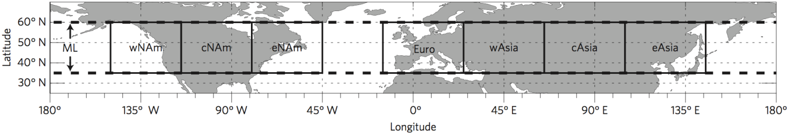
Temperature

Positive: abnormally high   Negative: abnormally low

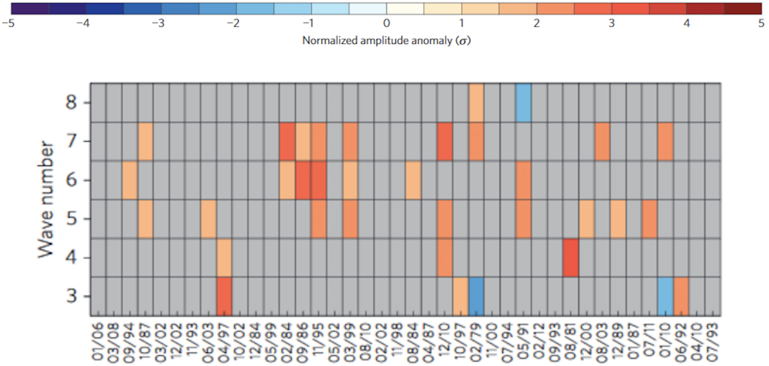
Precipitation

Positive: abnormally wet   Negative: abnormally dry

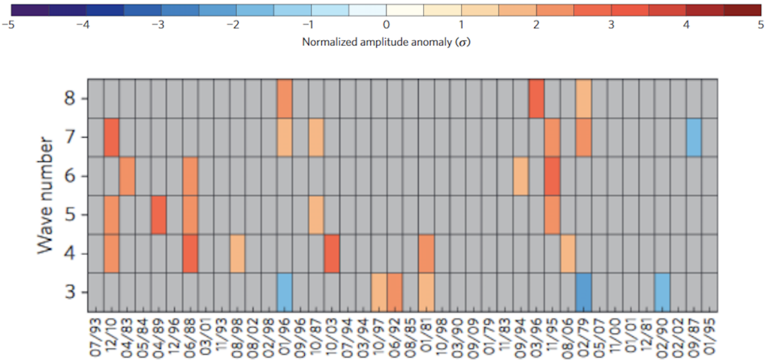
Mid latitude regions of the Northern hemisphere examined



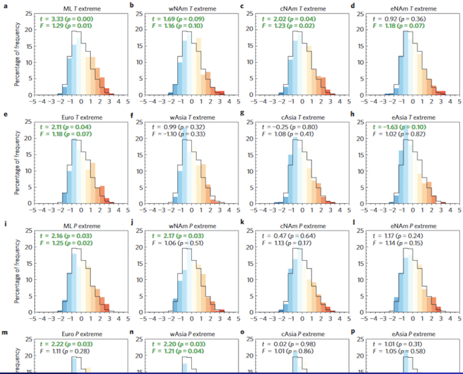
Charts to show the normalized amplitude anomalies for each extreme temperature event, in order of severity of weather, taking the most extreme events on the left.



Charts to show the normalized amplitude anomalies for each extreme precipitation event, in order of severity of weather, taking the most extreme events on the left.



Distributions comparing observed frequency of anomalous amplitudes of baroclinic Rossby waves during periods of extreme weather and the distributions expected from climatology across each examined region.



Future Forecast