Rossby Waves

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Shallow Water Equations

Starting from the equations:

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\mathbf{u} + f\hat{\mathbf{z}} \times \ \mathbf{u} = -g\nabla h \tag{1}$$

$$\frac{\partial h}{\partial t} + \nabla \cdot (h\mathbf{u}) = 0 \tag{2}$$

If we consider small amplitude motions on the surface $h = H + \eta$, $\mathbf{u} = (u', v', 0)$ where $\eta << 0,\ u', v' << 0$. Looking at the $O(1)$ terms, we get these equations.

$$\frac{\partial u'}{\partial t} - fv' = -g\frac{\partial \eta}{\partial x} \tag{3}$$

$$\partial v' t + fu' = -g\frac{\partial \eta}{\partial y} \tag{4}$$

$$\frac{\partial \eta}{\partial t} + H\frac{\partial u'}{\partial x} + H\frac{\partial v'}{\partial y} = 0 \tag{5}$$

Gravity Waves

The simplest case of the shallow water equations is in the absence of rotation $(f = 0)$. If we then take $\frac{\partial(3)}{\partial x}$, $\frac{\partial(4)}{\partial y}$, $\frac{\partial(5)}{\partial t}$, we get the wave equation

$$\frac{\partial^2 \eta}{\partial t^2} - gH\nabla^2\eta = 0 \tag{6}$$

This is the wave equation so will have solutions of the form.

$$\eta = h_0 \exp(i(kx + \ell y + \omega t)) \tag{7}$$

which by plugging this into Equation (6), we get the dispersion relation

$$\omega = \pm\sqrt{gH(k^2 + \ell^2)} \tag{8}$$

Inertio-Gravity/ Poincaré waves

Now looking at the case where we have constant rotation $(f = f_0)$ By taking various partial derivatives of Equations (3), (4), (5), we obtain the equation.

$$\frac{\partial}{\partial t}\left(\frac{\partial^2}{\partial t^2} + f_0^2 - gH\nabla^2\right)\eta = 0 \tag{9}$$

Which when we plug in (7) gives us the dispersion relation

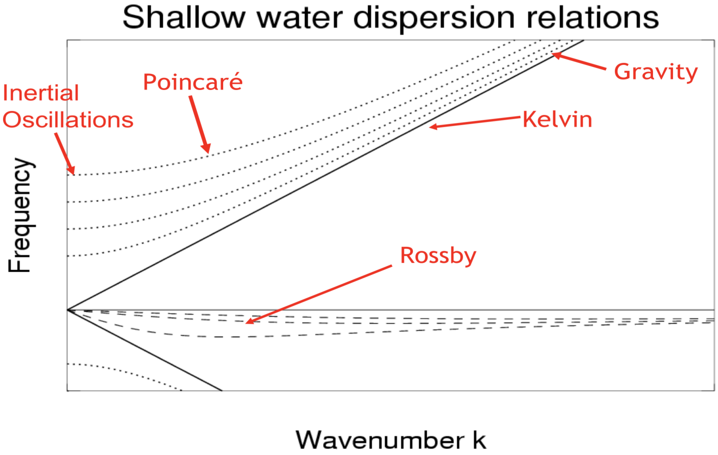
$$-i\omega(-\omega^2 + gH(k^2 + \ell^2) + f_0^2) \tag{10}$$

So has solutions

$$\omega = 0 \qquad \text{or} \qquad \omega = \pm\sqrt{f_0^2 + gH(k^2 + \ell^2)} \tag{11}$$

The zero frequency case is the time independent flow. The flow here oscillates at greater frequency than the standard gravity waves.

Comparing Shallow Water Waves



Quasi stationary synoptic Rossby waves high amplitude during weather events

by Petoukhov, Rahmstorf, Pteri and Schellnhuber 2013

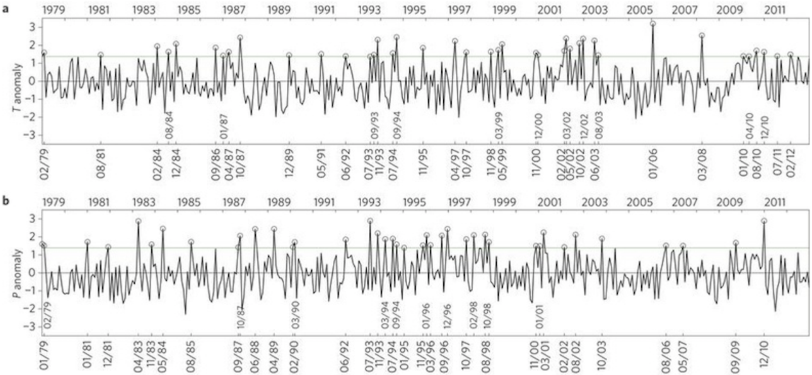
- Investigated physical model of quasi resonance effect.
- Forced wave trapping a free wave.
- Amplification of pressure system.
- Extreme weather results.

Amplified mid-latitude planetary waves favour particular regional weather extremes

by Screen and Simmonds

- Looked at distribution of quasi resonance events.
- Examined links between temperature and precipitation anomalies and abnormal quasi stationary wave amplitude from 1979 to 2012.

40 most extreme temperature and precipitation events in the mid latitudes between 1979 and 2012.



Anomalies: Prolonged periods over a large area

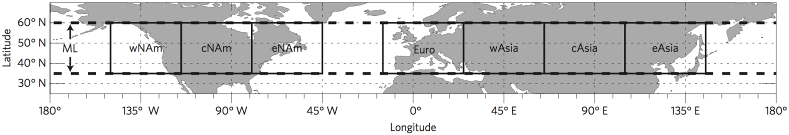
Temperature

Positive: abnormally high   Negative: abnormally low

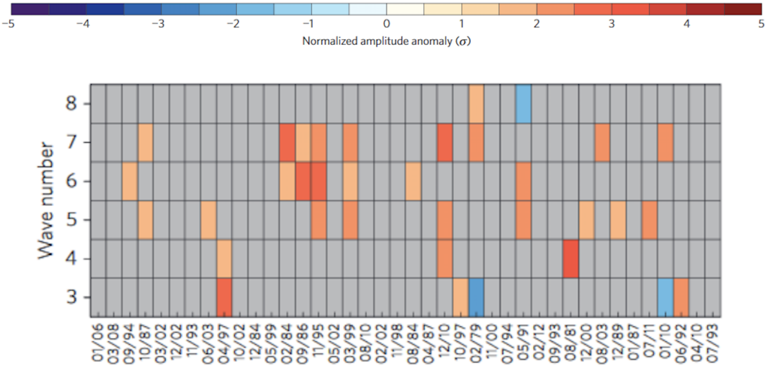
Precipitation

Positive: abnormally wet   Negative: abnormally dry

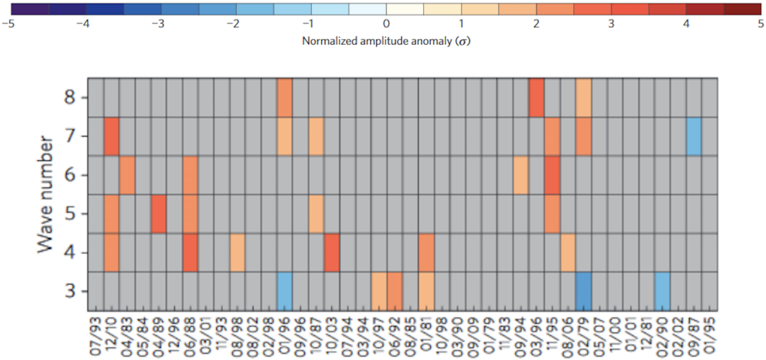
Mid latitude regions of the Northern hemisphere examined



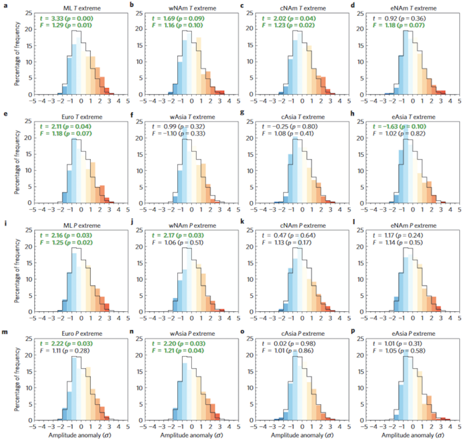
Charts to show the normalized amplitude anomalies for each extreme temperature event, in order of severity of weather, taking the most extreme events on the left.



Charts to show the normalized amplitude anomalies for each extreme precipitation event, in order of severity of weather, taking the most extreme events on the left.



Distributions comparing observed frequency of anomalous amplitudes of baroclinic Rossby waves during periods of extreme weather and the distributions expected from climatology across each examined region.



Future Forecast