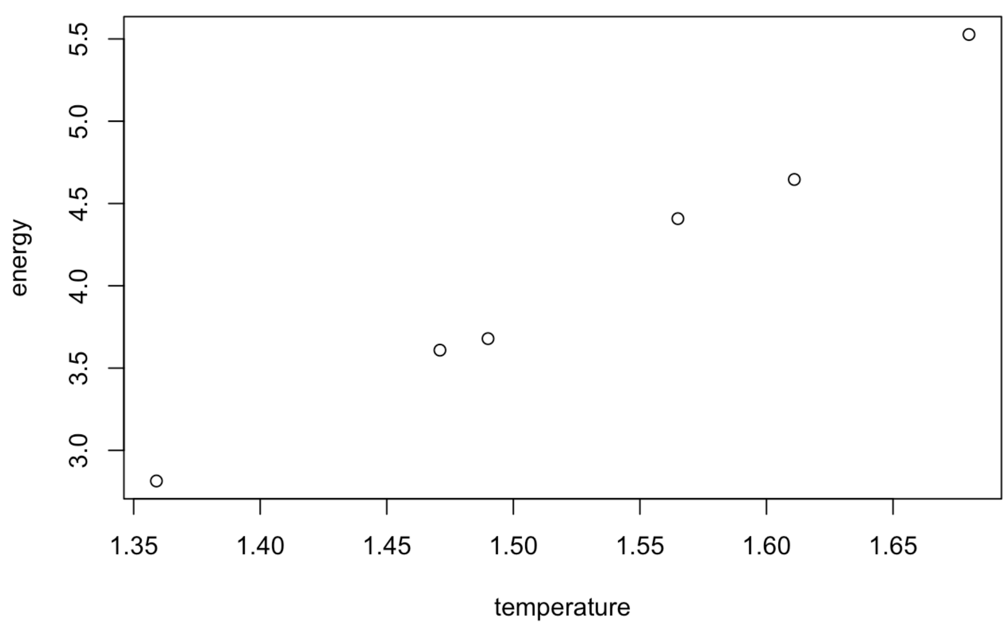
ST 512 Homework 3

```
library(dplyr)
```

```
Attaching package: 'dplyr'

The following objects are masked from 'package:stats':

    filter, lag

The following objects are masked from 'package:base':

    intersect, setdiff, setequal, union
```

```
lamp <- read.table("lamp.txt", header = TRUE)
with(lamp, plot(energy ~ temperature))
```




```
lamp.model <- lm(log(energy) ~ temperature, data = lamp)
```

```r
summary(lamp.model)
```

```
Call:
lm(formula = log(energy) ~ temperature, data = lamp)

Residuals:
        1         2         3         4         5         6 
-0.004769  0.012439 -0.007643  0.018037 -0.024505  0.006442 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) -1.77103    0.10459  -16.93 7.13e-05 ***
temperature  2.06800    0.06824   30.31 7.06e-06 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.01734 on 4 degrees of freedom
Multiple R-squared:  0.9957,	Adjusted R-squared:  0.9946 
F-statistic: 918.5 on 1 and 4 DF,  p-value: 7.061e-06
```

2.068 = estimate for O2 # #1 (2 points) Report the least-squares estimates of the parameters θ1 and θ2 for the lamp data. Show work or computer code.

θ1 = .171 θ2 = 2.066

```r
model1 <- nls(energy ~ th1 * exp(th2 * temperature), data = lam
summary(model1)
```

```
Formula: energy ~ th1 * exp(th2 * temperature)

Parameters:
    Estimate Std. Error t value Pr(>|t|)    
th1  0.17075    0.02099   8.135  0.00124 ** 
th2  2.06582    0.07810  26.452 1.21e-05 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.077 on 4 degrees of freedom

Number of iterations to convergence: 3 
Achieved convergence tolerance: 9.712e-09
```

#2

(1 point) Use your model fit from question 1 to predict the energy output of this lamp when the temperature is x = 1.4 (which corresponds to 1400 K). Show work or computer code.

3.084 energy per cm^2 per second

```
y_value <- .171 * exp(2.066 * 1.4)
```

#3

Using the model in equation 2, calculate an F-statistic and a corresponding p-value to test for differences among the seasons when comparing days with the same temperature, precipitation, and type of day. That is, in notation, test $H0 : \beta 4 = \beta 5 = 0$. Be sure to report both the F - statistic and the p-value, and be sure to give the associated df for the F-statistic. Show some work or computer code.

F-statistic = .213, p-value = .8086, DF = 2

```
trailuser <- read.table("trailuser.txt", header = TRUE, stringsA
trailuser$dayType <- as.factor(trailuser$dayType)
trailuser$season <- as.factor(trailuser$season)

contrasts(trailuser$dayType)
```

```
        weekend
weekday       0
weekend       1
```

```
contrasts(trailuser$season)
```

```
       spring summer
fall        0      0
spring      1      0
summer      0      1
```

```
additive_model <- lm(volume ~ avgtemp + precip + dayType + seaso
summary(additive_model)
```

```
Call:
lm(formula = volume ~ avgtemp + precip + dayType + season, data
= trailuser)

Residuals:
    Min      1Q  Median      3Q     Max
-238.22  -42.98   16.33   47.33  217.44

Coefficients:
               Estimate Std. Error t value Pr(>|t|)
(Intercept)     -28.186     83.261  -0.339 0.736027
avgtemp           6.923      1.796   3.855 0.000262 ***
precip         -165.044     43.371  -3.805 0.000309 ***
dayTypeweekend   86.200     29.380   2.934 0.004579 **
seasonspring     10.551     40.845   0.258 0.796948
seasonsummer    -11.404     63.647  -0.179 0.858336
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 97.94 on 67 degrees of freedom
Multiple R-squared:  0.4573,    Adjusted R-squared:  0.4168
F-statistic: 11.29 on 5 and 67 DF,  p-value: 6.428e-08
```

```r
full_model <- lm(volume ~ avgtemp + precip + dayType + season, d
reduced_model <- lm(volume ~ avgtemp + precip + dayType, data =
anova_results <- anova(reduced_model, full_model)
anova_results
```

```
Analysis of Variance Table

Model 1: volume ~ avgtemp + precip + dayType
Model 2: volume ~ avgtemp + precip + dayType + season
  Res.Df    RSS Df Sum of Sq      F Pr(>F)
1     69 646746
2     67 642657  2    4089.3 0.2132 0.8086
```

#4

(1 point) Choose the best interpretation of the outcome from your test in question 3.

A. There is mildly strong evidence that trail usage differs among the three seasons.

B. If we compare days with identical temperature, precipitation, and day type (that is, whether the day is a weekday or not), then there is mildly strong evidence that trail usage differs among the three seasons.

C. There is no evidence that trail usage differs among the three seasons.

D. If we compare days with identical temperature and precipitation, and day type (that is, whether the day is a weekday or not), then there is no evidence that trail usage differs among the three seasons.

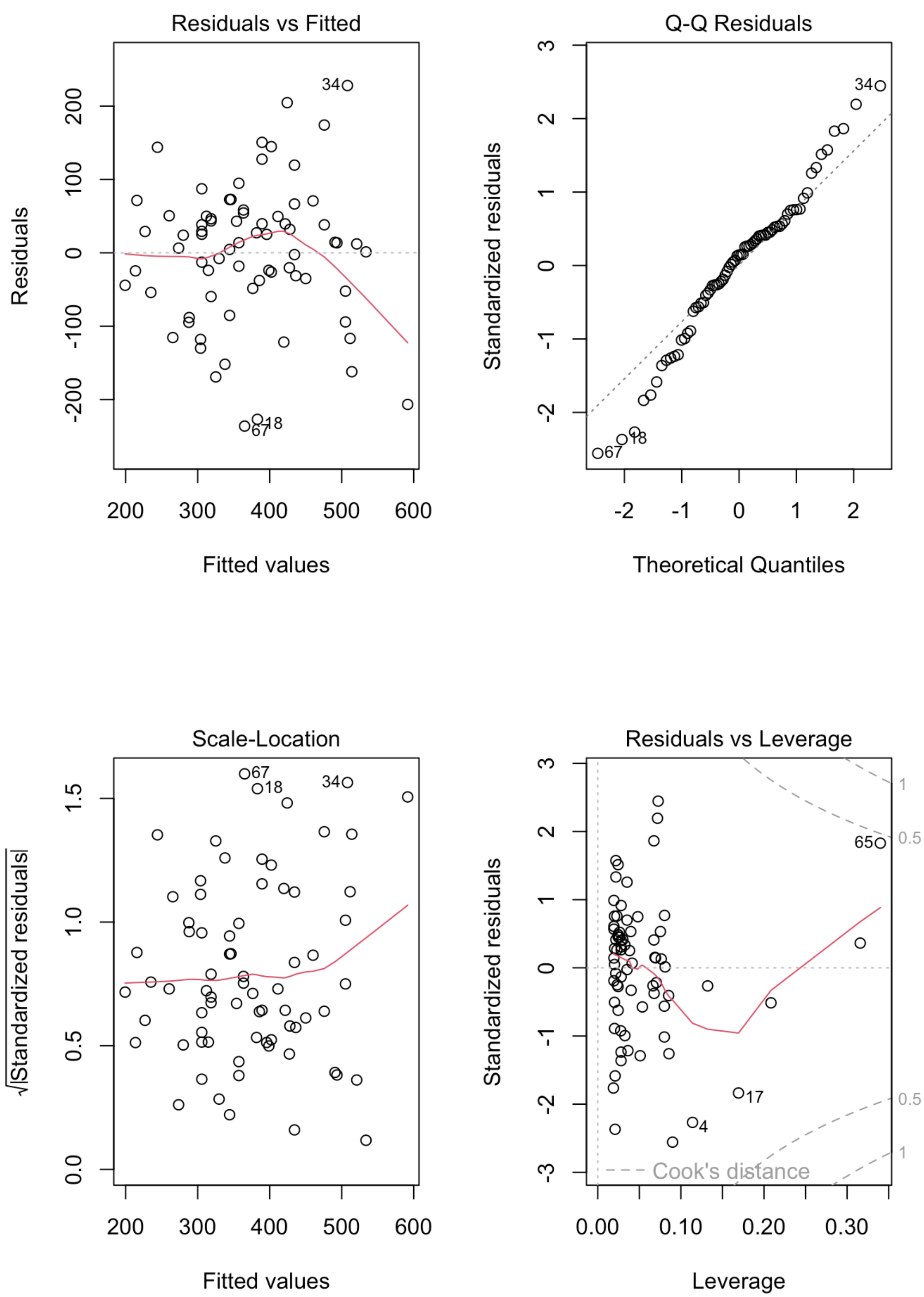
Answer = D

#5

(2 points) In the model in equation 3, provide a brief non-technical interpretation of why the estimated partial regression coefficient associated with the quadratic effect of avgtemp is negative. That is, what have we learned from the fact that $\hat{\beta} < 0$, and why does this make sense in the context of this particular data set?

While higher avgtemp initially leads to an increase in the number of trail users, there is a point in which further temperature increases result in decreased usage. This suggests that really high temperatures deter users.

```
par(mfrow = c(1, 2))
plot(reduced_model)
```




```
quadratic_model <- lm(volume ~ avgtemp + I(avgtemp^2) + precip
summary(quadratic_model)
```

```
Call:
lm(formula = volume ~ avgtemp + I(avgtemp^2) + precip + 
dayType,
    data = trailuser)

Residuals:
     Min       1Q   Median       3Q      Max 
-246.463  -43.313    7.456   47.170  209.489 

Coefficients:
                 Estimate Std. Error t value Pr(>|t|)    
(Intercept)    -599.94041  249.20172  -2.407 0.018784 *  
avgtemp          27.81275    8.64793   3.216 0.001990 ** 
I(avgtemp^2)     -0.18344    0.07369  -2.489 0.015248 *  
precip         -148.07875   41.43765  -3.574 0.000653 ***
dayTypeweekend   93.65017   27.43205   3.414 0.001083 ** 
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 93.36 on 68 degrees of freedom
Multiple R-squared:  0.4995,    Adjusted R-squared:   0.47 
F-statistic: 16.96 on 4 and 68 DF,  p-value: 1.086e-09
```

#6

Use the model in equation 3 to complete each of the statements below:

(a) (1 point) “When comparing two week-end days with the same temperature, a day on which it rains 0.25 inches will have an average of ***37.02 fewer*** trail users than a day on which it doesn’t rain.”

b) (1 point) “When comparing two days with the same temperature and precipitation, a weekday will have an average of ***93.65 fewer*** trail users than a weekend day.”

#7

(1 point) Use the model in equation 3 to predict the trail usage on a weekday when the avgtemp = 55∘ F and the precip = 0.15 in. Show some work or computer code.

532.643

```
beta0 <- -599.94041
beta1 <- 27.81275
beta2 <- -0.18344
beta3 <- -148.07875
dayType_weekend <- 0

avgtemp <- 55
precip <- 0.15

predicted_volume <- beta0 +
                   (beta1 * avgtemp) +
                   (beta2 * (avgtemp^2)) +
                   (beta3 * precip) +
                   (dayType_weekend * 93.65017)
predicted_volume
```

```
[1] 352.643
```

#8

(1 point) Use the model in equation 4 to predict the trail usage on a weekday when the avgtemp = 55∘ F and the precip = 0.15 in. Show some work or computer code.

314.292

```
trailuser2 <- trailuser |>
  mutate(sqrt_precip = sqrt(precip))
```

```
model4 <- lm(volume ~ avgtemp + I(avgtemp^2) + sqrt_precip + day

sqrt_precip_value <- sqrt(0.15)
new_data <- data.frame(avgtemp = 55,
                       I.avgtemp.2 = 55^2, sqrt_precip = sqrt_p
predicted_usage <- predict(model4, newdata = new_data)
predicted_usage
```

```
       1
314.2928
```

#9

I would favor reporting model 4....

#10

a. (1 point) Which of the following statements provides the best interpretation of the fact that β3 > 0? Choose the best answer. A. As BP increases, the slope of the association between Y and BMI increases. B. As BP increases, the slope of the association between Y and BMI decreases. C. BMI and BP are positively correlated. D. Both choices B and C are reasonable interpretations. E. None of the above are reasonable interpretations.

   Answer = A

b) (2 points) Use the model to predict the response for someone who has a BMI of 25 and a blood pressure (BP) of 90.

139.35

```
intercept <- 66.45
beta_BMI <- -1.62
beta_BP <- -1.49
beta_interaction <- 0.11

BMI <- 25
BP <- 90

interaction_term <- BMI * BP

predicted_response <- intercept + (beta_BMI * BMI) + (beta_BP *
predicted_response
```

```
[1] 139.35
```

c. (4 points) Which of the statements below are supported by the fit and analysis of the multiple regression model? Choose 'supported' if the statement is supported by the analysis of the multiple regression model, and 'not supported' if it is not supported.

i. Using the conventional threshold for statistical signif- icance, there is statistically significant evidence that the association between BMI and the response depends on the value of BP.

SUPPORTED

ii. The average value of the response is 66.45.

NOT SUPPORTED

iii. The multiple regression model provides a significantly better fit than a model that includes only an intercept.

SUPPORTED

iv. The multiple regression model explains roughly 40% of the variation in the response.

SUPPORTED