Machine Learning to model a developing boundary layer
Project Proposal

Physical Sciences

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1 Motivation

Boundary layer flows are common in nature and often pose a significant problem in efficiently calculating the fluid flow for a given geometry. The small scales and fluctuating terms often need to be either refined or accurately modeled using an approximation. Refining the small scales can be computationally expensive and using models can lead to unknown errors between the real world and the simulation. Thus, having good models to predict the fluctuation terms can be instrumental in a good simulation software package. These fluctuation models typically come from physical arguments and on observations of what seems to work well.

1.1 Navier-Stokes equations and the 2D turbulent boundary layer equations

The incompressible 2D turbulent boundary layer equations can be expressed as

$$\frac{\partial \overline{u_1}}{\partial x_1} + \frac{\partial \overline{u_2}}{\partial x_2} = 0$$

$$\overline{u_1}\frac{\partial \overline{u_1}}{\partial x_1} + \overline{u_2}\frac{\partial \overline{u_1}}{\partial x_2} + \frac{1}{\rho}\frac{\partial \overline{P}}{\partial x_1} - \nu\left(\frac{\partial^2 \overline{u}}{\partial x_2^2}\right) = -\overline{u_j'\frac{\partial u_1'}{\partial x_j}}$$

$$\frac{1}{\rho}\frac{\partial \overline{P}}{\partial x_2} = -\overline{u_j'\frac{\partial u_2'}{\partial x_j}}$$

Where the velocity can be expressed as the steady state and fluctuation part ($u_i = \overline{u_i} + u_i'$). This is derived from the Navier-Stokes equations where we have assumed steady-state, incompressible, constant density, and some neglected terms due to boundary layer growth assumptions. The continuity and momentum equations in the streamwise (x_1) and wall-normal (x_2) coordinates are presented above.

As stated previously, the fluctuation quantities (e.g. $\overline{u_j'\frac{\partial u_1'}{\partial x_j}}$) are modeled using the Eddy Viscosity model and the Mixing-Length Theory. This leads to an algebraic form of these unknown terms such that $-\overline{u_1'u_2'} = (\text{const})\left(l_1\frac{\partial \overline{u_1}}{\partial x_2}\right)\left(l_2\frac{\partial \overline{u_1}}{\partial x_2}\right)$.

The idea of this project is to come out with an approximating model for these fluctuating terms that lead to an alternative model to be used in marching the turbulent boundary layer equations.

This project is an application that is tackling a model formulation for fluctuation stresses in a boundary layer fluid flow.

2 Method

There are many methods that could be used to approximate these fluctuation terms. For a starting example a linear regression could be applied where the average base flow, the respective gradients, and the Reynolds number are the features and the output quantities will include the averaged fluctuation terms needed in the two dimensional turbulent boundary layer equations.

In addition to a linear regression, another approach to explore could be applying a random forest type technique to output the same quantities.

3 Inteded experiments

Exploring the inputs and outputs will be a bit of the intended application to tackle the problem of modeling the unknown quantities the best.

The data set available will include a few boundary layer flow fields modeled using a refined model where all the physics of interest is accurately resolved.

Some of the available data sets will be used for training purposes, and the others will be used for validation. The typical quantitative assessments of error between the model and the refined simulation will be used to evaluate the machine learning algorithm.

Data obtained from the JHTDB at http://turbulence.pha.jhu.edu will be useful in terms of training and validation. Additionally, neural networks have been applied with some success in CFD as seen in [Ling et al., 2016]

References

[Ling et al., 2016] Ling, J., Kurzawski, A., and Templeton, J. (2016). Reynolds averaged turbulence modelling using deep neural networks with embedded invariance. *Journal of Fluid Mechanics*, 807:155–166.