# Project 2

Build Unsteady Model of “Pike” .. Use Integrator of your choice

*Calculate:*

- *Chamber pressure profile*
- *Regression rate profile*
- *Massflow rate (compare to choking massflow)*
- *Mass depletion vs time*
- *plot Thrust profile*
- *plot Total Impulse profile*
- *Effective Mean Specific Impulse*

*Allow:*

- *St. Robert’s Parameter Input*
- *Variable Step Size*
- *Variable Thermodynamic Properties* ***(as inputs to the problem)***
- ***Erosive burn model for cylindrical port (Not Bates grain)***

# Project 2 (2)

## *Part 1, cylindrical port*

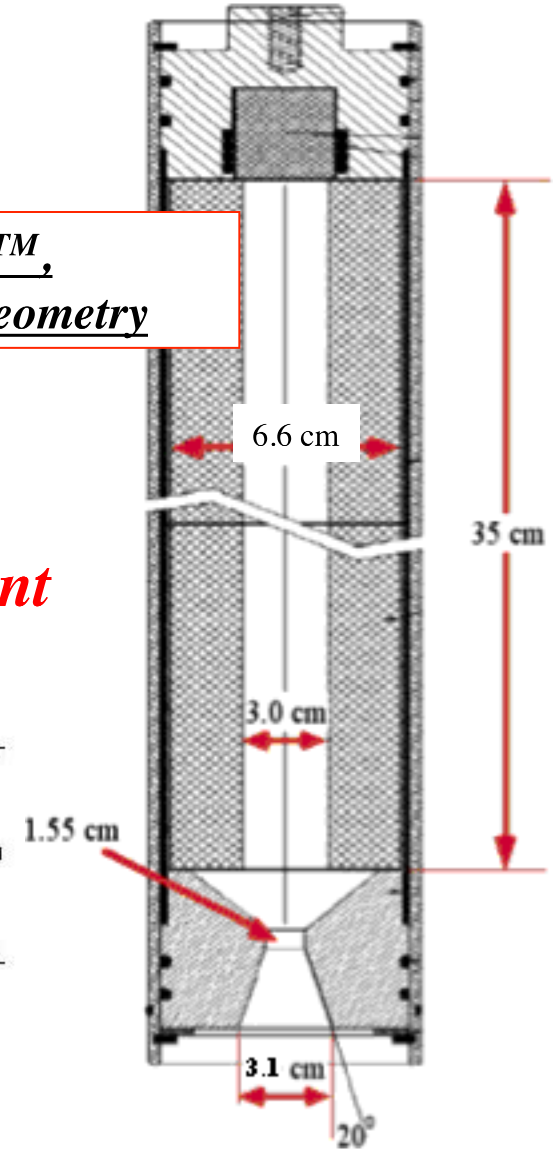
### Fuel Grain Geometry

$L_0 = 35$ cm
$D_0 = 6.6$ cm
$D_0 = 3$ cm
$\rho_{propellant} = 1260\ kg/M^3$

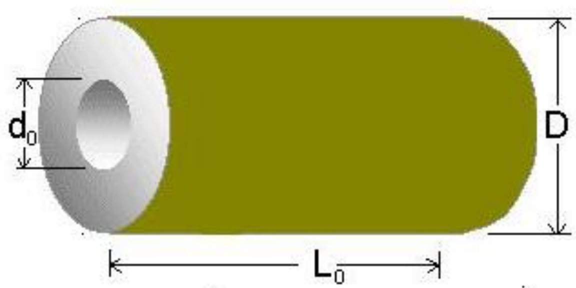
### Nozzle Geometry

$A^* = 1.887\ cm^2$
$A_{exit}/A^* = 4.0$
$\theta_{exit} = 20$ deg.

**Animal Works™, L700 Motor Geometry**




*Single propellant segment*




*Assume ends are burn inhibited*

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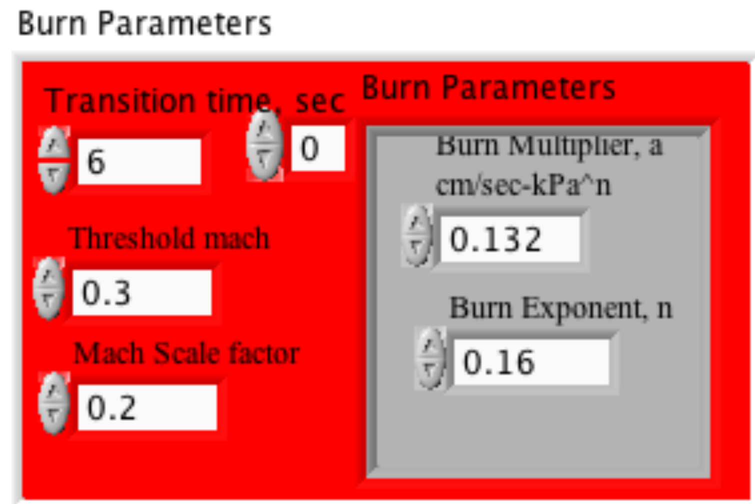
Mechanical & Aerospace Engineering

# Project 2 (3)




## *Combustion Gas Properties*

$\gamma = 1.18$

$M_W = 23$ *kg/kg-mol*

$T_0 = 2900$ K

## *Burn Parameters*

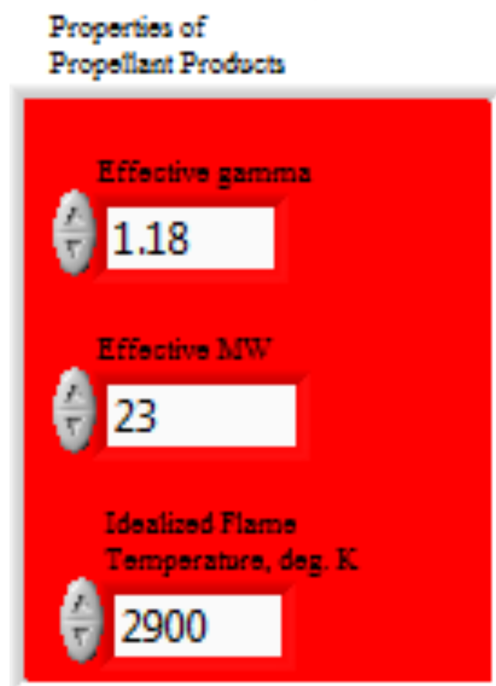
$a = 0.132$ *cm/(sec-kPa $^n$)*

$n = 0.16$

$M^{crit} = 0.3$

$k = 0.2$

*(cylindrical port only)*



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# Project 2 (4)

**Part 1, cylindrical port**

## Fuel Grain Geometry

$L_0 = 35$ cm

$D_0 = 6.6$ cm

$D_0 = 3$ cm

$\rho_{propellant} = 1260$ kg/M$^3$

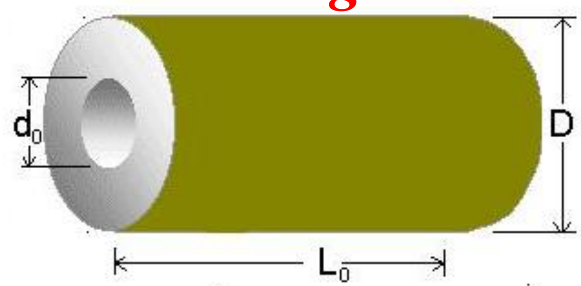
## Nozzle Geometry

$A^* = 1.887$ cm $^2$

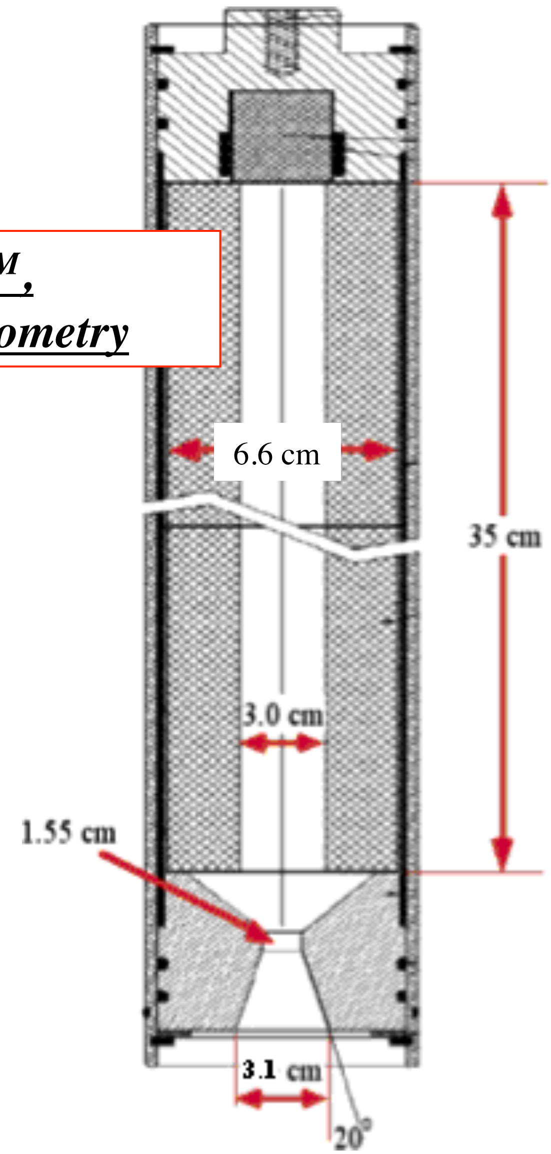
$A_{exit}/A^* = 4.0$

$\theta_{exit} = 20$ deg.

**Repeat results**
**Using bates grain**
**With 3 segments**




**Animal Works™, L700 Motor Geometry**




**Assume ends are not! burn inhibited**

# Project 2 (5)

## *Combustion Gas Properties*

$\gamma = 1.18$

$M_W = 23$ *kg/kg-mol*

$T_0 = 2900$ *K*

## *Burn Parameters*

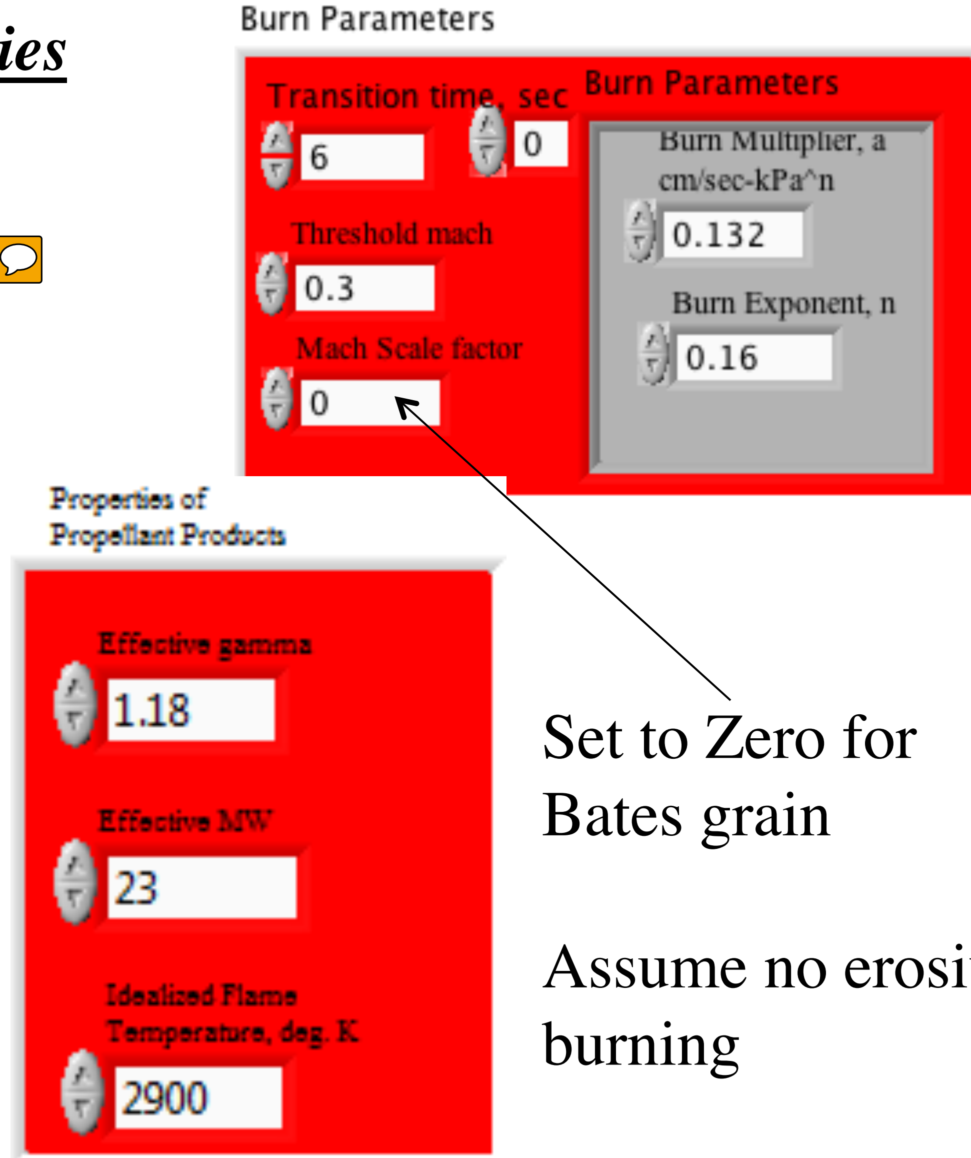
$a = 0.132$ *cm/(sec-kPa* $^n$*)*

$n = 0.16$

$M^{crit} = 0.3$

$k = 0.0$

***(Bates grain only)***




Set to Zero for Bates grain

Assume no erosive' burning

# Project 2 (6)

Examine sensitivity of calculations to burn rate parameters, Critical Mach number (for erosion) … cylindrical port Only, Assume bates grain does not burn erosively

What is the effect of Flame temperature ($T_0$)

Plot Regression rate versus Chamber pressure

Prepare report stating your results and conclusions

# State Equation Formulation of Problem

$$\begin{bmatrix} \dot{P}_0 \\ \dot{r} \end{bmatrix} = \begin{bmatrix} \left(\frac{A_{burn} \cdot \dot{r}}{V_c}\right) \cdot \left(\rho_{propellant} \cdot R_g \cdot T_0 - P_0\right) - \left(\frac{A^*}{V_c}\right) \cdot P_0 \cdot \sqrt{\gamma \cdot R_g \cdot T_0 \cdot \left(\frac{2}{\gamma+1}\right)^{\left(\frac{\gamma+1}{\gamma-1}\right)}} \\ a \cdot P_0^n \end{bmatrix}$$

$$\begin{bmatrix} P_0 \\ r \end{bmatrix}_{t=0} = \begin{bmatrix} P_{ambient} \\ \frac{d_0}{2} \end{bmatrix} \rightarrow \boxed{s(t) = \int_0^t \dot{r} \cdot dt \approx r(t) - \frac{d_0}{2}}$$

$$\begin{bmatrix} k \equiv Erosion\ Constant\ (\text{GRAIN DEPENDENT}) \\ M_{crit} \equiv Critical\ Port\ Mach\ Number \end{bmatrix}$$

$\rightarrow$ *State Equations for Erosive Burning :*

$$\begin{bmatrix} \dot{P}_0 \\ \dot{r} \end{bmatrix} = \begin{bmatrix} \left(\frac{A_{burn} \cdot \dot{r}}{V_c}\right) \cdot \left(\rho_{propellant} \cdot R_g \cdot T_0 - P_0\right) - \left(\frac{A^*}{V_c}\right) \cdot P_0 \cdot \sqrt{\gamma \cdot R_g \cdot T_0 \cdot \left(\frac{2}{\gamma+1}\right)^{\left(\frac{\gamma+1}{\gamma-1}\right)}} \\ \left(1 + k \cdot \frac{M_{port}}{M_{crit}}\right) \cdot a \cdot P_0^n / (1+k) \end{bmatrix}$$

$$\begin{bmatrix} P_0 \\ r \end{bmatrix}_{t=0} = \begin{bmatrix} P_{ambient} \\ \frac{d_0}{2} \end{bmatrix} \rightarrow \boxed{s(t) = \int_0^t \dot{r} \cdot dt \approx r(t) - \frac{d_0}{2}}$$

# State Equation Formulation of Problem (2)

$\rightarrow$ *Cylindrical Port :*

$$\boxed{\begin{aligned} A_{burn} &= 2 \cdot \pi \cdot r \cdot L_{port} \\ V_c &= \pi \cdot r^2 \cdot L_{port} \end{aligned}} \rightarrow \begin{bmatrix} r \equiv Port\ Radius \\ L_{port} \equiv Port\ Length \end{bmatrix}$$

$\rightarrow$ *Bates Grain :*

$$A_{burn} = N \cdot \pi \cdot \left\{ \left[ \frac{D_0^2 - (d_0 + 2 \cdot s)^2}{2} \right] + (L_0 - 2 \cdot s) \cdot (d_0 + 2 \cdot s) \right\}$$

$$V_c = \frac{N \cdot \pi}{4} \cdot \left\{ (d_0 + 2 \cdot s)^2 \cdot (L_0 - 2 \cdot s) + D_0^2 \cdot 2s \right\}$$

Do NOT! Use Erosive Burning for Bates Grain

# State Equation Formulation of Problem (3)

Calculating Chamber Mach Number

*Erosive Burning*

$$\rightarrow \frac{V_c / L_{port}}{A^*} = \frac{1}{M_{port}} \cdot \left[ \left( \frac{2}{\gamma + 1} \right) \cdot \left( 1 + \left( \frac{\gamma - 1}{2} \right) \cdot M_{port}^2 \right) \right]^{\left( \frac{\gamma+1}{2 \cdot (\gamma - 1)} \right)}.$$

*... Subsonic Branch Solution*!

Do NOT! Use Erosive Burning for Bates Grain

# Cylindrical Port: Decoupled Model

- Use Trapezoidal rule or Runge-Kutta to integrate

$$\frac{\partial P_0}{\partial t} = \frac{A_{burn} a P_o^{\,n}}{V_c}\left[\rho_p R_g T_0 - P_0\right] - P_0\left[\frac{A^*}{V_c}\sqrt{\gamma R_g T_0\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{(\gamma-1)}}}\right]$$

- Recursive propagation of chamber diameter

$$R_{burn_{k+1}} = R_{i_{initial}} + \int_0^{(k+1)\Delta t} \dot{r}\,dt = R_{i_{initial}} + \int_0^{(k)\Delta t} \dot{r}\,dt + \int_{(k)\Delta t}^{(k+1)\Delta t} \dot{r}\,dt \rightarrow$$

$$R_{burn_{k+1}} = R_{burn_k} + \int_{(k)\Delta t}^{(k+1)\Delta t} \dot{r}\,dt \approx R_{burn_k} + \dot{r}\,\Delta t = R_{burn_k} + a P_{o_k}^{\,n}\,\Delta t$$

# Bates grain Port: Decoupled Model

- Use Trapezoidal rule or Runge-Kutta to integrate

$$\frac{\partial P_0}{\partial t} = \frac{A_{burn} a P_o^{\,n}}{V_c}\left[\rho_p R_g T_0 - P_0\right] - P_0\left[\frac{A^*}{V_c}\sqrt{\gamma R_g T_0\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{(\gamma-1)}}}\right]$$

$$\left.\begin{array}{l}\dot{r} = a \cdot P_o^{\,n} \\ s_{regression} = \int_t \dot{r} \cdot dt\end{array}\right| \rightarrow$$

$$\left(A_{burn}\right)_{total} = N \cdot \pi \cdot \left[\frac{\left(D_0^{\,2} - \left(d_0 + 2\cdot s\right)^2\right)}{2} + \left(L_0 - 2\cdot s\right)\cdot\left(d_0 + 2\cdot s\right)\right]$$

$$\left(V_{ol}\right)_{total} = \frac{N\cdot\pi}{4}\left[\left(d_0 + 2\cdot s\right)^2 \cdot \left(L_0 - 2\cdot s\right) + D_0^{\,2}\cdot\left(2\cdot s\right)\right]$$