Gov 2001: Assignment 5 Solutions

Due Thursday, March 20, 5:00pm EST

1 Problem 1: The Logit Model

On the night of January 27, 1986, the night before the space shuttle Challenger accident, there was a teleconference among people at Morton Thiokol (manufacturer of the solid rocket motor) and NASA engineers. The discussion centered on the forecast of a 31°F temperature for launch time the next morning, and the effects of low temperatures on the performance of O-rings. Some participants recommended postponing the launch until the temperature rose above 53°F, the lowest temperature experienced in previous launches, believing that there was a relationship between temperature and O-ring performance. In spite of these recommendations, Mortion Thiokol recommended launching the Challenger on schedule. The next morning, 72 seconds after take-off, Challenger exploded in mid-fight. The Presidential Commission on the Challenger Accident later concluded that the accident was caused by a combustion gas leak through a joint in one of the booster rockets (which were supposed to be sealed by the O-rings). The Commission also concluded that O-rings do not seal properly at low temperatures. In this problem set, we will analyze this claim using data from the 23 space shuttle fights conducted prior to the Challenger accident. The dataset `data_ps5.RData` consists of 23 observations and 4 variables. Incident denotes whether any of the 6 O-rings was damaged during a flight. Temperature denotes the temperature at take-off. Pressure denotes the pressure used in preparation for the flight to test whether the O-rings would seal properly. Date denotes the date of the launch.[1]

a) Gary derives the log-likelihood for the Logit model in the lecture notes. Implement the log-likelihood for a logit model as a `R` function. Carefully comment your code and include it in your write-up The log-likelihood implemented in `R` is

```
logit.ll <- function(par, X, Y){#par is the parameter, X is the matrix of
   #covariates with a column of 1s, Y is the observed dependent variable
   betas <- par[1:ncol(X)]
   mu <- X %*% betas
   out <- -sum(log(1 + (exp((1-2*Y)*mu))))
  return(out)
}
```

[1]If this problem set piques your curiosity about the accident, take a look at Diane Vaughan's (1996) classic study *The Challenger Launch Decision: Risky Technology, Culture, and Deviance at NASA.* University of Chicago Press.

b) Use your new function to evaluate the claim that there is a relationship between temperature and the probability of O-ring failure, controlling for joint pressure. Use the Hessian to calculate standard errors. Then use Zelig to check your results.[2]

The maximum likelihood estimates and standard errors are

	Estimate	Standard Error
Intercept	13.312	7.657
Temperature	-0.229	0.110
Pressure	0.010	0.009

c) Use simulation to calculate the difference in the *expected* probability of O-ring failure when going from 53°F (the minimum temperature for launch recommended by some of the engineers before the Challenger accident) to 31°F, holding Pressure at its observed values. Don't forget to report a measure of uncertainty. (You are not allowed to use Zelig in this part.)

The first difference in the probability of failure when going from 53°F to 31°F is 0.106. The .95 confidence interval around this estimate is (0.001; 0.334).

d) Use simulation to calculate the *expected* values of O-ring failure for temperatures between 31°F and 81°F, holding Pressure at its observed values. (Do not use Zelig except to check your results.) Plot the expected values in a publication quality graph. Do not forget to add confidence bounds to report the uncertainty of your predictions. Also add a `rug()` representation of the temperature data to the plot. Why is it important to add a rug in this case?

Figure 1 shows the expected values of O-ring failure . A rug plot is important to reveal the places where the data are sparse. It shows that the data are sparse around the temperatures we are interested in, so we should be cautious about making inferences in that region.

e) Use simulation to calculate the *predicted* values of O-ring failure for temperatures between 31°F and 81°F, holding Pressure at its observed values. (Do not use Zelig except to check your results.) Plot the predicted values in a publication quality graph.

Because our dependent variable is dichotomous, a predicted value is a prediction of failure or not. Predicted values at each temperature are shown in Figure 2. Another interesting predicted quantity may be the predicted *probability* of failure, which we could estimate by drawing many predicted values at each temperature and taking an average for each temperature. This predicted quantity is shown in Figure 3. Note that the predicted probabilities are very similar to expected values in this case- convince yourself of why this makes sense by thinking of the last few steps in simulating each quantity.

f) What is the difference between expected and predicted values? How should each be interpreted in this case?

[2]The `zelig()` function in the Zelig library lets you specify `model = "logit"`.

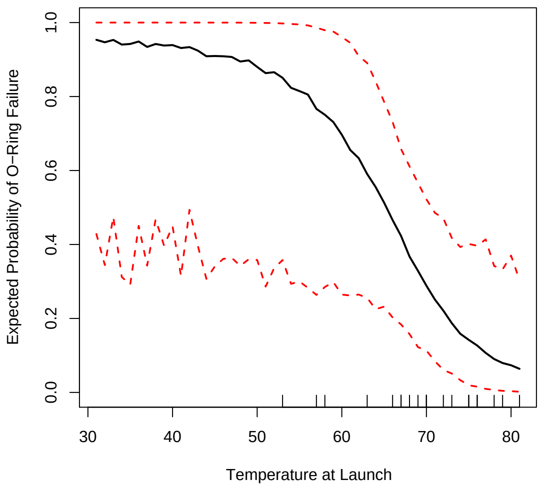

Figure 1: Expected values of O-ring failure for temperatures between 31°F and 81°F, holding Pressure at its observed values.

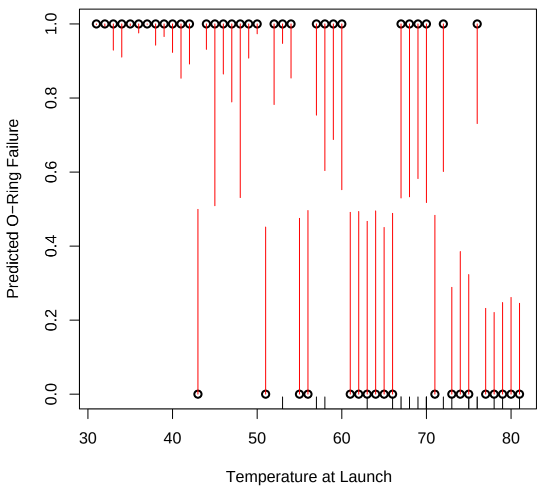

Figure 2: Predicted values of O-ring failure for temperatures between 31°F and 81°F, holding Pressure at its observed values. One standard deviation around each prediction is shown in red to demonstrate relative certainty (calculated from the definition of a binomial distribution).

Expected values and predicted values are both quantities which contain information about outcome variables which are not observed in our dataset. Expected values are the values of the outcome variable y that are expected to occur given what we've learned from our data and model assumptions which account for estimation uncertainty. Predicted values are the values of y predicted to occur given our data and model which account for both estimation uncertainty and fundamental uncertainty.

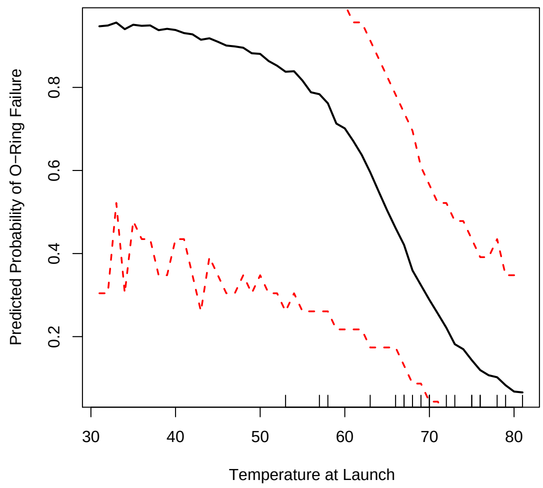

Figure 3: Predicted probabilities of O-ring failure for temperatures between 31°F and 81°F, holding Pressure at its observed values.

In this case, the predicted value of y is a prediction of whether an o-ring will fail or not at a particular temperature given observed pressure. The expected value of y is the value y is expected to take if we repeated launches at particular values of temperature over and over. The expected value is not constrained to be a 0 or 1, but instead gives an average, which in this case is a probability, of failure.

g) Does age of the program matter? Implement a Likelihood ratio test to evaluate the alternative hypotheses that the space shuttle program learned over time or that the equipment degraded over time. (There are several ways to address this question; there is no single correct approach. Be creative.)

   To address whether the age of the program matters, we can implement a likelihood ratio test. One simple way to include time in our model is to include a covariate for mission number. Comparing a model which includes temperature, pressure and mission number as covariates to a restricted model which only includes temperature and pressure as covariates results in the likelihood test statistic of 0.047. This test statistic is distributed chi-square. We would expect to see a test statistic this large in 82% of repeated samples if the null were true (where the null is that age is irrelevant). So we cannot reject the null hypothesis that age is irrelevant- age may matter.