Math 446/546: Midterm Exam Spring, 2022

This exam consists of three questions. You are required to answer all three questions in one hour and 15 minutes (75 minutes). Each question carries equal marks. Write your name clearly on each sheet of paper that you hand in and show all intermediate calculations. This is a closed book exam. Laptops and calculators permitted only for calculation purposes. Reference to class materials is forbidden. Use of the internet, cellular devices or any form of communication other than with the invigilator's permission is forbidden. The maximum number of partial marks is shown in square parentheses in the questions.

Question 1 [10]. Determine which of the following ARMA processes are causal and which of them are invertible. (In each case $\{Z_t\}$ denotes white noise.)

(1) $X_t + 0.6X_{t-1} = Z_t - 0.8Z_{t-1}$. [3]
(2) $X_t - 0.6X_{t-1} = Z_t + 0.2Z_{t-1} + 0.88Z_{t-2}$. [3]
(3) $X_t + 1.8X_{t-1} + 0.81X_{t-2} = Z_t - 0.4Z_{t-2}$. [3]

In practice, why should we avoid using ARMA processes which are not causal and not invertible? [1]

Question 2 [10]. Give the form of a noisy discrete time approximation of the equation of a 1D pendulum:

$$\frac{d^2}{dt^2}\theta(t) + \frac{g}{L} sin\theta(t) = 0,$$

where θ is the angle of the pendulum string from the vertical, g is the gravitational constant and L is the length of the string [3]. For simplicity, assume that $g = L$ and $\Delta t = 1$. Under what further approximations can you write this equation as a AR(2) process? [2] Determine whether the AR(2) process is causal [5].

Question 3 [10]. Using the likelihood ratio test, with the Wilk's approximation, which model would you select from the following ARMA(p,q) models at the 99% significance level? [5]

AIC	$p = 0$	$p = 1$
$q = 0$	-200.4	-201.9
$q = 1$	-201.9	-206.0

Is the difference between the AIC for ARMA(0,0) and ARMA(1,1) consistent with the Wilk's approximation? Explain your answer with reference to the assumptions behind the Wilk's approximation? [5]

Hint: You may find the following quantiles of the chi-squared distribution useful: $F_X^{-1}(0.99) = 6.634897$, where $F_X(x)$ is the CDF and $X \sim \chi^2_{d=1}$. $F_X^{-1}(0.99) = 9.21034$, where $F_X(x)$ is the CDF and $X \sim \chi^2_{d=2}$.